

Probing Gravitational Waves from First-Order Phase Transitions at eLISA

Germano Nardini
(Bern Univ.)

29/2/2016 ... two weeks after THE announcement

Based on *arXiv:1512.06357*

[Huber, Stephan (Sussex Univ.)
Konstandin, Thomas (DESY)
Rues, Ingo (DESY)]

+

arXiv:1512.06239

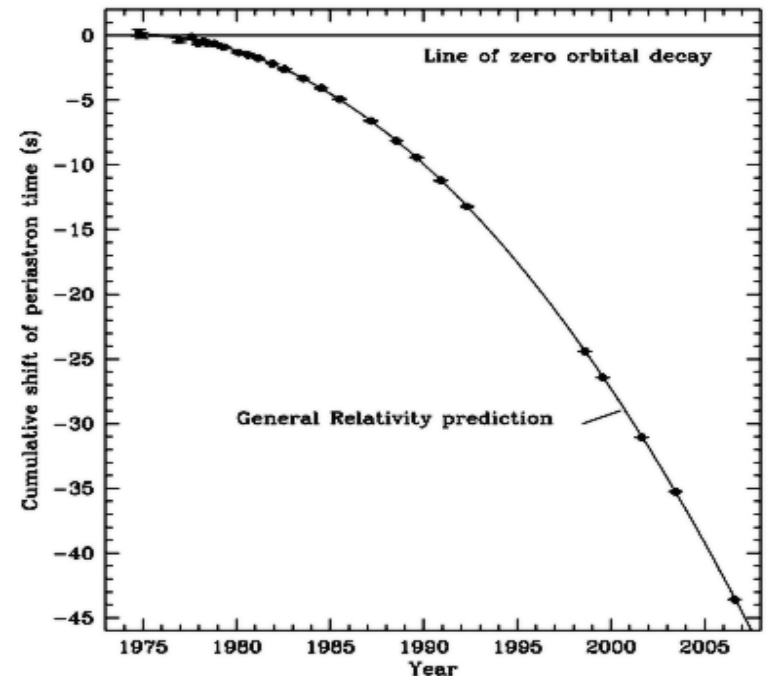
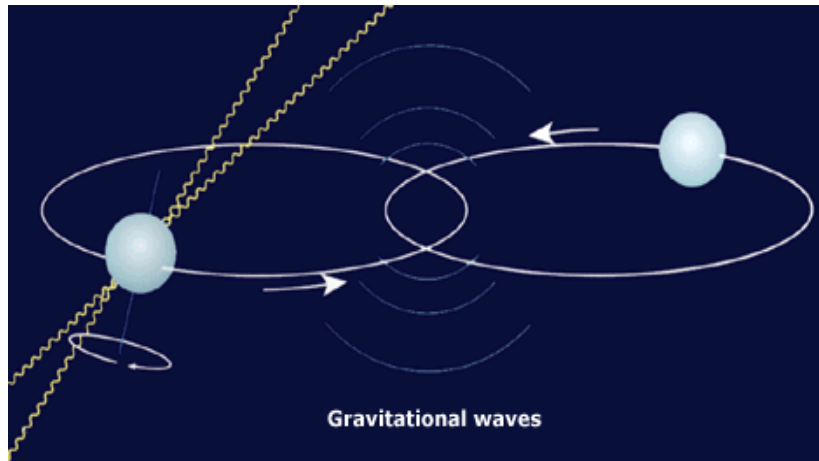
[subset of the eLISA cosmo working group]

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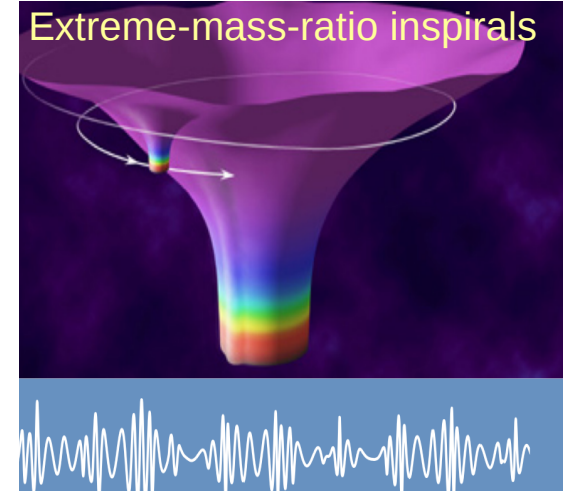
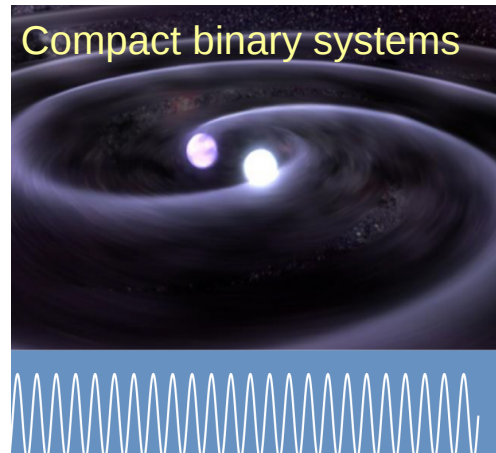
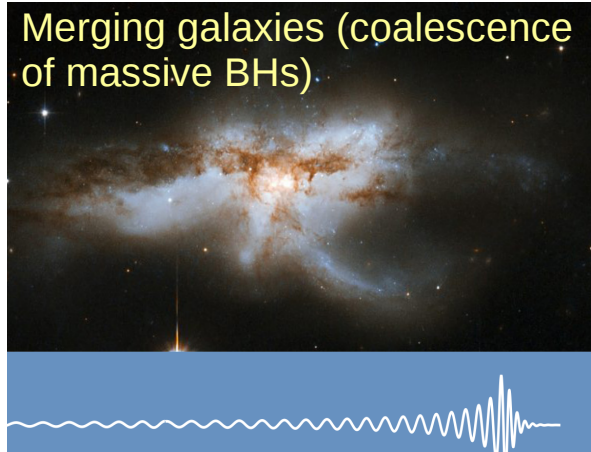
Gravitational Waves

- > Gravitational Waves (GWs) are a prediction of General Relativity: accelerating masses produce a spacetime perturbation that propagates (“ripples in spacetime”).
- > The Hulse-Taylor binary system provided the first *indirect* evidence that GWs exist.



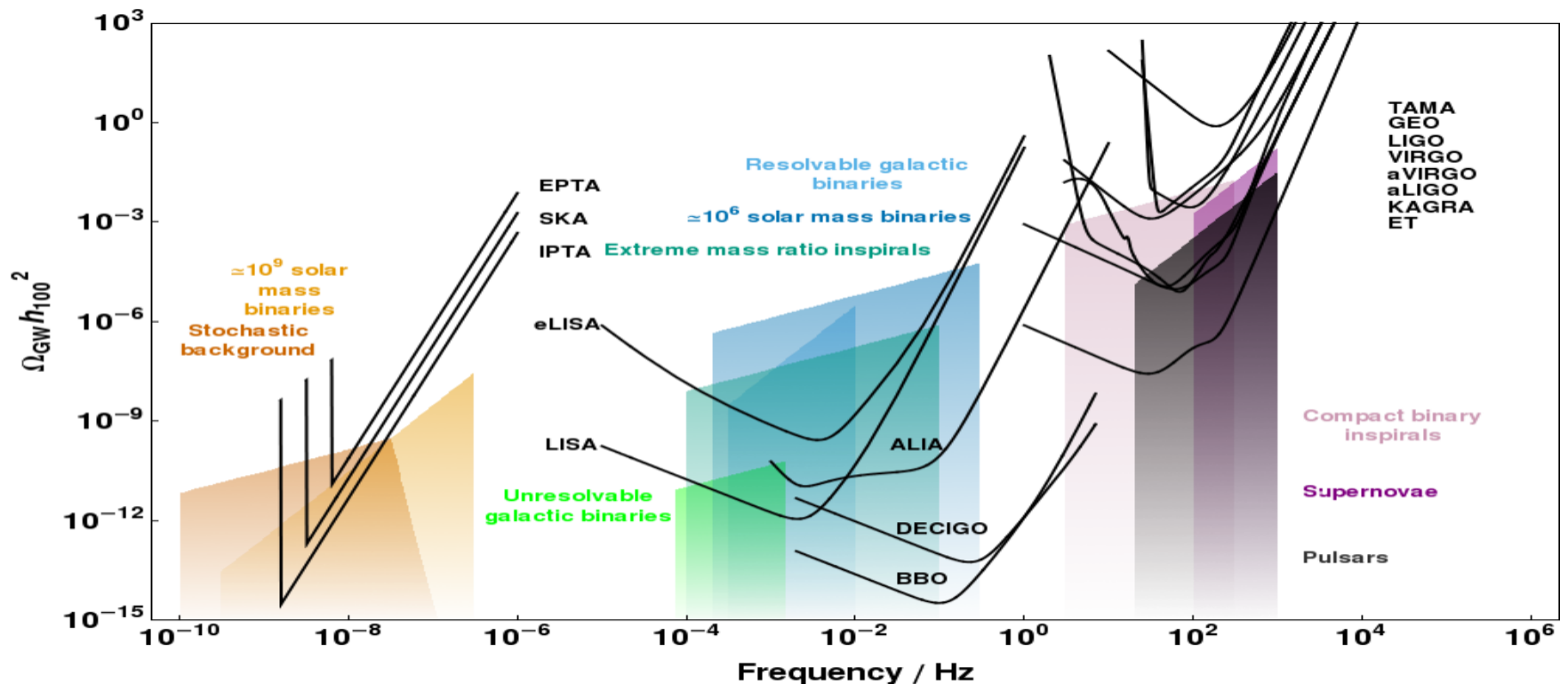
Gravitational Waves

- > Nevertheless, we will feel more comfortable after several *direct* proofs
- > Many potential **localized** sources are expected there waiting for us...



Gravitational Waves

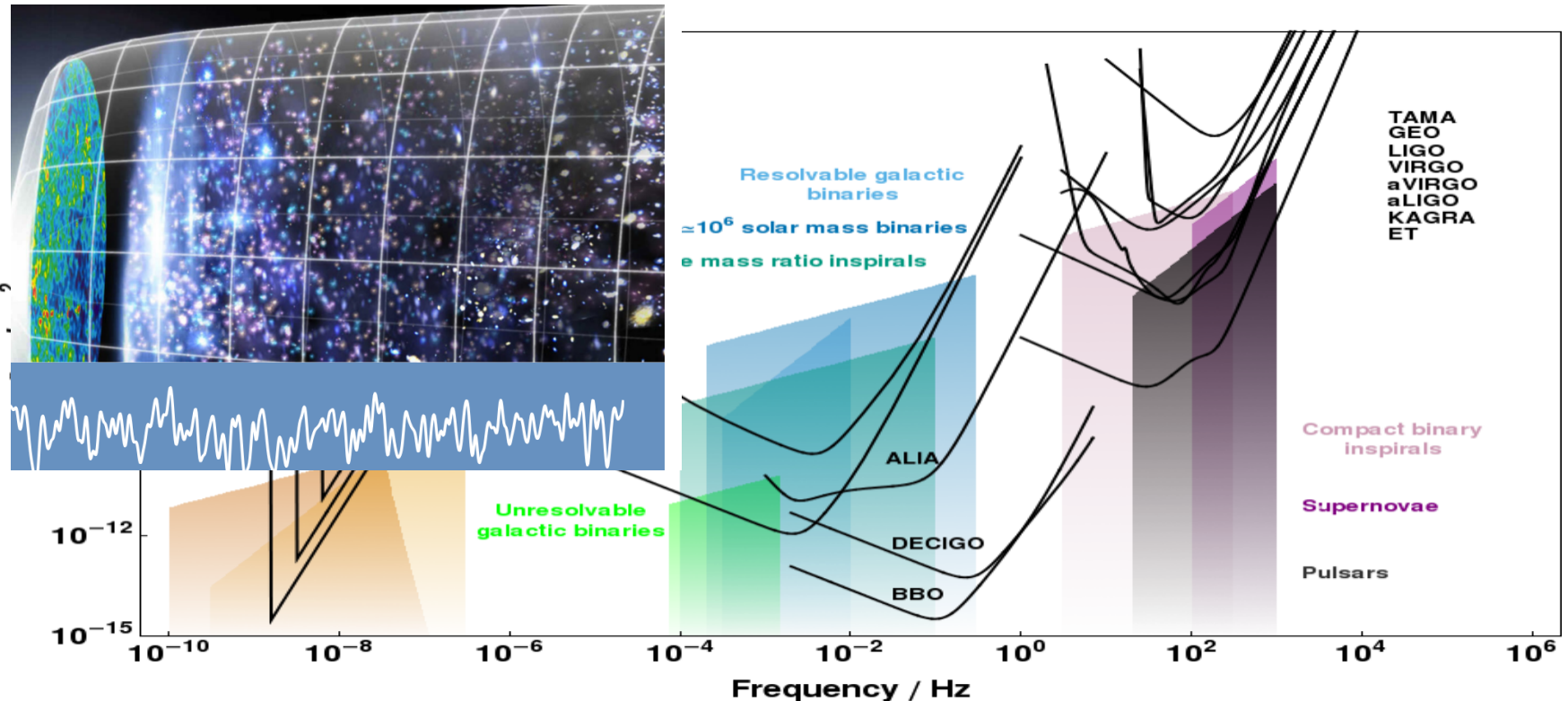
- > Nevertheless, we will feel more comfortable after several *direct* proofs
- > Many potential **localized** sources are expected there waiting for us...
- > ... and we are attempting to detect them (... and recently with success!!!)



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Gravitational Waves

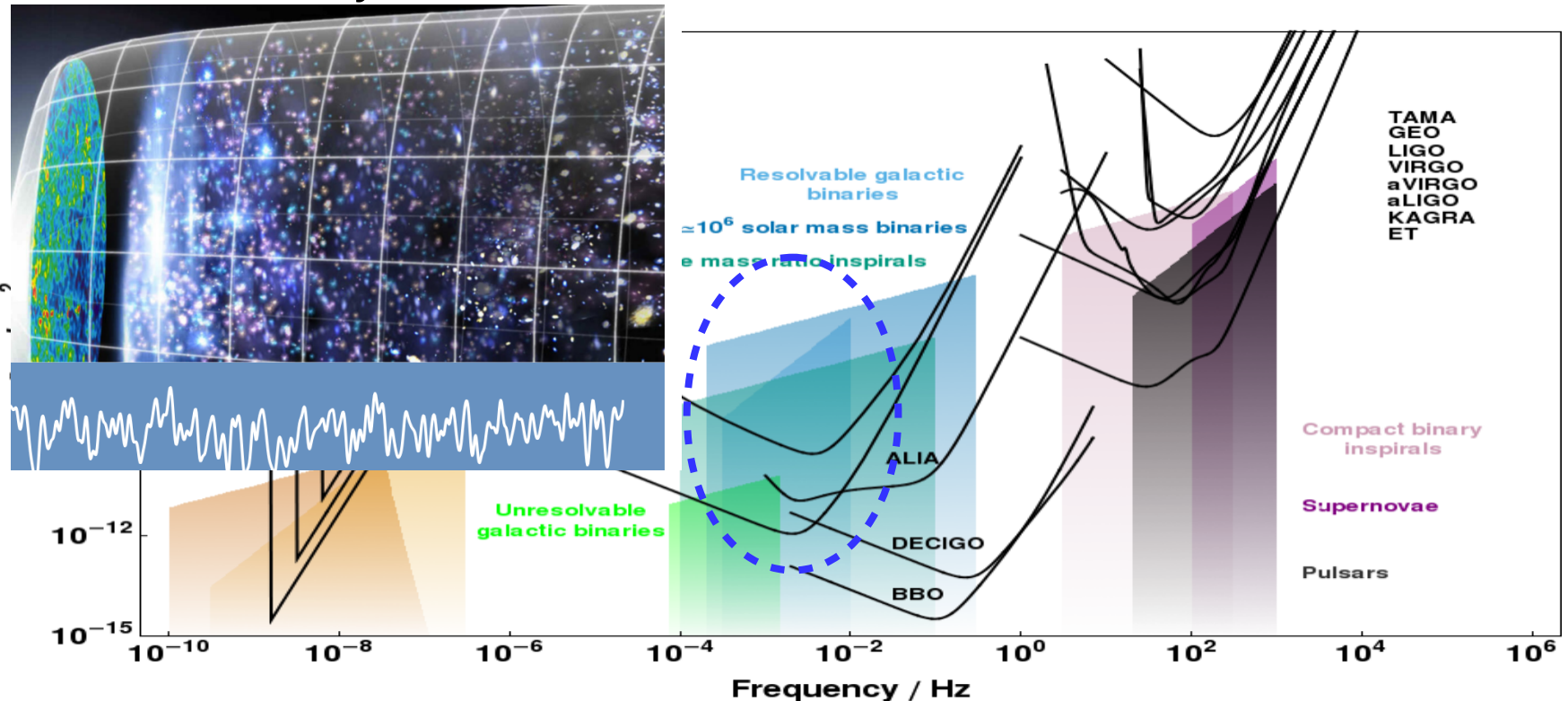
- Besides astrophysical sources, we also expect pre-BBN cosmological sources (inflationary epoch, topological defects, phase transitions, ...)
- These generate a stochastic (**non localized**) GW background



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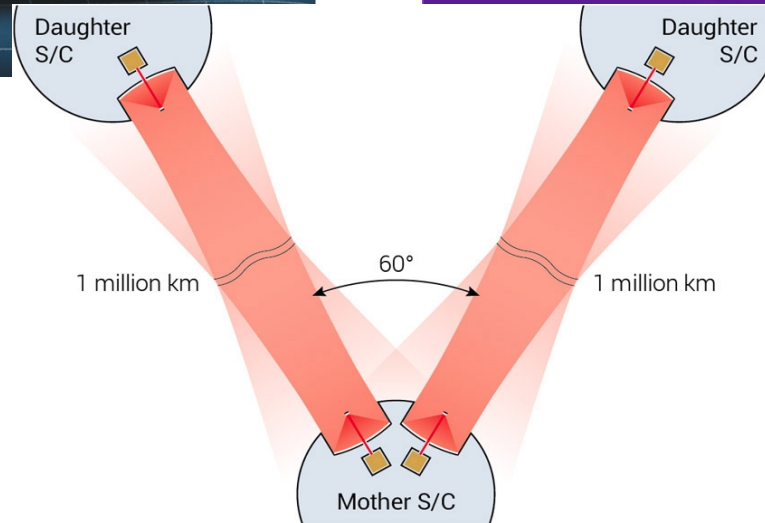
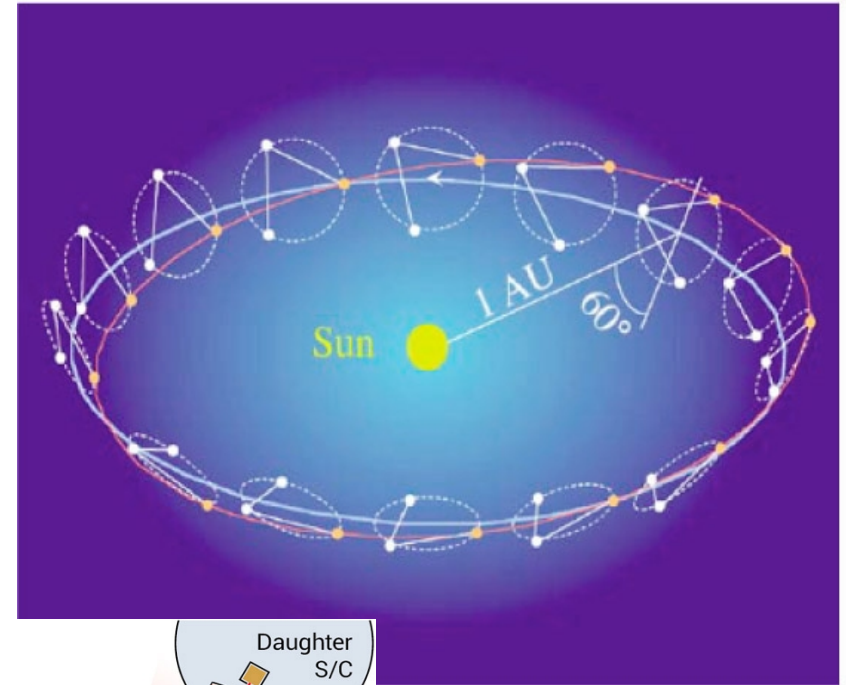
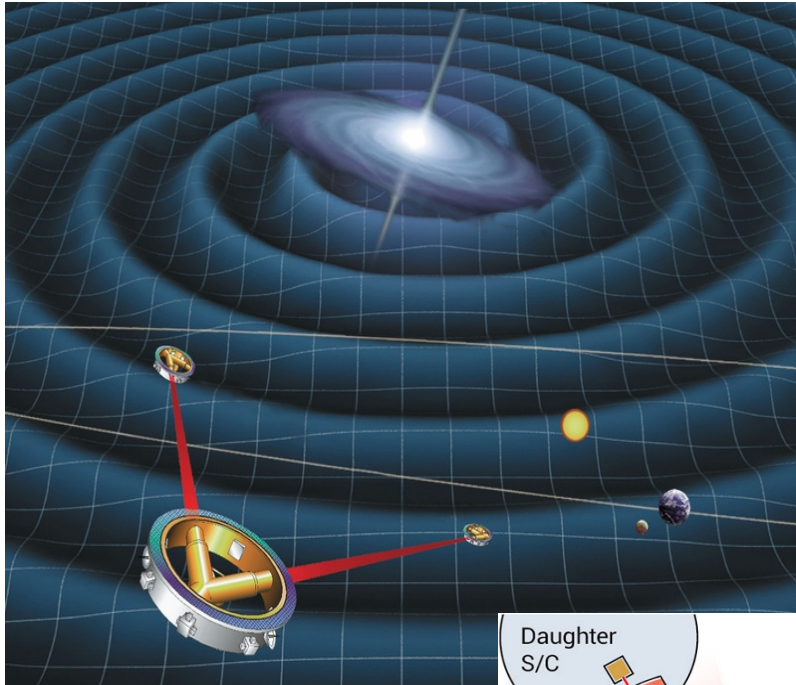
Gravitational Waves

- The ELECTROWEAK phase transition (EWPT) is particularly interesting: energy scale of pp collision at LHC and “a bit” beyond heavy-ion collisions at RHIC and LHC
- Moreover it may be testable at eLISA!



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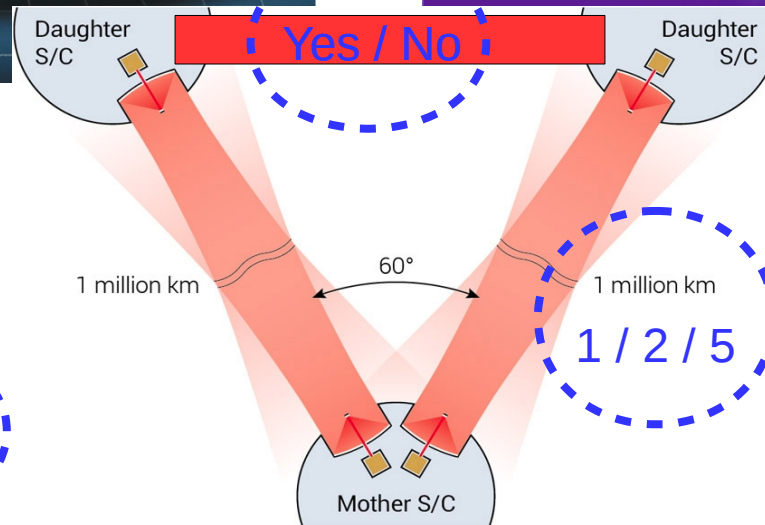
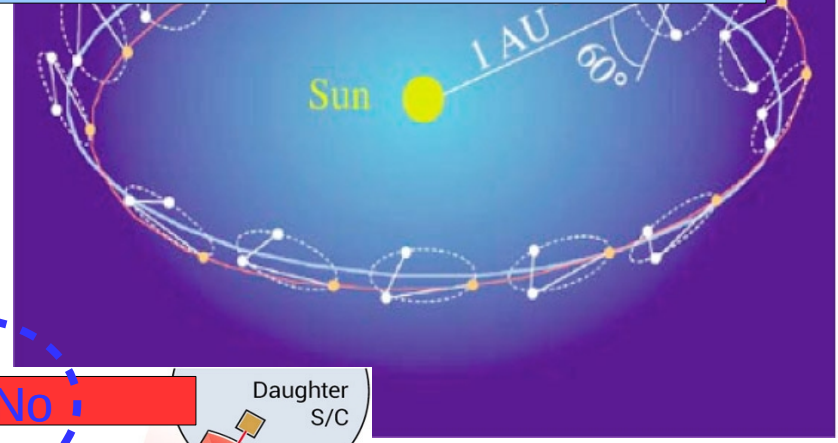
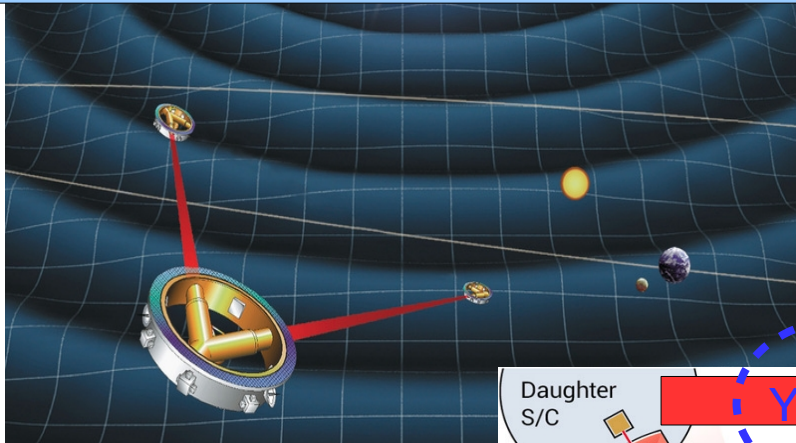
The eLISA interferometer (open issues)



The eLISA interferometer (open issues)

Belgium, Denmark, France, Germany, Italy, Netherlands, Spain, Switzerland, UK

Extra budget: partially because of reanalyses of costs, partially because of NASA+Japan, ... , and since two weeks a field with guaranteed “new” physics (B and CH recently became member states)



2 / 5 years of data taking

The eLISA interferometer (agenda)

- > Crucial period: from **2/12/2015** (Pathfinder launch) to **1/6/2016** (Pathfinder results)
- > eLISA designs vs. physics return (arXiv:1511.05581, 1512.06239, 1601.071, internal reports): **15/3/2016**
- > eLISA design decided: **~7/9/2016** (eLISA symposium in Zurich)
- > ESA decision: **2017 (?)**
- > eLISA launch: **2025 – 2032**



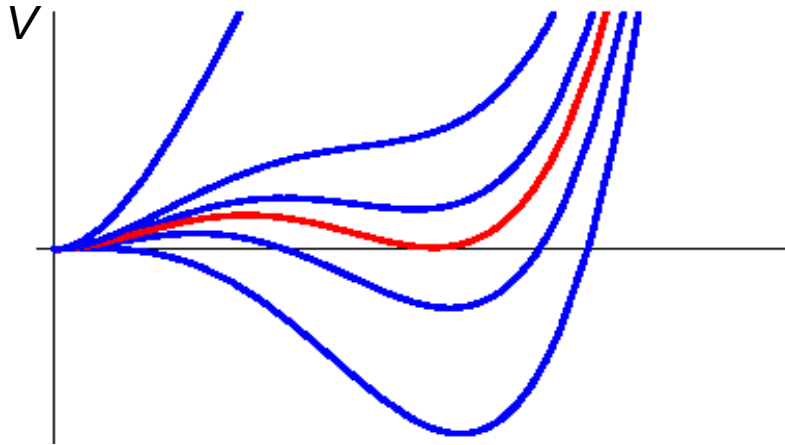
<https://www.elisascience.org>



- > Gravitational Waves from 1st-Order Phase Transitions
- > Mapping the EWPT of your model into the eLISA detection curves
- > (Q1) Do well-motivated models overcoming the LHC bounds and producing strong GW signals exist?
- > (Q2) Can eLISA probe these signals? Any preferable eLISA design?

Gravitational Waves from 1st-Order EWPT

- > Let us assume that the EWPT is of first order, i.e.

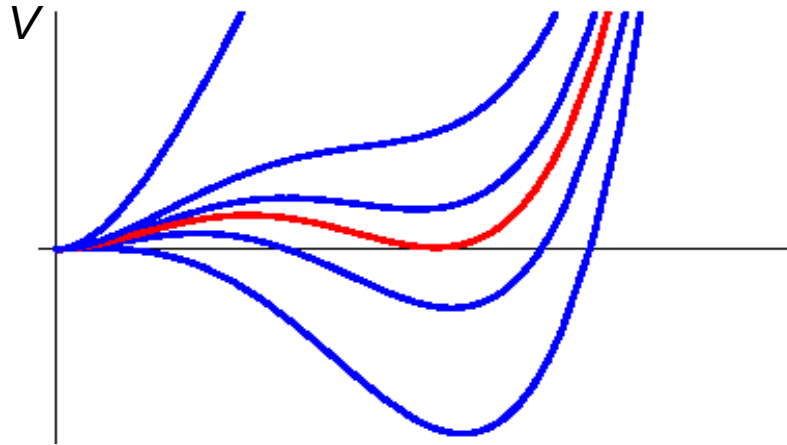


$$V(\phi, T) = D(T^2 - T_0^2)\phi^2 - ET\phi^3 + \lambda(T)\phi^4$$

- > The phase transition occurs via tunneling. In the place where the tunneling happens, a bubble of EW broken phase ($\langle \phi \rangle = \phi_{broken}$) nucleates.
- > Conventionally, the EWPT starts in the Universe when statistically we have 1 nucleated bubble per Hubble volume and time. The temperature of the Universe at this time is called T_n
- > The tunneling rate is $\Gamma(t) = \Gamma_0 \exp[-S(t)]$. If $\beta = -dS/dt|_{t=t_n}$ is large (small), many (a few) bubbles have nucleated by the time the first bubbles have \mathbf{u}^b expanded, i.e. the phase transition ends with many little (a few large) bubbles.

Gravitational Waves from 1st-Order EWPT

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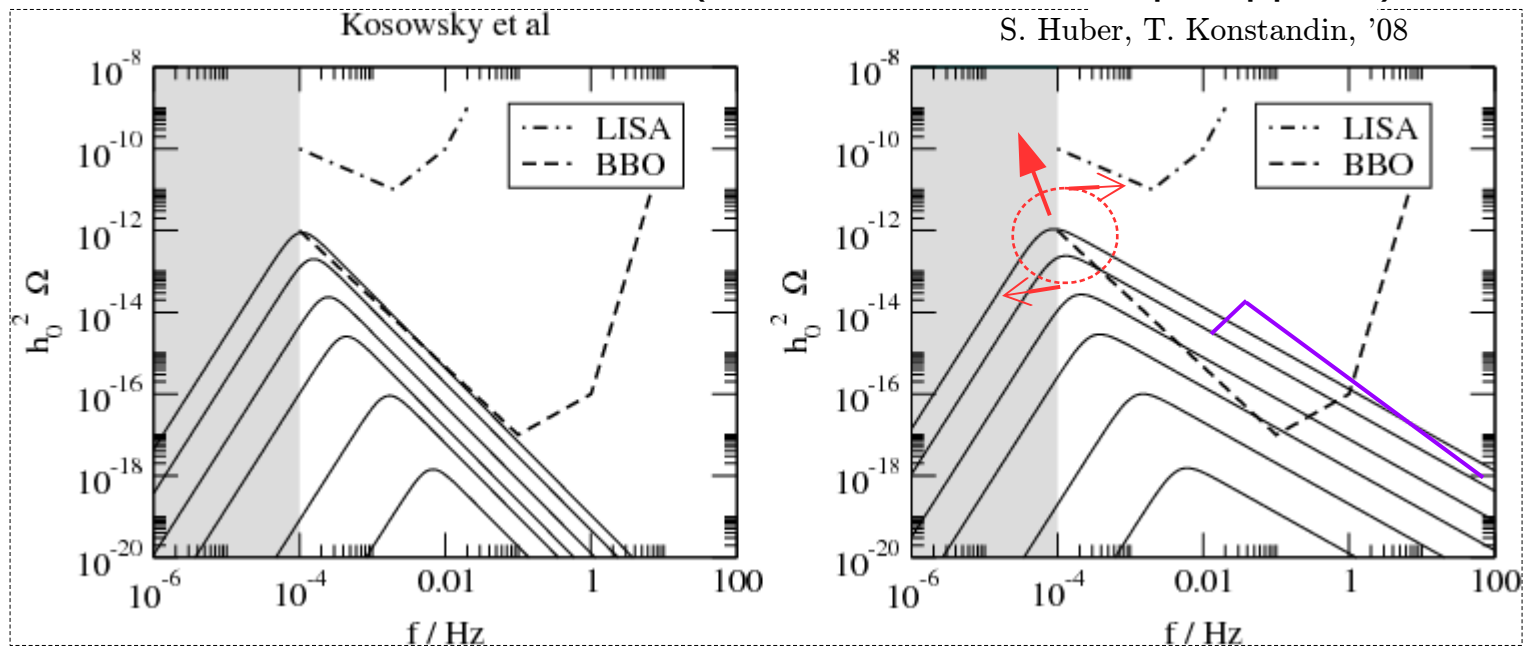
- > When bubbles collide, they convert part of their kinetic energy (of the expanding wall + turbulent fluid) into gravitational waves (GWs)! M. Kamionkowski et al., '94
- > So, the more energy is available (\rightarrow **supercooling**), the stronger the GW signal
- > This available energy is the latent heat

$$\epsilon(T_n) = \Delta V(T_n) - T \frac{\partial \Delta V(T_n)}{\partial T}, \quad \Delta V(T_n) = V(\phi_{sym}, T_n) - V(\phi_{brok}, T_n)$$

\mathbf{u}^b which we normalize to the radiation energy: $\alpha = \epsilon(T_n) / \left(\frac{\pi^2}{30} g_* T_n^4 \right)$

Gravitational Waves from 1st-Order EWPT

- Simulations on bubble collisions (based on the “envelope approx”) show



where (for $v_w \simeq 1$)

$$h_0^2 \Omega_{GW} \approx 10^{-10} \kappa^2(\alpha) \left(\frac{100}{\beta/H} \right)^2 \left(\frac{\alpha}{\alpha + 1} \right)^2$$

$$f_{peak} \approx \text{mHz} \left(\frac{\beta/H}{100} \right) \left(\frac{T_n}{100 \text{ GeV}} \right)$$

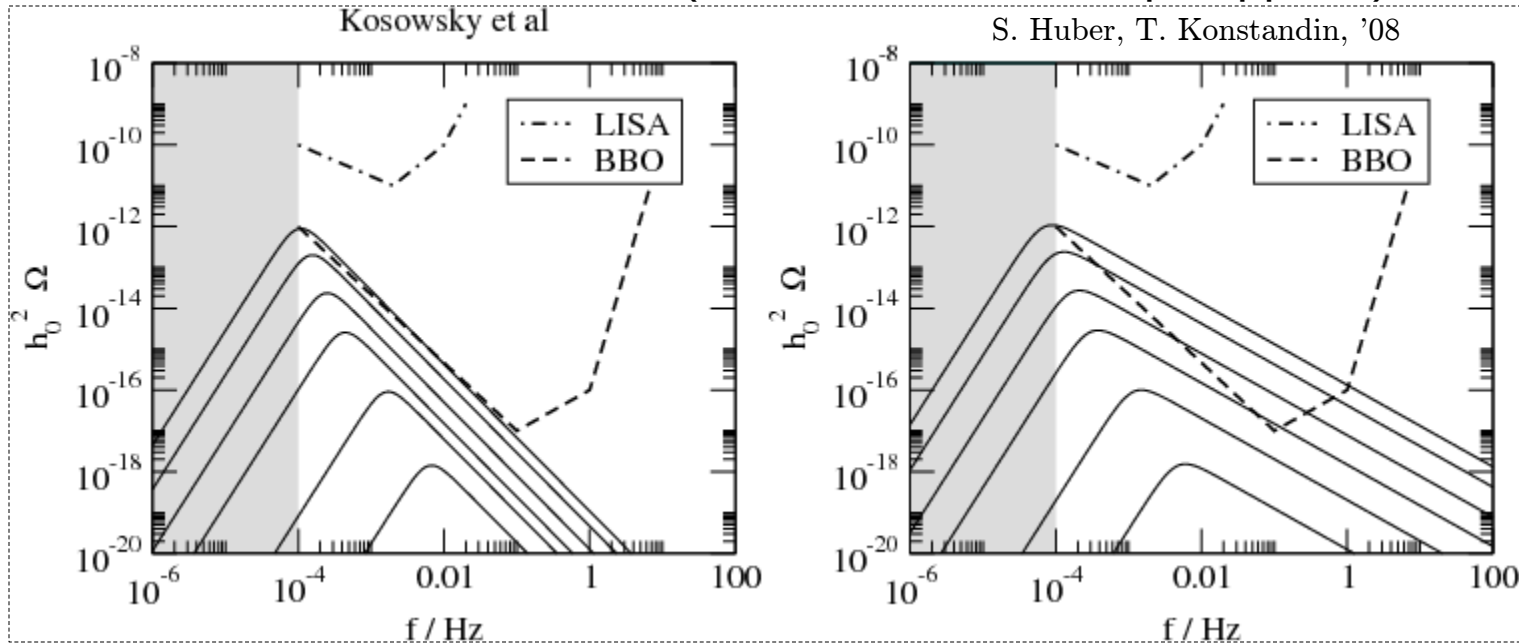
- On the top of the “envelope” result, there are corrections:

Magnetic HD

Sound Waves

Gravitational Waves from 1st-Order EWPT

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- Moreover the actual GW spectrum is sensitive to the interactions of the field of the tunneling with the plasma (non-runaway/runaway/vacuum cases) [[arXiv:1512.06239](https://arxiv.org/abs/1512.06239)] For simplicity in this talk we restrict ourselves to the runaway case.

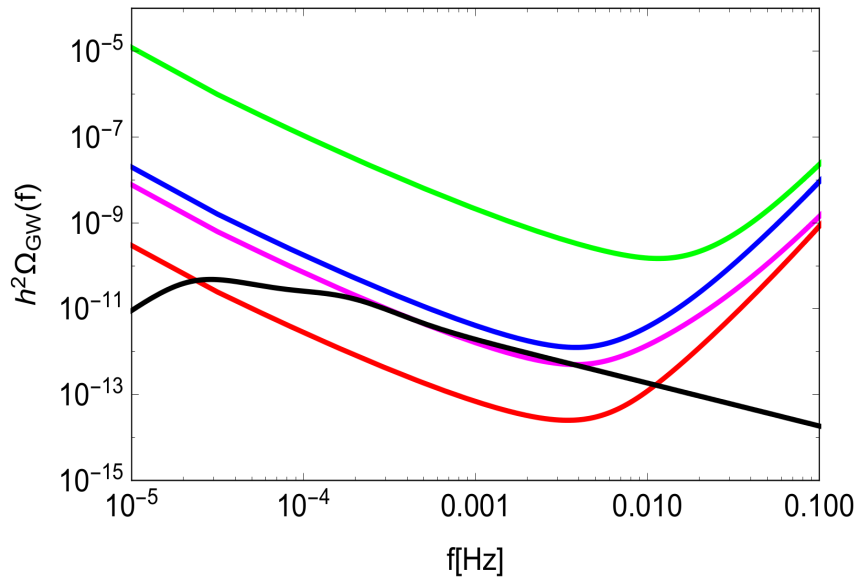
Mapping models into the eLISA detection curves

Subset of eLISA cosmology working group, 1512.06239

- > The eLISA sensitivity curves for a stochastic background are much better than what one could naively think.
- > The dominant enhancement is that the signal has a continuous (broken power-law) frequency spectrum over which you can integrate.
- > Moreover the signal has to be enough above the sensitivity curve to have a discovery ($\text{SNR} > \text{SNR}_{\text{thresh}}$).
- > The stochastic-background sensitivity curves will be public soon

Thrane,Romano, '13

A.Petiteau, 160x.xxxxx

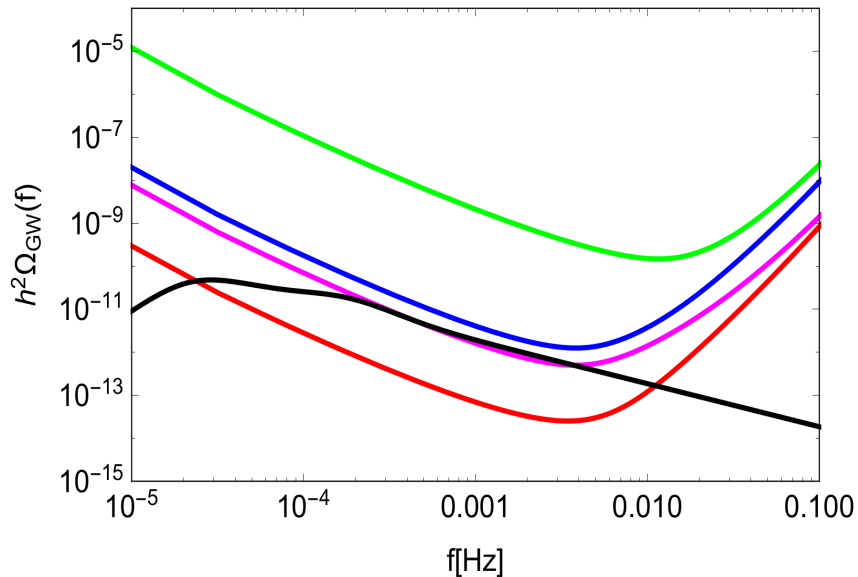


Name	C1	C2	C3	C4
Full name	N2A5M5L6	N2A1M5L6	N2A2M5L4	N1A1M2L4
# links	6	6	4	4
Arm length [km]	5M	1M	2M	1M
Duration [years]	5	5	5	2
Noise level	N2	N2	N2	N1

Mapping your model into the eLISA detection curves

- > The procedure to determine “ $\text{SNR} > \text{SNR}_{\text{thresh}}$ ” for a given eLISA sensitivity curve is quite involved.
- > The procedure can be absorbed into a detection map: you determine $(\alpha, \beta/H_n, T_n, \text{case})$ in your model, and then...

Subset of eLISA cosmology working group, 1512.06239



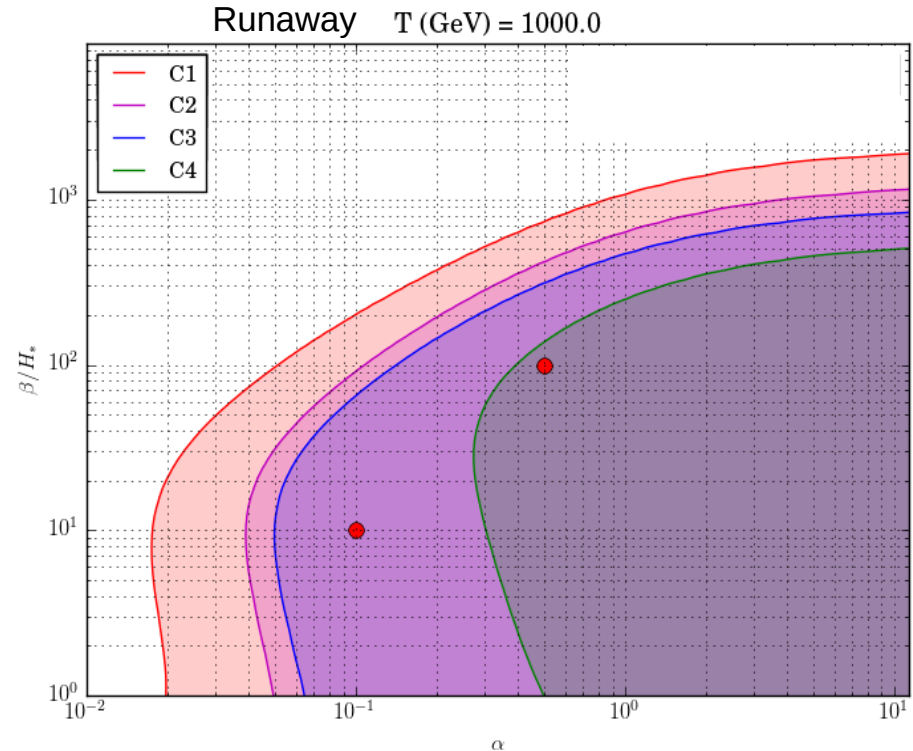
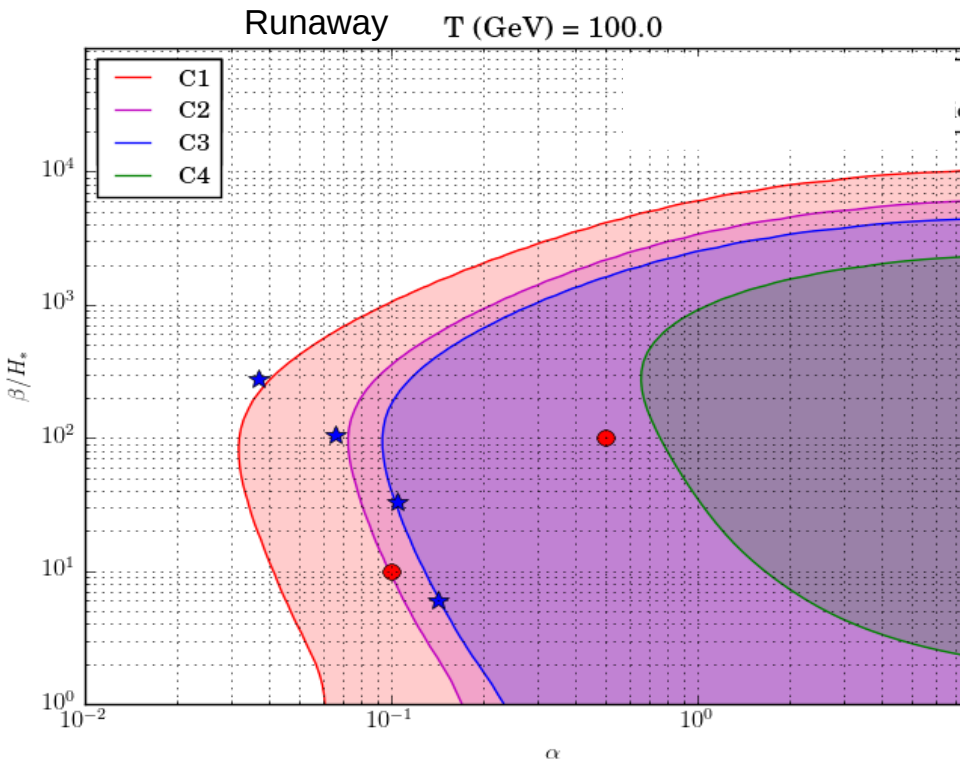
— C1
— C2
— C3
— C4

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Question

- > These bubble simulations show that a sizable GW spectrum from EWPTs is possible for some values of α and β/H taken as inputs
- > A wide region of α and β/H , which correspond to a supercooled EWPTs, can be probed at eLISA
- > But α and β/H are features of the potential that strongly depend on the particle physics model !!!

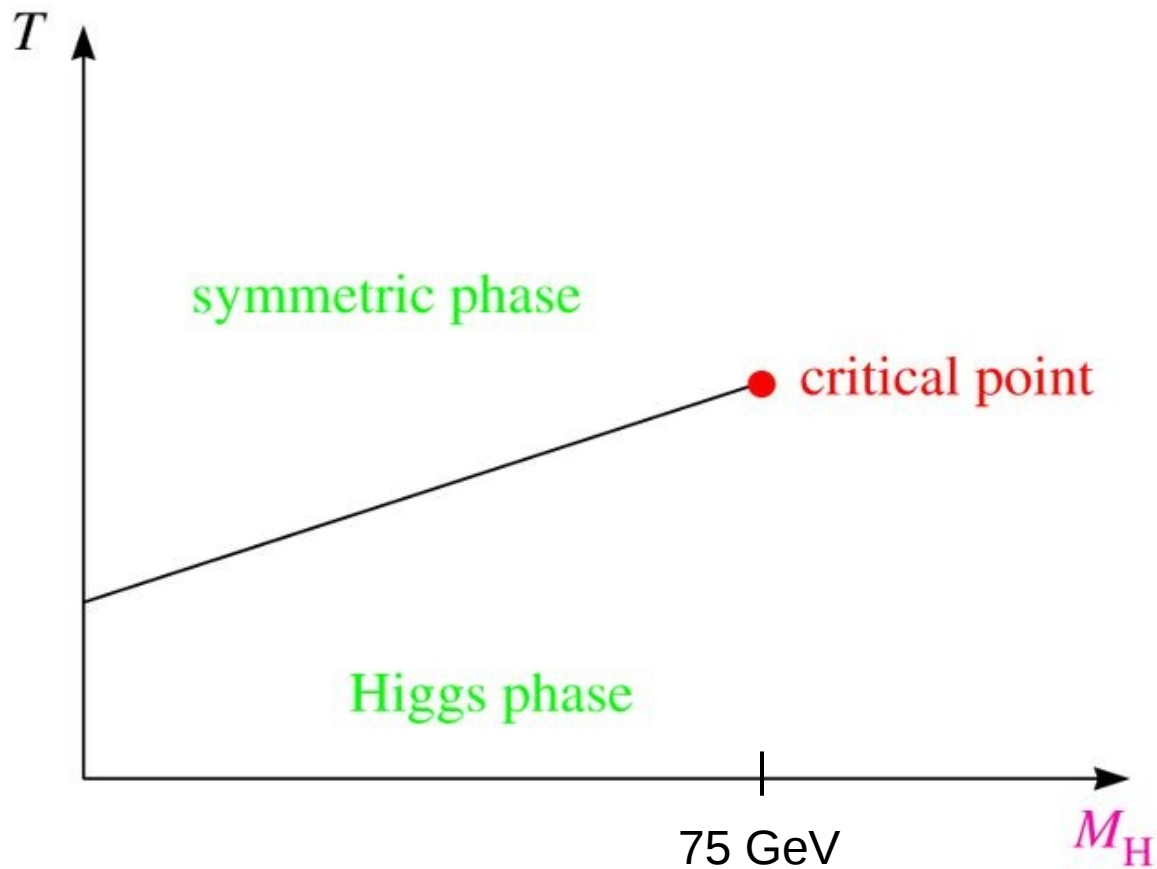
Is this region of α and β feasible in well-motivated UV theories?

- > Remark: some low-energy extensions of the SM seem to be able to reproduce such values (see e.g. 1512.06239)

Answer

- In the SM the EWPT is not of first-order

Kajantie, Laine, Rummukainen, Shaposhnikov, '96;
Karsh, Neuhaus, Patkos '96; Csikor, Fodor, Hietger '98.



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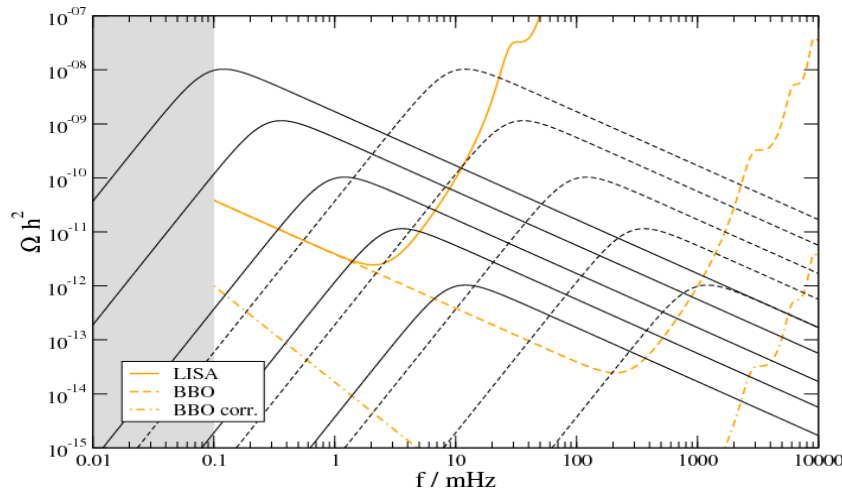
Kajantie,Laine,Rummukainen,Shaposhnikov, '96;
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- To change this feature we need to modify the EW sector by means of either finite-temperature radiative corrections or/and new Higgs fields. In practice both options imply new scalar fields below the \sim TeV scale (therefore testable)

- ◆ (Reminder) Exceptions to this criterion exist: RS models. If the Higgs start “existing” only at low temperature, its phase transition is strongly distorted although the Higgs potential is SM-like (in fact the EWPT can be linked to the transition of the radion).

Creminelli,Nicolis,Rattazzi, '02; Randall,Servant, '07;
GN,Quiros,Wulzer, '07; Konstandin,GN,Quiros, '10,
Konstandin, Servant, '11.

- ◆ YES in RS models



Answer 1bis (in supersymmetry)

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- SUSY models naturally satisfy both features.
- Let us start with the minimal supersymmetric extension...

Minimal option: MSSM

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BAD

Several uncertainties, but after the LHC the GWs produced by the MSSM EWPT are tiny (if any)

Carena,Quiros,Wagner, '96; Delepine,Gerard,Gonzalez Felipe, Weyers, '96;
Cline,Kainulainen '96.
Carena, GN, Quiros,Wagner, '09, '13; Laine,GN,Rummukainen, '13
Cohen,Morrissey,Pierce '12; Curtin,Jaiswal,Meade '12;
Carena, GN, Quiros,Wagner,'13;
Profumo et al.,'15.

Minimal option: MSSM

- To modify the $T \sim 100$ GeV effective potential of the SM-like Higgs, we need a particle in the thermal bath ($m/T < 3$) with sizable couplings to the Higgs. This particle is a light right-hand stop. Carena, Quiros, Wagner, '96; Delepine, Gerard, Gonzalez Felipe, Weyers, '96; Cline, Kainulainen '96.
- In the MSSM case the tunneling involves only the SM-like Higgs (no other directions but $V(h)$ are relevant; m_A is typically large).
- Perturbative and non-perturbative estimates show that, in order to induce large enough corrections to the Higgs potential, for $m_h = 125$ GeV the lighter stop has to be very light ($m_{\tilde{t}_R} \ll 200$ GeV) and with large coupling to the Higgs.

Carena, GN, Quiros, Wagner, '09, '13; Laine, GN, Rummukainen, '13

GENERAL LESSON (but not a theorem!)

If the tunneling involves only the SM-like Higgs direction, a large barrier is possible only by means of new large radiative corrections to the Higgs. These tend to spoil the Higgs signal strengths.

Answer 1bis (in supersymmetry)

Is it possible to avoid this issue in extensions of the MSSM ?

OUR AIM IS TO PROVE

Despite the parameter constraints due to supersymmetry and the LHC bounds, in principle, in extensions of the MSSM we can have striking GW signals from the EWPT

Since our aim is a “proof of principle”:

- 📦 We do not try to find the most minimal scenario exhibiting large GW signals
- 📦 We do not perform an exhaustive analysis of the full parameter space
- We choose a sensible model; we focus on a parameter region that is promising for GW and safe from collider constraints; we calculate the GW prediction

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General NMSSM

(MSSM + singlet, no additional discrete symmetry)

R. Apreda, M. Maggiore, A. Nicolis, A. Riotto, '02;
M. Jiang, L. Bian, W. Huang, J. Shu, '15;
Kozaczuk, Profumo, Haskin, Wainwright, '15.

Scalar potential

> The tree-level potential of the Higgs sector is

$$V_0 = \frac{1}{2}m_{H_d}^2 h_d^2 + \frac{1}{2}m_{H_u}^2 h_u^2 + \frac{1}{2}(B_S + m_S^2)s^2 + \frac{1}{3\sqrt{2}}T_\kappa s^3 - B_\mu h_d h_u - \frac{1}{\sqrt{2}}T_\lambda h_d h_u s \\ + \frac{1}{32}(g_1^2 + g_2^2)(h_d^2 - h_u^2)^2 + \frac{2}{\sqrt{2}}\xi_1 s + \left(L_1 + \frac{1}{\sqrt{2}}M_S s + \frac{\kappa}{2}s^2 - \frac{\lambda}{2}h_d h_u \right)^2 \\ + \frac{1}{2}(h_d^2 + h_u^2) \left(\frac{1}{\sqrt{2}}\lambda s + \mu \right)^2$$

> We assume B_μ large. We fix the parameters to have an approx. Z_2 symm., i.e. the tree-level potential has the minima

$$\langle \{h, s\} \rangle_{brok} = \{v_h, 0\} \qquad \langle \{h, s\} \rangle_{sym} = \{0, \pm \bar{v}_s\}$$

separated by a tree-level barrier. By modulated this barrier we might achieve very strong EWPT. Let us see... (similar idea used for EWBG, but either much before LHC or not pushed to the regime useful for GW signals)

R. Apreda, M. Maggiore, A. Nicolis, A. Riotto, '02;
J.M. No, Ramsey-Musolf, '14;
Profumo, Ramsey-Musolf, Wainwright, Winslow, '15;
M. Jiang, L. Bian, W. Huang, J. Shu, '15;
Kozaczuk, Profumo, Haskin, Wainwright, '15.

- > The light degrees of freedom:
 - Higgsino-like chargino and neutralinos
 - SM-like Higgs
 - singlet-like CP-even scalar

Scalars are OK with LHC, LEP, ... because of the tiny mixing

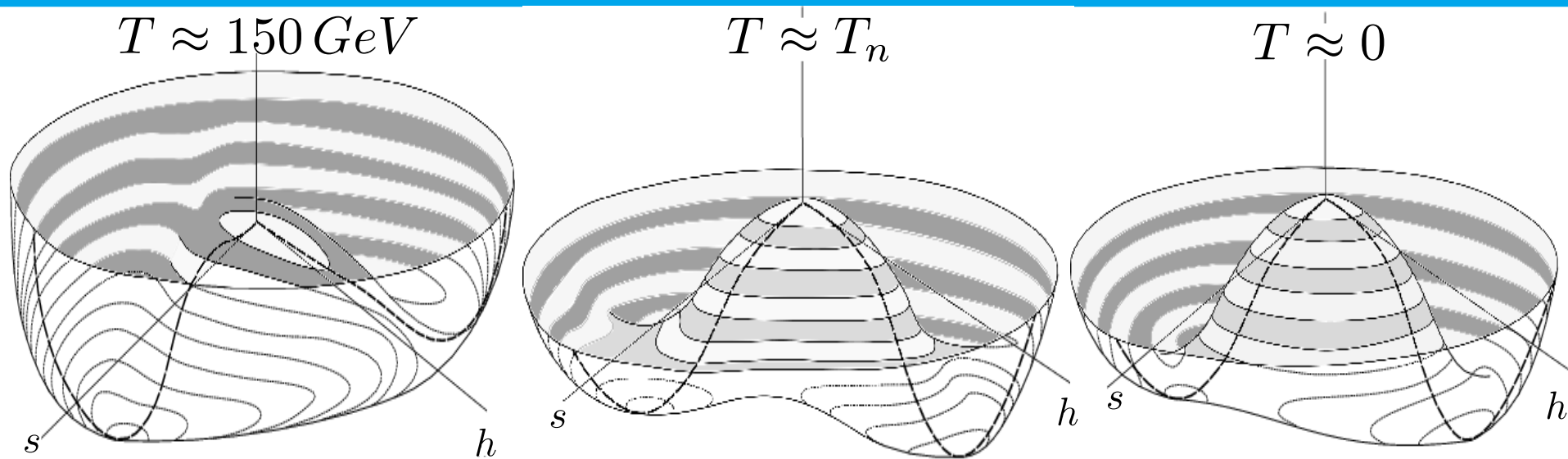
Higgsino-like states are compressed, so decays are too soft for the LHC

- > The rest of the particles (squarks, sleptons, gauginos, singlino, ...) are assumed at the TeV scale to easily overcome the LHC bounds

Heavy fields and the transition

- > The heavy fields induce large radiative corrections to the Higgs potential.
- > The modulation of the barrier that was transparent at tree level, now becomes obscure as depends on many inputs.
- > Indeed the parameter region that was leading to the strong EWPT is now reshuffled.
- > The usual solution is *brute-force*: the interesting parameter region is found by means of cumbersome and time-consuming parameter scans. *Scans likely miss small regions.*
- > Here we propose a different approach. We adopt a renormalization scheme that keeps the positions of all minima fixed. Particularly suitable for EWPT analyses.

Heavy fields and the transition



Graphics from: N.Blinov, J.Kozaczuk, D.Morrissey, C.Tamarit, '15

- > At high T (but not too high), there is only the minimum along the singlet direction
- > Nearby the critical T and below, there are minima in the singlet and Higgs orthogonal directions. They are separated by a barrier.
- > The tunnelling involves more than one field: **2-step phase transition**

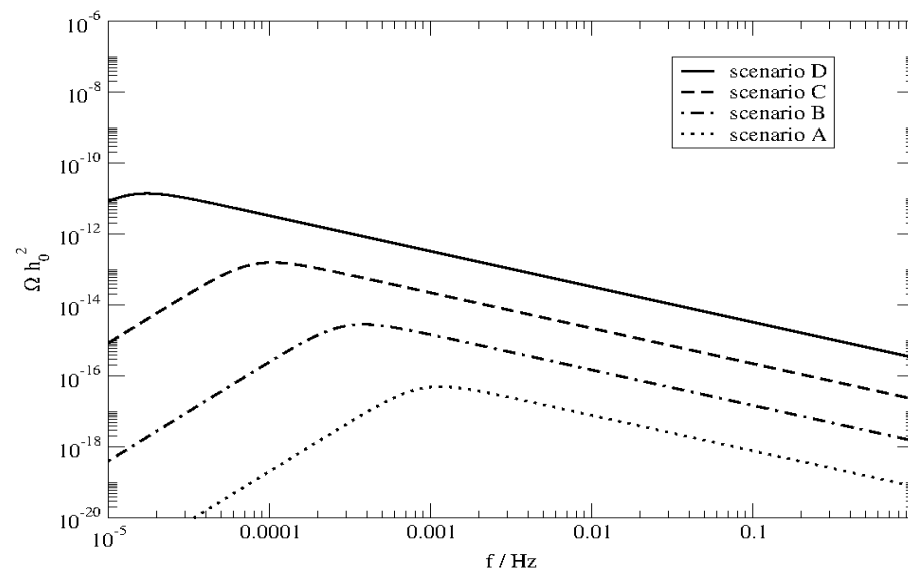
Beyond the minimal model, very strong EWPT are feasible!!!

Good news !!!

In principle we can have sizable GW spectrum from supersymmetric extensions of the SM.

Within supersymmetry, the most promising scenarios seem those where the EWPT occurs in 2 steps. (but see also M.Garcia-Pepin,M.Quiros,'16)

Likely, the presented results are quite generic and cover other SUSY models (if they do not depart too much from minimality).



Huber, Konstandin, GN, Rues'15

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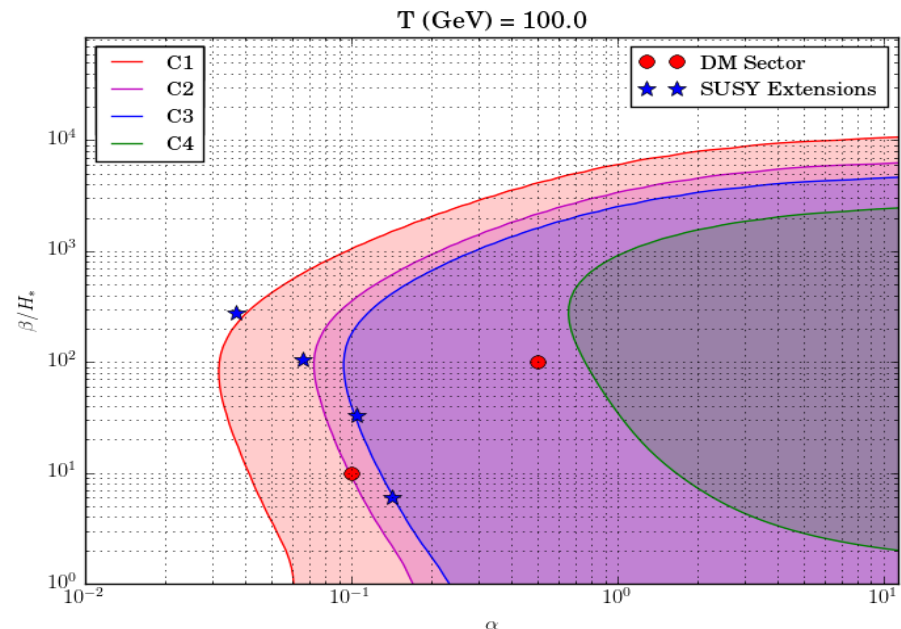
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... and detectable at eLISA
if the third arm is
approved !!!



Conclusions

- > GW experiments are starting detecting unexplored physics
- > GW detectors are opening a window also towards the early Universe

eLISA, possibly with the three-arm design, has a big potential to test cosmology

- > There are well-motivated models exhibiting 1st – order phase transitions (*at $T > 10$ GeV or even above the TeV scale*) with GW signatures in the eLISA sensitivity region

The phase transition of the Higgs sector falls exactly at the frequencies of eLISA

- > In some of these models (e.g. with an approx Z_2 symmetry) the parameter region interesting for GWs is very hard to probe at the LHC-14TeV or FCC.
- > If built, the FCC will not take data before ~2028, when also eLISA might be operating

For particle/cosmology physics, the complementarity of FCC and eLISA would be amazing

IF OF YOUR INTEREST:

The eLISA collaboration is still expanding. Many things need to be done. If you are interested in any of the working groups below, contact the corresponding coordinators

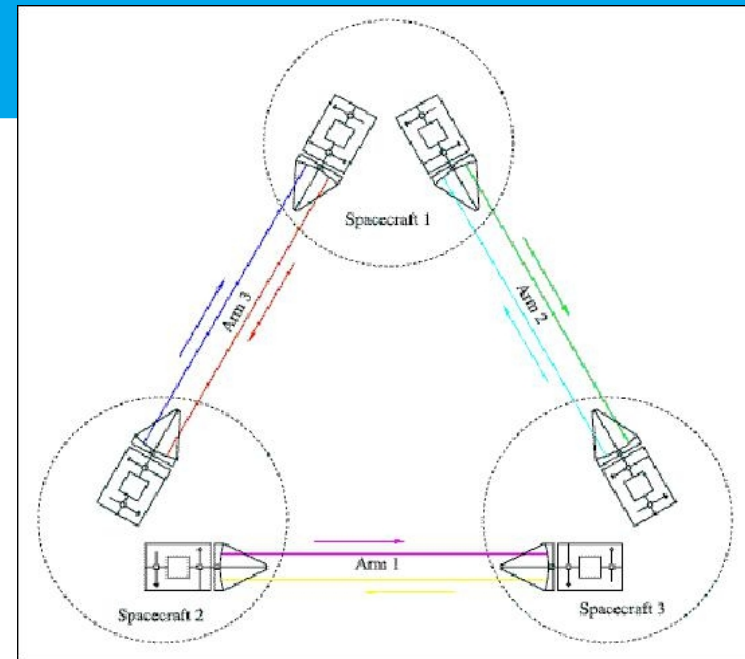
The screenshot shows the eLISA website with the following content:

- Navigation Menu:** Home, News, Whitepaper, Publications, Conferences, People, Multimedia, Positions, Book a scientist, Visit our labs, Contact, Working Group Activities, f, g+
- Left Sidebar:** Consortium Members, Consortium Structure, Working Groups (WG), Astrophysical black holes WG, Cosmology WG, Data analysis WG, Extreme mass ratio inspiral WG, LISA Pathfinder WG, Science of measurement WG, Tests of fundamental laws WG, Ultra-compact binaries WG, Working Group Activities, Publications, Conferences, Member Profiles, Join the community!, Book a scientist!, Visit our labs!
- Central Content:**
 - Announcement:** The new eLISA Consortium has set up several working groups. Scientists wishing to contribute are welcome to **register to the consortium and to join the working groups.**
 - Join the working groups:** There are currently **six science working groups**, each associated with a different low-frequency gravitational-wave source (plus data analysis):
 - **Astrophysical black holes** (Alberto Sesana, Monica Colpi)
 - **Cosmology** (Chiara Caprini, Germano Nardini)
 - **Data analysis** (Stas Babak, Martin Hewitson, Mauro Hueller, Ed Porter)
 - **Extreme mass ratio inspiral EMRI** (Pau Amaro-Seoane, Carlos Sopuerta)
 - **Tests of fundamental laws** (Jonathan Gair, Philippe Grandclement)
 - **Ultra-compact binaries** (Gijs Nelemans)
 - Instrument Working Group:** There is currently **one instrument working group**:
 - **Science of measurement** (Gerhard Heinzl, Bill Weber, Hubert Halloin)
 - LISA Pathfinder Group:** In addition, there is a **working group focussing on LISA Pathfinder**:
 - **LISA Pathfinder Group**
- Right Sidebar (Working Group Activities):**
 - Nov 30, 2015: A Century of General Relativity, November 30 – December 2, 2015, Harnack House Berlin
 - Sep 22, 2015: Workshop of the eLISA Cosmology Working Group (Stavanger)
 - Jun 21, 2015: The 11th Edoardo Amaldi Conference on Gravitational Waves, June 21-26, 2015, Gwangju, Korea
 - Apr 13, 2015: 1st eLISA Cosmology Working Group Workshop
 - Nov 24, 2014: Position for a eLISA/LISA Pathfinder Data Analysis /Instrumentation Engineer at APC (Paris)
 - Nov 15, 2014: Postdoctoral and Doctoral Openings at the AEI in Potsdam
 - Nov 15, 2014: CIERA Postdoctoral Fellowships, Northwestern

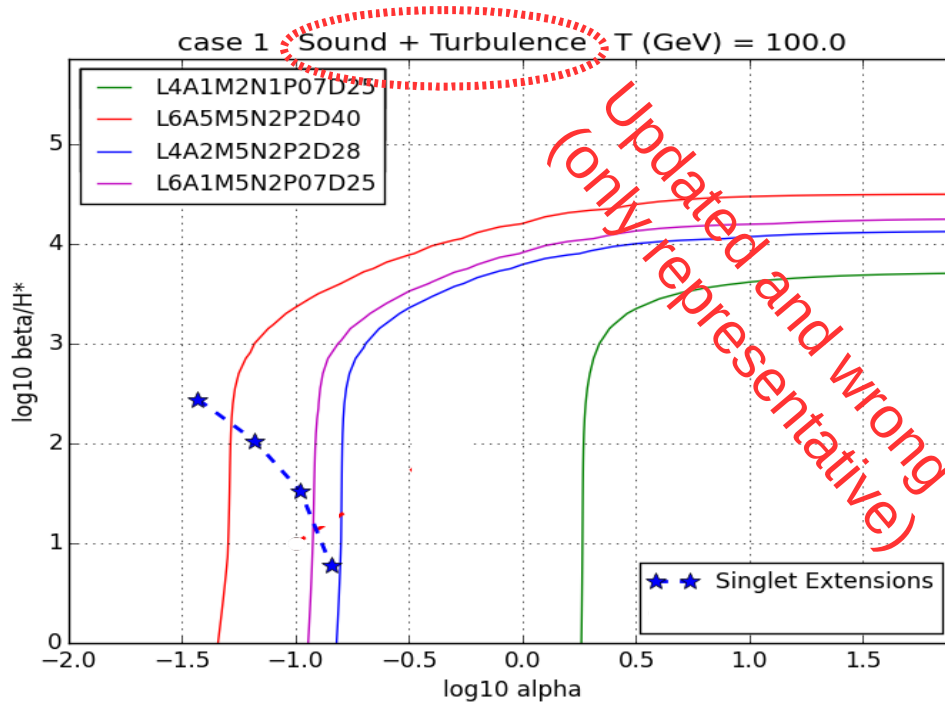
Backup slides

Answer 2

- > To answer, we need to know the sensitivity curves of eLISA, which depend on options that are still under discussion:
 - # of arms: 2 or 3 ?
 - Length of the arms: 1, 2 or 5 Gm ?
 - # of years of data taking: 2 or 5 ?
 - Noise: pathfinder success or expected ?
- > Reminder: the sensitivity curves for our spectra (i.e. stochastic GW background) are more sensitive than those for isolated sources Thrane,Romano, '13
- > We consider 4 possible configurations:
 - 1) 2 arms of 1 Gm, 2 years, noise pathfinder success (Design 1)
 - 2) 2 arms of 2 Gm, 5 years, noise expected (Design 2)
 - 3) 3 arms of 1 Gm and 5 years, noise expected (Design 3)
 - 4) 3 arms of 5 Gm and 5 years, noise expected (Design 4)



Answer 2 (straightforward using the cosmology WG map)



Non-runaway
case

The third arm is very
beneficial for allowing eLISA
to detect SUSY GW signals

- 1) 2 arms of 1 Gm, 2 years, noise of pathfinder success (Design 1)
- 2) 2 arms of 2 Gm, 5 years, noise expected (Design 2)
- 3) 3 arms of 1 Gm and 5 years, noise expected (Design 3)
- 4) 3 arms of 5 Gm and 5 years, noise expected (Design 4)

Conclusions

- > GW measurements can be a way to probe the EWPT and, in turn, the Higgs sector
- > However, in the forthcoming experiments only supercooled EWPT might be probed
- > Detectable GWs are possible within warped models
- > In supersymmetry:
 - in the MSSM the LHC bounds make the EWPT too weak; it seems difficult to modify these results in extensions where only the SM-like Higgs plays a role in the tunneling
 - in the general NMSSM a very strong EWPT is viable. Similar result should be possible in other extensions where the tunneling involves more fields
 - The GW signal is border line with the preliminary sensitivity curves.
 - **The third arm is very useful**

Benchmark points

	A	B	C	D
$\tan \beta$	5	-	-	-
\bar{v}_s [GeV]	307.5	319.8	323.5	324.0
λ	0.7	-	-	-
κ	0.015	-	-	-
L_1	0	-	-	-
B_S [GeV ²]	-250 ²	-	-	-
μ [GeV]	300	-	-	-

tree	A	B	C	D
m_{h_1}	93	-	-	-
m_{h_2}	96	-	-	-
m_{A_1}	373	-	-	-
$m_{\chi_1^0}$	286	-	-	-
$m_{\chi_2^0}$	310	-	-	-
$m_{\chi_1^\pm}$	296	-	-	-

1-loop	A	B	C	D
m_{h_1}	91	-	-	-
m_{h_2}	125.6	-	-	-
$\sin^2 \gamma$	10 ⁻³	-	-	-

- > We choose a parameter point where the symmetric and broken EW minima are separated by a sizeable tree-level barrier
- > We modulate the size of the barrier (mostly the width) by keeping all parameters constant but the singlet VEV
- > We employ the usual bounce method to determine the tunneling action
- > From the action we have β/H and α (via T_n) which can be plugged into the GW formulas

S. Huber, T. Konstandin, '08

Benchmark points

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	A	B	C	D
T_n [GeV]	112.3	94.7	82.5	76.4
α	0.037	0.066	0.105	0.143
β/H	277	105.9	33.2	6.0
$v_h(T_n)/T_n$	1.89	2.40	2.83	3.12

