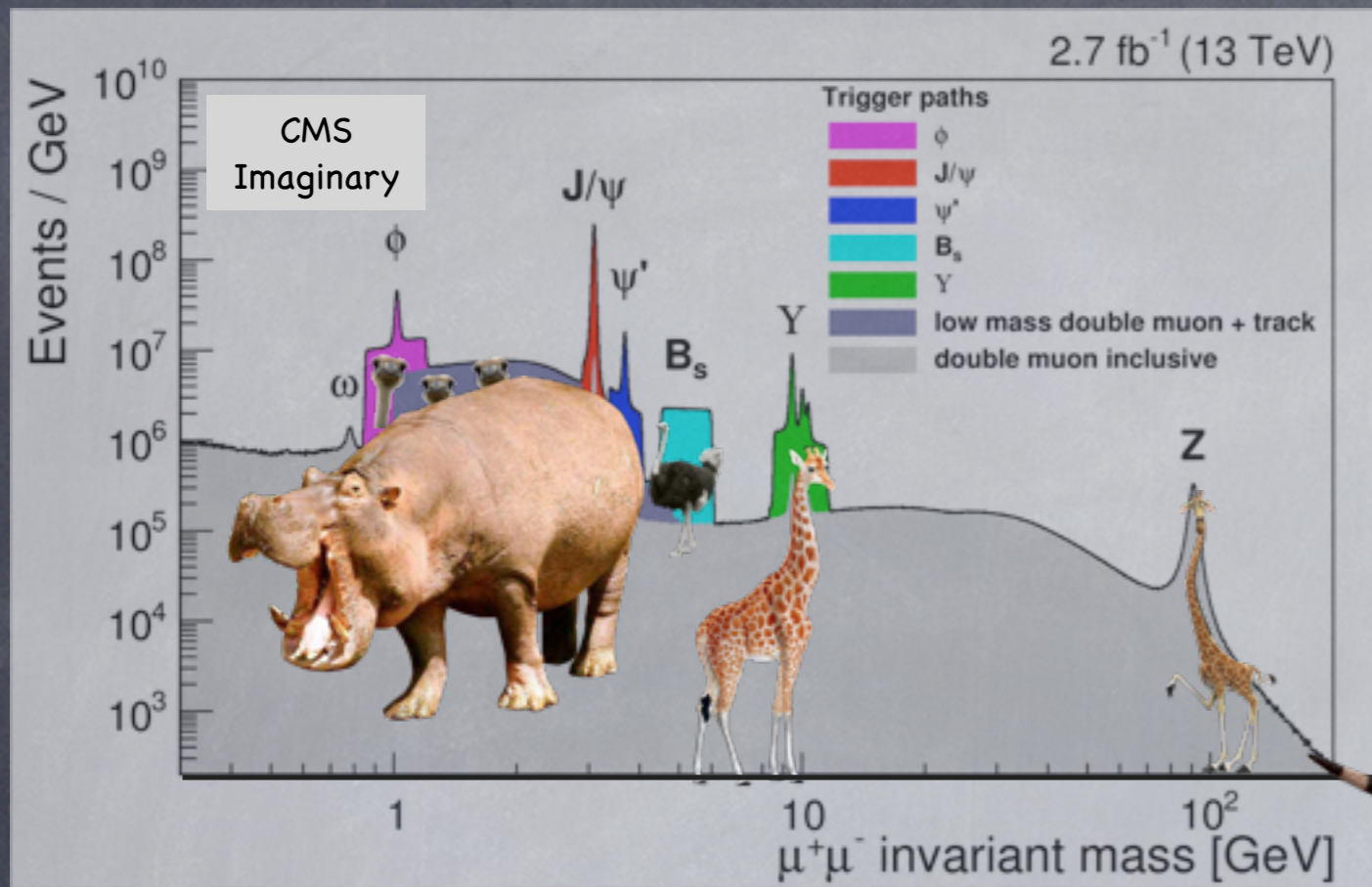


On the SM EFT and its deformations

Barcelona, 27 January 2017



Fantastic Beasts and Where To Find Them



- It is likely that mass scale Λ of BSM particles is beyond kinematic reach of the LHC
- If that is true, effective field theory (EFT) approach may be only way to collect partial information about BSM structure (much like Fermi theory taught us something about W and Z before they could be produced)
- Even if new particles can be reached directly, EFT is useful and compact framework for practical calculations at $E \ll \Lambda$ (much like we still use Fermi effective theory to calculate weak decays of particles with $m \ll m_Z$)

SM EFT

SM EFT Approach to BSM

Basic assumptions

- Much as in SM, relativistic QFT with linearly realized $SU(3) \times SU(2) \times U(1)$ local symmetry spontaneously broken by VEV of Higgs doublet field
- Mass scale Λ of new particles separated from characteristic energy scale E of experiment, $\Lambda \gg E$, such that experimental observables can be expanded in powers of E/Λ

SM EFT Lagrangian expanded in inverse powers of Λ , equivalently in operator dimension D

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda} \mathcal{L}^{D=5} + \frac{1}{\Lambda^2} \mathcal{L}^{D=6} + \frac{1}{\Lambda^3} \mathcal{L}^{D=7} + \frac{1}{\Lambda^4} \mathcal{L}^{D=8} + \dots$$

Lepton number or B-L violating,
hence too small to probed at present
and near-future colliders

Generated by integrating out
heavy particle with mass scale Λ
In large class of BSM models,
describe leading effects of new physics
on collider observables at $E \ll \Lambda$

By assumption,
subleading
to $D=6$

Advantages of SM EFT

- Framework general enough to describe leading effects of a large class (though not all!) of BSM scenarios
- Theoretical correlations between signal and background and different signal channels taken into account
- Very easy to recast SM EFT results as constraints on specific BSM models
- SM EFT is consistent QFT, so that calculations and predictions can be systematically improved (higher-loops, higher order terms in EFT expansion if needed). In particular, SM EFT is renormalizable at each order in $1/\Lambda$ expansion
- Some tools to assess validity of $1/\Lambda$ expansion

Many possible D=6 operators!

One example of non-redundant set,
so-called SILH basis

Giudice et al [hep-ph/0703164](#)
Contino et al [1303.3876](#)

Table 97: Bosonic $D=6$ operators in the SILH basis.

Bosonic CP-even		Bosonic CP-odd	
O_H	$\frac{1}{2v^2} [\partial_\mu (H^\dagger H)]^2$		
O_T	$\frac{1}{2v^2} (H^\dagger \overleftrightarrow{D}_\mu H)^2$		
O_6	$-\frac{\lambda}{v^2} (H^\dagger H)^3$		
O_g	$\frac{g_s^2}{m_W^2} H^\dagger H G_{\mu\nu}^a G_{\mu\nu}^a$	\tilde{O}_g	$\frac{g_s^2}{m_W^2} H^\dagger H \tilde{G}_{\mu\nu}^a G_{\mu\nu}^a$
O_γ	$\frac{g'^2}{m_W^2} H^\dagger H B_{\mu\nu} B_{\mu\nu}$	\tilde{O}_γ	$\frac{g'^2}{m_W^2} H^\dagger H \tilde{B}_{\mu\nu} B_{\mu\nu}$
O_W	$\frac{ig}{2m_W^2} (H^\dagger \sigma^i \overleftrightarrow{D}_\mu H) D_\nu W_{\mu\nu}^i$		
O_B	$\frac{ig'}{2m_W^2} (H^\dagger \overleftrightarrow{D}_\mu H) \partial_\nu B_{\mu\nu}$		
O_{HW}	$\frac{ig}{m_W^2} (D_\mu H^\dagger \sigma^i D_\nu H) W_{\mu\nu}^i$	\tilde{O}_{HW}	$\frac{ig}{m_W^2} (D_\mu H^\dagger \sigma^i D_\nu H) \tilde{W}_{\mu\nu}^i$
O_{HB}	$\frac{ig'}{m_W^2} (D_\mu H^\dagger D_\nu H) B_{\mu\nu}$	\tilde{O}_{HB}	$\frac{ig'}{m_W^2} (D_\mu H^\dagger D_\nu H) \tilde{B}_{\mu\nu}$
O_{2W}	$\frac{1}{m_W^2} D_\mu W_{\mu\nu}^i D_\rho W_{\rho\nu}^i$		
O_{2B}	$\frac{1}{m_W^2} \partial_\mu B_{\mu\nu} \partial_\rho B_{\rho\nu}$		
O_{2G}	$\frac{1}{m_W^2} D_\mu G_{\mu\nu}^a D_\rho G_{\rho\nu}^a$		
O_{3W}	$\frac{g^3}{m_W^2} \epsilon^{ijk} W_{\mu\nu}^i W_{\nu\rho}^j W_{\rho\mu}^k$	\tilde{O}_{3W}	$\frac{g^3}{m_W^2} \epsilon^{ijk} \tilde{W}_{\mu\nu}^i W_{\nu\rho}^j W_{\rho\mu}^k$
O_{3G}	$\frac{g_s^3}{m_W^2} f^{abc} G_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c$	\tilde{O}_{3G}	$\frac{g_s^3}{m_W^2} f^{abc} \tilde{G}_{\mu\nu}^a G_{\nu\rho}^b G_{\rho\mu}^c$

Table 98: Two-fermion dimension-6 operators in the SILH basis. They are the same as in the Warsaw basis, except that the operators $[O_{H\ell}]_{11}$, $[O'_{H\ell}]_{11}$ are absent by definition. We define $\sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2$. In this table, e, u, d are always right-handed fermions, while ℓ and q are left-handed. For complex operators the complex conjugate operator is implicit.

	Vertex		Yukawa and Dipole
$[O_{H\ell}]_{ij}$	$\frac{i}{v^2} \bar{\ell}_i \gamma_\mu \ell_j H^\dagger \overleftrightarrow{D}_\mu H$	$[O_e]_{ij}$	$\frac{\sqrt{2m_{e_i} m_{e_j}}}{v^3} H^\dagger H \bar{\ell}_i H e_j$
$[O'_{H\ell}]_{ij}$	$\frac{i}{v^2} \bar{\ell}_i \sigma^k \gamma_\mu \ell_j H^\dagger \sigma^k \overleftrightarrow{D}_\mu H$	$[O_u]_{ij}$	$\frac{\sqrt{2m_{u_i} m_{u_j}}}{v^3} H^\dagger H \bar{q}_i \tilde{H} u_j$
$[O_{He}]_{ij}$	$\frac{i}{v^2} \bar{e}_i \gamma_\mu e_j H^\dagger \overleftrightarrow{D}_\mu H$	$[O_d]_{ij}$	$\frac{\sqrt{2m_{d_i} m_{d_j}}}{v^3} H^\dagger H \bar{q}_i H d_j$
$[O_{Hq}]_{ij}$	$\frac{i}{v^2} \bar{q}_i \gamma_\mu q_j H^\dagger \overleftrightarrow{D}_\mu H$	$[O_{eW}]_{ij}$	$\frac{g}{m_W^2} \frac{\sqrt{2m_{e_i} m_{e_j}}}{v} \bar{\ell}_i \sigma^k H \sigma_{\mu\nu} e_j W_{\mu\nu}^k$
$[O'_{Hq}]_{ij}$	$\frac{i}{v^2} \bar{q}_i \sigma^k \gamma_\mu q_j H^\dagger \sigma^k \overleftrightarrow{D}_\mu H$	$[O_{eB}]_{ij}$	$\frac{g'}{m_W^2} \frac{\sqrt{2m_{e_i} m_{e_j}}}{v} \bar{\ell}_i H \sigma_{\mu\nu} e_j B_{\mu\nu}$
$[O_{Hu}]_{ij}$	$\frac{i}{v^2} \bar{u}_i \gamma_\mu u_j H^\dagger \overleftrightarrow{D}_\mu H$	$[O_{uG}]_{ij}$	$\frac{g_s}{m_W^2} \frac{\sqrt{2m_{u_i} m_{u_j}}}{v} \bar{q}_i \tilde{H} \sigma_{\mu\nu} T^a u_j G_{\mu\nu}^a$
$[O_{Hd}]_{ij}$	$\frac{i}{v^2} \bar{d}_i \gamma_\mu d_j H^\dagger \overleftrightarrow{D}_\mu H$	$[O_{uW}]_{ij}$	$\frac{g}{m_W^2} \frac{\sqrt{2m_{u_i} m_{u_j}}}{v} \bar{q}_i \sigma^k \tilde{H} \sigma_{\mu\nu} u_j W_{\mu\nu}^k$
$[O_{Hud}]_{ij}$	$\frac{i}{v^2} \bar{u}_i \gamma_\mu d_j \tilde{H}^\dagger D_\mu H$	$[O_{uB}]_{ij}$	$\frac{g'}{m_W^2} \frac{\sqrt{2m_{u_i} m_{u_j}}}{v} \bar{q}_i \tilde{H} \sigma_{\mu\nu} u_j B_{\mu\nu}$
		$[O_{dG}]_{ij}$	$\frac{g_s}{m_W^2} \frac{\sqrt{2m_{d_i} m_{d_j}}}{v} \bar{q}_i H \sigma_{\mu\nu} T^a d_j G_{\mu\nu}^a$
		$[O_{dW}]_{ij}$	$\frac{g}{m_W^2} \frac{\sqrt{2m_{d_i} m_{d_j}}}{v} \bar{q}_i \sigma^k H \sigma_{\mu\nu} d_j W_{\mu\nu}^k$
		$[O_{dB}]_{ij}$	$\frac{g'}{m_W^2} \frac{\sqrt{2m_{d_i} m_{d_j}}}{v} \bar{q}_i H \sigma_{\mu\nu} d_j B_{\mu\nu}$

Table 99: Four-fermion operators in the SILH basis. They are the same as in the Warsaw basis [614], except that the operators $[O_{\ell\ell}]_{1221}$, $[O_{\ell\ell}]_{1122}$, $[O_{uu}]_{3333}$ are absent by definition. In this table, e, u, d are always right-handed fermions, while ℓ and q are left-handed. A flavour index is implicit for each fermion field. For complex operators the complex conjugate operator is implicit.

$(\bar{L}L)(\bar{L}L)$ and $(\bar{L}R)(\bar{L}R)$	$(\bar{R}R)(\bar{R}R)$	$(\bar{L}L)(\bar{R}R)$			
$O_{\ell\ell}$	$\frac{1}{v^2} (\bar{\ell}\gamma_\mu\ell)(\bar{\ell}\gamma_\mu\ell)$	O_{ee}	$\frac{1}{v^2} (\bar{e}\gamma_\mu e)(\bar{e}\gamma_\mu e)$	O_{le}	$\frac{1}{v^2} (\bar{\ell}\gamma_\mu\ell)(\bar{e}\gamma_\mu e)$
O_{qq}	$\frac{1}{v^2} (\bar{q}\gamma_\mu q)(\bar{q}\gamma_\mu q)$	O_{uu}	$\frac{1}{v^2} (\bar{u}\gamma_\mu u)(\bar{u}\gamma_\mu u)$	O_{lu}	$\frac{1}{v^2} (\bar{\ell}\gamma_\mu\ell)(\bar{u}\gamma_\mu u)$
O'_{qq}	$\frac{1}{v^2} (\bar{q}\gamma_\mu\sigma^i q)(\bar{q}\gamma_\mu\sigma^i q)$	O_{dd}	$\frac{1}{v^2} (\bar{d}\gamma_\mu d)(\bar{d}\gamma_\mu d)$	O_{ld}	$\frac{1}{v^2} (\bar{\ell}\gamma_\mu\ell)(\bar{d}\gamma_\mu d)$
$O_{\ell q}$	$\frac{1}{v^2} (\bar{\ell}\gamma_\mu\ell)(\bar{q}\gamma_\mu q)$	O_{eu}	$\frac{1}{v^2} (\bar{e}\gamma_\mu e)(\bar{u}\gamma_\mu u)$	O_{eq}	$\frac{1}{v^2} (\bar{q}\gamma_\mu q)(\bar{e}\gamma_\mu e)$
$O'_{\ell q}$	$\frac{1}{v^2} (\bar{\ell}\gamma_\mu\sigma^i\ell)(\bar{q}\gamma_\mu\sigma^i q)$	O_{ed}	$\frac{1}{v^2} (\bar{e}\gamma_\mu e)(\bar{d}\gamma_\mu d)$	O_{qu}	$\frac{1}{v^2} (\bar{q}\gamma_\mu q)(\bar{u}\gamma_\mu u)$
O_{quqd}	$\frac{1}{v^2} (\bar{q}^j u) \epsilon_{jk} (\bar{q}^k d)$	O_{ud}	$\frac{1}{v^2} (\bar{u}\gamma_\mu u)(\bar{d}\gamma_\mu d)$	O'_{qu}	$\frac{1}{v^2} (\bar{q}\gamma_\mu T^a q)(\bar{u}\gamma_\mu T^a u)$
O'_{quqd}	$\frac{1}{v^2} (\bar{q}^j T^a u) \epsilon_{jk} (\bar{q}^k T^a d)$	O'_{ud}	$\frac{1}{v^2} (\bar{u}\gamma_\mu T^a u)(\bar{d}\gamma_\mu T^a d)$	O_{qd}	$\frac{1}{v^2} (\bar{q}\gamma_\mu q)(\bar{d}\gamma_\mu d)$
O_{lequ}	$\frac{1}{v^2} (\bar{\ell}^j e) \epsilon_{jk} (\bar{q}^k u)$			O'_{qd}	$\frac{1}{v^2} (\bar{q}\gamma_\mu T^a q)(\bar{d}\gamma_\mu T^a d)$
O'_{lequ}	$\frac{1}{v^2} (\bar{\ell}^j \sigma_{\mu\nu} e) \epsilon_{jk} (\bar{q}^k \sigma^{\mu\nu} u)$				
O_{ledq}	$\frac{1}{v^2} (\bar{\ell}^j e)(\bar{d}q^j)$				

Full set has 2499 distinct operators,
including flavor structure and CP conjugates

Alonso et al 1312.2014, Henning et al 1512.03433

Observable effects of D=6 operators

- Corrections to Higgs self-couplings

$$\mathcal{L} \supset \frac{m_h^2}{2v} (1 + \delta\lambda_3) h^3$$

- Corrections to SM Z and W boson couplings to fermions (so-called vertex corrections)

$$\mathcal{L}_{vff} = \frac{g_L}{\sqrt{2}} \left(W_\mu^+ \bar{u} \bar{\sigma}_\mu (V_{CKM} + \delta g_L^{Wq}) d + W_\mu^+ u^c \sigma_\mu \delta g_R^{Wq} \bar{d}^c + W_\mu^+ \bar{\nu} \bar{\sigma}_\mu (I + \delta g_L^{W\ell}) e + \text{h.c.} \right) \\ + \sqrt{g_L^2 + g_Y^2} Z_\mu \left[\sum_{f \in u, d, e, \nu} \bar{f} \bar{\sigma}_\mu (T_f^3 - s_\theta^2 Q_f + \delta g_L^{Zf}) f + \sum_{f^c \in u^c, d^c, e^c} f^c \sigma_\mu (-s_\theta^2 Q_f + \delta g_R^{Zf}) \bar{f}^c \right]$$

- Corrections to SM Higgs couplings to matter and new tensor structures of these interactions

$$\mathcal{L}_{hvv} = \frac{h}{v} [2(1 + \delta c_w) m_W^2 W_\mu^+ W_\mu^- + (1 + \delta c_z) m_Z^2 Z_\mu Z_\mu \\ + c_{ww} \frac{g_L^2}{2} W_{\mu\nu}^+ W_{\mu\nu}^- + \tilde{c}_{ww} \frac{g_L^2}{2} W_{\mu\nu}^+ \tilde{W}_{\mu\nu}^- + c_{w\Box} g_L^2 (W_\mu^- \partial_\nu W_{\mu\nu}^+ + \text{h.c.}) \\ + c_{gg} \frac{g_s^2}{4} G_{\mu\nu}^a G_{\mu\nu}^a + c_{\gamma\gamma} \frac{e^2}{4} A_{\mu\nu} A_{\mu\nu} + c_{z\gamma} \frac{eg_L}{2c_\theta} Z_{\mu\nu} A_{\mu\nu} + c_{zz} \frac{g_L^2}{4c_\theta^2} Z_{\mu\nu} Z_{\mu\nu} \\ + c_{z\Box} g_L^2 Z_\mu \partial_\nu Z_{\mu\nu} + c_{\gamma\Box} g_L g_Y Z_\mu \partial_\nu A_{\mu\nu} \\ + \tilde{c}_{gg} \frac{g_s^2}{4} G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a + \tilde{c}_{\gamma\gamma} \frac{e^2}{4} A_{\mu\nu} \tilde{A}_{\mu\nu} + \tilde{c}_{z\gamma} \frac{eg_L}{2c_\theta} Z_{\mu\nu} \tilde{A}_{\mu\nu} + \tilde{c}_{zz} \frac{g_L^2}{4c_\theta^2} Z_{\mu\nu} \tilde{Z}_{\mu\nu}]$$

- Corrections to triple and quartic gauge couplings and new tensor structures of these interactions

$$\mathcal{L}_{tgc} = ie [(W_{\mu\nu}^+ W_\mu^- - W_{\mu\nu}^- W_\mu^+) A_\nu + (1 + \delta\kappa_\gamma) A_{\mu\nu} W_\mu^+ W_\nu^-] \\ + ig_L c_\theta [(1 + \delta g_{1,z}) (W_{\mu\nu}^+ W_\mu^- - W_{\mu\nu}^- W_\mu^+) Z_\nu + (1 + \delta\kappa_z) Z_{\mu\nu} W_\mu^+ W_\nu^-] \\ + i \frac{e}{m_W^2} \lambda_\gamma W_{\mu\nu}^+ W_{\nu\rho}^- A_{\rho\mu} + i \frac{g_L c_\theta}{m_W^2} \lambda_z W_{\mu\nu}^+ W_{\nu\rho}^- Z_{\rho\mu}$$

- Contact 4-fermion interactions

One flavor ($I = 1 \dots 3$)	Two flavors ($I < J = 1 \dots 3$)
$[O_{\ell\ell}]_{IIII} = \frac{1}{2} (\bar{\ell}_I \bar{\sigma}_\mu \ell_I) (\bar{\ell}_I \bar{\sigma}_\mu \ell_I)$	$[O_{\ell\ell}]_{IIJJ} = (\bar{\ell}_I \bar{\sigma}_\mu \ell_I) (\bar{\ell}_J \bar{\sigma}_\mu \ell_J)$
$[O_{\ell e}]_{IIII} = (\bar{\ell}_I \bar{\sigma}_\mu \ell_I) (e_I^c \sigma_\mu \bar{e}_I^c)$	$[O_{\ell e}]_{IIJJ} = (\bar{\ell}_I \bar{\sigma}_\mu \ell_I) (e_J^c \sigma_\mu \bar{e}_J^c)$
	$[O_{\ell e}]_{JJII} = (\bar{\ell}_J \bar{\sigma}_\mu \ell_J) (e_I^c \sigma_\mu \bar{e}_I^c)$
	$[O_{\ell e}]_{IJJI} = (\bar{\ell}_I \bar{\sigma}_\mu \ell_J) (e_J^c \sigma_\mu \bar{e}_I^c)$
$[O_{ee}]_{IIII} = \frac{1}{2} (e_I^c \sigma_\mu \bar{e}_I^c) (e_I^c \sigma_\mu \bar{e}_I^c)$	$[O_{ee}]_{IIJJ} = (e_I^c \sigma_\mu \bar{e}_I^c) (e_J^c \sigma_\mu \bar{e}_J^c)$

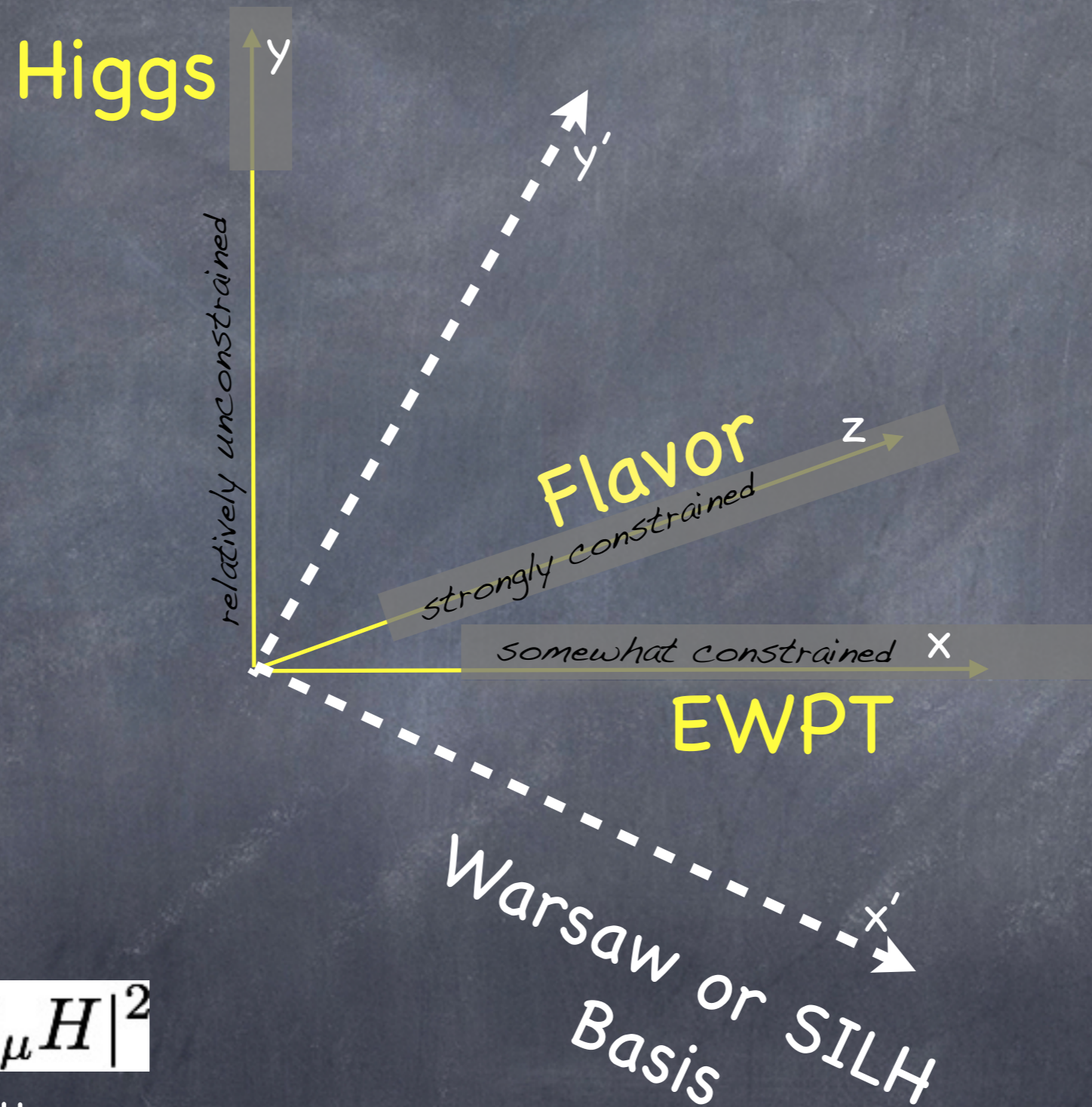
- ... and much more

Important: correlations between different parameters describing deviations from SM

SM EFT in practice

- At first sight, working with a theory with 2499 parameters seems hopeless
- However, typically a much smaller set of operators relevant for given process. This is especially true if EFT corrections are calculated at tree level only
- Moreover, using constraints from previous experiments (e.g. from low-energy precision experiments) may further reduce number of parameters relevant for given experimental observable
- Less generally, imposing flavor symmetries greatly reduces (by $O(100)$) number of independent dimension-6 coefficients
- Importance of convenient parametrization of space of dimension-6 operators that makes explicit directions that are very well constrained and those that are poorly constrained
- Importance of global fits to make full use of experimental constraints

Bird's eye view of dimension-6 space



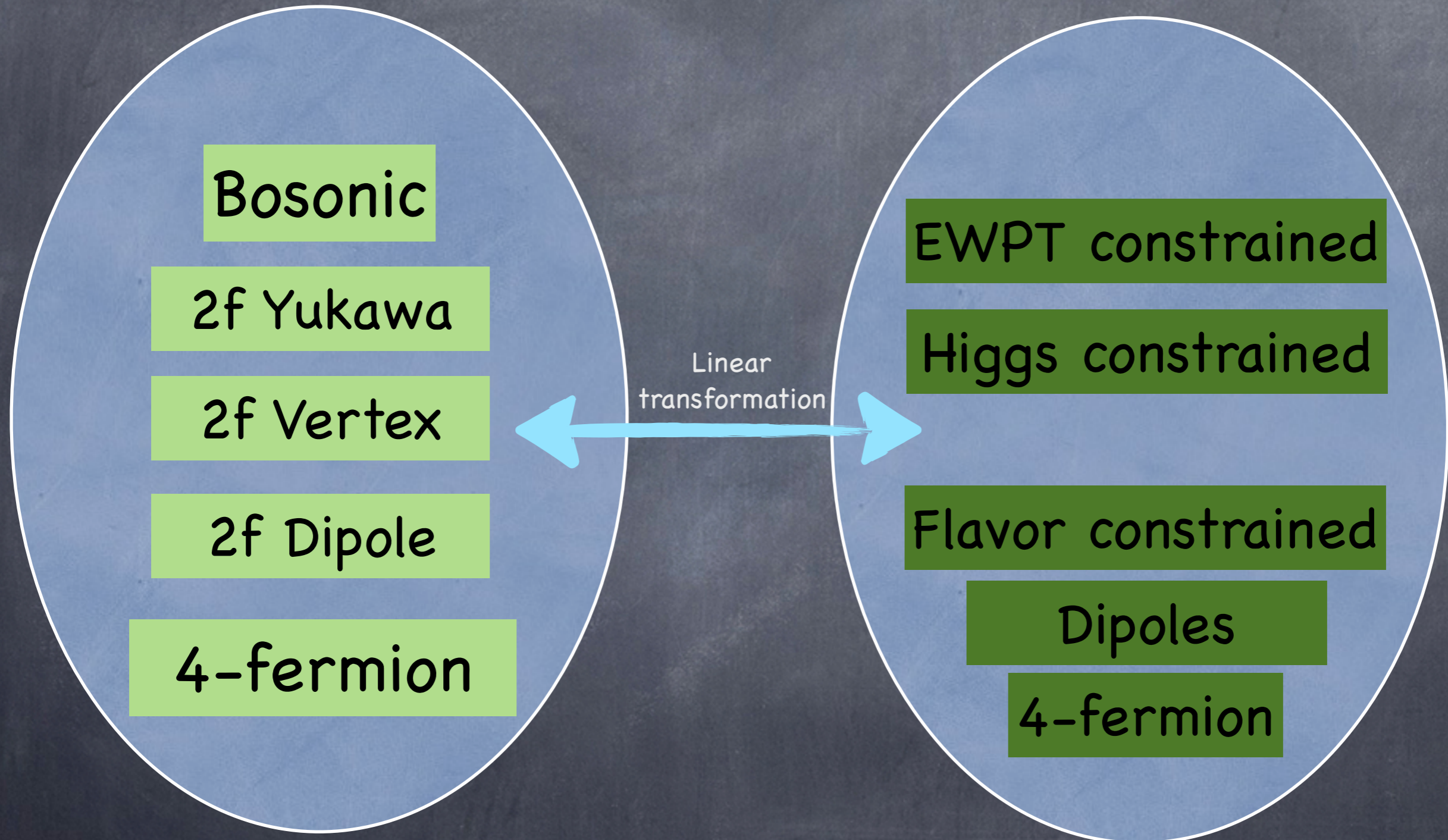
e.g.

$$O_{HD} = |H^\dagger D_\mu H|^2$$

contributes both
to Higgs couplings and
to W/Z mass difference

EFT primaries

To characterize dimension-6 parameter space, more transparent to rotate basis and use linear combination of Wilson coefficients that map directly to particular "measurable" couplings in mass eigenstate Lagrangian



SILH or Warsaw or another set of 2499 Wilson coefficients of dimension-6 operators

Set of 2499 couplings in mass eigenstate Lagrangian describing independent deformations of SM Lagrangian

Pole observables - constraints

All diagonal vertex corrections except for δg_{WqR} and δg_{ZtR} simultaneously constrained in a completely model-independent way

$$\mathcal{L}_{vff} = \frac{g_L}{\sqrt{2}} \left(W_\mu^+ \bar{u} \bar{\sigma}_\mu (V_{CKM} + \delta g_L^{Wq}) d + W_\mu^+ u^c \sigma_\mu \delta g_R^{Wq} \bar{d}^c + W_\mu^+ \bar{\nu} \bar{\sigma}_\mu (I + \delta g_L^{W\ell}) e + \text{h.c.} \right) \\ + \sqrt{g_L^2 + g_Y^2} Z_\mu \left[\sum_{f \in u, d, e, \nu} \bar{f} \bar{\sigma}_\mu (T_f^3 - s_\theta^2 Q_f + \delta g_L^{Zf}) f + \sum_{f^c \in u^c, d^c, e^c} f^c \sigma_\mu (-s_\theta^2 Q_f + \delta g_R^{Zf}) \bar{f}^c \right]$$

$$[\delta g_L^{We}]_{ii} = \begin{pmatrix} -1.00 \pm 0.64 \\ -1.36 \pm 0.59 \\ 1.95 \pm 0.79 \end{pmatrix} \times 10^{-2},$$

$$[\delta g_L^{Ze}]_{ii} = \begin{pmatrix} -0.26 \pm 0.28 \\ 0.1 \pm 1.1 \\ 0.16 \pm 0.58 \end{pmatrix} \times 10^{-3}, \quad [\delta g_R^{Ze}]_{ii} = \begin{pmatrix} -0.37 \pm 0.27 \\ 0.0 \pm 1.3 \\ 0.39 \pm 0.62 \end{pmatrix} \times 10^{-3},$$

$$[\delta g_L^{Zu}]_{ii} = \begin{pmatrix} -0.8 \pm 3.1 \\ -0.16 \pm 0.36 \\ -0.28 \pm 3.8 \end{pmatrix} \times 10^{-2}, \quad [\delta g_R^{Zu}]_{ii} = \begin{pmatrix} 1.3 \pm 5.1 \\ -0.38 \pm 0.51 \\ \times \end{pmatrix} \times 10^{-2},$$

$$[\delta g_L^{Zd}]_{ii} = \begin{pmatrix} -1.0 \pm 4.4 \\ 0.9 \pm 2.8 \\ 0.33 \pm 0.16 \end{pmatrix} \times 10^{-2}, \quad [\delta g_R^{Zd}]_{ii} = \begin{pmatrix} 2.9 \pm 16 \\ 3.5 \pm 5.0 \\ 2.30 \pm 0.82 \end{pmatrix} \times 10^{-2}.$$

- Z coupling to charged leptons constrained at 0.1% level, W couplings to leptons constrained at 1% level. Some couplings to quarks (bottom, charm) also constrained at 1% level
- Some couplings very weakly constrained in a model-independent way, in particular Z couplings to light quarks (though their combination affecting *total* hadronic Z-width is strongly constrained)

Pole constraints - flavor blind

$$[\delta g^{Vf}]_{ij} = \delta g^{Vf} \delta_{ij}$$

$$\begin{pmatrix} \delta g_L^{W\ell} \\ \delta g_L^{Ze} \\ \delta g_R^{Ze} \\ \delta g_L^{Zu} \\ \delta g_R^{Zu} \\ \delta g_L^{Zd} \\ \delta g_R^{Zd} \end{pmatrix} = \begin{pmatrix} -0.89 \pm 0.84 \\ -0.20 \pm 0.23 \\ -0.20 \pm 0.24 \\ -1.7 \pm 2.1 \\ -2.3 \pm 4.6 \\ 2.8 \pm 1.5 \\ 19.9 \pm 7.7 \end{pmatrix} \times 10^{-3}$$

- All leptonic couplings constrained at per-mille level, all quark couplings constrained at 1% level or better

Off-Pole constraints on 4-lepton observables

AA, Mimouni
1511.07434

One flavor ($I = 1 \dots 3$)	Two flavors ($I < J = 1 \dots 3$)
$[O_{\ell\ell}]_{IIII} = \frac{1}{2}(\bar{\ell}_I \bar{\sigma}_\mu \ell_I)(\bar{\ell}_I \bar{\sigma}_\mu \ell_I)$	$[O_{\ell\ell}]_{IIJJ} = (\bar{\ell}_I \bar{\sigma}_\mu \ell_I)(\bar{\ell}_J \bar{\sigma}_\mu \ell_J)$
$[O_{\ell e}]_{IIII} = (\bar{\ell}_I \bar{\sigma}_\mu \ell_I)(e_I^c \sigma_\mu \bar{e}_I^c)$	$[O_{\ell e}]_{IJJI} = (\bar{\ell}_I \bar{\sigma}_\mu \ell_J)(\bar{\ell}_J \bar{\sigma}_\mu \ell_I)$
	$[O_{\ell e}]_{IIJJ} = (\bar{\ell}_I \bar{\sigma}_\mu \ell_I)(e_J^c \sigma_\mu \bar{e}_J^c)$
	$[O_{\ell e}]_{JJII} = (\bar{\ell}_J \bar{\sigma}_\mu \ell_J)(e_I^c \sigma_\mu \bar{e}_I^c)$
	$[O_{\ell e}]_{IJJJ} = (\bar{\ell}_I \bar{\sigma}_\mu \ell_J)(e_J^c \sigma_\mu \bar{e}_I^c)$
$[O_{ee}]_{IIII} = \frac{1}{2}(e_I^c \sigma_\mu \bar{e}_I^c)(e_I^c \sigma_\mu \bar{e}_I^c)$	$[O_{ee}]_{IIJJ} = (e_I^c \sigma_\mu \bar{e}_I^c)(e_J^c \sigma_\mu \bar{e}_J^c)$

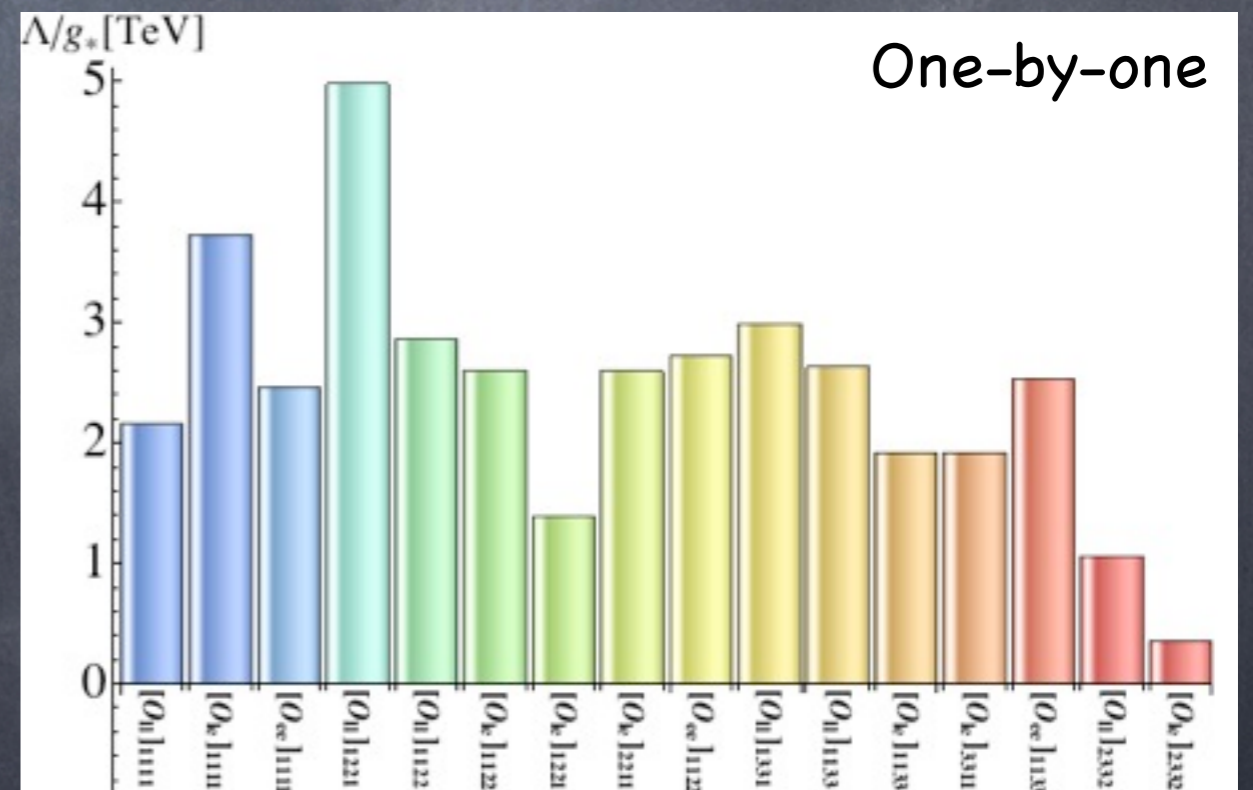
- There's 27 lepton-flavor conserving 4-lepton operators, 3 of which are complex, however not all are currently probed by experiment
- Using $e+e- \rightarrow ll$ scattering in LEP-2, low-energy neutrino scattering on electrons, W mass measurement, low-energy parity violating Moller scattering, and muon and tau decays
- All these observables depend also on leptonic vertex corrections, so combination with previous pole constraints is necessary

$$\delta m = \frac{\delta g_L^{W_e} + \delta g_L^{W_\mu}}{2} - \frac{[c_{\ell\ell}]_{1221}}{4}.$$

Off-Pole + Pole constraints combined

Model-independent		
δg_L^{We}	-0.37 ± 0.43	$\times 10^{-2}$.
$\delta g_L^{W\mu}$	-1.43 ± 0.59	
$\delta g_L^{W\tau}$	1.46 ± 0.70	
δg_L^{Ze}	-0.029 ± 0.028	
$\delta g_L^{Z\mu}$	0.01 ± 0.11	
$\delta g_L^{Z\tau}$	0.016 ± 0.058	
δg_R^{Ze}	-0.035 ± 0.027	
$\delta g_R^{Z\mu}$	0.00 ± 0.13	
$\delta g_R^{Z\tau}$	0.037 ± 0.062	
δg_L^{Zu}	-0.6 ± 3.0	
δg_L^{Zc}	-0.16 ± 0.36	
δg_L^{Zt}	-0.3 ± 3.8	
δg_R^{Zu}	1.3 ± 5.0	
δg_R^{Zc}	-0.37 ± 0.51	
δg_L^{Zd}	-1.0 ± 3.7	
δg_L^{Zs}	1.2 ± 1.7	
δg_L^{Zb}	0.33 ± 0.16	
δg_R^{Zd}	3 ± 15	
δg_R^{Zs}	2.9 ± 4.8	
δg_R^{Zb}	2.3 ± 0.8	
<hr/>		
$\delta g_{1,z}$	-62 ± 37	
$\delta \kappa_\gamma$	-23 ± 23	
λ_z	65 ± 40	
$[c_{\ell\ell}]_{1111}$	1.00 ± 0.39	
$[c_{\ell e}]_{1111}$	-0.23 ± 0.22	
$[c_{ee}]_{1111}$	0.23 ± 0.39	
$[c_{\ell\ell}]_{1221}$	-3.7 ± 1.4	
$[c_{\ell\ell}]_{1122}$	2.0 ± 2.3	
$[c_{\ell e}]_{1122}$	1.0 ± 2.3	
$[c_{\ell e}]_{2211}$	-0.9 ± 2.2	
$[c_{ee}]_{1122}$	1.5 ± 2.6	
$[c_{\ell\ell}]_{1331}$	1.8 ± 1.3	
$[c_{\ell\ell}]_{1133}$	140 ± 170	
$[c_{\ell e}]_{1133} + [c_{\ell e}]_{3311}$	-0.55 ± 0.64	
$[c_{ee}]_{1133}$	-150 ± 180	
$[c_{\ell\ell}]_{2332}$	1.9 ± 2.1	

- Full correlation matrix also calculated
- Typical constraints at 1% level
- Flat directions for electron-tau operators: no additional observables to break LEP-2 degeneracy



Higgs Basis - parameters

EFT parameters along EWPT unconstrained directions
affecting LHC Higgs observables at leading order

Higgs couplings to
gauge bosons

CP even : δc_z $c_{z\Box}$ c_{zz} $c_{z\gamma}$ $c_{\gamma\gamma}$ c_{gg}

CP odd : \tilde{c}_{zz} $\tilde{c}_{z\gamma}$ $\tilde{c}_{\gamma\gamma}$ \tilde{c}_{gg}

Higgs couplings to
fermions

CP even : δy_u δy_d δy_e

CP odd : ϕ_u ϕ_d ϕ_e

Higgs couplings to
itself

CP even : $\delta\lambda_3$

$$\mathcal{L}_{h,\text{self}} = -(\lambda + \delta\lambda_3)vh^3.$$

Assuming Minimal Flavor Violation, and that
parameters strongly constrained at LO by
EWPT can be ignored,
we have 10 CP-even and 6 CP-odd
parameters to be constrained by LHC
Higgs analyses

$$\begin{aligned} \mathcal{L}_{\text{hvv}} = & \frac{h}{v} [2(1 + \delta c_w)m_W^2 W_\mu^+ W_\mu^- + (1 + \delta c_z)m_Z^2 Z_\mu Z_\mu \\ & + c_{ww} \frac{g_L^2}{2} W_{\mu\nu}^+ W_{\mu\nu}^- + \tilde{c}_{ww} \frac{g_L^2}{2} W_{\mu\nu}^+ \tilde{W}_{\mu\nu}^- + c_{w\Box} g_L^2 (W_\mu^- \partial_\nu W_{\mu\nu}^+ + \text{h.c.}) \\ & + c_{gg} \frac{g_s^2}{4} G_{\mu\nu}^a G_{\mu\nu}^a + c_{\gamma\gamma} \frac{e^2}{4} A_{\mu\nu} A_{\mu\nu} + c_{z\gamma} \frac{eg_L}{2c_\theta} Z_{\mu\nu} A_{\mu\nu} + c_{zz} \frac{g_L^2}{4c_\theta^2} Z_{\mu\nu} Z_{\mu\nu} \\ & + c_{z\Box} g_L^2 Z_\mu \partial_\nu Z_{\mu\nu} + c_{\gamma\Box} g_L g_Y Z_\mu \partial_\nu A_{\mu\nu} \\ & + \tilde{c}_{gg} \frac{g_s^2}{4} G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a + \tilde{c}_{\gamma\gamma} \frac{e^2}{4} A_{\mu\nu} \tilde{A}_{\mu\nu} + \tilde{c}_{z\gamma} \frac{eg_L}{2c_\theta} Z_{\mu\nu} \tilde{A}_{\mu\nu} + \tilde{c}_{zz} \frac{g_L^2}{4c_\theta^2} Z_{\mu\nu} \tilde{Z}_{\mu\nu}] \end{aligned}$$

$$\mathcal{L}_{\text{hff}} = -\frac{h}{v} \sum_{f=u,d,e} m_f f^c (I + \delta y_f e^{i\phi_f}) f + \text{h.c.}$$

- BSM corrections to Higgs couplings in mass eigenstate Lagrangian can be related by linear transformation to Wilson coefficients of any basis of D=6 operators
- Unexpected dependence of fermionic operators due to rescaling of SM couplings
- Corrections to Higgs and other SM couplings are $O(1/\Lambda^2)$ in EFT expansion.

Example:
Higgs couplings
expressed by
SILH Wilson coefficients

$$\begin{aligned}
 c_{gg} &= \frac{16}{g^2} \bar{c}_g, & \delta c_w &= -\frac{1}{2} \bar{c}_H - \frac{1}{g^2 - g'^2} \left[4g'^2 (\bar{c}_W + \bar{c}_B + \bar{c}_{2B} + c_{2W}) - 2g^2 \bar{c}_T + \frac{3g^2 + g'^2}{2} [\bar{c}'_{H\ell}]_{22} \right] \\
 c_{\gamma\gamma} &= \frac{16}{g^2} \bar{c}_\gamma, & \delta c_z &= -\frac{1}{2} \bar{c}_H - \frac{3}{2} [\bar{c}'_{H\ell}]_{22}. \\
 c_{zz} &= -\frac{4}{g^2 + g'^2} \left[\bar{c}_{HW} + \frac{g'^2}{g^2} \bar{c}_{HB} - 4 \frac{g'^2}{g^2} s_\theta^2 \bar{c}_\gamma \right], \\
 c_{z\Box} &= \frac{2}{g^2} \left[\bar{c}_W + \bar{c}_{HW} + \bar{c}_{2W} + \frac{g'^2}{g^2} (\bar{c}_B + \bar{c}_{HB} + \bar{c}_{2B}) - \frac{1}{2} \bar{c}_T + \frac{1}{2} [\bar{c}'_{H\ell}]_{22} \right], \\
 c_{z\gamma} &= \frac{2}{g^2} (\bar{c}_{HB} - \bar{c}_{HW} - 8s_\theta^2 \bar{c}_\gamma), \\
 c_{\gamma\Box} &= \frac{2}{g^2} (\bar{c}_{HW} - \bar{c}_{HB}) + \frac{4}{g^2 - g'^2} \left[\bar{c}_W + \bar{c}_{2W} + \frac{g'^2}{g^2} (\bar{c}_B + \bar{c}_{2B}) - \frac{1}{2} \bar{c}_T + \frac{1}{2} [\bar{c}'_{H\ell}]_{22} \right] \\
 c_{ww} &= -\frac{4}{g^2} \bar{c}_{HW}, \\
 c_{w\Box} &= \frac{2\bar{c}_{HW}}{g^2} + \frac{2}{g^2 - g'^2} \left[\bar{c}_W + \bar{c}_{2W} + \frac{g'^2}{g^2} (\bar{c}_B + \bar{c}_{2B}) - \frac{1}{2} \bar{c}_T + \frac{1}{2} [\bar{c}'_{H\ell}]_{22} \right], \\
 \delta\lambda_3 &= \lambda \left(\bar{c}_6 - \frac{3}{2} \bar{c}_H - \frac{1}{2} [\bar{c}'_{H\ell}]_{22} \right)
 \end{aligned}
 \tag{I}$$

See

LHCHXSWG-INT-2015-001

for full dictionary and other bases

Higgs Run-2 results coming!

- For Higgs analyses, the energy gain from 8 to 13 TeV is less relevant than for heavy new physics searches: cross section increases only by factor of 2. Therefore, progress with respect to run-1 is less spectacular.
- Nevertheless, already enough data analyzed to rediscover the Higgs boson at 13 TeV, and rates are measured with similar precision as in Run-1
- So far, Higgs rediscovered in $\gamma\gamma$ and ZZ decay channels, and interesting results also available for bb decays and tth production

Channel	Production	Run-1	ATLAS Run-2	CMS Run-2
$\gamma\gamma$	ggh	$1.10^{+0.23}_{-0.22}$	$0.62^{+0.30}_{-0.29}$ [4]	$0.77^{+0.25}_{-0.23}$ [5]
	VBF	$1.3^{+0.5}_{-0.5}$	$2.25^{+0.75}_{-0.75}$ [4]	$1.61^{+0.90}_{-0.80}$ [5]
	Wh	$0.5^{+1.3}_{-1.2}$	-	-
	Zh	$0.5^{+3.0}_{-2.5}$	-	-
	Vh	-	$0.30^{+1.21}_{-1.12}$ [4]	-
	$t\bar{t}h$	$2.2^{+1.6}_{-1.3}$	$-0.22^{+1.26}_{-0.99}$ [4]	$1.9^{+1.5}_{-1.2}$ [5]
$Z\gamma$	incl.	$1.4^{+3.3}_{-3.2}$	-	-
ZZ^*	ggh	$1.13^{+0.34}_{-0.31}$	$1.34^{+0.39}_{-0.33}$ [4]	$0.96^{+0.40}_{-0.33}$ [6]
	VBF	$0.1^{+1.1}_{-0.6}$	$3.8^{+2.8}_{-2.2}$ [4]	$0.67^{+1.61}_{-0.67}$ [6]
WW^*	ggh	$0.84^{+0.17}_{-0.17}$	-	-
	VBF	$1.2^{+0.4}_{-0.4}$	$1.7^{+1.2}_{-0.9}$	-
	Wh	$1.6^{+1.2}_{-1.0}$	$3.2^{+4.4}_{-4.2}$	-
	Zh	$5.9^{+2.6}_{-2.2}$	-	-
	$t\bar{t}h$	$5.0^{+1.8}_{-1.7}$	-	-
	incl.	-	-	0.3 ± 0.5 [7]
$\tau^+\tau^-$	ggh	$1.0^{+0.6}_{-0.6}$	-	-
	VBF	$1.3^{+0.4}_{-0.4}$	-	-
	Wh	$-1.4^{+1.4}_{-1.4}$	-	-
	Zh	$2.2^{+2.2}_{-1.8}$	-	-
	$t\bar{t}h$	$-1.9^{+3.7}_{-3.3}$	-	-
$b\bar{b}$	VBF	-	$-3.9^{+2.8}_{-2.9}$ [8]	$-3.7^{+2.4}_{-2.5}$ [9]
	Wh	$1.0^{+0.5}_{-0.5}$	-	-
	Zh	$0.4^{+0.4}_{-0.4}$	-	-
	Vh	-	$0.21^{+0.51}_{-0.50}$ [10]	-
	$t\bar{t}h$	$1.15^{+0.99}_{-0.94}$	$2.1^{+1.0}_{-0.9}$ [11]	$-0.19^{+0.80}_{-0.81}$
$\mu^+\mu^-$	incl.	$0.1^{+2.5}_{-2.5}$	$-0.8^{+2.2}_{-2.2}$ [13]	-
multi- ℓ	cats.	-	$2.5^{+1.3}_{-1.1}$ [14]	$2.3^{+0.9}_{-0.8}$ [15]

LO EFT parameter fits

- In SM EFT, assuming MFV, only 9 CP-even parameters unconstrained by LEP affect Higgs signal strength observables at LO. CP-odd parameters enter only at quadratic order and they are less relevant unless one studies certain differential distributions
- All these 9 parameters are already constrained in a non-trivial way by LHC Run1 and Run2 results
- Currently, some 2.5 sigma tension because of excess in observed $t\bar{t}h$ production rate and deficit in observed higgs decay to bottom quarks

	Higgs Run1&2
δc_z	-0.13 ± 0.11
c_{zz}	-0.56 ± 0.33
$c_{z\Box}$	0.21 ± 0.12
$c_{\gamma\gamma}$	0.0072 ± 0.0073
$c_{z\gamma}$	-0.015 ± 0.074
c_{gg}	-0.0040 ± 0.0009
δy_u	0.17 ± 0.13
δy_d	-0.51 ± 0.18
δy_e	-0.13 ± 0.13

+ full 9x9
correlation matrix

Channel	Production	Run-1	ATLAS Run-2	CMS Run-2
$\gamma\gamma$	ggh	$1.10^{+0.23}_{-0.22}$	$0.62^{+0.30}_{-0.29}$ [4]	$0.77^{+0.25}_{-0.23}$ [5]
	VBF	$1.3^{+0.5}_{-0.5}$	$2.25^{+0.75}_{-0.75}$ [4]	$1.61^{+0.90}_{-0.80}$ [5]
	Wh	$0.5^{+1.3}_{-1.2}$	-	-
	Zh	$0.5^{+3.0}_{-2.5}$	-	-
	Vh	-	$0.30^{+1.21}_{-1.12}$ [4]	-
	$t\bar{t}h$	$2.2^{+1.6}_{-1.3}$	$-0.22^{+1.26}_{-0.99}$ [4]	$1.9^{+1.5}_{-1.2}$ [5]
$Z\gamma$	incl.	$1.4^{+3.3}_{-3.2}$	-	-
ZZ^*	ggh	$1.13^{+0.34}_{-0.31}$	$1.34^{+0.39}_{-0.33}$ [4]	$0.96^{+0.40}_{-0.33}$ [6]
	VBF	$0.1^{+1.1}_{-0.6}$	$3.8^{+2.8}_{-2.2}$ [4]	$0.67^{+1.61}_{-0.67}$ [6]
WW^*	ggh	$0.84^{+0.17}_{-0.17}$	-	-
	VBF	$1.2^{+0.4}_{-0.4}$	$1.7^{+1.2}_{-0.9}$	-
	Wh	$1.6^{+1.2}_{-1.0}$	$3.2^{+4.4}_{-4.2}$	-
	Zh	$5.9^{+2.6}_{-2.2}$	-	-
	$t\bar{t}h$	$5.0^{+1.8}_{-1.7}$	-	-
	incl.	-	-	0.3 ± 0.5 [7]
$\tau^+\tau^-$	ggh	$1.0^{+0.6}_{-0.6}$	-	-
	VBF	$1.3^{+0.4}_{-0.4}$	-	-
	Wh	$-1.4^{+1.4}_{-1.4}$	-	-
	Zh	$2.2^{+2.2}_{-1.8}$	-	-
	$t\bar{t}h$	$-1.9^{+3.7}_{-3.3}$	-	-
$b\bar{b}$	VBF	-	$-3.9^{+2.8}_{-2.9}$ [8]	$-3.7^{+2.4}_{-2.5}$ [9]
	Wh	$1.0^{+0.5}_{-0.5}$	-	-
	Zh	$0.4^{+0.4}_{-0.4}$	-	-
	Vh	-	$0.21^{+0.51}_{-0.50}$ [10]	-
	$t\bar{t}h$	$1.15^{+0.99}_{-0.94}$	$2.1^{+1.0}_{-0.9}$ [11]	$-0.19^{+0.80}_{-0.81}$
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multi- ℓ	cats.	-	$2.5^{+1.3}_{-1.1}$ [14]	$2.3^{+0.9}_{-0.8}$ [15]

On deforming SM EFT

Higgs boson in SM

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \dots \\ v + h + \dots \end{pmatrix}$$

$$\mathcal{L}_{\text{SM}} = D_\mu H^\dagger D_\mu H + m_H^2 H^\dagger H - \lambda (H^\dagger H)^2 + \left(\frac{y_{ij}}{\sqrt{2}} H \bar{\psi}_i \psi_j + \text{h.c.} \right) + (\dots \text{Higgs})$$

Couplings to
EW gauge
bosons

$$\left(\frac{h}{v} + \frac{h^2}{2v^2} \right) (2m_W^2 W_\mu^+ W_\mu^- + m_Z^2 Z_\mu Z_\mu)$$

Ensures unitarity of
VV→hh scattering

Ensures unitarity of
VV→VV scattering

Self-
Couplings

$$-\frac{m_h^2}{2v} h^3 - \frac{m_h^2}{8v^2} h^4$$

Couplings
to fermions

$$-\frac{h}{v} \sum_f m_f \bar{f} f$$

Ensures unitarity of
VV→ff scattering

What are Higgs
self-couplings for?

Triple Higgs coupling in SM EFT

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \dots \\ v + h + \dots \end{pmatrix}$$

$$\mathcal{L}_{\text{SM}} = D_\mu H^\dagger D_\mu H + m_H^2 H^\dagger H - \lambda (H^\dagger H)^2 + \left(\frac{y_{ij}}{\sqrt{2}} H \bar{\psi}_i \psi_j + \text{h.c.} \right) + (\text{no Higgs})$$

Couplings to
EW gauge
bosons

Self-
Couplings

Couplings
to fermions

It is clear what goes wrong when self-couplings are modified in framework of SM EFT where SM Lagrangian is extended by higher-dimensional operators. New scale M suppressing $D>4$ operators sets maximum validity range Λ of SM EFT

$$\mathcal{L} = \mathcal{L}_{\text{SM}} - \frac{c_6 (H^\dagger H)^3}{M^2}$$

$$\mathcal{L} \supset -\frac{m_h^2}{2v} (1 + \delta\lambda_3) h^3 - \frac{m_h^2}{8v^2} (1 + \delta\lambda_4) h^4 - \frac{\lambda_5}{v} h^5 - \frac{\lambda_6}{v^2} h^6$$

$$\delta\lambda_3 = \frac{2v^4 c_6}{m_h^2 M^2} \quad \delta\lambda_4 = \frac{12v^4 c_6}{m_h^2 M^2} \quad \lambda_5 = \frac{3v^2 c_6}{4M^2} \quad \lambda_6 = \frac{v^2 c_6}{8M^2}$$

$$\Lambda \lesssim \frac{4\pi M}{\sqrt{|c_6|}} = \frac{4\pi v}{\sqrt{|\delta\lambda_3|}} \frac{\sqrt{2}v}{m_h}$$

E.g. $hh \rightarrow 3h$, or $hh \rightarrow 4h$ scattering loses perturbative unitarity at scale Λ .

Important feature: in SM EFT with $|\delta\lambda_3| \ll 1$ validity range can be parametrically separated from TeV scale $4\pi v$

Triple Higgs coupling in SM EFT

$$\mathcal{L} = \mathcal{L}_{\text{SM}} - \frac{c_6 (H^\dagger H)^3}{M^2}$$

By SM gauge invariance, there are higher-point vertices with Goldstone bosons, thus also scattering of longitudinal W and Z becomes non-unitary

$$\mathcal{L}_{\text{EFT}} \supset -\frac{c_6}{M^2} (2G^+G^- + G_z^2) \left(\frac{3vh^3}{2} + \frac{3h^4}{8} \right) - \frac{c_6}{M^2} (2G^+G^- + G_z^2)^2 \left(\frac{3vh}{4} + \frac{3h^2}{8} \right) - \frac{c_6}{8M^2} (2G^+G^- + G_z^2)^3$$

Consider isospin-0 scattering $VV \rightarrow VVh$, and $VV \rightarrow VVhh$

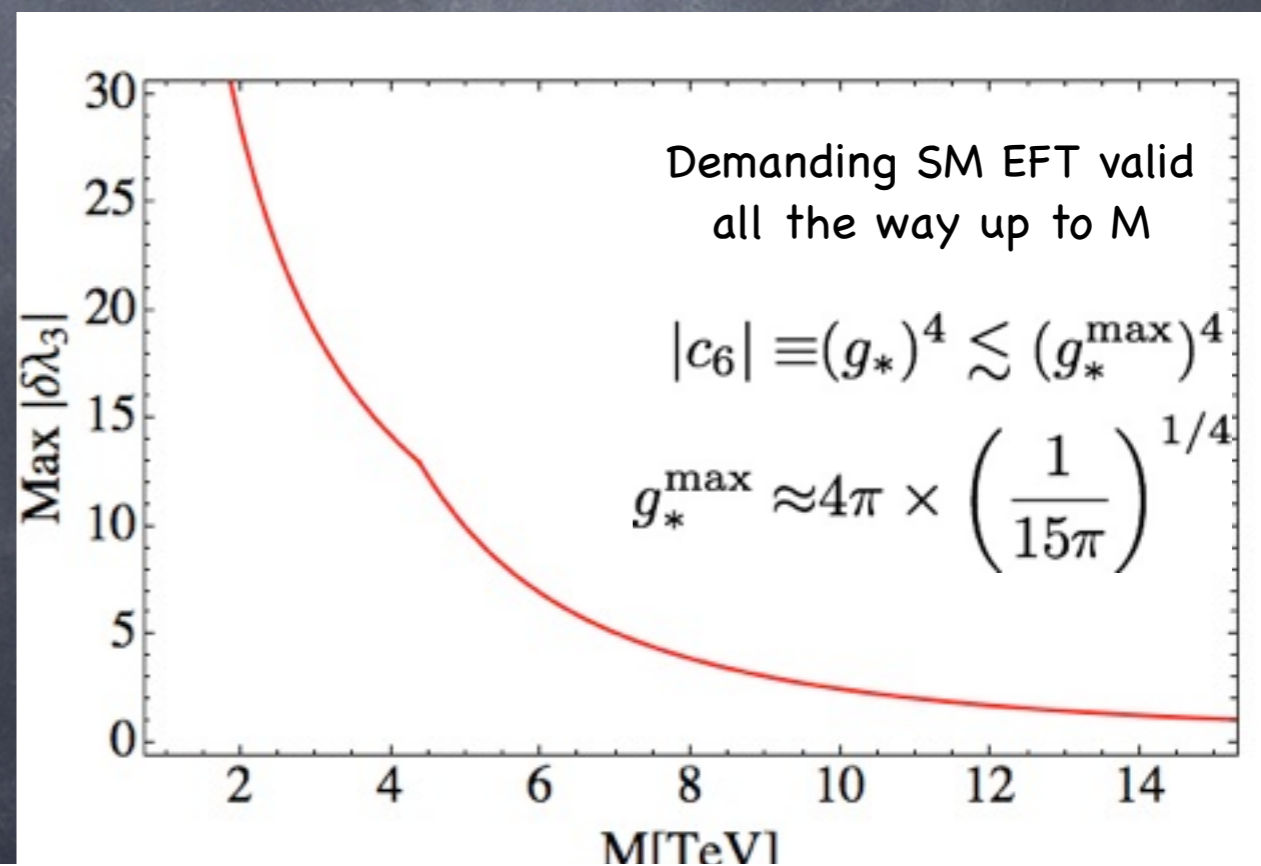
Unitarity limit on inelastic channels follows from

$$2\text{Im}\mathcal{M}(p_1, p_2 \rightarrow p_1, p_2) = S_2 \int d\Pi_2 |\mathcal{M}_{\text{el.}}(p_1, p_2 \rightarrow k_1, k_2)|^2 + \sum S_n \int d\Pi_n |\mathcal{M}_{\text{inel.}}(p_1, p_2 \rightarrow k_1 \dots k_n)|^2$$

Assuming $VV \rightarrow VV$ amplitude dominated by s-wave at high energy: $S_n \sum \int d\Pi_n |\mathcal{M}_{\text{inel.}}|^2 \leq \frac{8\pi}{S_2}$

Actually, bounds from $VVh \rightarrow VVh$ better by $O(1)$ numerical factor

$ \delta\lambda_3 $	$\Lambda_{\text{SM EFT}} [\text{TeV}]$
0.01	160
0.1	50
1	16
10	5.0
20	2.8
40	1.4



h^3 -deformed SM

Here I address a different question: what goes wrong in a theory where only triple Higgs coupling is deformed away from SM and no other interactions are affected (in particular, there's no h^5 or h^6 terms in the Lagrangian)

Answer: multibody $V_L V_L \rightarrow (n \times h)(m \times V_L)$ (and crossed) scattering with $n+m > 2$ loses perturbative unitarity around the scale $\Lambda \sim 4\pi v \sim 3 \text{ TeV}$

Consider $V_L V_L \rightarrow hhh$ which depends on triple and other Higgs couplings. Diagrams with one triple Higgs vertex contribute

$$\mathcal{M} \sim \frac{m_W^2}{v^2} \frac{m_h^2}{v} (1 + \delta\lambda_3) \left(\frac{\sqrt{s}}{m_W} \right)^2 \frac{1}{s - m_h^2} + \dots$$

hhWW
vertex

Triple Higgs vertex

Longitudinal
polarization

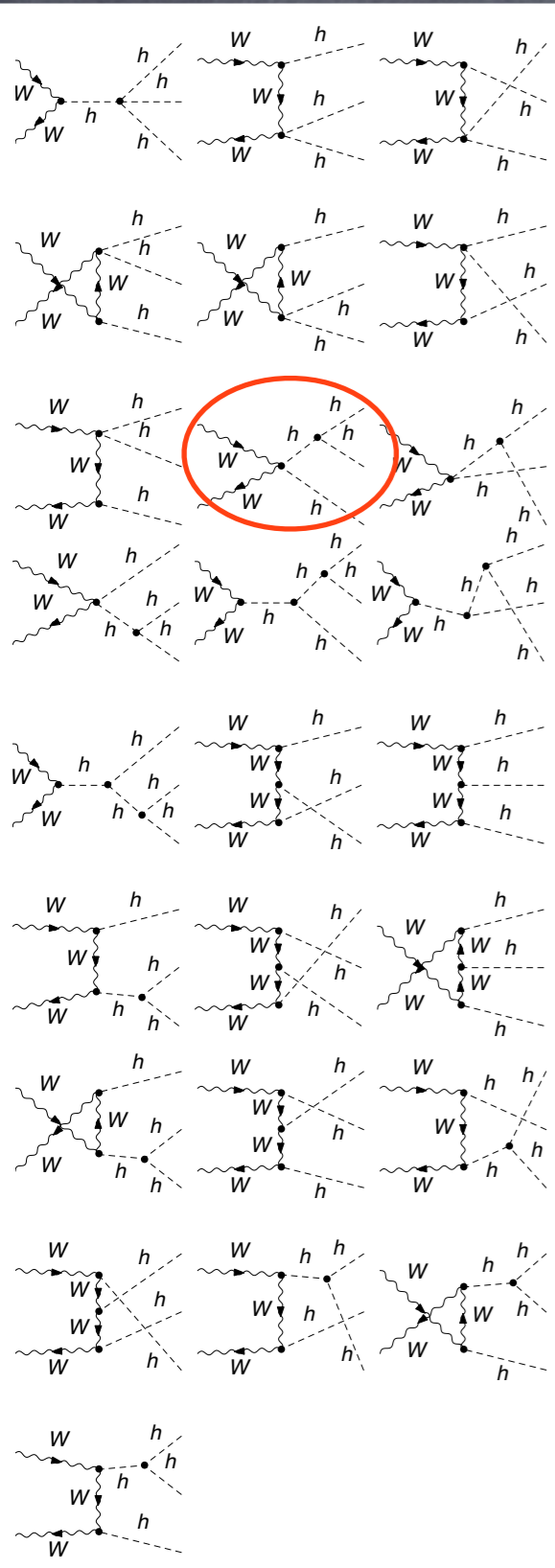
Propagator

In SM, various contributions that go like E^0 cancel against each other so that full amplitude behaves as $1/E$ at high energy, consistently with perturbative unitarity

However, as soon as $\delta\lambda_3 \neq 0$, cancellation is no longer happening, and then tree level $V_L V_L \rightarrow hhh$ cross section explodes at high energies

Perturbative unitarity of $V_L V_L \rightarrow hhh$ is lost at scale

$$\Lambda \sim \frac{4\pi v}{|\delta\lambda_3|}$$



h^3 -deformed SM

Much as in SM EFT, one can derive this result via **equivalence theorem**

Given Lagrangian for Higgs boson h , one can always uplift it to manifestly gauge invariant form by replacing

$$h \rightarrow \sqrt{2H^\dagger H} - v$$

$$\frac{m_h^2}{2} h^2 + \frac{m_h^2}{2v} (1 + \delta\lambda_3) h^3 + \frac{m_h^2}{8v^2} h^4 \quad \Lambda_3 = \frac{m_h^2}{2v} \delta\lambda_3$$

$$\rightarrow m^2 H^\dagger H + \lambda (H^\dagger H)^2 + 3\Lambda_3 v^2 (2H^\dagger H)^{1/2} + \Lambda_3 (2H^\dagger H)^{3/2}$$

Non-analytic terms lead to infinite series of n -point Goldstone and Higgs boson interactions

$$\mathcal{L} \supset \mathcal{L}_{G^2} + \mathcal{L}_{G^4} + \mathcal{L}_{G^6} + \dots$$

$$\mathcal{L}_{G^2} = -m_h^2 (2G_+ G_- + G_z^2) \left[\frac{h}{2v} + \frac{1 + 3\delta\lambda_3}{4} \frac{h^2}{v^2} - \frac{3\delta\lambda_3}{4} \frac{h^3}{v^3} + \dots \right]$$

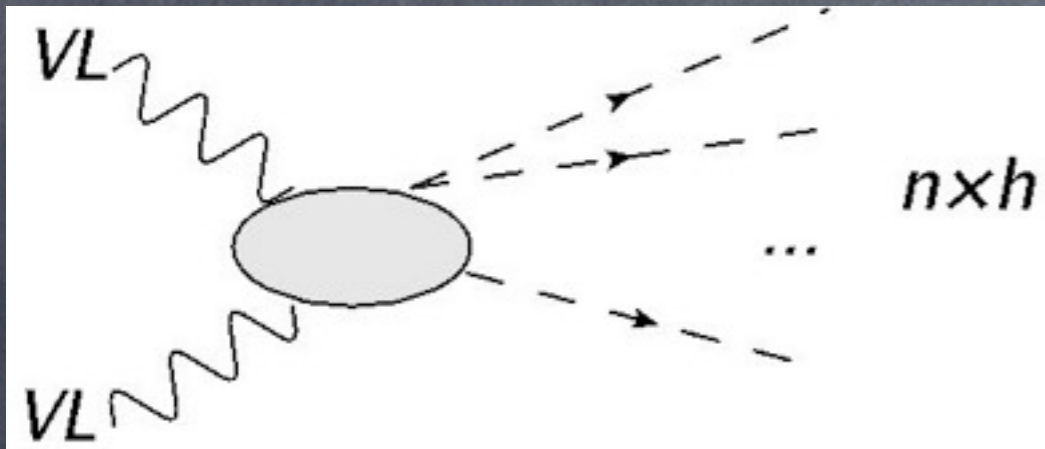
$$\mathcal{L}_{G^4} = -m_h^2 (2G_+ G_- + G_z^2)^2 \left(\frac{1}{8v^2} + \frac{3\delta\lambda_3}{8} \frac{h}{v^3} - \frac{15\delta\lambda_3}{16} \frac{h^2}{v^4} + \dots \right)$$

$$H = \begin{pmatrix} iG_+ \\ \frac{v+h-iG_z}{\sqrt{2}} \end{pmatrix}$$

Consequence: in deformed SM with $\delta\lambda_3 \neq 0$, not only $VV \rightarrow 3h$, but also $VV \rightarrow n \times h$, $VV \rightarrow VV + n \times h$, ..., lose unitarity at some high-energy scale

multi-Higgs production in h^3 -deformed SM

High-energy limit of scattering amplitude of isospin-0 longitudinal gauge 2-body state into multi-Higgs state



$$\mathcal{M}([GG]_0 \rightarrow h^n) \rightarrow -\frac{3\sqrt{3}m_h^2}{2v^n} n! \delta\lambda_3$$

Unitarity limit

$$\frac{1}{2!n!} \int d\Pi_n |\mathcal{M}([GG]_0 \rightarrow h^n)|^2 = \left(\frac{s}{(4\pi v)^2} \right)^{n-2} \frac{27m_h^4}{64\pi v^4} \frac{n}{(n-2)!} (\delta\lambda_3)^2$$

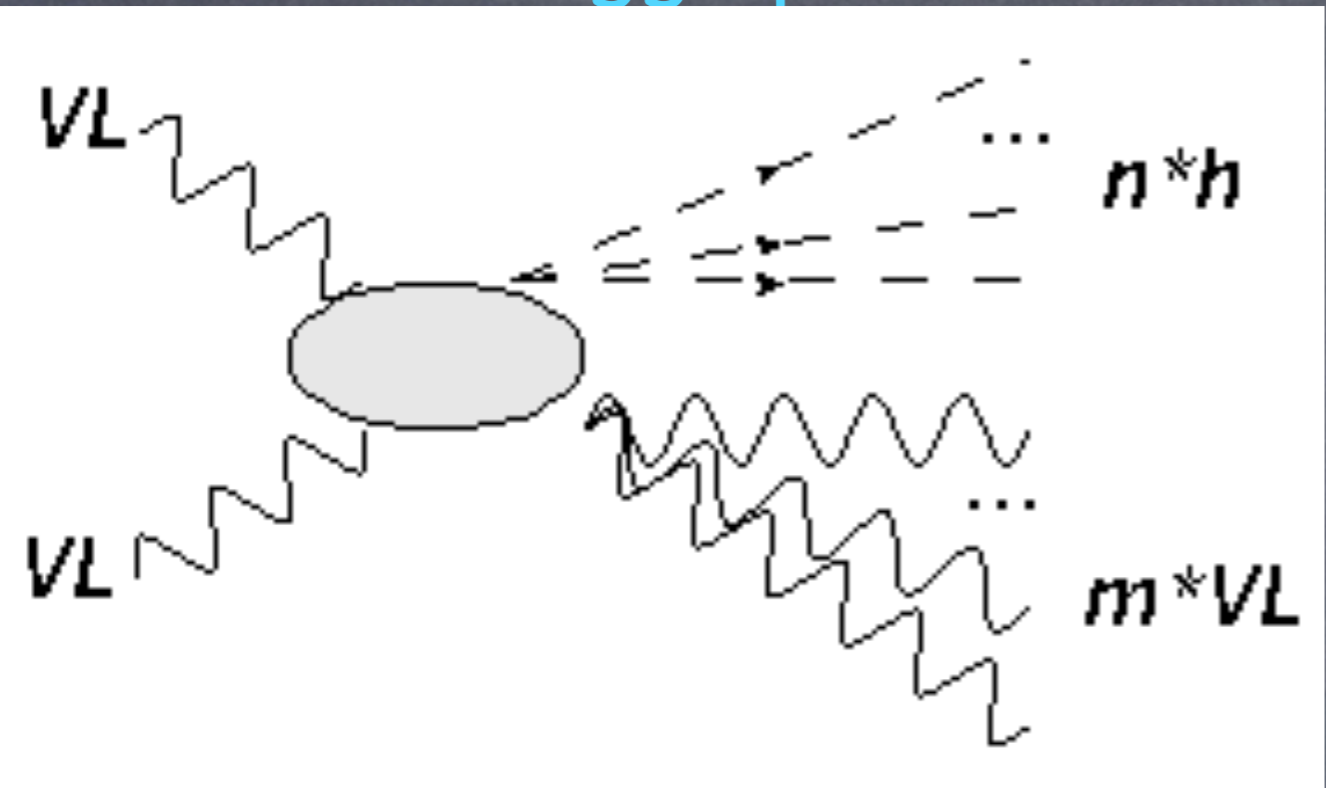
$$\lesssim 2! * 8\pi$$

$$\Lambda \lesssim 4\pi v \left(\frac{32\pi \sqrt{(n-2)!} v^2}{\sqrt{27n} m_h^2 \delta\lambda_3} \right)^{\frac{1}{n-2}} \approx \frac{4\pi v}{(\delta\lambda_3)^{\frac{1}{n-2}}} \sqrt{\frac{n-2}{e}} \left(\frac{32\pi (2\pi(n-2))^{1/4} v^2}{\sqrt{27n} m_h^2} \right)^{\frac{1}{n-2}}$$

For small enough $\delta\lambda_3$, stronger bound on Λ may be obtained from scattering with $n > 3$

$\delta\lambda_3$	$n=3$	$n=n_{\text{best}}$	\sum_n
0.01	13000	12.8 @ 20	12.0
0.1	1300	11.0 @ 15	10.1
1	130	8.9 @ 11	7.9
10	13.4	6.1 @ 6	4.9
40	3.3	3.3 @ 3	2.6

multi-Higgs production in h^3 -deformed SM

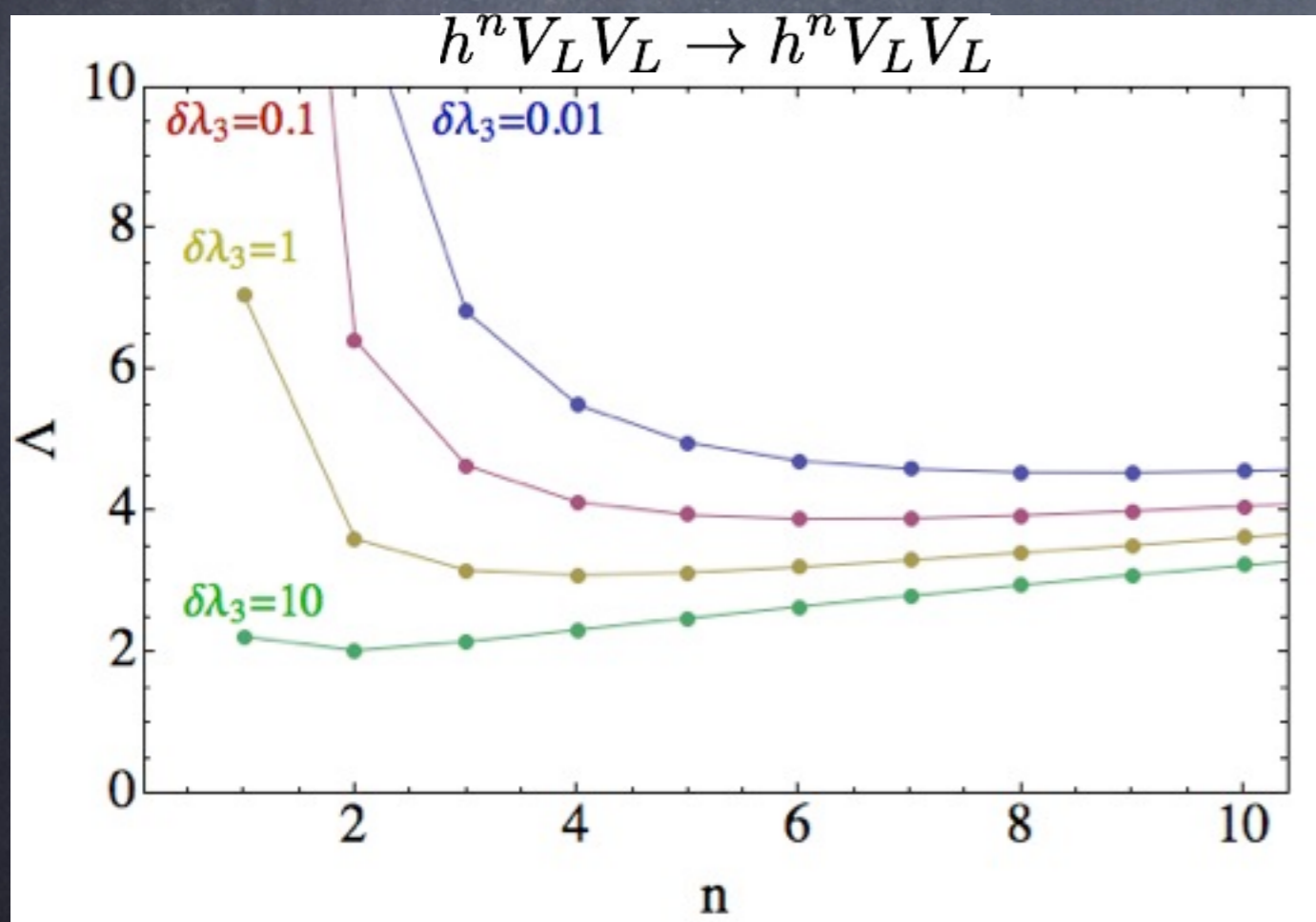


Numerically, slightly better bounds from scattering with longitudinal W and Z

For small $|\delta\lambda_3|$, cutoff approximately

$$\Lambda \sim 2\pi v \sqrt{|\log |\delta\lambda_3||}$$

in practice, never parametrically above $4\pi v$



$h^n V_L V_L \rightarrow h^n V_L V_L$

$ \delta\lambda_3 $	Λ [TeV]	n_{best}	Λ_{SMEFT} [TeV]
0.01	4.5	9	160
0.1	3.9	6	50
1	3.1	4	16
10	2.0	2	5.0
20	1.6	1	2.8
40	1.1	1	1.4

SM EFT vs NH EFT

More generally: NH EFT = SM + non-analytic terms

$$\mathcal{L}_{\text{NH EFT}} = \frac{1}{2} f_h(h) \partial_\mu h \partial_\mu h - V(h) + \frac{v^2}{4} f_1(h) \text{Tr}[\partial_\mu U^\dagger \partial_\mu U] + v^2 f_2(h) (\text{Tr}[U^\dagger \partial_\mu U \sigma_3])^2 + \dots$$

$$U = e^{i\pi^a \sigma^a / v}, \quad U \rightarrow e^{ig_L \alpha_L^a \sigma^a / 2} U e^{-ig_Y \alpha_Y \sigma^3 / 2}$$

Question: what are conditions on functions $f(h)$ such that this Lagrangian is really SM EFT in disguise?

One can always lift non-linear symmetry to linearly realized SM gauge symmetry by replacing

$$U \rightarrow \frac{(\tilde{H}, H)}{\sqrt{H^\dagger H}}$$

$$H = \begin{pmatrix} iG_+ \\ \frac{v+h-iG_z}{\sqrt{2}} \end{pmatrix}, \quad \tilde{H} = i\sigma^2 H^* = \begin{pmatrix} \frac{v+h+iG_z}{\sqrt{2}} \\ iG_- \end{pmatrix}$$

NH EFT Lagrangian belongs to SM EFT class when, after this replacement, Lagrangian is analytic at $v=0$

SM EFT vs NH EFT

Example: matching to dimension-6 EFT

$$\mathcal{L}_{\text{NH EFT}} = \frac{1}{2} f_h(h) \partial_\mu h \partial_\mu h - V(h) + \frac{v^2}{4} f_1(h) \text{Tr}[\partial_\mu U^\dagger \partial_\mu U] + v^2 f_2(h) (\text{Tr}[U^\dagger \partial_\mu U \sigma_3])^2 + \dots$$

NH EFT is dimension-6 SM EFT when f-functions have following form

$$f_h = \frac{2b_0 - b_1}{2} + \frac{b_1}{2} \left(1 + \frac{h}{v}\right)^2,$$

$$f_1 = \frac{2b_0 - b_1}{2} \left(1 + \frac{h}{v}\right)^2 + \frac{2 - 2b_0 + b_1}{2} \left(1 + \frac{h}{v}\right)^4,$$

$$f_2 = d_0 \left(1 + \frac{h}{v}\right)^4$$

2-parameter redundancy here, as one can always redefine h such that fh=1

If f functions are different polynomials (of the same order) then non-analytic terms appear on the H-side, resulting in unitarity loss at scale $4\pi v$

This corresponds to dimension-6 Lagrangian

$$\mathcal{L} = \left(b_0 - \frac{b_1}{2}\right) |D_\mu H|^2$$

$$+ \frac{b_0 - 1}{2} \frac{[\partial_\mu (H^\dagger H)]^2}{v^2}$$

$$+ (2 - 2b_0 + b_1) \frac{H^\dagger H |D_\mu H|^2}{v^2}$$

$$+ d_0 \frac{(D_\mu H^\dagger H - H^\dagger D_\mu H)^2}{v^2}$$

Two-parameter redundancy on SM EFT, as H can be rescaled, and one operator can be eliminated by field redefinition

Summary

- SM EFT is currently a useful bookkeeping device to understand constraint on heavy BSM physics. Many dimension-6 operators are constrained in a model-independent way using low-energy, electroweak precision, LHC Higgs, and other experiments
- The h^3 -deformed SM (the theory with the SM field content and interactions except for the triple Higgs boson coupling deformed away from the SM value) is similar to Higgsless theories in that it loses perturbative unitarity around the scale $4\pi v$, even if the deformation is small. Same conclusions if the quartic Higgs coupling is deformed
- Such set-up does not belong to the SM EFT class, and is not an effective theory obtained by integrating out heavy BSM particles. In fact, it corresponds to an effective theory where masses of integrated-out particles vanish in the limit of no electroweak symmetry breaking
- Similar discussion applies for other Higgs couplings deformations that are not described by SM EFT