Cosmological production of dark matter axions

Ken'ichi Saikawa (DESY)

Based on
T. Hiramatsu, M. Kawasaki, KS, T. Sekiguchi, PRD85, 105020 (2012) [1202.5851]
T. Hiramatsu, M. Kawasaki, KS, T. Sekiguchi, JCAP01, 001 (2013) [1207.3166]
M. Kawasaki, KS, T. Sekiguchi, PRD91, 065014 (2015) [1412.0789]
A. Ringwald, KS, PRD93, 085031 (2016) [1512.06436]

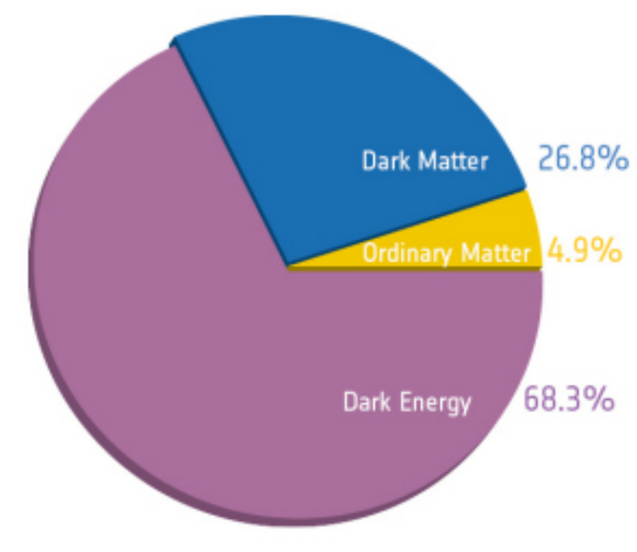
5 May 2017, IFAE, Barcelona

Plan

- Brief review of axion cosmology
  - Pre-inflationary symmetry breaking scenario
  - Post-inflationary symmetry breaking scenario
- Axion production from strings and domain walls
  - Estimate of dark matter abundance
  - Constraints on models
- Conclusion

Dark matter

- Recent astrophysical observations imply that about 27% of the total energy of the universe is occupied by unknown matter.
  - Stable in cosmological timescales
  - Collisionless (“invisible”)
  - “Cold” (velocity dispersion is sufficiently small at the beginning of structure formation)



Credit: ESA and the Planck Collaboration

- Physics beyond the Standard Model
  - A well motivated candidate: axion

Peccei-Quinn extension of the standard model

  - Solution to the strong CP problem

Strong CP problem and axion

- Strong CP problem
  - Quantum chromodynamics (QCD) allows a CP violating term:

$$\mathcal{L} \supset \frac{\alpha_s}{8\pi} \theta G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$

Physical observable: $\bar{\theta} = \theta + \arg \det M_q$

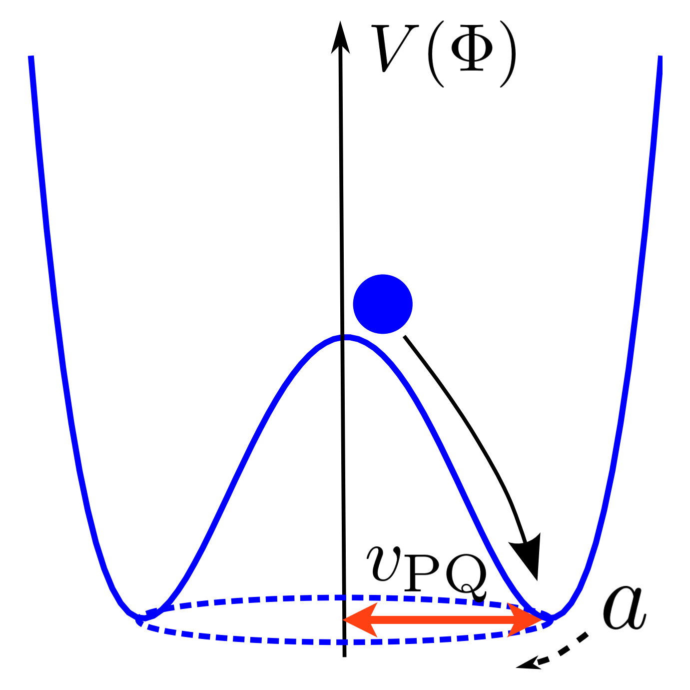
  - Non-observation of neutron electric dipole moment implies

$$|\bar{\theta}| < \mathcal{O}(10^{-11})$$ "Why it is so small ?"

- Peccei-Quinn (PQ) mechanism
  - Take $\bar{\theta}$ as a dynamical variable that explains its smallness, i.e. $\bar{\theta} \to \bar{\theta}_{\rm eff}(x) = a(x)/F_a$
  - Predicts the existence of light particle $a(x)$ = axion .

Axion as a Nambu-Goldstone boson

- Axions can be identified as Nambu-Goldstone bosons arising from breaking of global symmetry. (Peccei-Quinn (PQ) symmetry)

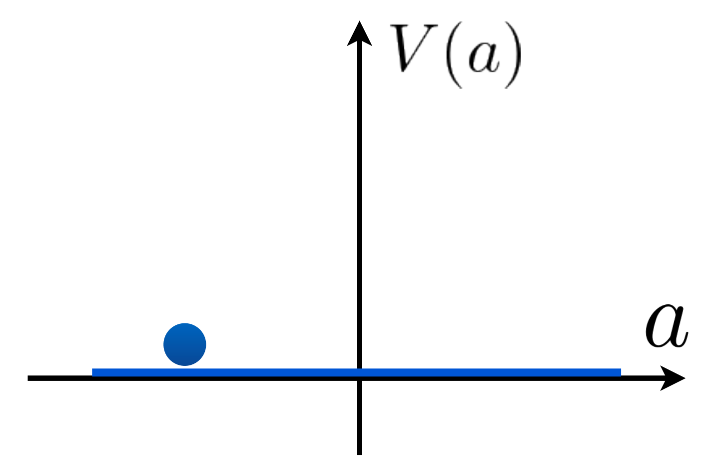


- Hidden scalar field:

$$\Phi(x) = \frac{1}{\sqrt{2}}\left[v_{\rm PQ} + \rho(x)\right] e^{ia(x)/v_{\rm PQ}}$$

Massive modulus, massless phase:

$$m_\rho \sim v_{\rm PQ}, \quad m_a = 0$$



- Interactions with standard model particles are suppressed by a large symmetry breaking scale.

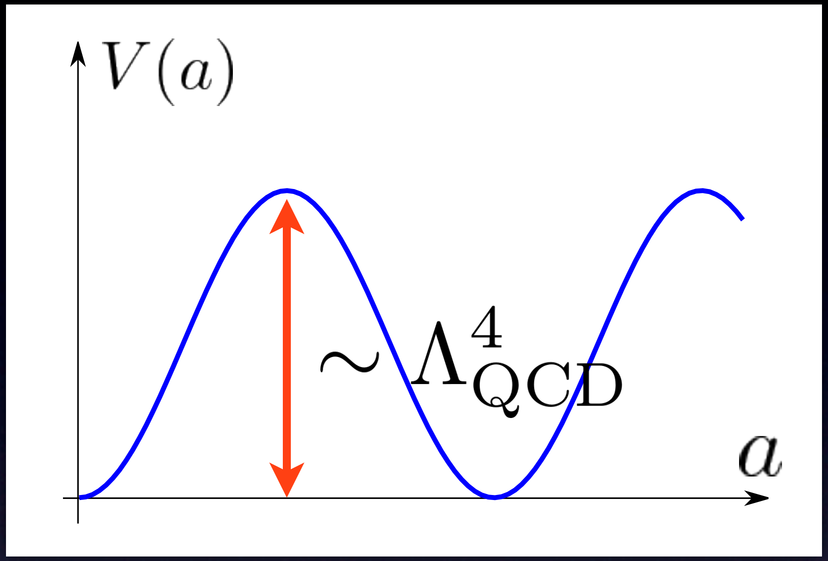
$$v_{\rm PQ} \gg v_{\rm electroweak} \approx \mathcal{O}(100)\,{\rm GeV}$$

Properties of axion

- Axions can couple to gluons via

$$\mathcal{L} \supset -\frac{\alpha_s}{8\pi}\frac{a}{F_a} G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

$F_a \propto v_{\rm PQ}$: axion decay constant



- Below the QCD scale $\Lambda_{\rm QCD} \sim \mathcal{O}(100\,{\rm MeV})$, topological charge fluctuations in QCD vacuum induce the potential energy:

$$V(a) \sim \Lambda_{\rm QCD}^4 \left(1 - \cos\frac{a}{F_a}\right)$$

➡ $\langle a \rangle = 0$ at the minimum, solving strong CP problem

- Mass of QCD axions $m_a \sim \Lambda_{\rm QCD}^2/F_a$:

$$m_a = \frac{m_\pi F_\pi}{F_a}\frac{\sqrt{z}}{1+z} \simeq 6\,{\rm meV}\left(\frac{10^9\,{\rm GeV}}{F_a}\right) \qquad z = m_u/m_d = 0.48(3)$$

- Tiny coupling with matter + non-thermal production

→ good candidate of cold dark matter

How axions are produced ?

$$\Omega_a \equiv \frac{\rho_a}{\rho_c} = \Omega_a(F_a), \quad m_a \simeq 6\,\mathrm{meV}\left(\frac{10^9\,\mathrm{GeV}}{F_a}\right)$$

ρ_c : total energy density of the universe today

- What is the "typical mass" of the axion (or the "typical value" for the axion decay constant), if axions explain 100% of dark matter abundance ?
- Predictions strongly depend on the early history of the universe.
- Two possibilities:
  - PQ symmetry is never restored after inflation
  - PQ symmetry is restored during/after inflation

Pre-inflationary PQ symmetry breaking scenario

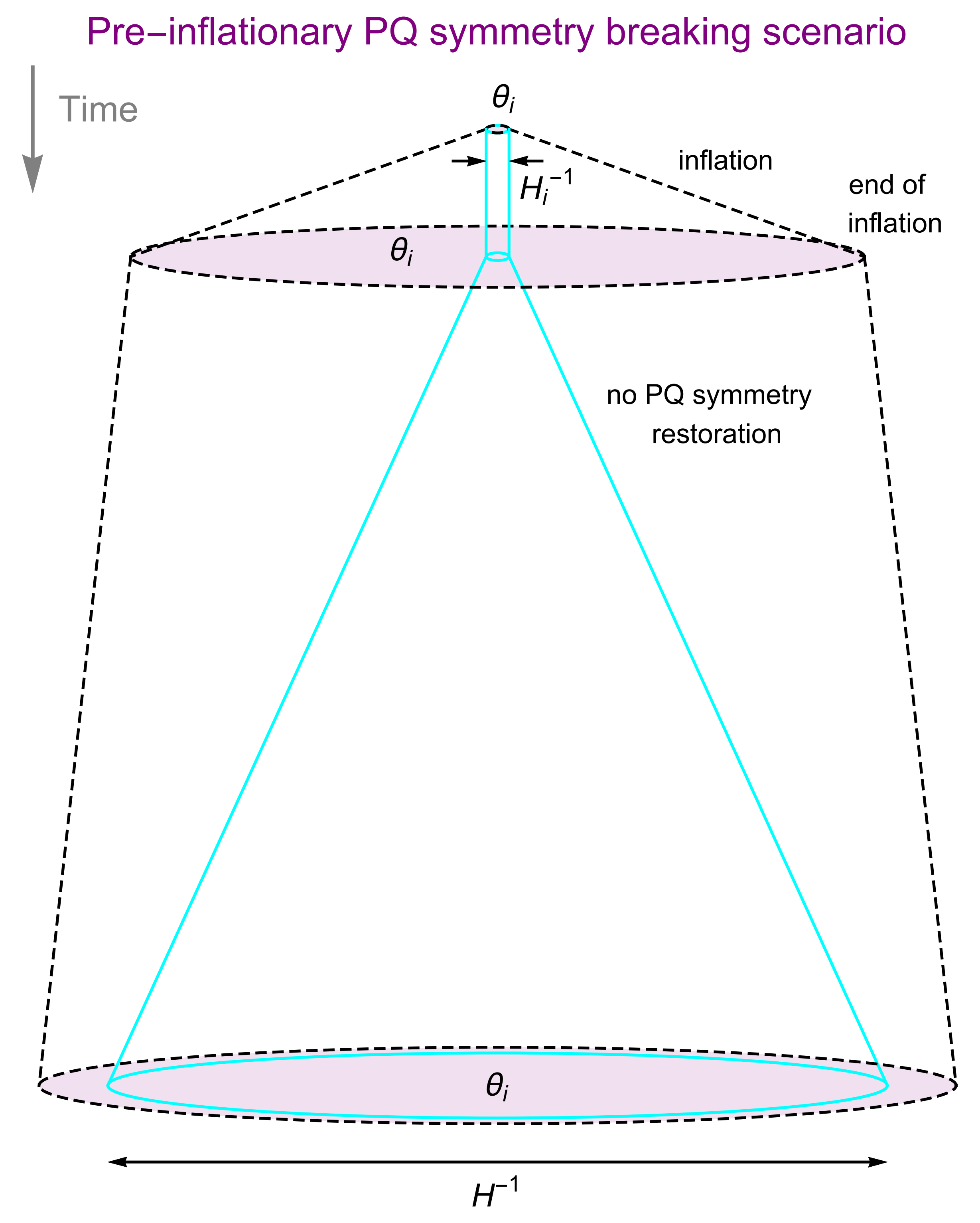
- How the spatial distribution of an angular field

$$\theta(x) = \frac{a(x)}{F_a}$$

evolves over time ?

- The size of the patch of universe in which θ takes a certain value θ_i can be much larger than the Hubble radius at the present time.

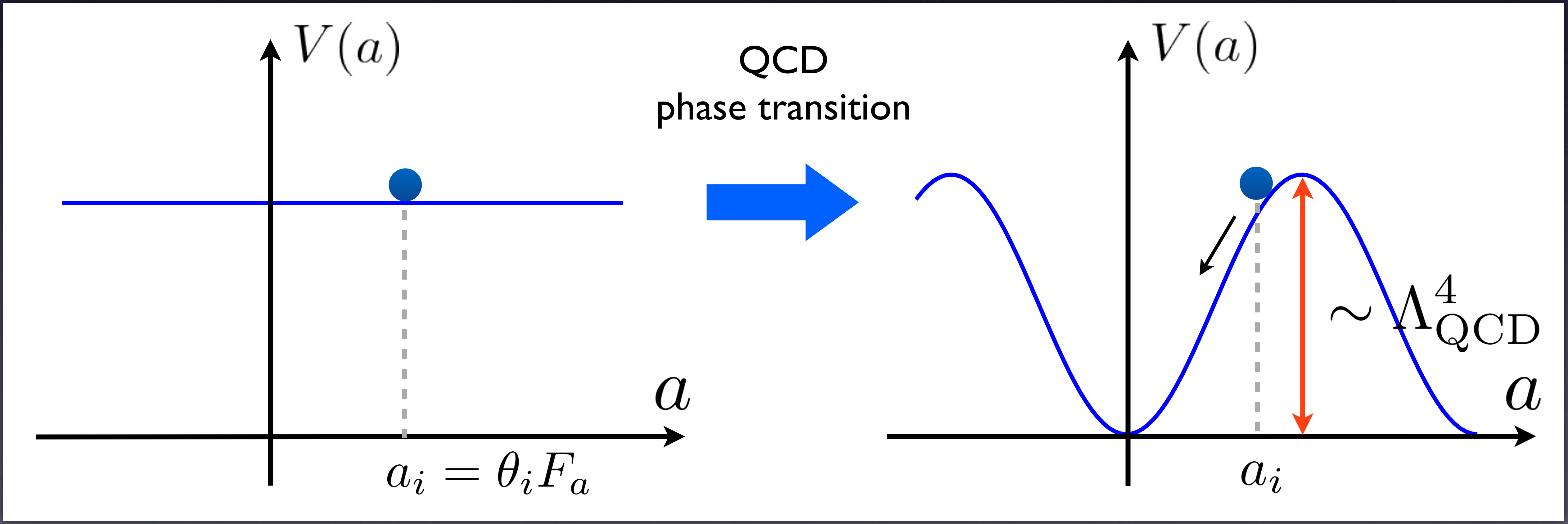
- Relic axion abundance depends on F_a and initial angle θ_i .



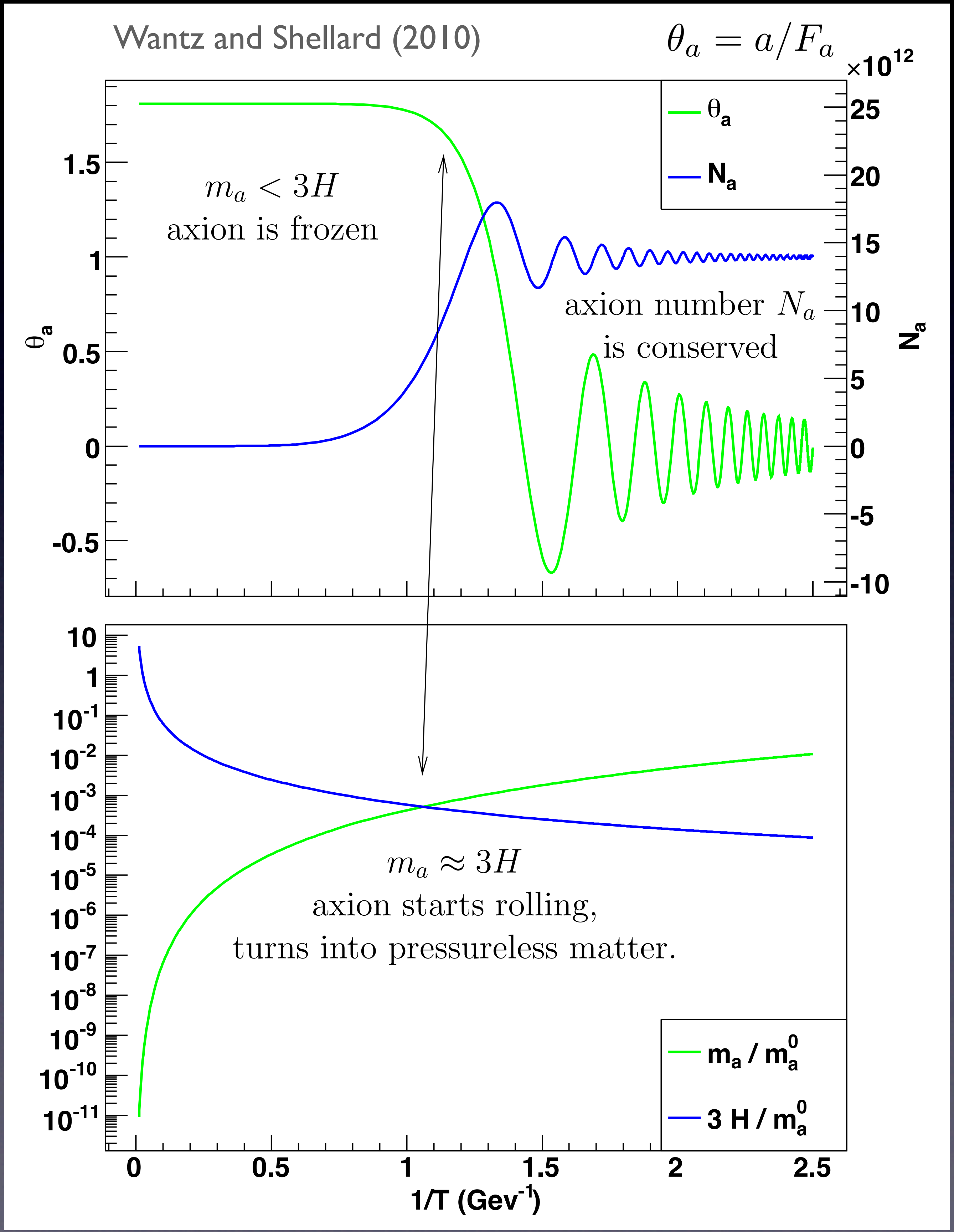
Realignment mechanism

Preskill, Wise and Wilczek (1983); Abbott and Sikivie (1983); Dine and Fischler (1983)

- In the early universe, axion fields are spatially homogeneous, taking some initial value a_i .
- The initial field value is generically different from the location of the minimum of the potential $\langle a \rangle = 0$.
- The classical axion fields start to oscillate around the minimum after QCD phase transition.



Realignment mechanism



- Field equation

$$\left(\frac{d^2}{dt^2} + 3H\frac{d}{dt} + m_a^2\right) a = 0$$

$$H = \frac{1}{R}\frac{dR}{dt} \propto 1/t$$

: Hubble parameter
(expansion rate of the universe)

$$R(t) \propto t^{1/2}$$

: Scale factor of the universe

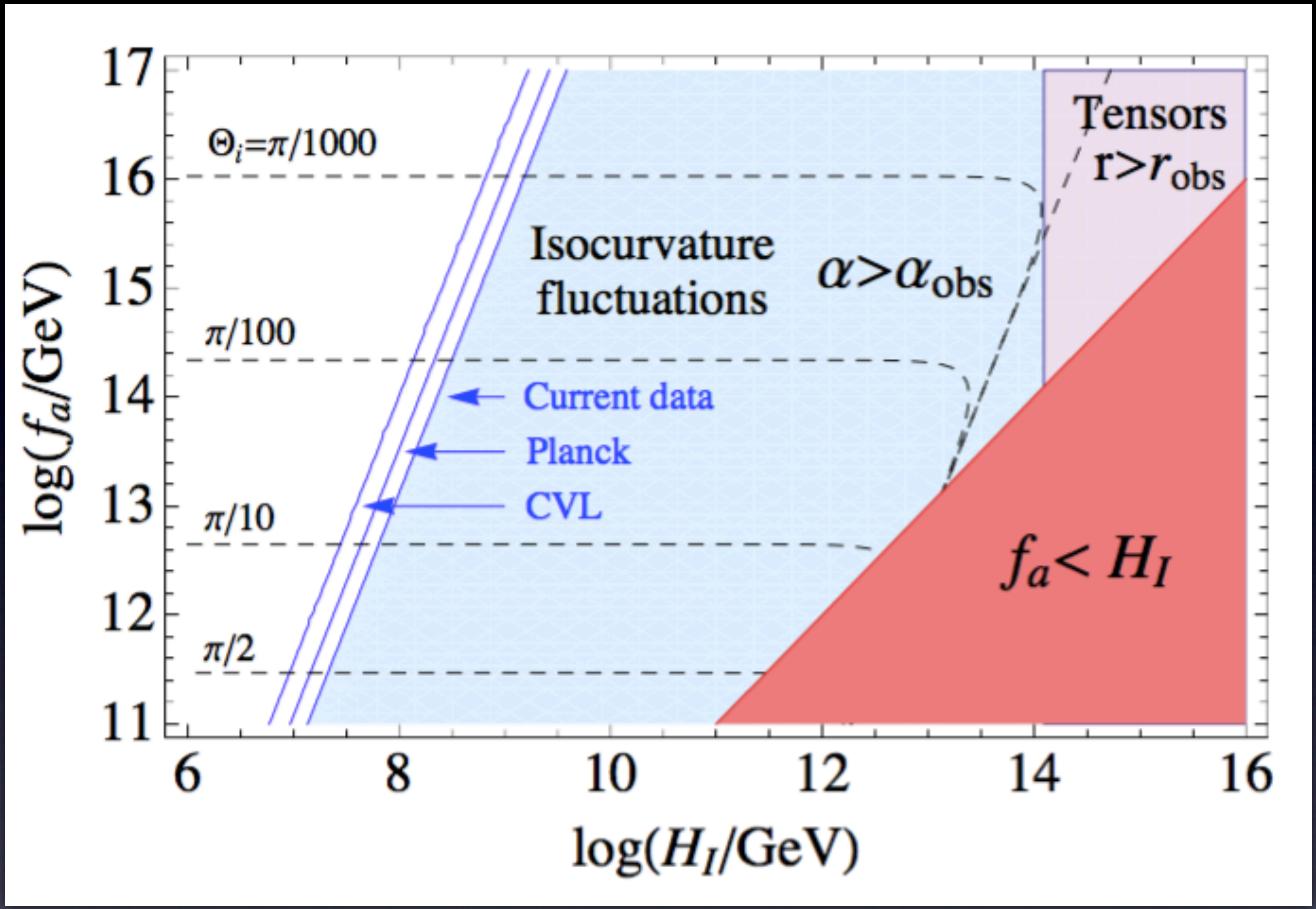
- Energy density of axion fields

$$\rho_a = \frac{1}{2}\left(\frac{da}{dt}\right)^2 + \frac{1}{2}m_a^2 a^2$$

- Axion number is conserved for $m_a \gtrsim 3H$:

$$N_a = \frac{\rho_a R^3}{m_a} = \text{const.}$$

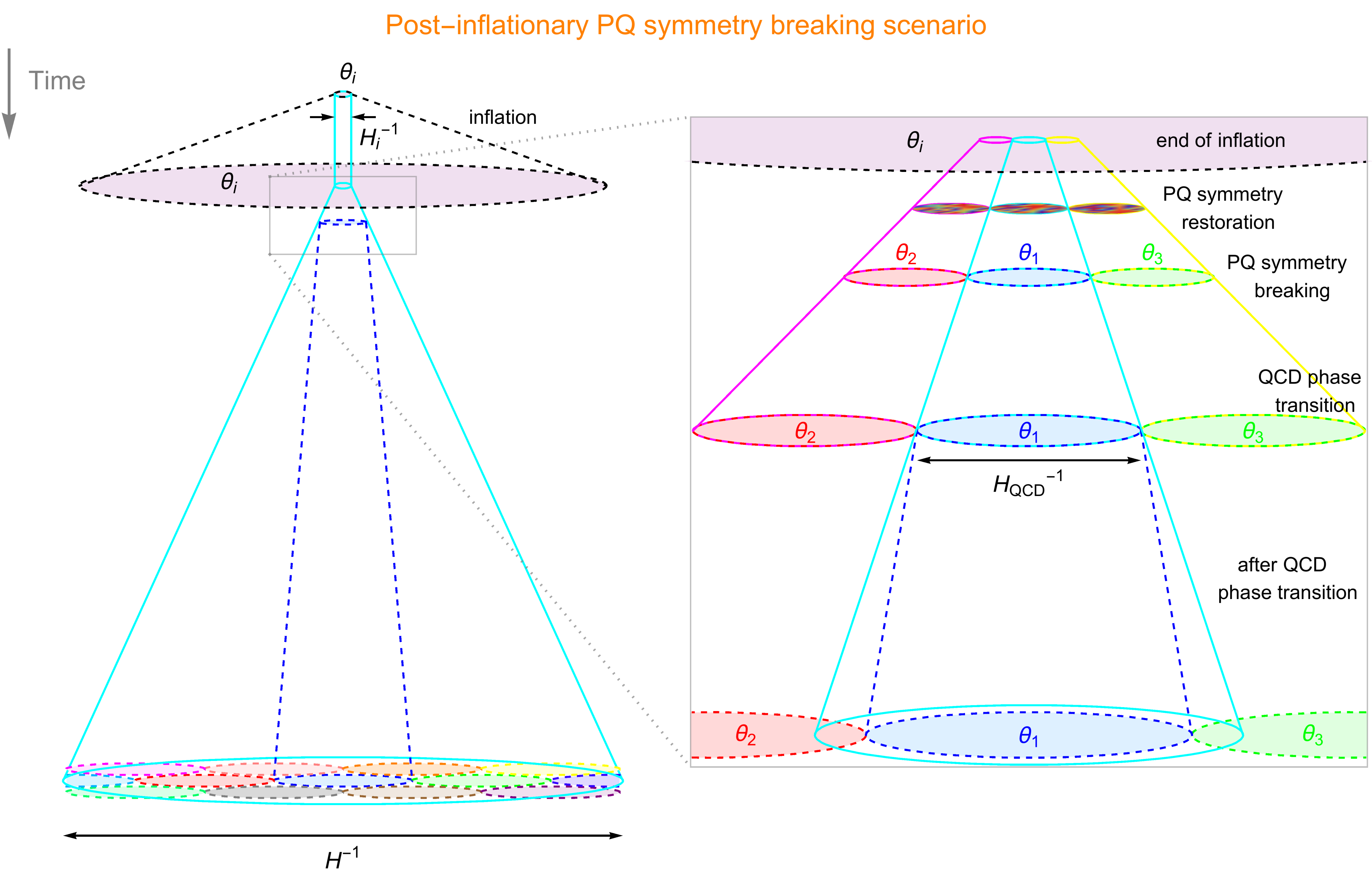
Axion dark matter and the energy scale of inflation



Hamann, Hannestad, Raffelt and Wong (2009)

- Severe constraints from isocurvature fluctuations if inflationary scale is sufficiently high.
- Tuning of the initial field value (“anthropic window”)

Post-inflationary PQ symmetry breaking scenario

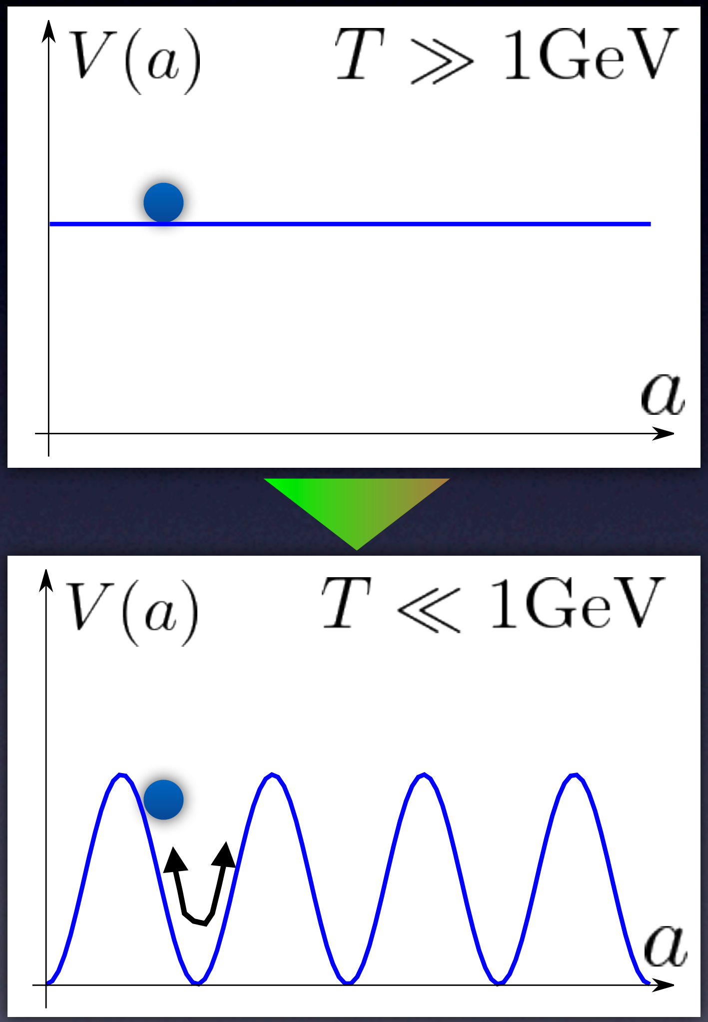


- Present observable universe contains many different patches with different values of θ .
- Topological defects (strings and domain walls) are formed.

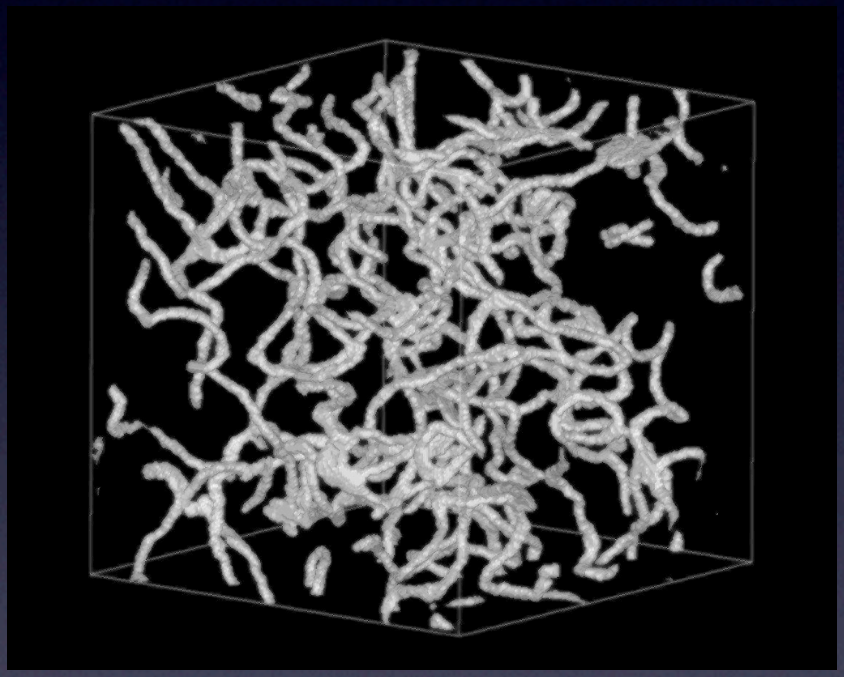
How axions are produced ?

If PQ symmetry is broken after inflation,
there are three contributions:

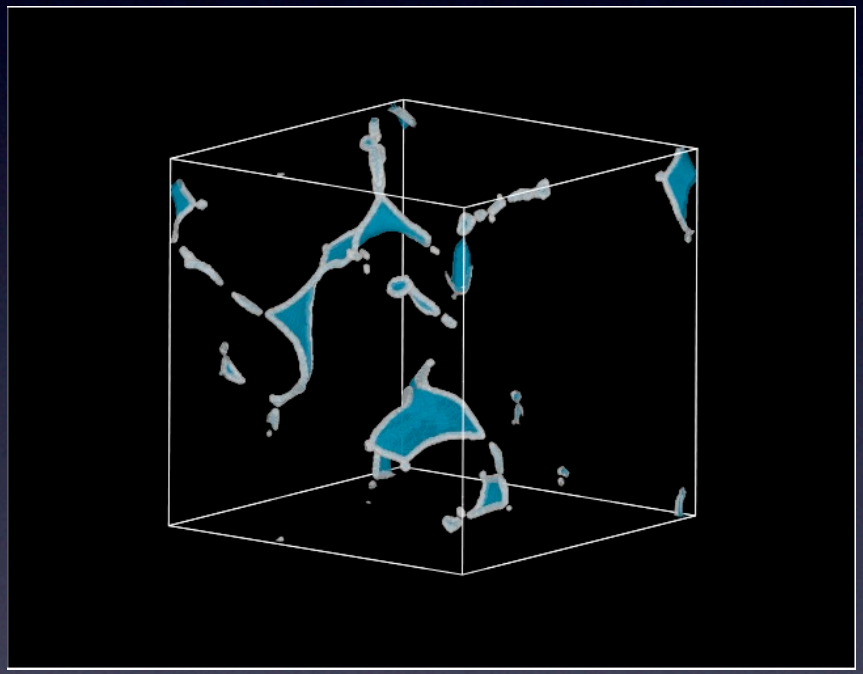
(1) Realignment mechanism



(2) Radiation from strings



(3) Collapse of string-wall systems



- Total abundance is sum of all these contributions.
- All these effects have to be quantitatively taken into account.

Axionic string

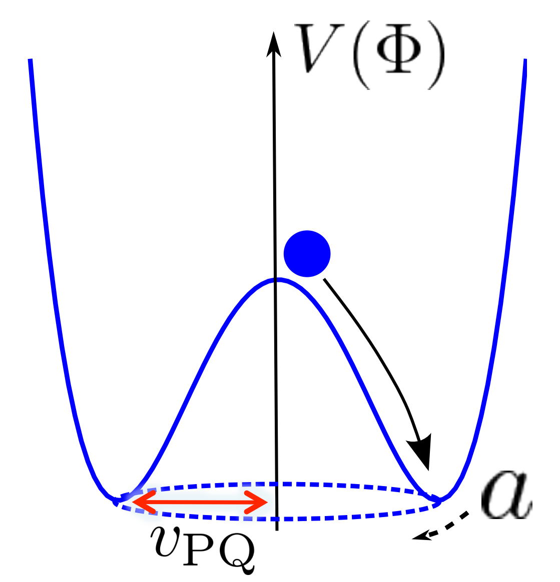
- Peccei-Quinn field (complex scalar field)

$$\Phi = |\Phi| e^{ia(x)/v_{\rm PQ}}$$

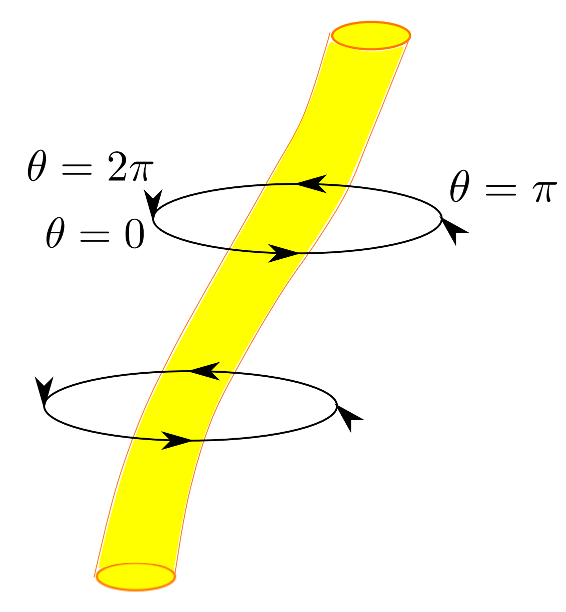
$a(x)$: axion field

- Spontaneous breaking of global U(1) symmetry

$$V(\Phi) = \lambda \left(|\Phi|^2 - \frac{v_{\rm PQ}^2}{2} \right)^2$$



field space

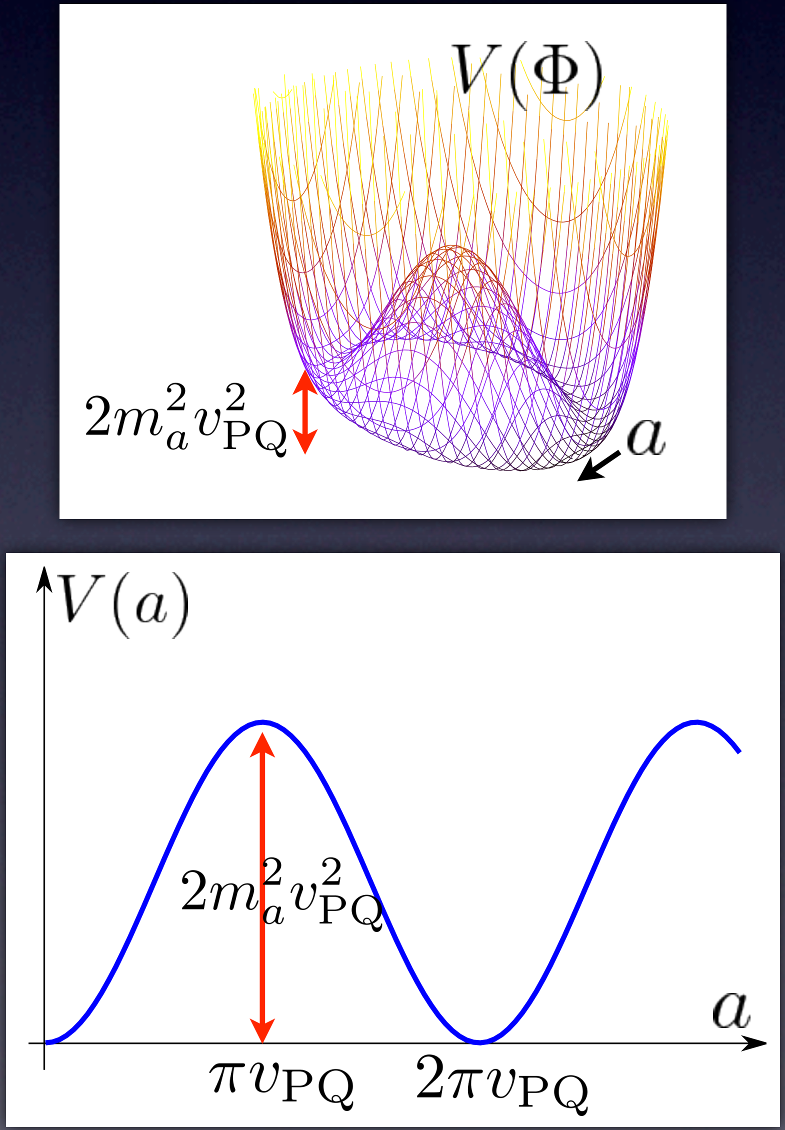


coordinate space

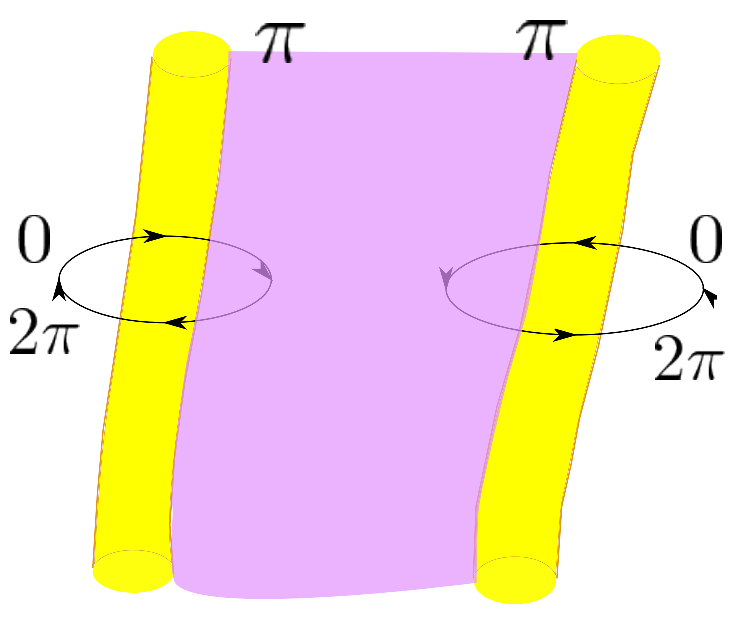
Axionic domain wall

- Mass of the axion (QCD effect ; $T \lesssim 1\mathrm{GeV}$)

$$V(\Phi) = \lambda \left(|\Phi|^2 - \frac{v_{\mathrm{PQ}}^2}{2} \right)^2 + \boxed{m_a^2 v_{\mathrm{PQ}}^2 (1 - \cos(a/v_{\mathrm{PQ}}))}$$



field space



coordinate space

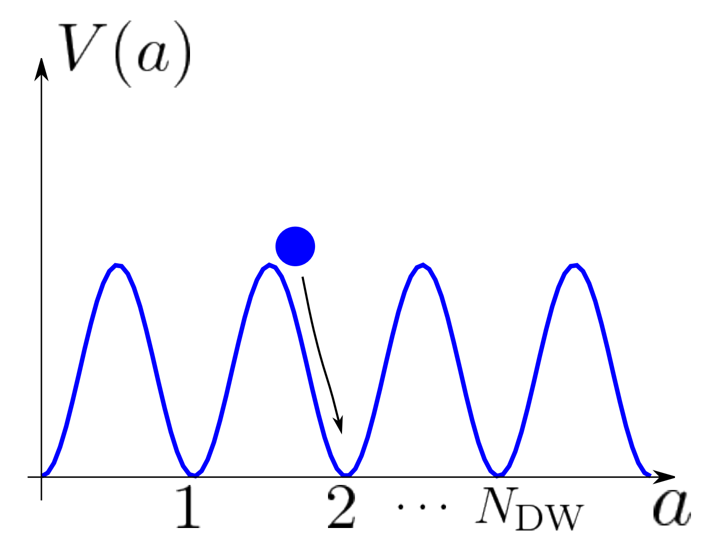
Strings attached by domain walls

Domain wall problem

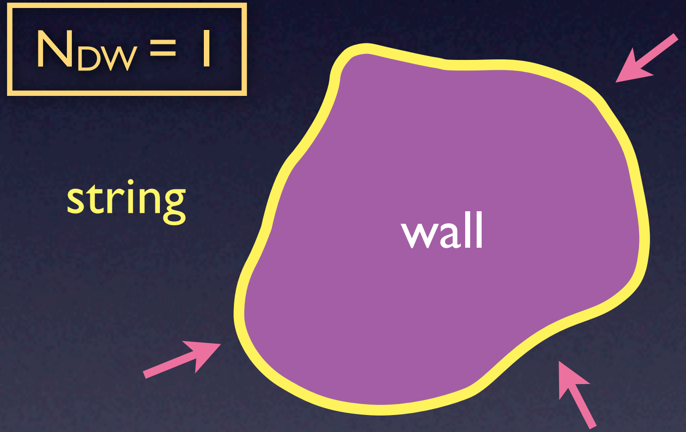
- Domain wall number NDW
  - NDW degenerate vacua

$$\boxed{V(a) = \frac{m_a^2 v_{\rm PQ}^2}{N_{\rm DW}^2}\left(1-\cos\left(N_{\rm DW}\frac{a}{v_{\rm PQ}}\right)\right)}$$

$N_{\rm DW}$: integer determined by QCD anomaly



- If NDW = 1, string-wall systems are unstable.
  - They collapse soon after the formation.



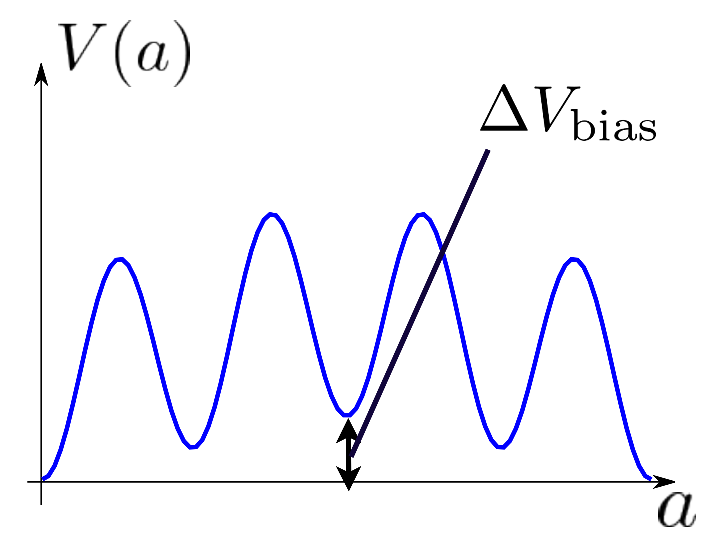
- If NDW > 1, string-wall systems are stable.
  - coming to overclose the universe.

Zel'dovich, Kobzarev and Okun (1975)

  - We may avoid this problem by introducing an energy bias (walls become unstable). Sikivie (1982)

$$V(a) = \frac{m_a^2 v_{\rm PQ}^2}{N_{\rm DW}^2}\left(1-\cos\left(\frac{N_{\rm DW}a}{v_{\rm PQ}}\right)\right) + \underline{\Delta V_{\rm bias}}$$

lifts degenerate vacua

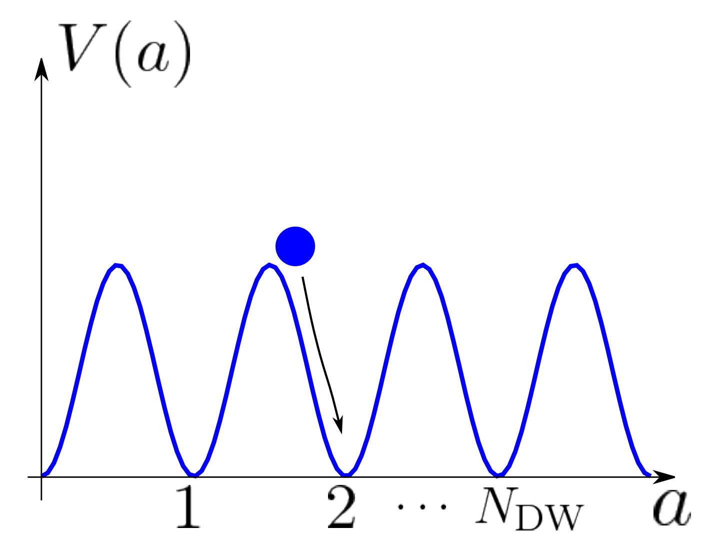


Domain wall problem

- Domain wall number $N_{\rm DW}$
  - $N_{\rm DW}$ degenerate vacua

$$\boxed{V(a) = \frac{m_a^2 v_{\rm PQ}^2}{N_{\rm DW}^2}\left(1-\cos\left(N_{\rm DW}\frac{a}{v_{\rm PQ}}\right)\right)}$$

$N_{\rm DW}$: integer determined by QCD anomaly



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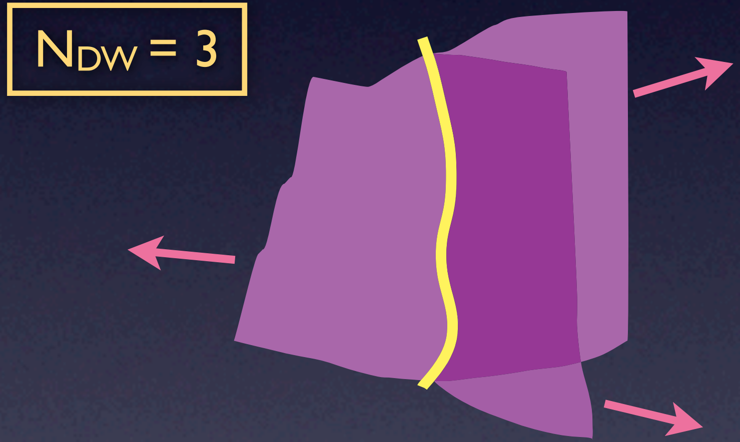

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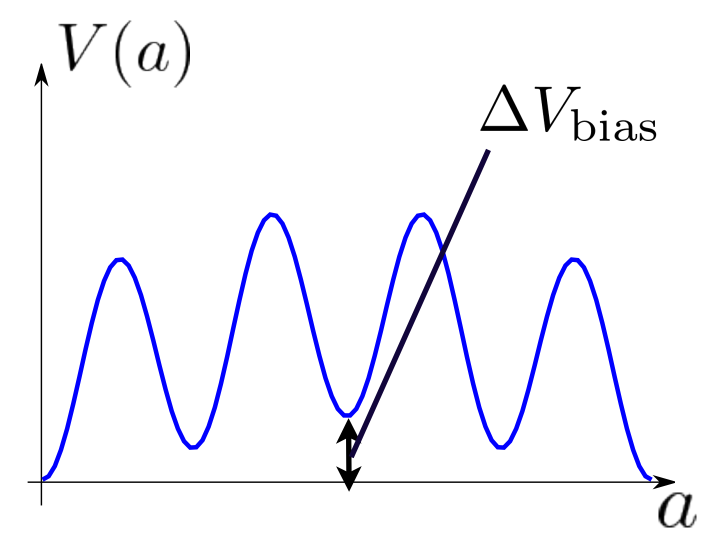
lifts degenerate vacua



Annihilation mechanism of domain walls

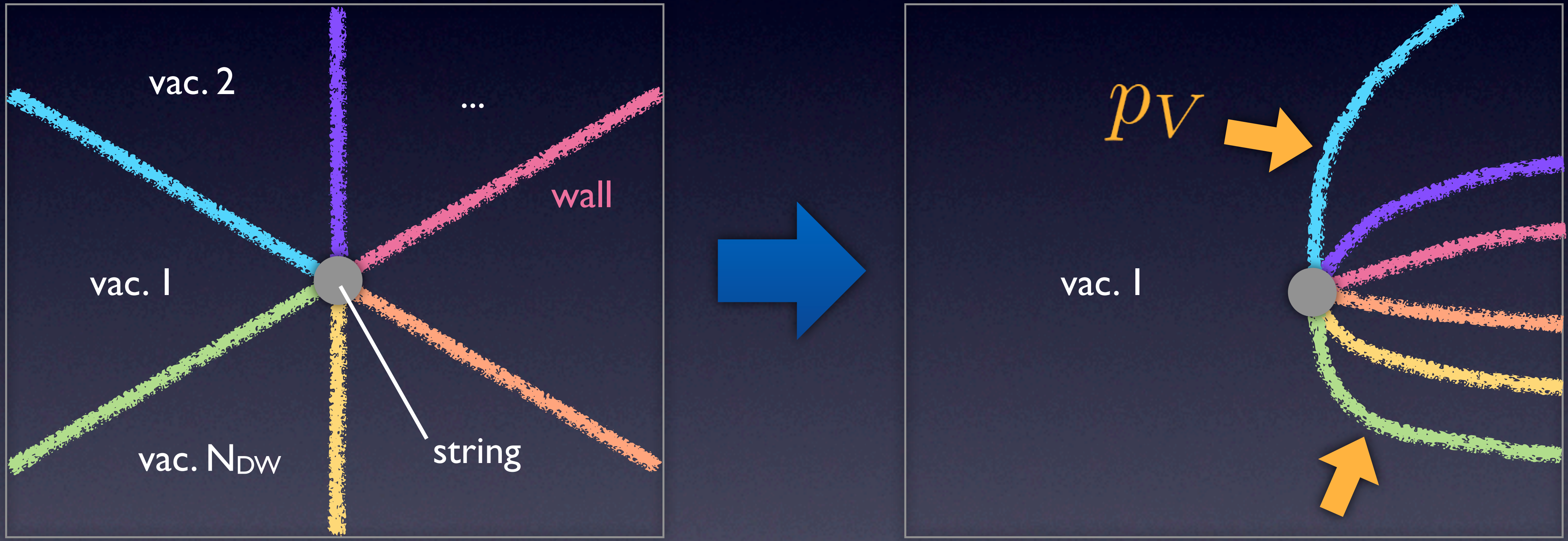
The energy bias acts as a pressure force p_V on the wall

$$p_V \sim \Delta V_{\rm bias}$$

Annihilation occurs when the tension p_T becomes comparable with the pressure p_V

$$p_T \sim \sigma_{\rm wall}/R \sim m_a v_{\rm PQ}^2/N_{\rm DW}^2 R$$

R : curvature radius of walls
$\sigma_{\rm wall}$: surface mass density of walls



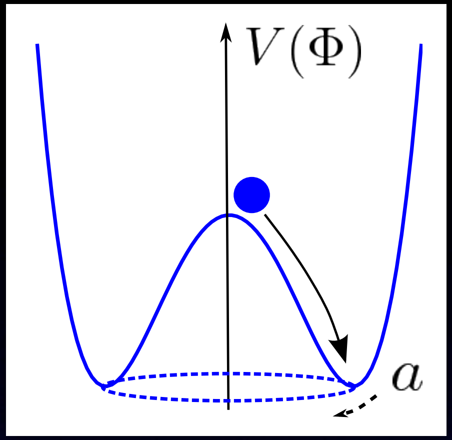
Annihilation time

$$t_{\rm ann} \sim R|_{p_V=p_T} \sim \frac{m_a v_{\rm PQ}^2}{N_{\rm DW}^2 \Delta V_{\rm bias}}$$

$$\sim \mathcal{O}(10^{-4})\,{\rm sec} \left(\frac{6}{N_{\rm DW}}\right)^4 \left(\frac{10^{-51}}{\Delta V_{\rm bias}/v_{\rm PQ}^4}\right) \left(\frac{10^9\,{\rm GeV}}{F_a}\right)^3$$

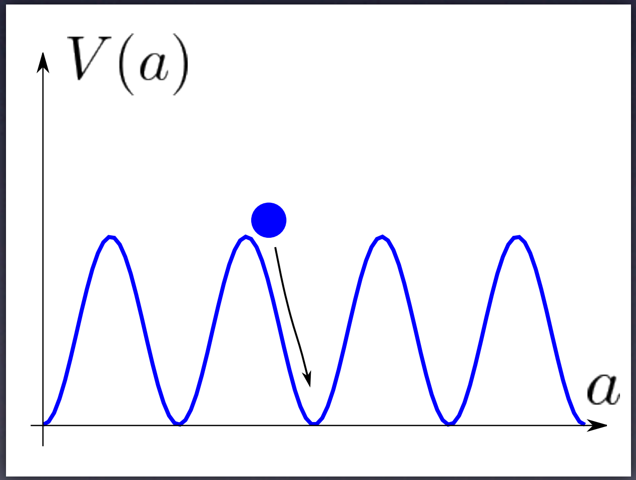
Production of axions in the early universe

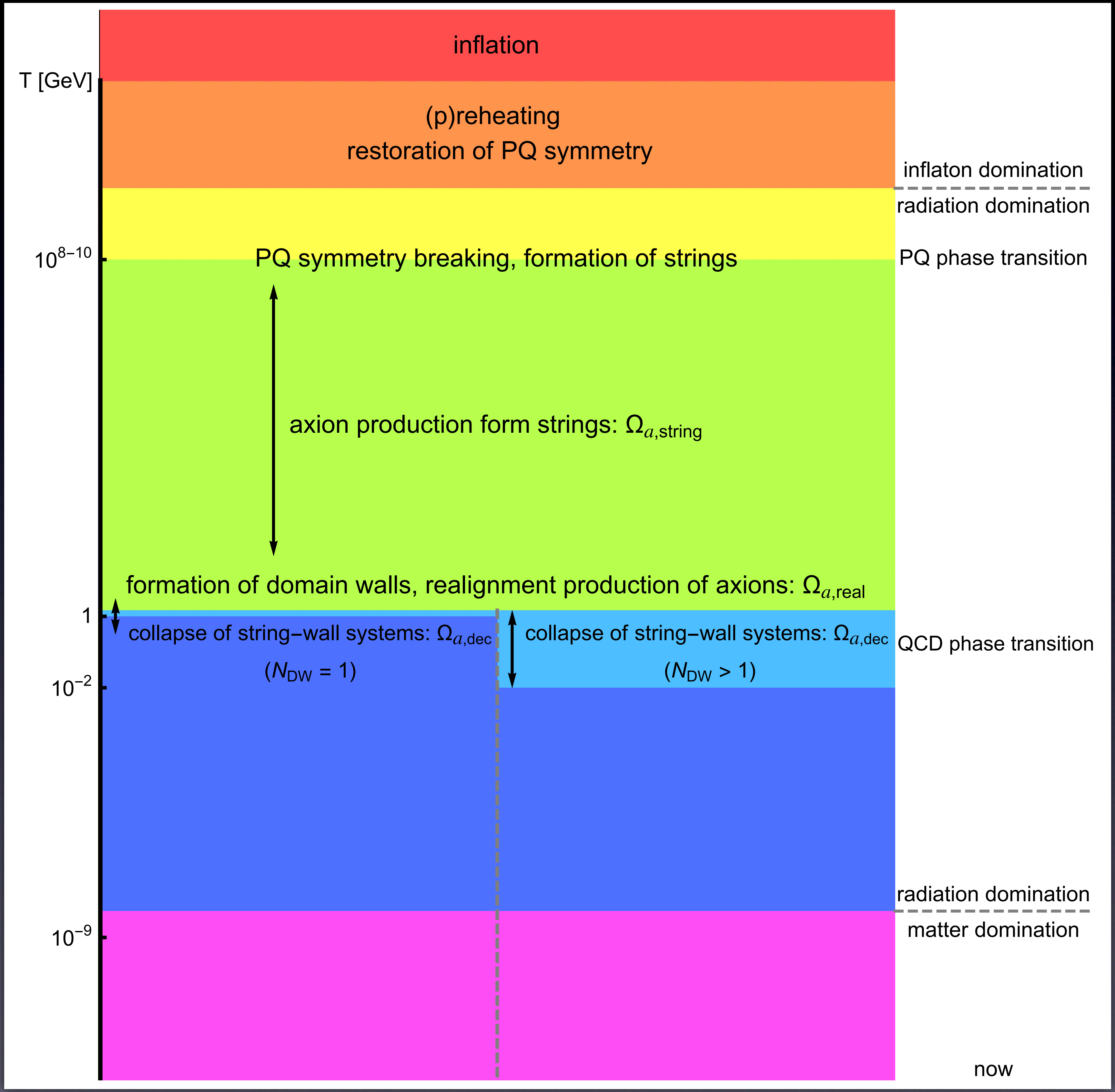
(post-inflationary PQ symmetry breaking scenario)



$$T \lesssim F_a \simeq 10^{8-11}\,\mathrm{GeV}$$

$$T \lesssim 1\,\mathrm{GeV}$$





T [GeV]

10^{8-10}

1

10^{-2}

10^{-9}

inflation

(p)reheating
restoration of PQ symmetry

PQ symmetry breaking, formation of strings

axion production form strings: $\Omega_{a,\text{string}}$

formation of domain walls, realignment production of axions: $\Omega_{a,\text{real}}$

collapse of string–wall systems: $\Omega_{a,\text{dec}}$
($N_{\text{DW}} = 1$)

collapse of string–wall systems: $\Omega_{a,\text{dec}}$
($N_{\text{DW}} > 1$)

inflaton domination

radiation domination

PQ phase transition

QCD phase transition

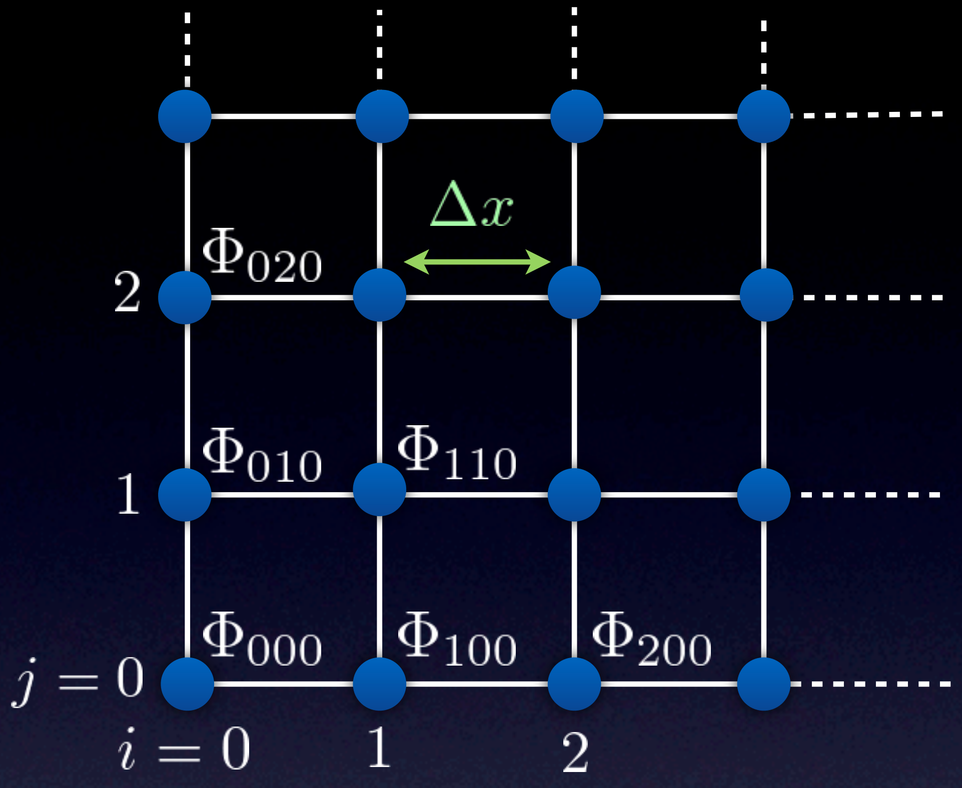
radiation domination

matter domination

now

Numerical simulation

- Discretize the spatial coordinate



$$\vec{\mathbf{x}} \quad \rightarrow \quad (i,j,k)$$

$$i,j,k = 0,1,\ldots,N-1$$

$$\Phi(\vec{\mathbf{x}}) \quad \rightarrow \quad \Phi_{i,j,k}$$

$$\nabla^2\Phi(\vec{\mathbf{x}}) \;\rightarrow\; (\nabla^2\Phi)_{i,j,k}$$

$$= \frac{1}{12(\Delta x)^2}\left[16(\Phi_{i+1,j,k}+\Phi_{i-1,j,k}+\Phi_{i,j+1,k}+\Phi_{i,j-1,k}+\Phi_{i,j,k+1}+\Phi_{i,j,k-1})\right.$$
$$\left.-(\Phi_{i+2,j,k}+\Phi_{i-2,j,k}+\Phi_{i,j+2,k}+\Phi_{i,j-2,k}+\Phi_{i,j,k+2}+\Phi_{i,j,k-2})-90\Phi_{i,j,k}\right]$$

- Solve the classical EOM for complex scalar $\Phi = \phi_1 + i\phi_2$ on lattice

$$\ddot{\Phi} + 3H\dot{\Phi} - \frac{\nabla^2}{R^2(t)}\Phi = -\frac{\partial V}{\partial \Phi^*}$$

- Number of grids in simulation box : $N^3 = 512^3$ (3D)

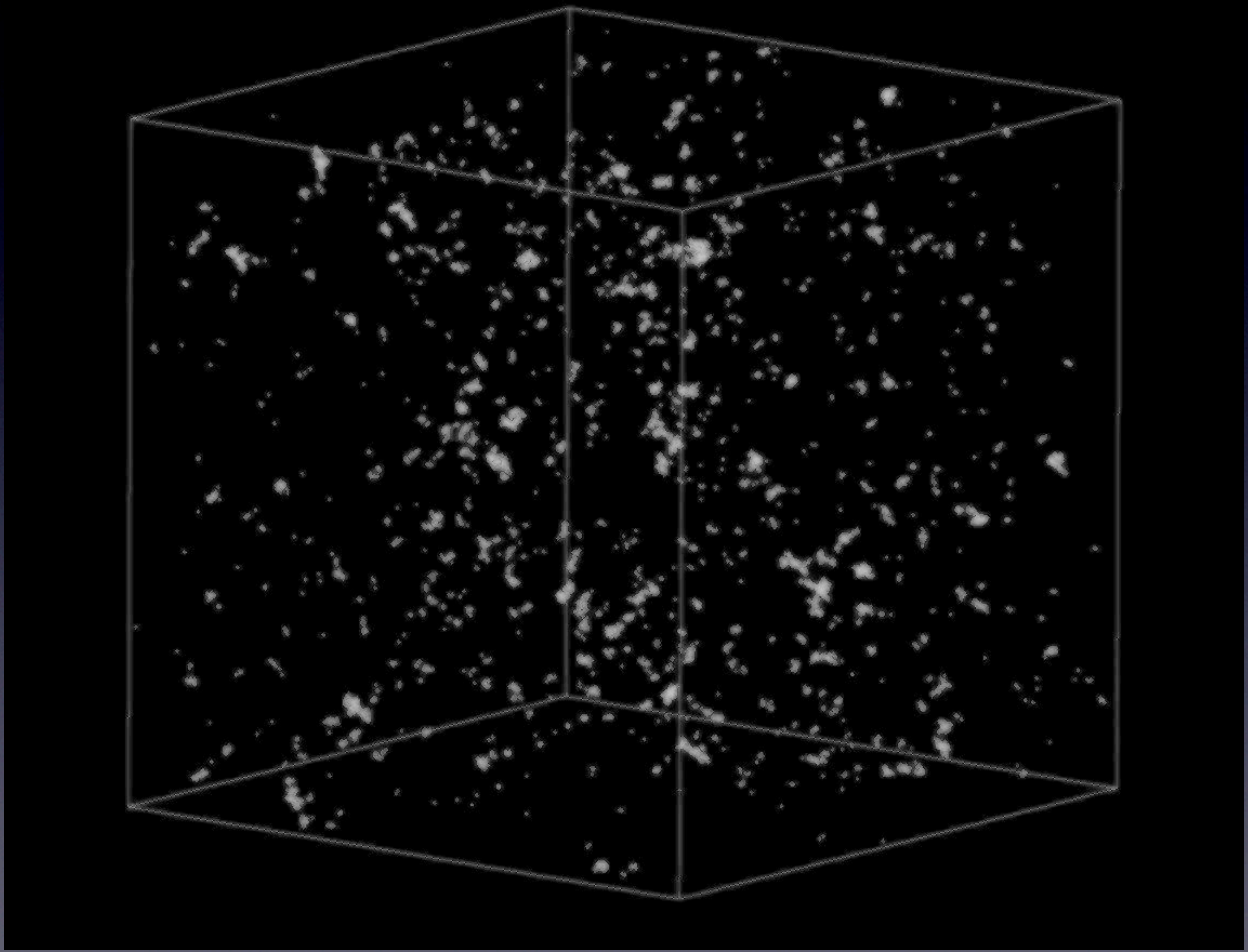
  $N^2 = 8192^2, 16384^2, 32768^2$ (2D)

Numerical simulation : N_{DW} = 1

Hiramatsu, Kawasaki, KS and Sekiguchi (2012)

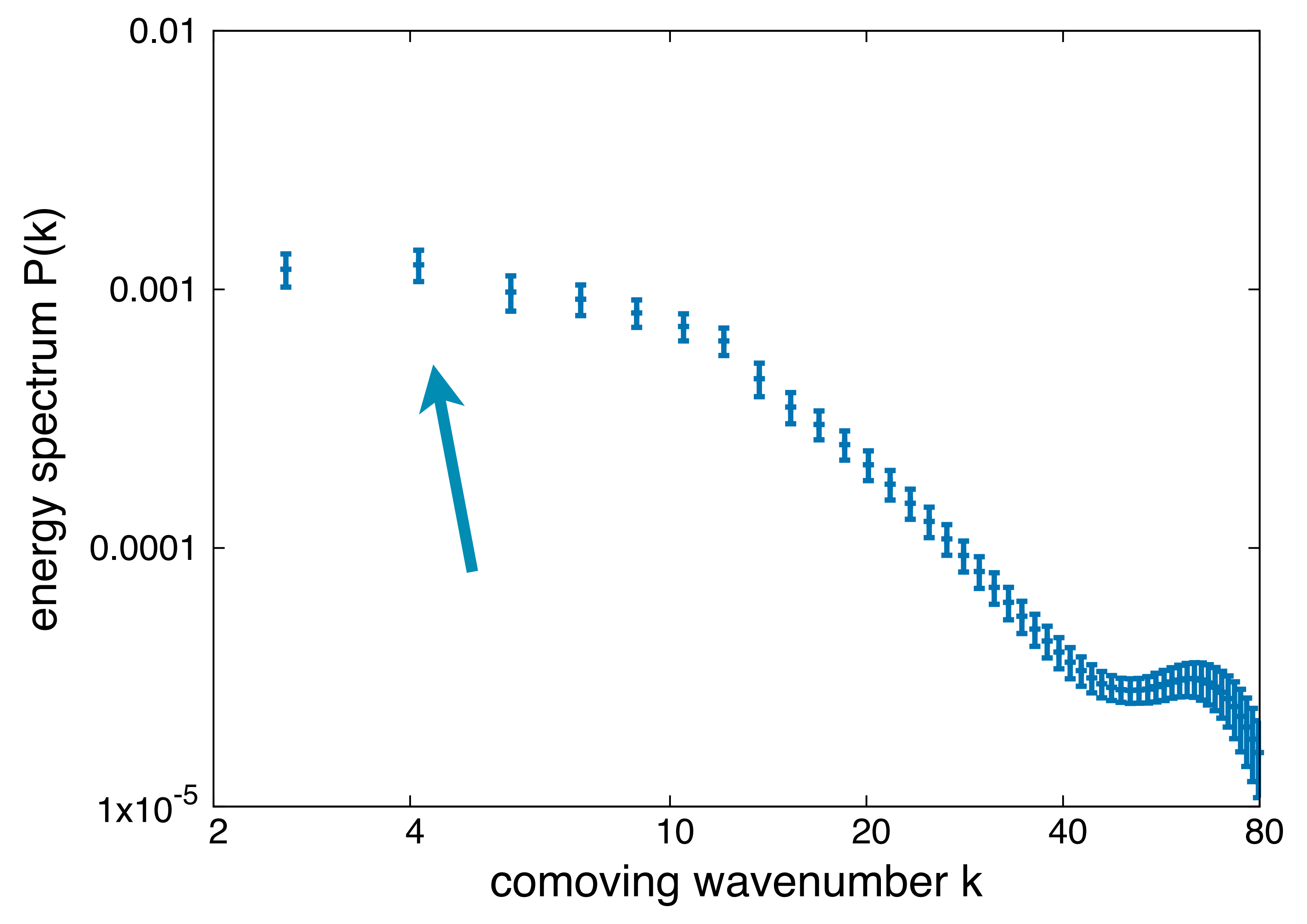
Numerical simulation : N_{DW} = 1

Hiramatsu, Kawasaki, KS and Sekiguchi (2012)



Spectrum of radiated axions

Hiramatsu, Kawasaki, KS and Sekiguchi (2012)
Kawasaki, KS and Sekiguchi (2015)



Mean energy

$$\frac{\langle \omega_a \rangle}{m_a}(t_{\rm decay}) = 3.23 \pm 0.18$$

$$\omega_a = \sqrt{m_a^2 + k^2/R(t)^2}$$

$R(t)$: scale factor of the universe

Contribution to the relic abundance

$$\rho_a(t_{\rm today}) = m_a \frac{\rho_a(t_{\rm decay})}{\langle \omega_a \rangle}\left(\frac{R(t_{\rm decay})}{R(t_{\rm today})}\right)^3$$

$$\rho_a(t_{\rm decay}) \approx \rho_{\rm defects}(t_{\rm decay})$$

Axion dark matter abundance ($N_{DW} = 1$)

- Re-alignment mechanism Borsanyi et al. (2016); Ballesteros, Redondo, Ringwald and Tamarit (2016)

$$\Omega_{a,\mathrm{real}}h^2 \approx (3.8 \pm 0.6) \times 10^{-3} \left(\frac{F_a}{10^{10}\,\mathrm{GeV}} \right)^{1.165}$$

- Production from string-wall systems

$$\Omega_{a,\text{string-wall}}h^2 \approx 1.2^{+0.9}_{-0.7} \times 10^{-2} \left(\frac{F_a}{10^{10}\,\mathrm{GeV}} \right)^{1.165}$$

- Total axion abundance

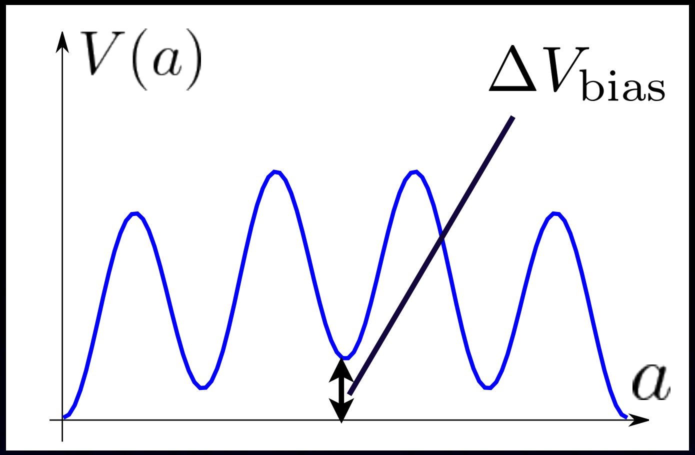
$$\Omega_{a,\mathrm{tot}}h^2 \approx 1.6^{+1.0}_{-0.7} \times 10^{-2} \left(\frac{F_a}{10^{10}\,\mathrm{GeV}} \right)^{1.165}$$

$$\Omega_{a,\mathrm{tot}} \leq \Omega_{\mathrm{CDM}} \quad \Rightarrow \quad \boxed{\begin{aligned} F_a &\lesssim (3.8\text{–}9.9) \times 10^{10}\,\mathrm{GeV} \\ m_a &\gtrsim (0.6\text{–}1.5) \times 10^{-4}\,\mathrm{eV} \end{aligned}}$$

$$\Omega_{\mathrm{CDM}}h^2 \simeq 0.12$$

- Potentially large uncertainty from estimation of the string density in numerical simulations. Fleury and Moore (2016)

Models with $N_{\mathrm{DW}} > 1$: long-lived domain walls



- Domain walls are long-lived and eventually annihilated due to the energy bias.
- Origin of the bias term ?
  - $U(1)_{\mathrm{PQ}}$ may not be an exact symmetry: Global symmetry can be spoiled by gravitational effects.

    Holman et al. ; Kamionkowski and March-Russell; Barr and Seckel; Ghigna, Lusignoli and Roncadelli; Dine (1992)
  - We can assume that the PQ symmetry is not *ad hoc* but instead an accidental symmetry of an exact discrete Z_N symmetry (with large N).

    Choi, Nilles, Ramos-Sanchez and Vaudrevange (2009)
  - Planck-suppressed operators allowed by the Z_N symmetry work as the bias term.

$$\mathcal{L} \supset \frac{g}{M_{\mathrm{Pl}}^{N-4}} \Phi^N + \mathrm{h.c.}$$

$$g = |g| e^{i\Delta}$$

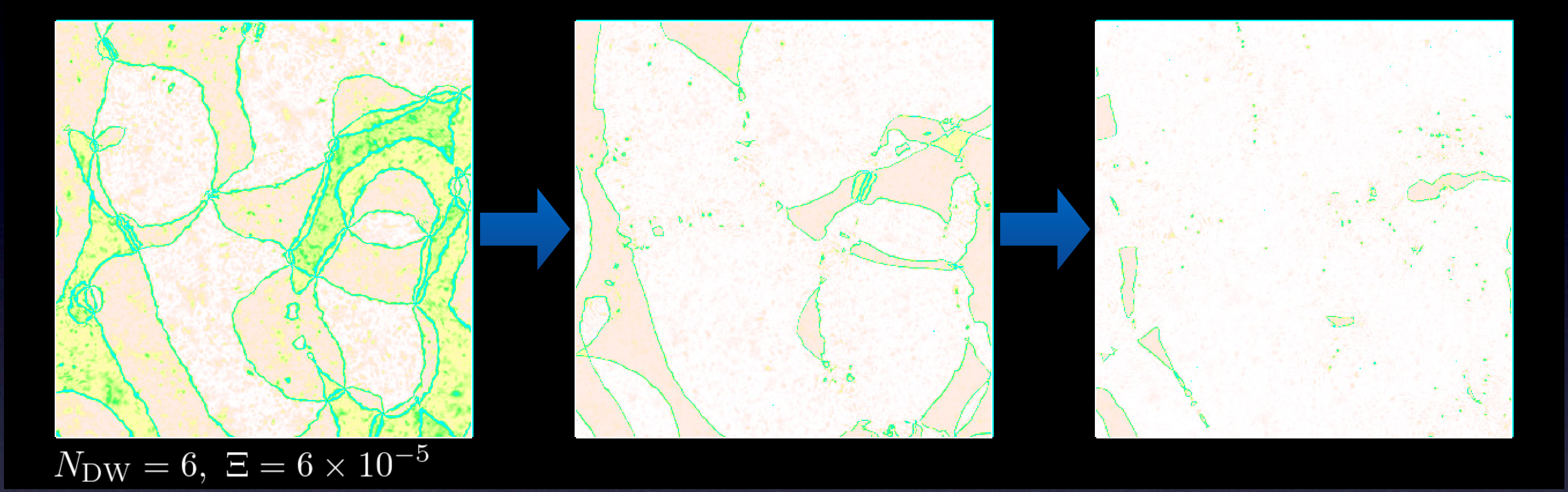
$$\Rightarrow$$

$$\boxed{\begin{aligned} \Delta V_{\mathrm{bias}}(a) &= -2\Xi v_{\mathrm{PQ}}^4 \cos\left(N \frac{a}{v_{\mathrm{PQ}}} + \Delta_D\right) \\ \Xi = \frac{|g| N_{\mathrm{DW}}^{N-4}}{(\sqrt{2})^N} \left(\frac{F_a}{M_{\mathrm{Pl}}}\right)^{N-4}, &\quad \Delta_D = \Delta - N\bar{\theta} \end{aligned}}$$

Numerical simulation : $N_{DW} > 1$

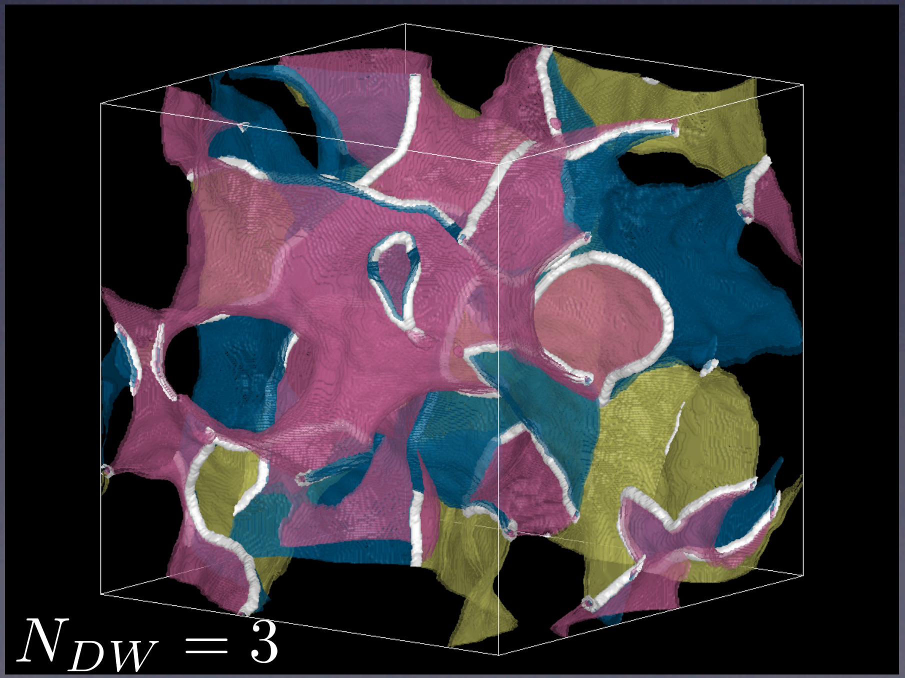
Hiramatsu, Kawasaki, KS and Sekiguchi (2013)
Kawasaki, KS and Sekiguchi (2015)

- 8192^2, 16384^2, 32768^2 (2D) → decay time of domain walls

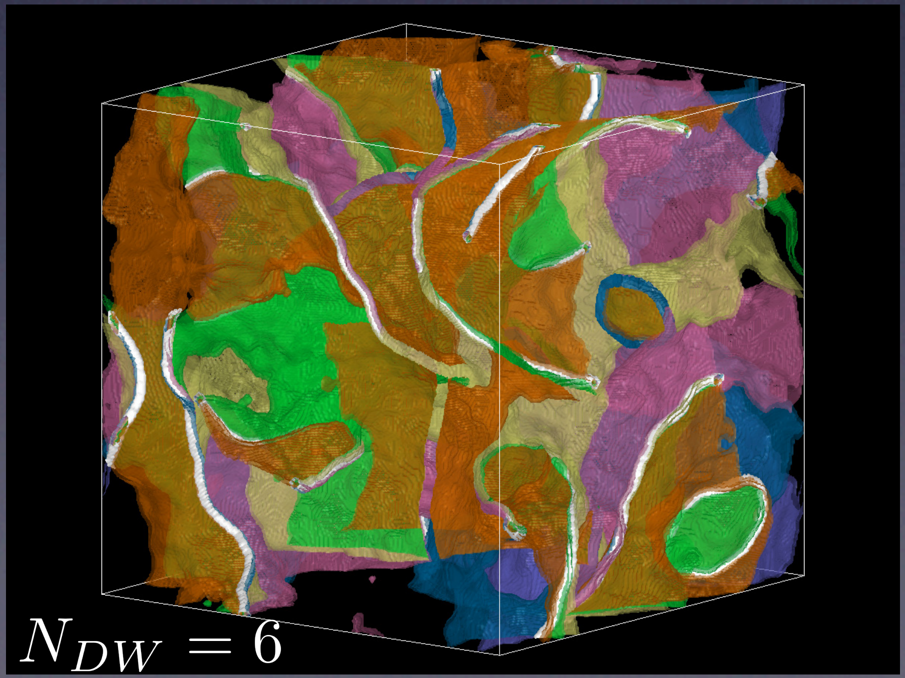


$N_{\rm DW} = 6, \;\; \Xi = 6 \times 10^{-5}$

- 512^3 (3D) → spectrum of radiated axions



$N_{DW} = 3$



$N_{DW} = 6$

Constraints

- CP violation

The higher dimensional operator shifts the minimum of the potential and spoils the original Peccei-Quinn solution to the strong CP problem.

$$\frac{\langle a \rangle}{F_a} \simeq \frac{\frac{N|g|N_{\rm DW}^{N-1}}{(\sqrt{2})^{N-2}} \left(\frac{F_a}{M_{\rm Pl}}\right)^{N-2} M_{\rm Pl}^2 \sin \Delta_D}{m_a^2 + \frac{N^2|g|N_{\rm DW}^{N-2}}{(\sqrt{2})^{N-2}} \left(\frac{F_a}{M_{\rm Pl}}\right)^{N-2} M_{\rm Pl} \cos \Delta_D} < 7 \times 10^{-12}$$

→ Large N is required

- Dark matter abundance

Long-lived domain walls produce too much cold axions.

Hiramatsu, Kawasaki, KS, and Sekiguchi (2013); Kawasaki, KS, and Sekiguchi (2015)

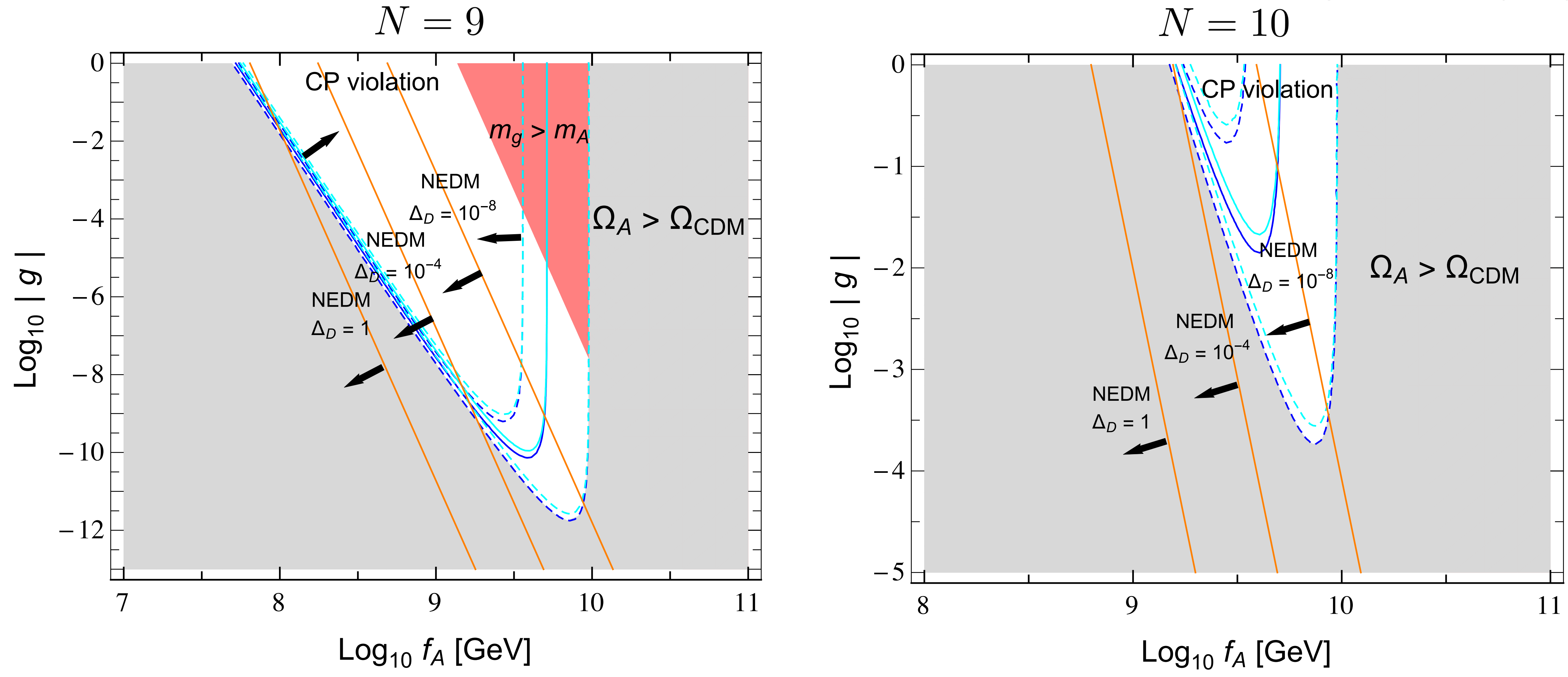
$$\Omega_a h^2 \simeq (3.4\text{–}6.2) \times N_{\rm DW}^{-2} \left(\frac{\Xi}{10^{-52}}\right)^{-1/2} \left(\frac{F_a}{10^9\,{\rm GeV}}\right)^{-1/2} \quad \text{with} \quad \Xi = \frac{|g|N_{\rm DW}^{N-4}}{(\sqrt{2})^N} \left(\frac{F_a}{M_{\rm Pl}}\right)^{N-4}$$

→ Small N is required

- Constraints on the energy bias (= on the coefficient g)

$$\Delta V_{\rm bias}(a) = -2\Xi v_{\rm PQ}^4 \cos\left(N\frac{a}{v_{\rm PQ}} + \Delta_D\right) \Longleftarrow \mathcal{L} \supset \frac{g}{M_{\rm Pl}^{N-4}}\Phi^N + {\rm h.c.}$$

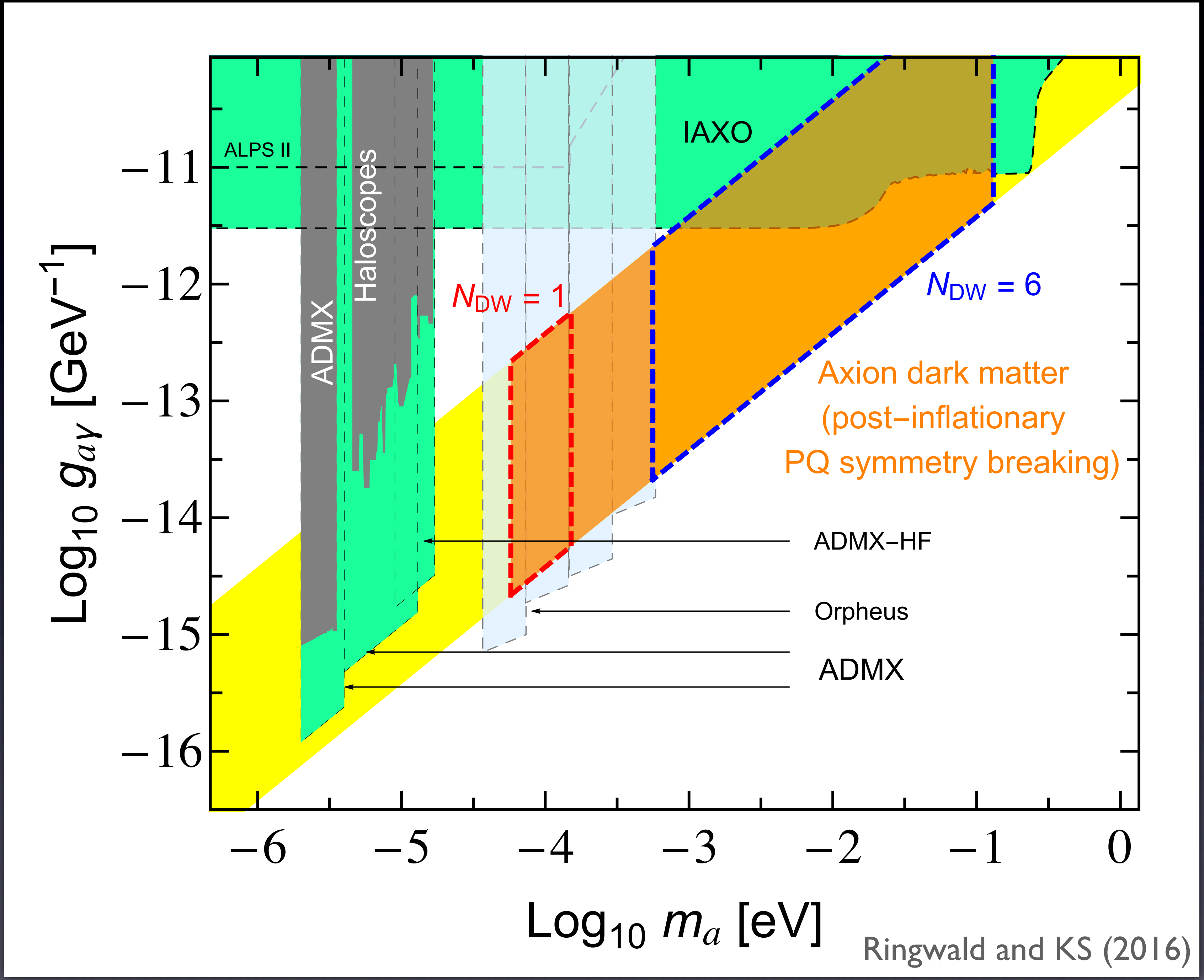
- Loopholes appear if the order of the discrete symmetry is N = 9 or 10, but some tuning of the phase parameter Δ_D is required.
- If we allow such a mild tuning, axions can explain total dark matter abundance in the small F_a range.



Ringwald and KS (2016)

Search for axion dark matter

Search space in photon coupling $g_{a\gamma} \sim \alpha/(2\pi F_a)$ vs. mass m_a



Ringwald and KS (2016)

Mass ranges predicted in the post-inflationary PQ symmetry breaking scenario can be probed by various future experimental studies.

Conclusion

- Predictions for axion dark matter strongly depend on the early history of the universe.
- If the PQ symmetry is broken after inflation, string-wall systems give additional contribution to the CDM abundance.
- Axion can be dominant component of dark matter if

$$\boxed{\begin{array}{ll} F_a \simeq \mathcal{O}(10^{10}\text{–}10^{11})\,\mathrm{GeV} & \\ & \text{for } N_{DW} = 1 \\ m_a \simeq \mathcal{O}(10^{-5}\text{–}10^{-4})\,\mathrm{eV} & \end{array}}$$

$$\boxed{\begin{array}{ll} F_a \simeq \mathcal{O}(10^{8}\text{–}10^{10})\,\mathrm{GeV} & \\ & \text{for } N_{DW} > 1 \\ m_a \simeq \mathcal{O}(10^{-4}\text{–}10^{-2})\,\mathrm{eV} & \end{array}}$$

- Mass ranges can be probed in the future experiments.