

Non-perturbative analysis of the spectrum of meson resonances in an ultraviolet-complete composite-Higgs model

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Motivations

- *TeV scale **little hierarchy problems** do not diminish **big hierarchy solutions***
- *If the Higgs is composite, **how heavy the other resonances** are predicted to be?*
- *Definite answers require an **ultraviolet complete theory***
- ***Gauge theories of fermions that confine** at the TeV scale can be studied with a plethora of QCD tools*
- ***Analytic results, holding in well-defined approximations,** may be very informative with little computational effort*

Outline

- *Higgs as a composite Goldstone: the electroweak sector*
- *Tools to study the global symmetry breaking*
- *Tools to study the spectrum of meson resonances*
- *Cross-checks and limitations of the results*
- *Composite top-quark partners: the colour sector*
- *Non-trivial interplay of the two sectors: anomalies, coupled gap equations, mixing of singlet scalars, ...*
- *Some perspectives*

Note I will stick to **the chiral limit** : no new results (for now) on SM Yukawas, nor on electroweak symmetry breaking.

Some remarks **beyond the chiral limit** : constituent fermion masses, and SM gauging.

The electroweak sector

Requirements:

- **The Higgs** as a composite Goldstone boson (with custodial symmetry): $H_F \supset SU(2)_L \times SU(2)_R$
- A minimal number of extra Goldstones:
 $G_F = SU(4)$ broken to $H_F = Sp(4) \rightarrow$ the Higgs + a singlet

Katz-Nelson-Walker 05, Gripaos-Pomarol-Riva-Serra 09, Frigerio-Pomarol-Riva-Urbano 12

- An explicit ultraviolet completion, with **no fundamental scalars**: a gauge theory of fermions that confines
- Need for 4 fermions ψ , in a pseudo-real representation of the gauge group: the fundamental of $G_{HC} = Sp(2N)$

Minimal technicolour (N=1):

Ryttov-Sannino 08, Galloway-Evans-Luty-Tacchi 10, Cacciapaglia-Sannino 14

Other possible minimal models of this sort classified in Ferretti-Karateev '13, Vecchi '15

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Katz-Nelson-Walker 05, Gripaio

Compare with QCD:

- An explicit ultraviolet completion as a gauge theory of fermions
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Combining constituent fermions

			COLOUR	FLAVOUR	
		Lorentz	$Sp(2N)$	$SU(4)$	$Sp(4)$
	ψ_i^a	$(1/2, 0)$	\square_i	4^a	4
	$\bar{\psi}_{ai} \equiv \psi_{aj}^\dagger \Omega_{ji}$	$(0, 1/2)$	\square_i	$\bar{4}_a$	4^*
spin-zero bilinears	$M^{ab} \sim (\psi^a \psi^b)$	$(0, 0)$	1	6^{ab}	$5 + 1$
	$\bar{M}_{ab} \sim (\bar{\psi}_a \bar{\psi}_b)$	$(0, 0)$	1	$\bar{6}_{ab}$	$5 + 1$
spin-one bilinears	$a^\mu \sim (\bar{\psi}_a \bar{\sigma}^\mu \psi^a)$	$(1/2, 1/2)$	1	1	1
	$(V^\mu, A^\mu)_a^b \sim (\bar{\psi}_a \bar{\sigma}^\mu \psi^b)$	$(1/2, 1/2)$	1	15_a^b	$10 + 5$

Hypercolour-invariant fermion bilinears have the quantum numbers of meson resonances

scalars : $\sigma + S^{\hat{A}} \sim 1 + 5$	pseudoscalars : $\eta' + G^{\hat{A}} \sim 1 + 5$
vectors : $V_\mu^A \sim 10$	axialvectors : $a_\mu + A_\mu^{\hat{A}} \sim 1 + 5$

The fate of the SU(4) symmetry (without computations)

- The model is a **vector-like gauge theory** : all fermions ψ can be made massive, while preserving $G_C = \text{Sp}(2N)$: $m_\psi \psi\psi$
- **Vafa-Witten theorem** : the G_F subgroup preserved by m_ψ cannot be spontaneously broken \rightarrow **Sp(4) unbroken**
- **'t Hooft anomaly-matching** : any global UV anomaly must be matched in the IR, either by massless spin-1/2 baryons or by Goldstone bosons \rightarrow **SU(4) necessarily breaks** because ψ 's cannot form any spin-1/2 baryon

$$d^{ABC\hat{C}} = 2\text{tr}(\{T^A, T^B\}T^{\hat{C}})$$

SU(4) broken and unbroken generators combine in non-zero anomaly coefficients

$$(\psi^a \psi^b) \equiv \psi_i^a \Omega_{ij} \psi_j^b$$

The unique invariant tensor of Sp(2N) is two-index antisymmetric

The fate of the SU(4) symmetry (effective potential from 4-fermion operators)

- **Nambu-Jona Lasinio approximation of strong dynamics:** 'decouple' hypergluons inducing effective 4-fermion interactions

$$\mathcal{L}_{scal}^{\psi} = \frac{\kappa_A}{2N} (\psi^a \psi^b) (\bar{\psi}_a \bar{\psi}_b) - \frac{\kappa_B}{8N} [\epsilon_{abcd} (\psi^a \psi^b) (\psi^c \psi^d) + h.c.]$$

- Introduce an **auxiliary scalar field** M_{ab} whose equation of motion is

$$M^{ab} = -\frac{\kappa_A + \kappa_B}{2N} (\psi^a \psi^b)$$

- Compute the effective potential $V_{\text{eff}}(M_{ab})$ induced by fermion loops and minimise

- Minimum is non-zero above **four-fermion critical coupling**

$$\langle M_{ab} \rangle = \frac{M_{\psi}}{2} \Sigma_0 = \frac{M_{\psi}}{2} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

$$M_{\psi} \neq 0 \Rightarrow SU(4) \rightarrow Sp(4)$$

The fate of the SU(4) symmetry (mass-gap from 4-fermion operators)

$$\mathcal{L}_{scal}^{\psi} = \frac{\kappa_A}{2N} (\psi^a \psi^b) (\bar{\psi}_a \bar{\psi}_b) - \frac{\kappa_B}{8N} [\epsilon_{abcd} (\psi^a \psi^b) (\psi^c \psi^d) + h.c.]$$

Mass-gap equation: self-consistent condition for **dynamical fermion mass M_{ψ}**

$$M_{\psi} = 4(\kappa_A + \kappa_B) M_{\psi} \tilde{A}_0(M_{\psi}^2)$$

(this amounts to resum all massless-fermion diagrams leading in $1/N$:
each additional tadpole come with a factor N for the loop and $1/N$ for the coupling)

Need for a cutoff Λ for the fermion loops, and for the whole NJL effective theory

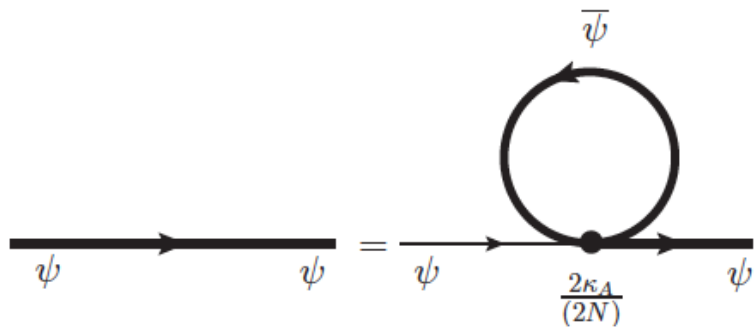
$$\xi \equiv \frac{\Lambda^2(\kappa_A + \kappa_B)}{4\pi^2} = \left[1 - \frac{M_{\psi}^2}{\Lambda^2} \ln \left(\frac{\Lambda^2 + M_{\psi}^2}{M_{\psi}^2} \right) \right]^{-1} \quad 0 < M_{\psi}/\Lambda \lesssim 1$$

critical coupling $1 < \xi \lesssim 3.25$ maximal coupling

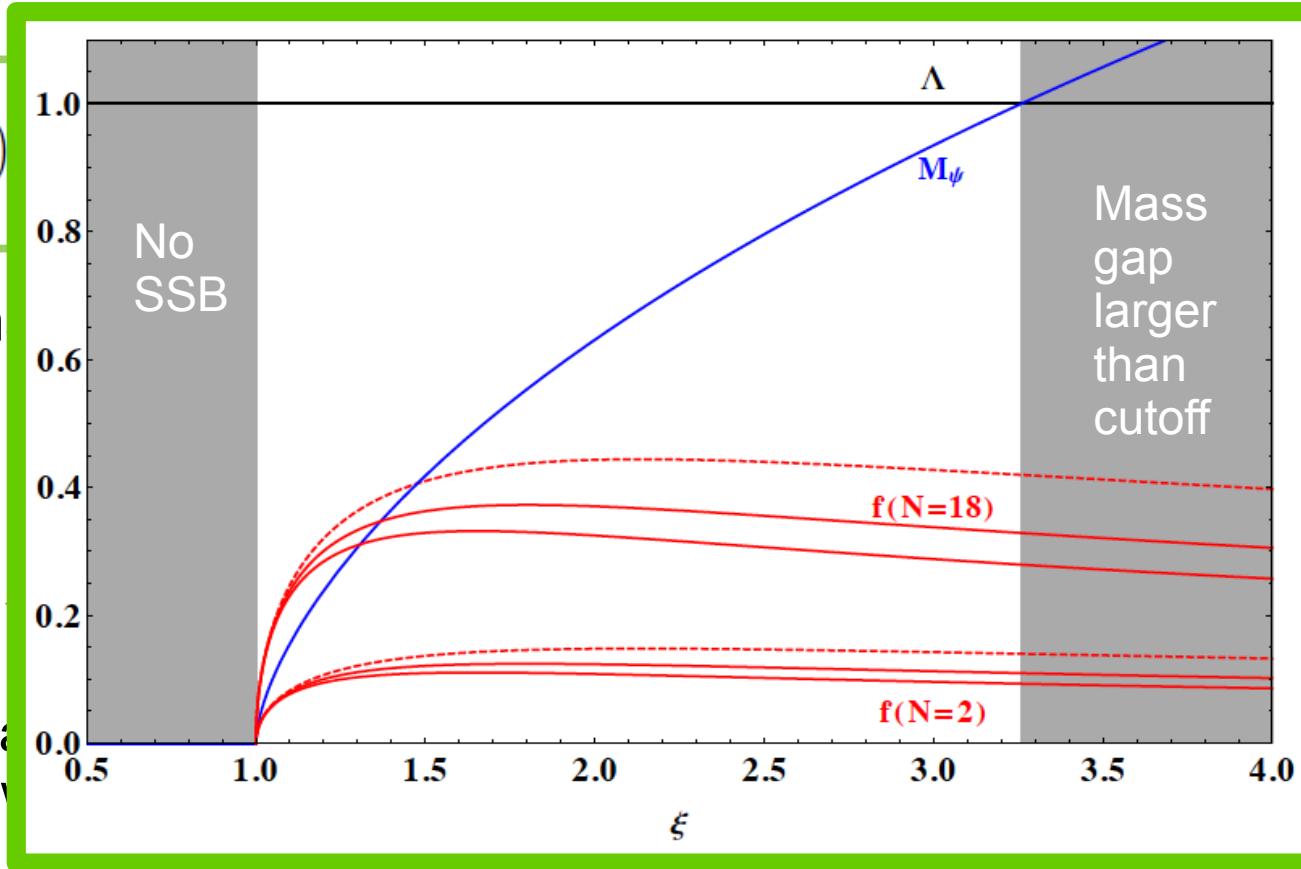
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$$0 < M_\psi / \Lambda \lesssim 1$$

critical
coupling

$$1 < \xi \lesssim 3.25$$

maximal
coupling

The spectrum of mesons (before computations)

$$\begin{array}{ll} \text{scalars : } \sigma + S^{\hat{A}} \sim 1 + 5 & \text{pseudoscalars : } \eta' + G^{\hat{A}} \sim 1 + 5 \\ \text{vectors : } V_{\mu}^A \sim 10 & \text{axialvectors : } a_{\mu} + A_{\mu}^{\hat{A}} \sim 1 + 5 \end{array}$$

- **Chiral limit:** G's are massless; for vanishing κ_B also η' massless
[κ_B induced by the $U(1)_{\psi}$ anomaly, that vanishes for large N]
- **Super-convergent spectral sum rules:** some V-A and S-P two-point correlators are order parameters for $SU(4)/Sp(4)$
→ they must vanish rapidly in the UV
→ associated spectral densities converge fast

$$\mathcal{J}_{\mu}^A = \Omega_{ij} \bar{\psi}_i \bar{\sigma}_{\mu} T^A \psi_j$$

$$\Pi_V(q^2) \delta^{AB} (q_{\mu} q_{\nu} - \eta_{\mu\nu} q^2) = i \int d^4x e^{iq \cdot x} \langle \text{vac} | T \{ \mathcal{J}_{\mu}^A(x) \mathcal{J}_{\nu}^B(0) \} | \text{vac} \rangle$$

$$\lim_{q^2 \rightarrow -\infty} (q^2)^2 \times (\Pi_V - \Pi_A)(q^2) = 0$$

$$\int_0^{\infty} dt \text{Im} \Pi_{V-A}(t) = 0 \qquad \int_0^{\infty} dt t \text{Im} \Pi_{V-A}(t) = 0$$

$$\langle 0 | \mathcal{J}_{\mu}^A(0) | V^B(p) \rangle \equiv f_V M_V \epsilon_{\mu}(p) \delta^{AB}$$

$$f_V^2 M_V^2 - f_A^2 M_A^2 - F_G^2 = 0, \qquad f_V^2 M_V^4 - f_A^2 M_A^4 = 0$$

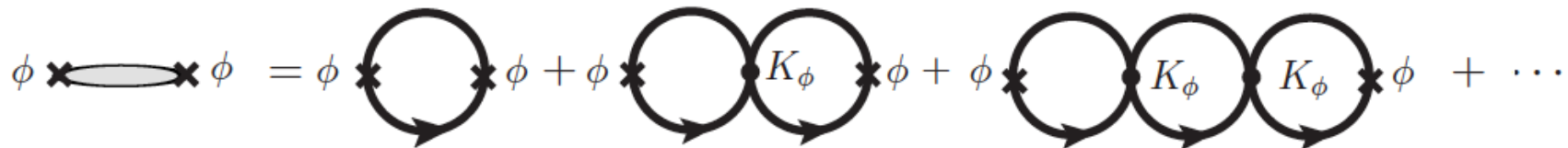
(narrow-width single-resonance approximation)

The spectrum of mesons

(from four-fermion operators)

Resummation of constituent fermion loops at leading order in $1/N$

→ two-point correlators develop a pole



$$\bar{\Pi}_\phi(q^2) \equiv \frac{\tilde{\Pi}_\phi(q^2)}{1 - 2K_\phi \tilde{\Pi}_\phi(q^2)}$$

The pole defines the meson mass

$$1 - 2K_\phi \tilde{\Pi}_\phi(q^2 = M_\phi^2) = 0$$

ϕ	K_ϕ	$\tilde{\Pi}_\phi(q^2)$
$G^{\hat{A}}$	$2(\kappa_A + \kappa_B)/(2N)$	$\tilde{\Pi}_P(q^2) = (2N) [\tilde{A}_0(M_\psi^2) - \frac{q^2}{2} \tilde{B}_0(q^2, M_\psi^2)]$
η	$2(\kappa_A - \kappa_B)/(2N)$	
$S^{\hat{A}}$	$2(\kappa_A - \kappa_B)/(2N)$	$\tilde{\Pi}_S(q^2) = (2N) [\tilde{A}_0(M_\psi^2) - \frac{1}{2}(q^2 - 4M_\psi^2) \tilde{B}_0(q^2, M_\psi^2)]$
σ	$2(\kappa_A + \kappa_B)/(2N)$	

and similarly for the spin-one channels V and A

The spectrum of mesons

(from four-fermion operators)

$$1 - 2K_\phi \tilde{\Pi}_\phi(q^2 = M_\phi^2) = 0$$

Inserting the gap-equation, one recovers consistently the Goldstone pole:

$$M_G = 0$$

Singlet pseudoscalar proportional to anomaly and mixes with axial vector:

$$M_{\eta'}^2 = -\frac{\kappa_B}{\kappa_A^2 - \kappa_B^2} \frac{1}{\tilde{B}_0(M_{\eta'}^2, M_\psi^2)} \left[1 - 2K_a \tilde{\Pi}_A^L(M_{\eta'}^2) \right]$$

Scalars proportional to the mass gap:

$$M_\sigma^2 = 4M_\psi^2$$

$$M_S^2 = 4M_\psi^2 + M_\eta^2 \frac{\tilde{B}_0(M_\eta^2, M_\psi^2)}{\tilde{B}_0(M_S^2, M_\psi^2)} \simeq M_\sigma^2 + M_\eta^2$$

Vector heavy even for a vanishing mass gap:

$$M_V^2 = -\frac{3}{4\kappa_D \tilde{B}_0(M_V^2, M_\psi^2)} + 2M_\psi^2 \frac{\tilde{B}_0(0, M_\psi^2)}{\tilde{B}_0(M_V^2, M_\psi^2)} - 2M_\psi^2$$

Axial vector generally the heaviest:

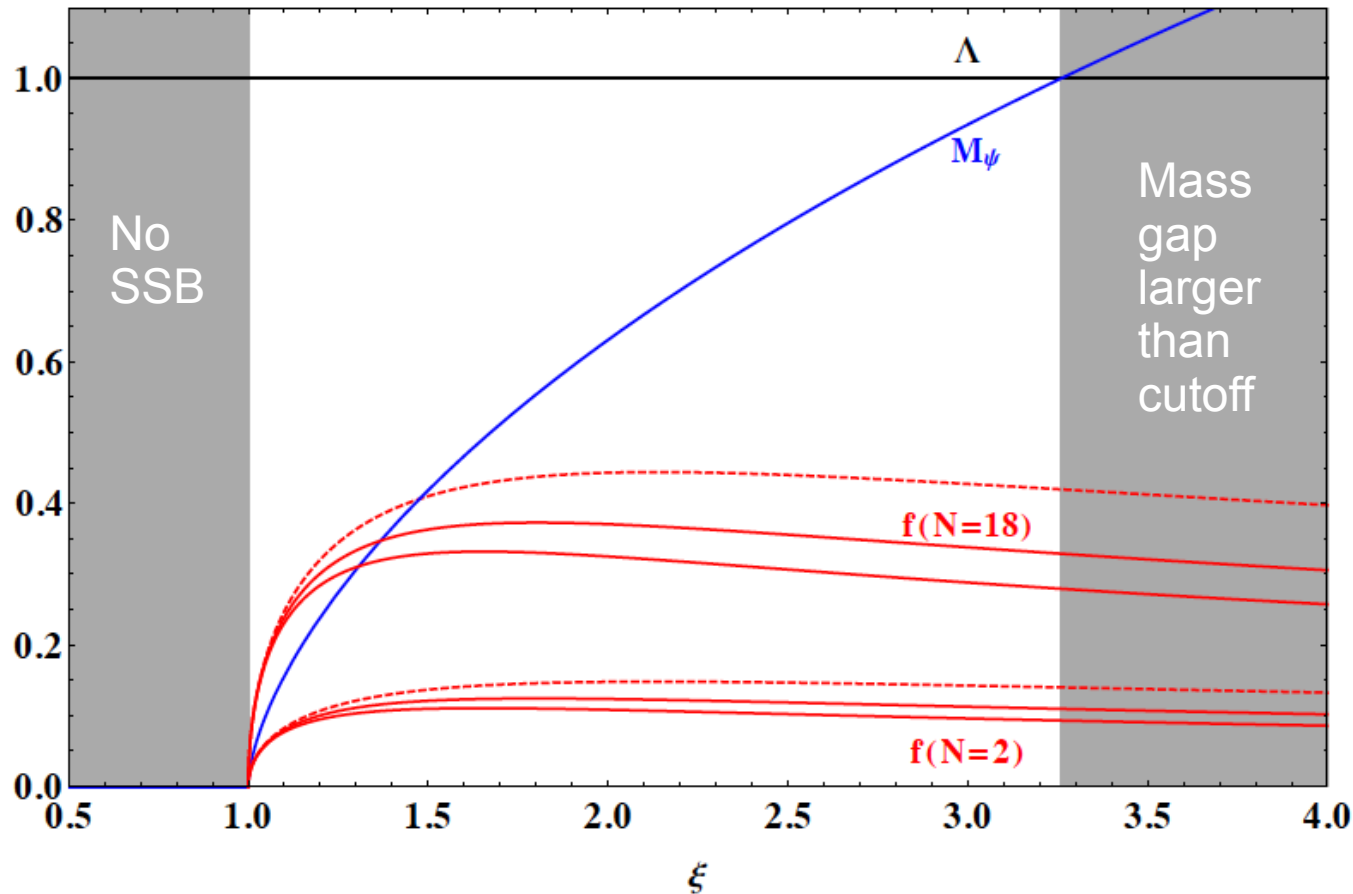
$$M_A^2 = -\frac{3}{4\kappa_D \tilde{B}_0(M_A^2, M_\psi^2)} + 2M_\psi^2 \frac{\tilde{B}_0(0, M_\psi^2)}{\tilde{B}_0(M_A^2, M_\psi^2)} + 4M_\psi^2 \simeq M_V^2 + 6M_\psi^2$$

The Goldstone decay constant

$$\langle \text{vac} | \mathcal{J}_\mu^{\hat{A}}(0) | G^{\hat{B}}(p) \rangle = ip_\mu \frac{f}{\sqrt{2}} \delta^{\hat{A}\hat{B}}$$

Electroweak precision observables receive order v^2 / f^2 corrections $\rightarrow f > (0.5-1)\text{TeV}$

$$\frac{f^2}{2} = \lim_{q^2 \rightarrow 0} (-q^2 \bar{\Pi}_A(q^2)) = \frac{\tilde{\Pi}_A(0)}{1 - 2K_D \tilde{\Pi}_A(0)}, \quad \tilde{\Pi}_A(0) = -2(2N) M_\psi^2 \tilde{B}_0(0, M_\psi^2)$$



f is the residue of the Goldstone boson pole in the resummed, transverse, axial-vector correlator

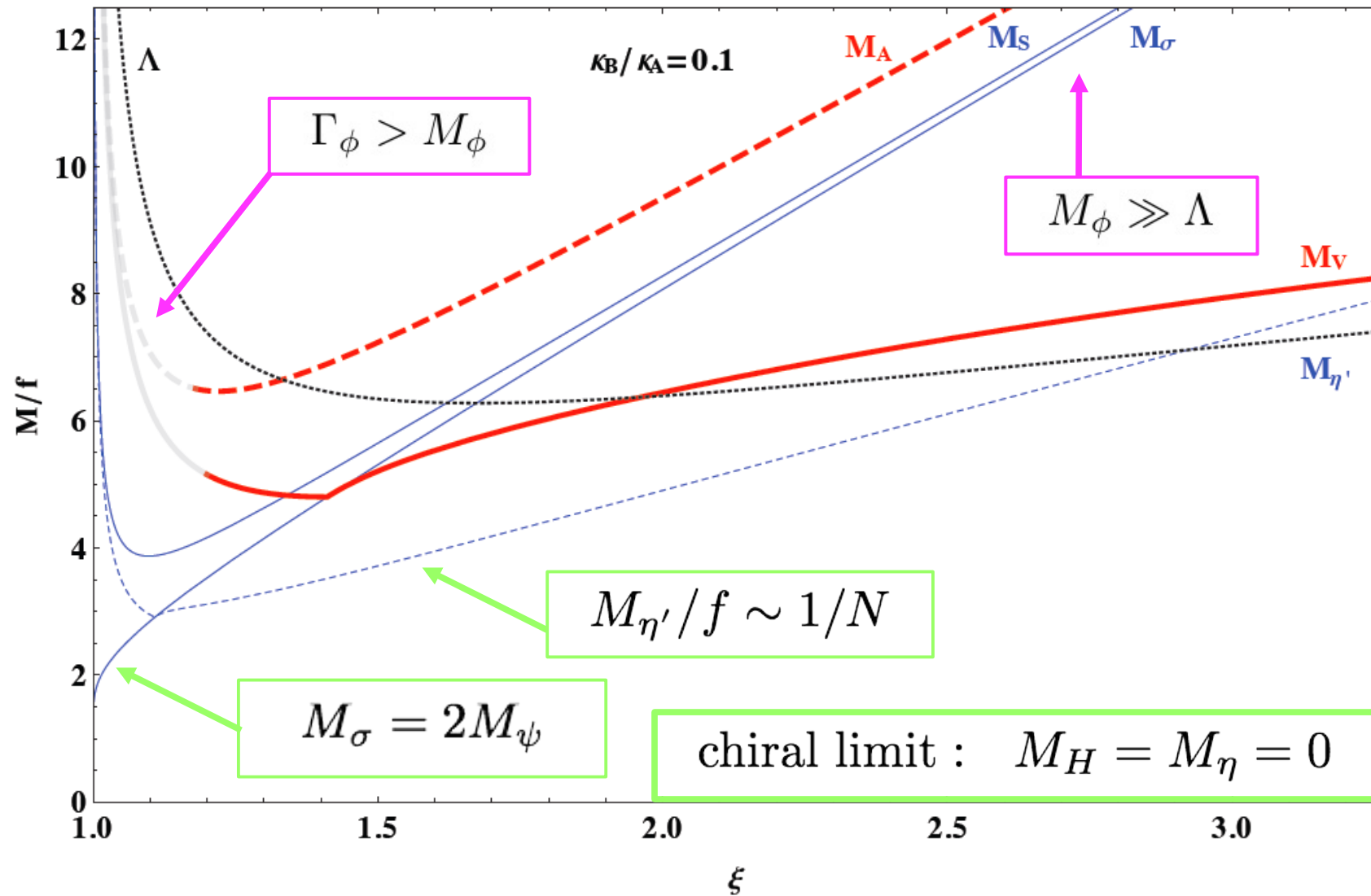
$$f \propto \sqrt{N}$$

f can be as small as a tenth of the NJL cutoff

Meson masses in units of f

$M_\phi/f \sim \sqrt{1/N}$ here $N = 4$

Spin-zero and spin-one 4-fermion operators have the same coupling strength, if single-hypergluon exchange dominates (expected when N is large)



$\kappa_C = \kappa_A$
 $\kappa_D = \kappa_A$

$\kappa_A + \kappa_B \sim \xi$
 κ_B/κ_A free

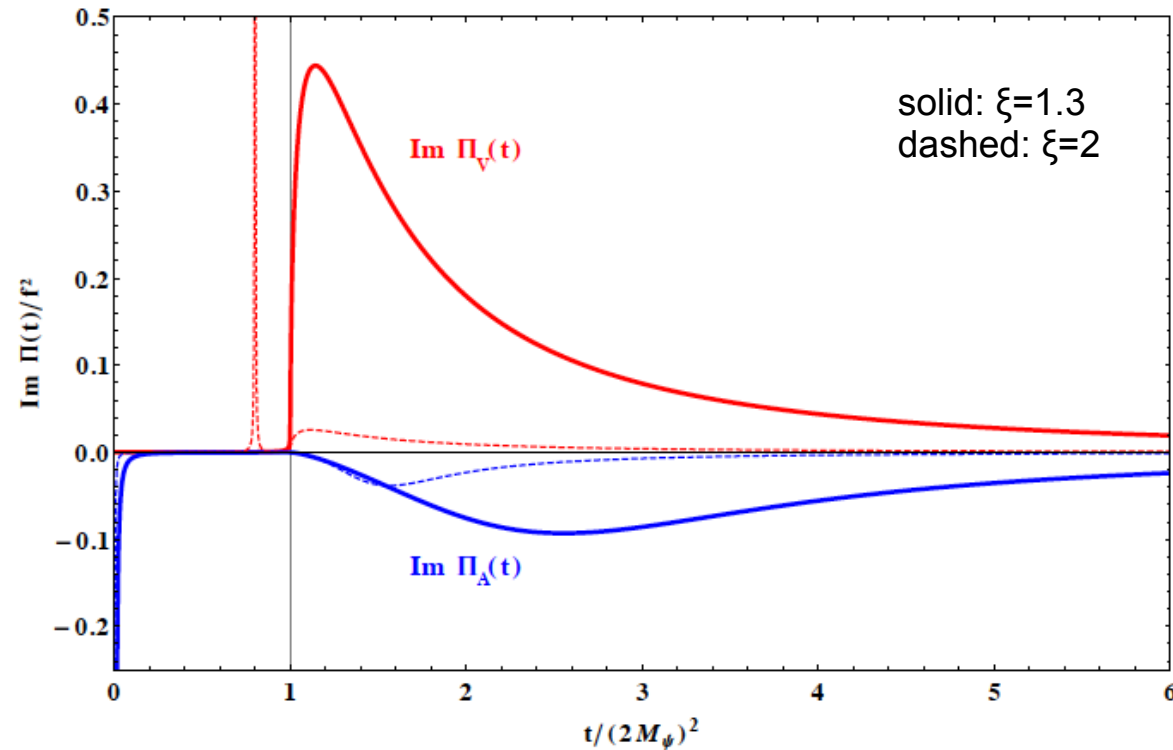
Degenerate $Sp(4)$ multiplets (electroweak splitting neglected)

$M_a = M_A$

Comparison with sum rules

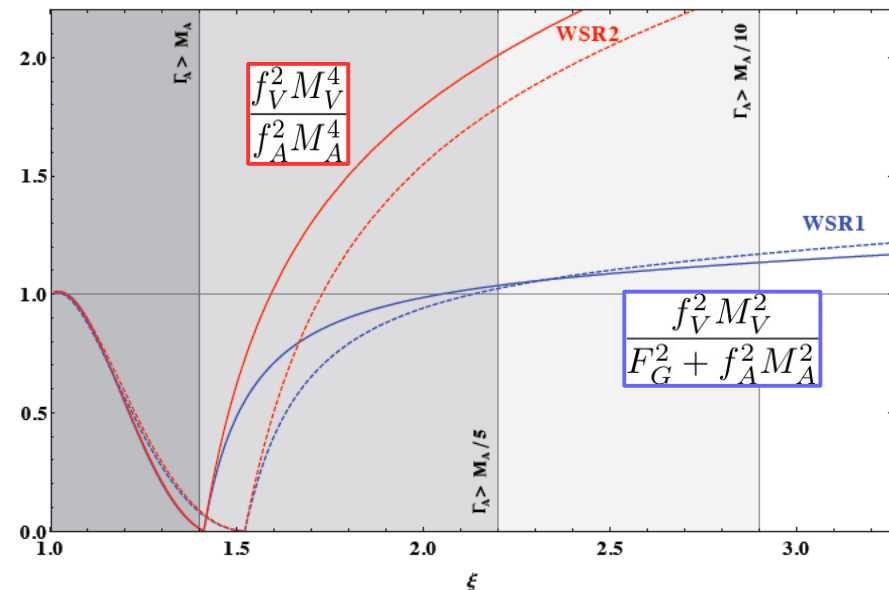
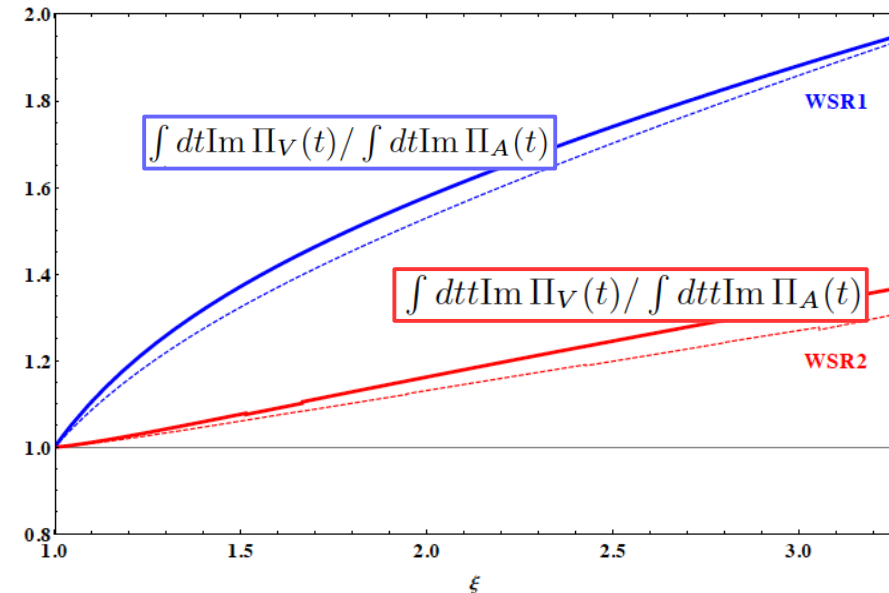
$$\int_0^\infty dt \operatorname{Im} \Pi_{V-A}(t) = 0 \quad \int_0^\infty dt t \operatorname{Im} \Pi_{V-A}(t) = 0$$

NJL effective theory ceases to be valid for $t > \Lambda^2$: this region is negligible only for small ξ



Narrow-width & single-resonance approximation
(masses and decay constants extracted from the resummed two-point correlators)

$$f_V^2 M_V^2 - f_A^2 M_A^2 - F_G^2 = 0, \quad f_V^2 M_V^4 - f_A^2 M_A^4 = 0$$



Comparison with lattice

Arthur-Drach-Hansen-Hietanen-Pica-Sannino, 1602.06559 & 1607.06654

Recent lattice study for $N=1$: $G_{HC} = Sp(2) \approx SU(2)$

$$\begin{array}{lll} M_V = 13.1(2.2)f & M_A = 14.5(3.6)f & \\ M_\sigma = 19.2(10.8)f & M_S = 16.7(4.9)f & M_{\eta'} = 12.8(4.7)f \end{array}$$

*possible $\sqrt{2}$ issue with the normalisation of f

Meson masses computed à la NJL scale as $1/\sqrt{N}$, and they are expected to be *more precise at large N* ...

Matching lattice and NJL predictions, one can roughly identify the *preferred values for 4-fermion couplings*:

- Small A-V splitting prefers ξ close to one, but then σ is too light: intermediate values $\xi \sim (1.5 - 2)$ seem to provide a better global agreement
- The heavy pseudoscalar requires $\kappa_B/\kappa_A > 0.2$

Introducing the colour sector

- To couple to the composite Higgs, the **SM fermions may mix linearly with composite resonances** (partial compositeness)
 - especially suitable to induce the large **top quark Yukawa coupling**
- Need to introduce **constituent fermions X^f** that (i) are coloured and (ii) can form spin-1/2 baryons
 - need to go beyond the $Sp(2N)$ -fundamental representation & to have $N \geq 2$

Barnard, Gherghetta, Sankar Ray, 1311.6562

	Lorentz	HYPER COLOUR $Sp(2N)$	FLAVOUR $SU(6)$	$SO(6)$
X_{ij}^f	$(1/2, 0)$	\square_{ij}	6^f	6
$\bar{X}_{fij} \equiv \Omega_{ik} X_{fkl}^\dagger \Omega_{lj}$	$(0, 1/2)$	\square_{ij}	$\bar{6}_f$	6
$M_c^{fg} \sim (X^f X^g)$	$(0, 0)$	1	21^{fg}	$20' + 1$
$\bar{M}_{cfg} \sim (\bar{X}_f \bar{X}_g)$	$(0, 0)$	1	$\bar{21}_{fg}$	$20' + 1$
$a_c^\mu \sim (\bar{X}^f \bar{\sigma}^\mu X_f)$	$(1/2, 1/2)$	1	1	1
$(V_c^\mu, A_c^\mu)_f^g \sim (\bar{X}_f \bar{\sigma}^\mu X^g)$	$(1/2, 1/2)$	1	35_f^g	$15 + 20'$

$$X^f \sim 6_{SU(6)} = (3 + \bar{3})_{SU(3)_c}$$

$$X_{ij}^f = -X_{ji}^f \quad X_{ij}^f \Omega_{ji} = 0$$

$$X^f \sim \bar{X}_f : \quad SU(6) \rightarrow SO(6)$$

} spin-zero mesons

} spin-one mesons

U(1) (anomalous) symmetries

Besides $SU(4)_\psi$ and $SU(6)_X$, it is important to consider **global fermion numbers** $U(1)_\psi$ and $U(1)_X$ with currents

$$\mathcal{J}_{\psi\mu}^0 = -\bar{\psi}\bar{\sigma}_\mu\psi \quad \mathcal{J}_{X\mu}^0 = \bar{X}\bar{\sigma}_\mu X$$

They are both **anomalous w.r.t. to $Sp(2N)$** [like $U(1)_A$ w.r.t. QCD]

$$\partial^\mu \mathcal{J}_{\psi, X\mu}^0 \propto \frac{g_{HC}^2}{32\pi^2} \sum_{I=1}^{N(2N+1)} \epsilon_{\mu\nu\rho\sigma} G_{HC}^{I,\mu\nu} G_{HC}^{I,\rho\sigma}$$

However **a linear combination U(1) is anomaly-free** and thus conserved

$$\mathcal{J}_\mu^0 = \mathcal{J}_{X\mu}^0 - 3(N-1)\mathcal{J}_{\psi\mu}^0 \quad q_\psi = -3(N-1)q_X$$

When either $\langle\psi\psi\rangle$ or $\langle XX\rangle$ condense, a **new Goldstone η_0** appears, while **one unique η' receives a mass** from the anomaly

An operator for the Sp(2N) anomaly

The analog of 't Hooft determinant in QCD : $2N_F$ - fermion operator, that breaks the anomalous $U(1)_A$ and explains the large η' mass

Let us build the minimal fermion operator that breaks the anomalous $U(1)$'s but preserves all exact symmetries. It incorporates the effect of the Sp(2N) anomaly!

Electroweak sector: Sp(2N) anomaly breaks $U(1)_\psi$ $\mathcal{O}_\psi = -\frac{1}{4}\epsilon_{abcd}(\psi^a\psi^b)(\psi^c\psi^d)$

Colour sector: Sp(2N) anomaly breaks $U(1)_X$ $\mathcal{O}_X = -\frac{1}{6!}\epsilon_{f_1\dots f_6}\epsilon_{g_1\dots g_6}(X^{f_1}X^{g_1})\dots(X^{f_6}X^{g_6})$

But, full theory preserves $U(1)_{X-3(N-1)\psi}$

$$\mathcal{L}_{\psi X} = A_{\psi X} \frac{\mathcal{O}_\psi}{(2N)^2} \left[\frac{\mathcal{O}_X}{[(2N+1)(N-1)]^6} \right]^{(N-1)} + h.c.$$

Four-fermion operators are recovered after spontaneous symmetry breaking

$$\langle \Psi\Psi \rangle \equiv -2(2N)M_\psi \tilde{A}_0(M_\psi^2) \quad \langle XX \rangle = -2(2N+1)(N-1)M_X \tilde{A}_0(M_X^2)$$

Non-standard large-N behaviour of the anomaly : $\dim_X \sim N^2 \Rightarrow M_{\eta'}^2 \sim A_{\psi X} \sim N^0$

The fate of $SU(4) \times SU(6) \times U(1)$ ('t Hooft anomaly matching with baryons)

$$\Psi^{abf} = (\psi^a \psi^b X^f), \quad \Psi_f^{ab} = (\psi^a \psi^b \bar{X}_f), \quad \Psi_b^{af} = (\psi^a \bar{\psi}_b X^f), \quad \Psi^{fgh} = (X^f X^g X^h), \quad \Psi_h^{fg} = (X^f X^g \bar{X}_h)$$

$$2\text{tr}(T^{\hat{A}}(r)\{T^B(r), T^C(r)\}) = A(r)d^{\hat{A}BC}$$

$$\sum_{i=\psi, X} n_i A(r_i) = \sum_{i=baryon} n'_i A(r_i)$$

$SU(4)^3$: matching impossible for $N \neq 8n \rightarrow SU(4)$ breaks to $Sp(4)$
 \rightarrow one expects a non-zero condensate $\langle \psi\psi \rangle \neq 0$

$SU(6)^3$: matching always possible $\rightarrow SU(6)$ may not break to $SO(6)$
 \rightarrow the condensate $\langle XX \rangle$ may vanish or not

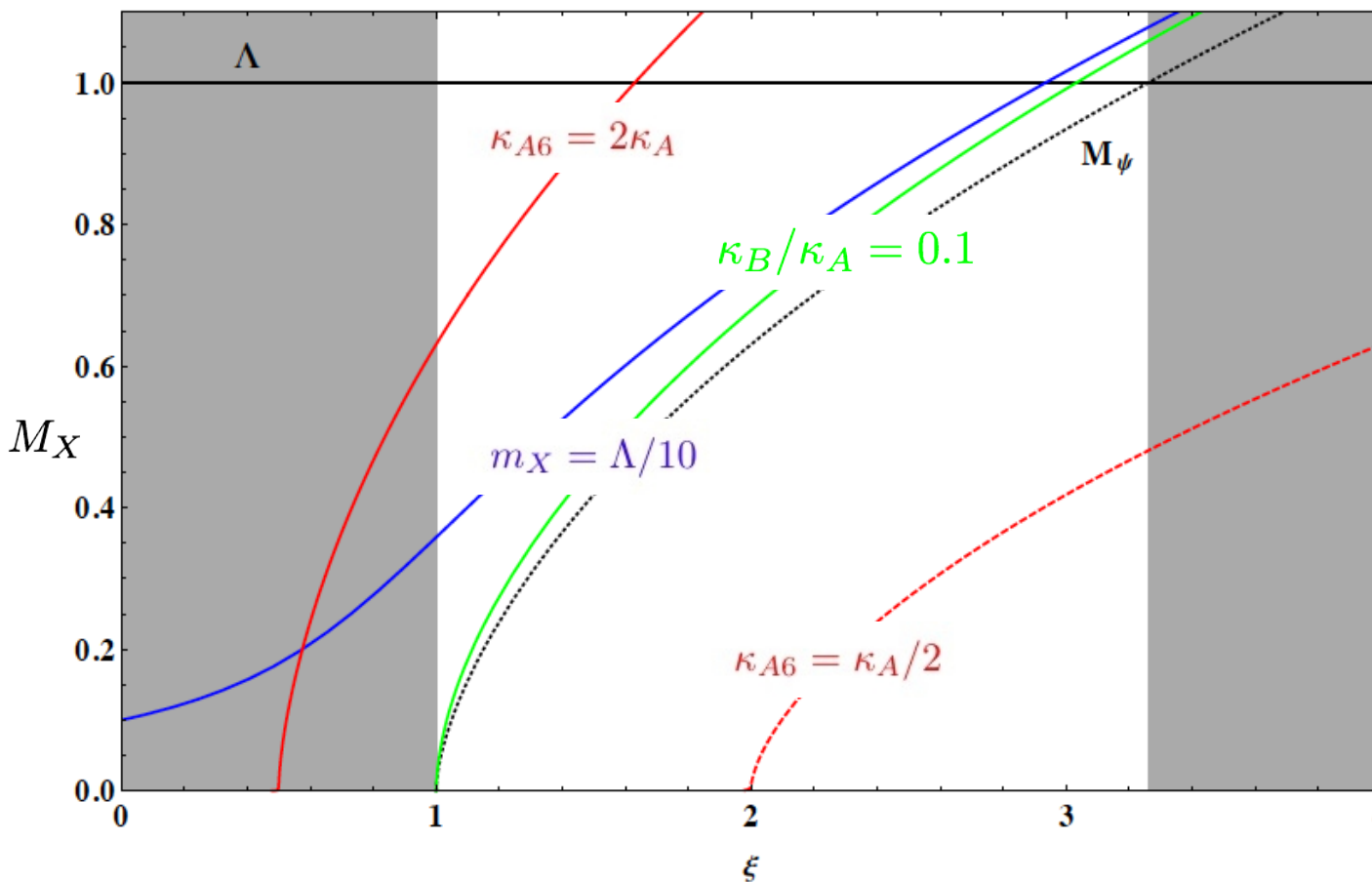
If it vanishes, light baryons may be top partners: Cacciapaglia-Parolini'15

$SU(6)^2 \times U(1)$, $SU(4)^2 \times U(1)$, $U(1)^3$: hard to match all by a common set of baryons; in any case, $U(1)$ is broken when $\langle \psi\psi \rangle \neq 0$

The fate of $SU(4) \times SU(6) \times U(1)$ (coupled mass-gap equations)

$$\begin{cases} M_\psi = 4[\kappa_A + \kappa_B(M_X^2)]M_\psi \tilde{A}_0(M_\psi^2) \\ M_X = 4[\kappa_{A6} + \kappa_{B6}(M_\psi^2, M_X^2)]M_X \tilde{A}_0(M_X^2) + m_X \end{cases}$$

$$\left. \begin{array}{l} \kappa_{B6} = \kappa_B = 0 \\ m_X = 0 \\ \kappa_{A6} = \kappa_A \end{array} \right\} \Rightarrow M_X = M_\psi$$



The colour sector window, between critical coupling, where $M_X = 0$, and maximal coupling, where $M_X = \Lambda$, shifts respect to the electroweak sector window

Strong dependence on the ratio κ_{A6} / κ_A

The large- N approximation does not determine this ratio uniquely

Coloured meson masses

$$M_\phi/f \sim \sqrt{1/N} \quad \text{here } N = 4$$

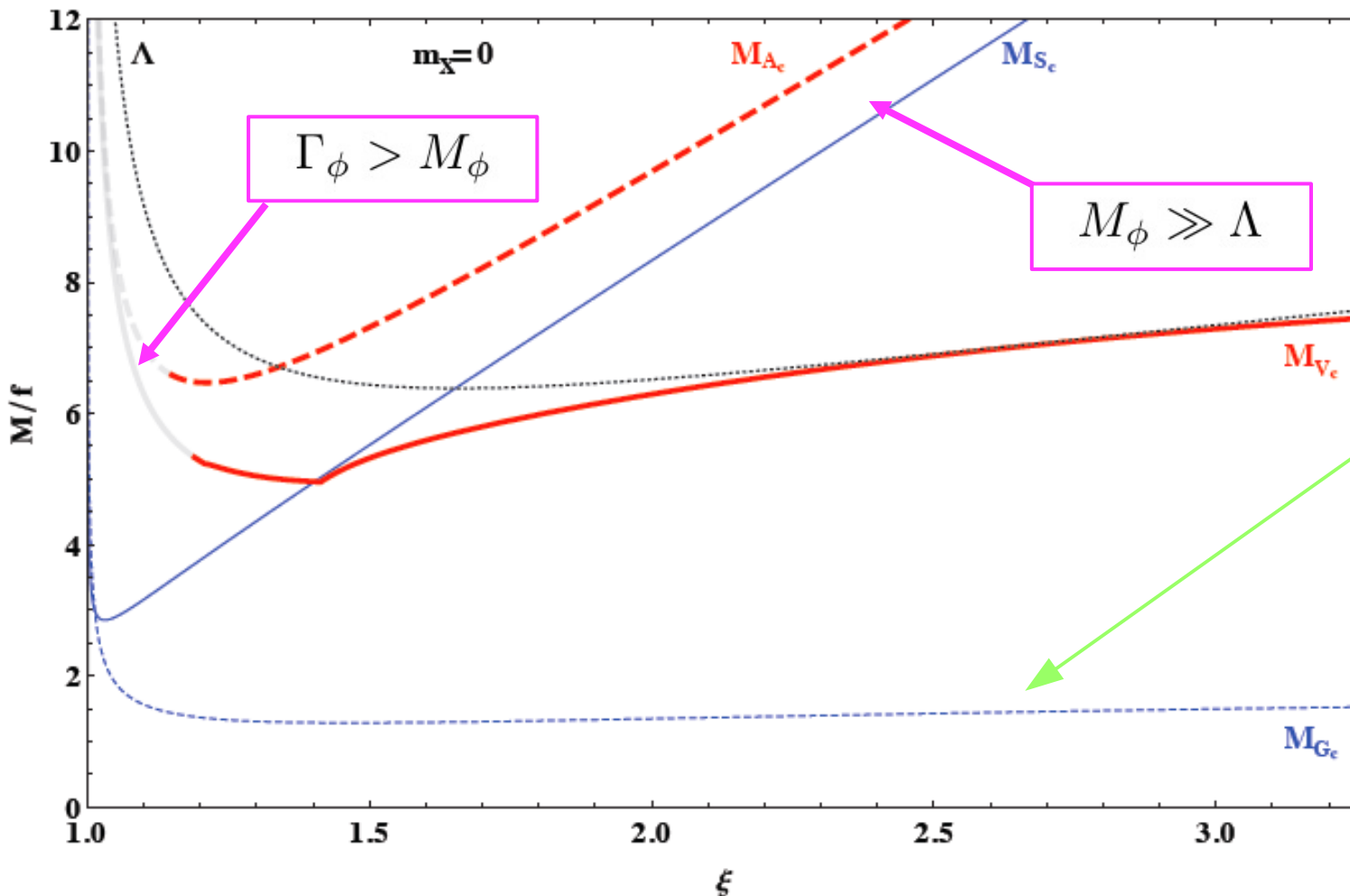
The ratio EW masses / coloured masses strongly depends on the ratio κ_{A6}/κ_A that unfortunately is a free parameter

$$\kappa_{A6} = \kappa_A$$

$$\kappa_B = \kappa_A/100$$

Coloured Goldstone bosons receive a mass from gluon loops: $M_{G_c} > 1.3 f$
Behind the corner at the LHC?

Goldstone mass may be raised by a non-zero m_x (confront with pions)



Singlet meson masses with mixing

The Sp(4) singlet mesons $\sigma_\psi, \eta_\psi, a^\mu_\psi$ may mix with the SO(6) singlet ones $\sigma_X, \eta_X, a^\mu_X$ as they are **all SM singlets**

- **Axial-vectors:** Sp(2N) current-current operators (leading in 1/N) do not mix Ψ and X sectors \rightarrow the mixing is subleading.
- **(Pseudo-)scalars:** **anomalous operator $A_{\psi X}$** induces a coupling $\Psi^2 X^2$ of the same order as the couplings $\Psi^4, X^4 \rightarrow$ the mixing is a **leading effect for pseudo-scalars:** one linear combination of η_ψ and η_X is massless (for $m_X=0$): **the U(1) Goldstone η_0**

$$\mathbf{\Pi}_{\sigma_\psi \sigma_X} = \begin{pmatrix} \tilde{\Pi}_S^\psi & 0 \\ 0 & \tilde{\Pi}_S^X \end{pmatrix}, \quad \mathbf{K}_{\sigma_\psi \sigma_X} = \begin{pmatrix} K_{\sigma_\psi} & K_{\psi X} \\ K_{\psi X} & K_{\sigma_X} \end{pmatrix}$$

Spectrum is very sensitive even to a small anomaly coefficient $A_{\psi X}$

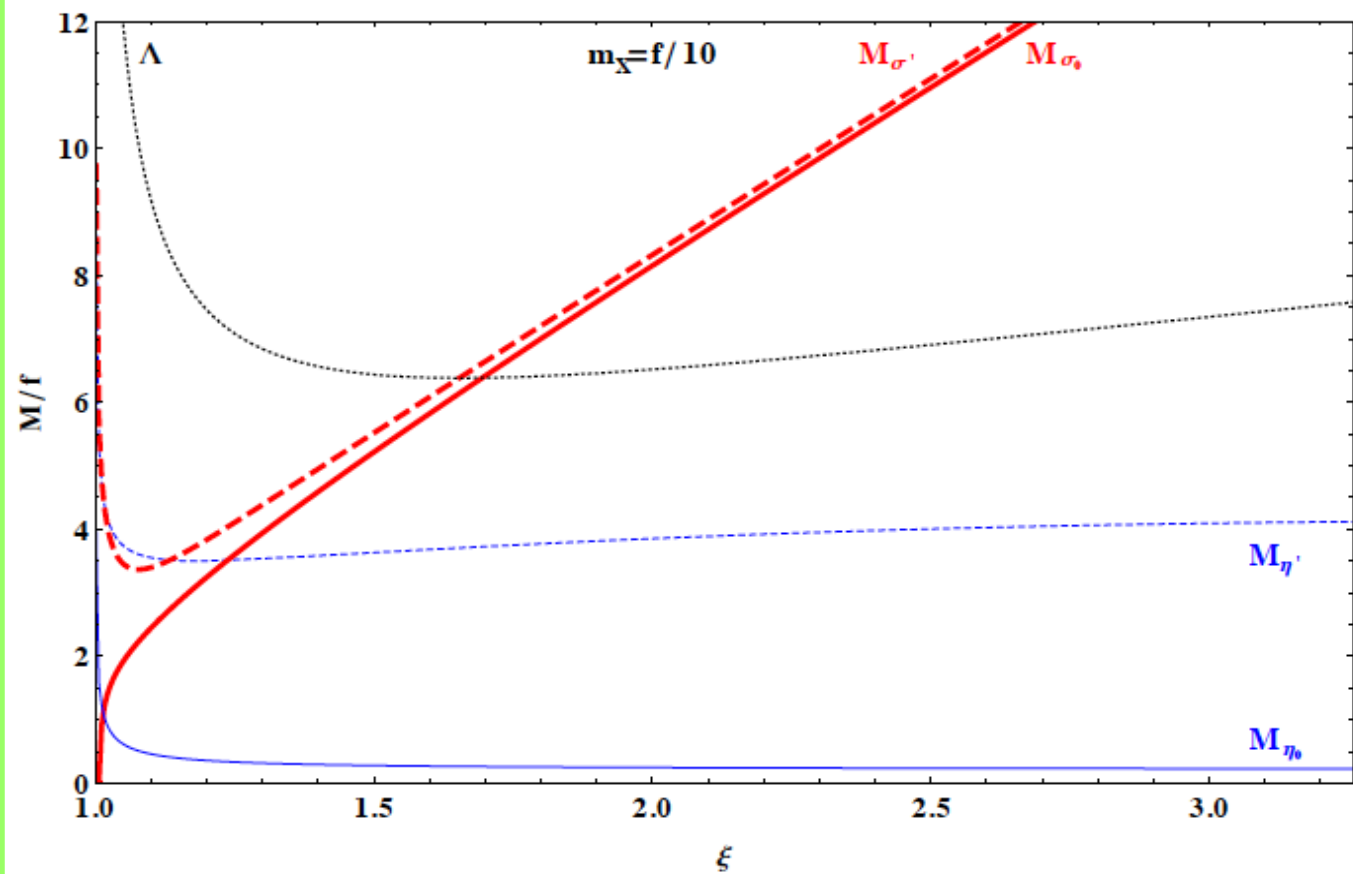
$$\mathbf{\Pi}_{\eta_\psi \eta_X} = \begin{pmatrix} \tilde{\Pi}_P^\psi & 0 & \sqrt{p^2} \tilde{\Pi}_{AP}^\psi & 0 \\ 0 & \tilde{\Pi}_P^X & 0 & \sqrt{p^2} \tilde{\Pi}_{AP}^X \\ \sqrt{p^2} \tilde{\Pi}_{AP}^\psi & 0 & \tilde{\Pi}_A^{L\psi} & 0 \\ 0 & \sqrt{p^2} \tilde{\Pi}_{AP}^X & 0 & \tilde{\Pi}_A^{LX} \end{pmatrix} \quad \mathbf{K}_{\eta_\psi \eta_X} = \begin{pmatrix} K_{\eta_\psi} & -K_{\psi X} & 0 & 0 \\ -K_{\psi X} & K_{\eta_X} & 0 & 0 \\ 0 & 0 & K_a & 0 \\ 0 & 0 & 0 & K_{a_c} \end{pmatrix}$$

Mixed states may couple both to electroweak bosons and gluons!

Singlet me

The Sp(4) singlet mesons σ as they are **all SM singlets**

- **Axial-vectors:** Sp(2N) cu X sectors \rightarrow the mixing is
- **(Pseudo-)scalars:** **anoma** order as the couplings Ψ^4 , one linear combination of



$$\mathbf{\Pi}_{\sigma_\psi \sigma_X} = \begin{pmatrix} \tilde{\Pi}_S^\psi & 0 \\ 0 & \tilde{\Pi}_S^X \end{pmatrix}, \quad \mathbf{K}_{\sigma_\psi \sigma_X} = \begin{pmatrix} K_{\sigma_\psi} & K_{\psi X} \\ K_{\psi X} & K_{\sigma_X} \end{pmatrix}$$

Spectrum is very sensitive even to a small anomaly coefficient $A_{\psi X}$

$$\mathbf{\Pi}_{\eta_\psi \eta_X} = \begin{pmatrix} \tilde{\Pi}_P^\psi & 0 & \sqrt{p^2} \tilde{\Pi}_{AP}^\psi & 0 \\ 0 & \tilde{\Pi}_P^X & 0 & \sqrt{p^2} \tilde{\Pi}_{AP}^X \\ \sqrt{p^2} \tilde{\Pi}_{AP}^\psi & 0 & \tilde{\Pi}_A^{L\psi} & 0 \\ 0 & \sqrt{p^2} \tilde{\Pi}_{AP}^X & 0 & \tilde{\Pi}_A^{LX} \end{pmatrix}, \quad \mathbf{K}_{\eta_\psi \eta_X} = \begin{pmatrix} K_{\eta_\psi} & -K_{\psi X} & 0 & 0 \\ -K_{\psi X} & K_{\eta_X} & 0 & 0 \\ 0 & 0 & K_a & 0 \\ 0 & 0 & 0 & K_{a_c} \end{pmatrix}$$

Mixed states may couple both to electroweak bosons and gluons!

Couplings to SM gauge bosons

The $SU(3)_c \times SU(2)_w \times U(1)_Y$ currents within $SO(6) \times Sp(4)$ are weakly coupled to the SM gauge bosons

$$\mathcal{L}_{\text{int}} = -ig_W \mathcal{W}^\mu J_\mu^W$$

This explicit symmetry breaking, when acting on the Goldstone-boson states, induces non-zero matrix elements, that can be computed perturbatively

Goldstones with SM charges acquire a positive mass from gauge boson loops

$$\Delta M_{\hat{A}}^2 = -\frac{3}{4\pi} \times \frac{1}{F_G^2} \times \frac{g_W^2}{4\pi} \times \int_0^\infty dQ^2 Q^2 \Pi_{V-A}(Q^2) \times C_2^{(H_W)}(R_W)$$

For our estimates, we integrated the V-A correlator, resummed à la NJL, up to $Q^2 = \Lambda^2$

Goldstones with SM anomalies decay into two SM gauge bosons with definite strength

$$\mathcal{L}_{\text{eff}}^{\text{WZW}} = -\frac{g_W^2 d_{HC}}{64\pi^2 F_G} \epsilon_{\mu\nu\rho\sigma} \mathcal{W}^{\mu\nu}(x) \mathcal{W}^{\rho\sigma}(x) \sum_{\hat{A}} d^{WW\hat{A}} G^{\hat{A}}(x)$$

This is relevant for the two singlet Goldstones, η and η_0 , and for the color octet O_0 :

Waiting for true resonances in di-photons, di-W/Z, di-jets ...

To-do list

- Study couplings among resonances: widths, decay chains, ...
- The Goldstone (Higgs) potential (mass) receives contributions from
 - SM gauging (determined up to a form factor)
 - SM Yukawas (dependent on the mixing of SM fermions with baryons)
 - non-SM parameters such as $m_{\psi, X}$ (arbitrary)
- Need to compute mass and interactions of the top partners
 - resummation of leading-1/N diagrams is possible, by generalising the meson procedure to baryons (baryon masses reasonably reproduced by the NJL model for QCD): in progress...
- With an improved control on the top-partner parameters, the top contribution to the Goldstone potential should be more predictable
 - study EWSB, determine the tuning needed for $m_h = 125$ GeV, ...

$$\Psi^{abf} = (\psi^a \psi^b X^f)$$

Summary

- Higgs bound state of a strongly-coupled gauge theory → accompanied by **a set of mesons with electroweak (and likely colour) quantum numbers**
- Non-perturbative QCD artillery allows to characterise the meson spectrum
- Large-N, Nambu-Jona Lasinio techniques: a crude approximation of real dynamics → predictions may carry **a large (a few over N) uncertainty**, but:
 - The model is clearly defined and precisely **computable**
 - The detailed symmetry structure is correctly accounted for → a number of **self-consistency relations hold**
- In the minimal model, the **meson masses lie above $(4-5)f > 4\text{TeV}$, with a few significant exceptions**: two singlet Goldstones η and η_0 , possibly the σ_0 and/or the η' , and the coloured Goldstones
- Need to estimate the top partner spectrum to quantify the little hierarchy