Multi-Meson Model applied to $D \to hhh$

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Seminar @ IFAE

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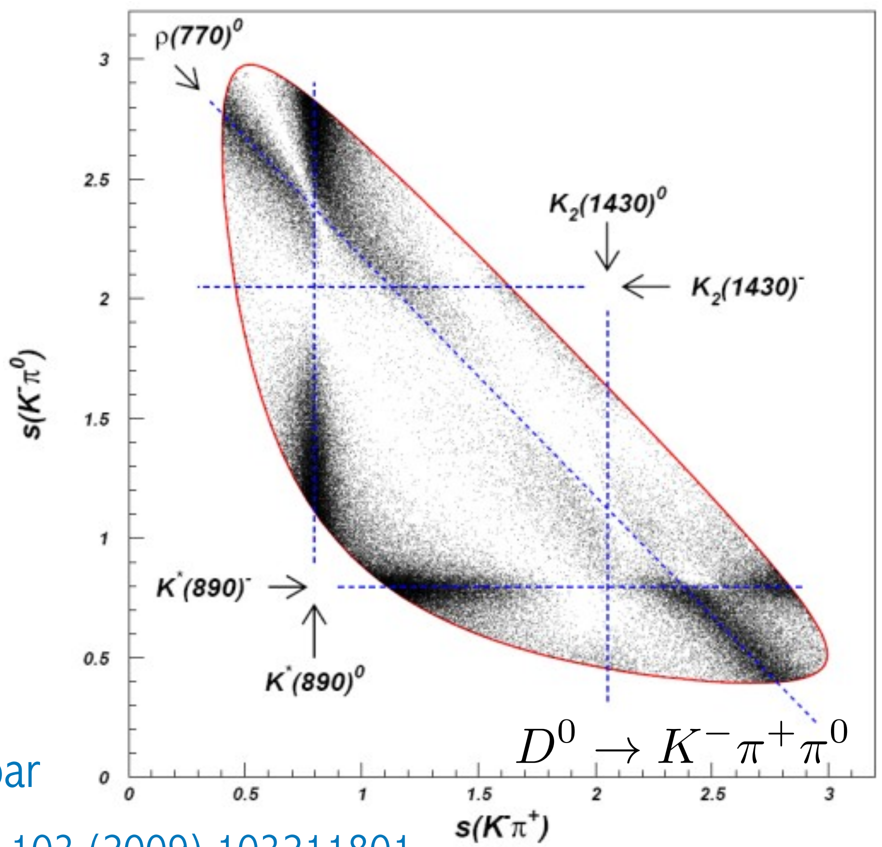
what can we learn from $D \to hhh$?

- D three-body HADRONIC decay are dominated by resonances



Babar

PRL 103 (2009) 103211801

→ spectroscopy low energy resonances σ, κ

→ underlying strong force behave

↳ meson-meson interactions and resonance structures

→ new large data sample from LHCb, Belle II, BES III + …

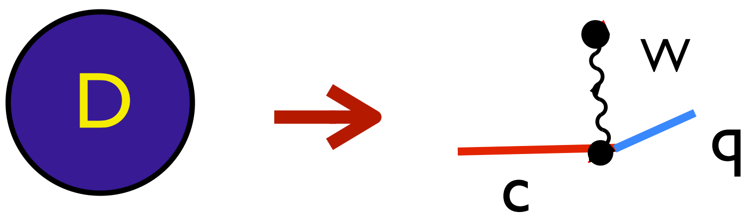
- CP-Violation
  - 1st observation in charm LHCb 2019 $A_{cp}(D^0 \to K^+K^-) - A_{cp}(D^0 \to \pi^+\pi^-)$

→ CPV on $D \to hhh$?

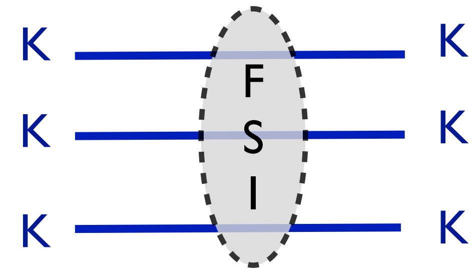
→ searches in many process

→ can lead to new physics

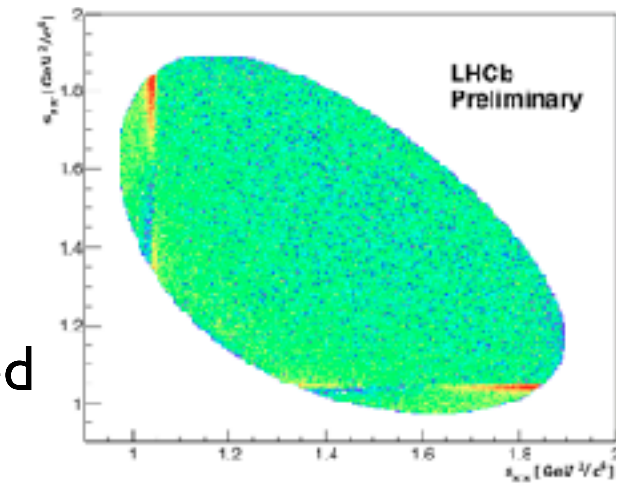
- ex: $D^+ \to K^- K^+ K^-$



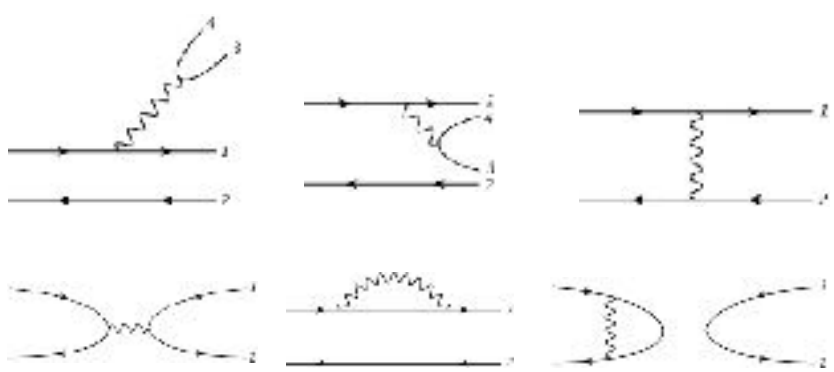
hadronize



observed

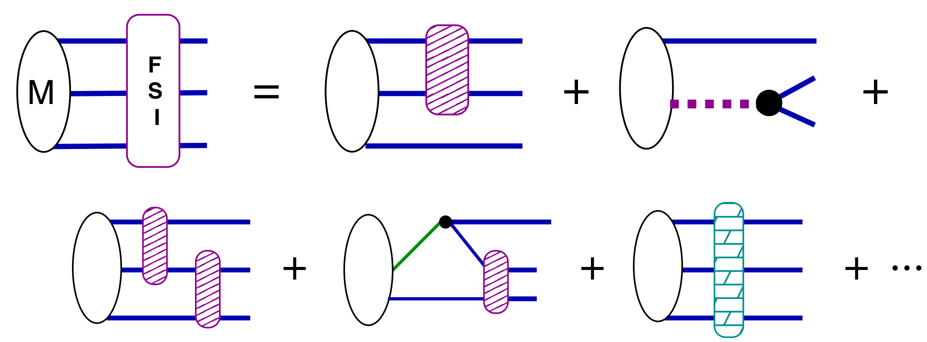


primary vertex
- weak -



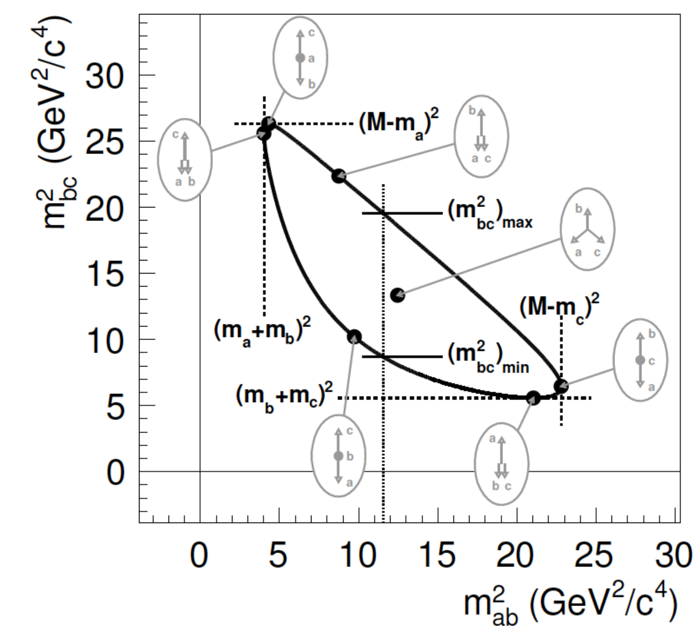
QCD, CKM coupling and phase

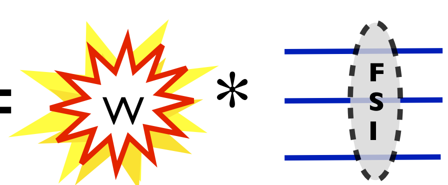
Final State Interactions
- strong -



(2+1) + 3-body interactions

Dalitz plot



A = 

$$\frac{d\Gamma}{ds_{12}ds_{23}} = \frac{1}{(2\pi)^3}\frac{1}{32M^3}|\mathcal{A}(s_{12},s_{23})|^2$$

dynamics

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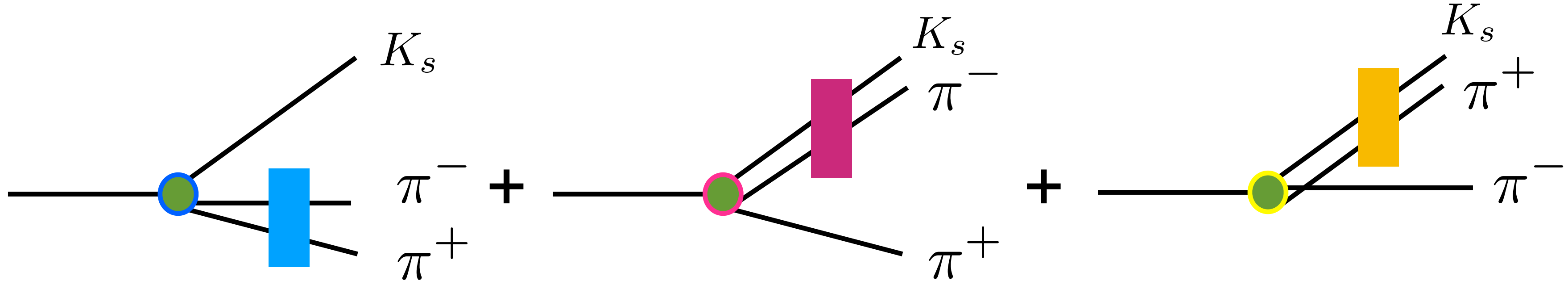
standard approach

- common cartoon to described 3-body decay

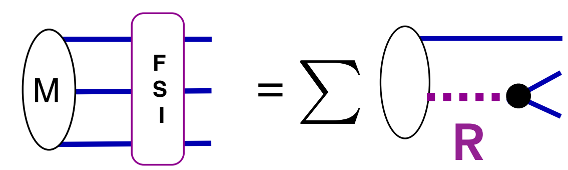
$D^0 \to K_s \pi^- \pi^+$

$\mathcal{A}(s_{12}, s_{23})$ =



- isobar model widely used by experimentalists:
  - (2+1) approximation → ignore the interaction with 3rd particle (bachelor)
  - $A = \sum c_k A_k$; + NR coherent sum of amplitude's in different parcial waves

! Warning: when A_k are single resonances



!

↳ defined as Breit-Wigner $\mathrm{BW}(s_{12}) = \dfrac{1}{m_R^2 - s_{12} - i m_R \Gamma(s_{12})}$,

- sum of BW violates two-body unitarity (close Rs in the same channel - scalars)
- resonance's mass and width are processes dependent

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- movement to use better 2-body (unitarity) inputs in data analysis

  - “K-matrix" : ππ S-wave 5 coupled-channel modulated by a production amplitude

    ↳ used by Babar, LHCb, BES III — Anisovich PLB653(2007)

  - rescattering $\pi\pi \to KK$ contribution in LHCb $\begin{cases} B^\pm \to \pi^+\pi^-\pi^\pm \\ B^\pm \to K^-K^+\pi^\pm \end{cases}$ [arXiv:1909.05212; 1909.05211] [arXiv:1905.09244]

    Pelaez, Yndurain PRD71(2005) 074016

    ↳ new parametrization Pelaez, Rodas, Elvira *Eur.Phys.J.C* 79 (2019) 12, 1008

→ Still not enough to described data

- from theory: list of scalar and vector form factors

$< \pi\pi|0 >$ Moussallam EPJ C 14, 111 (2000); Daub, Hanhart, and B. Kubis JHEP 02 (2016) 009. Hanhart, PL B715, 170 (2012). Dumm and Roig EPJ C 73, 2528 (2013).

$< K\pi|0 >$ Moussallam EPJ C 53, 401 (2008) Jamin, Oller and Pich, PRD 74, 074009 (2006) Boito, Escribano, and Jamin EPJ C 59, 821 (2009).

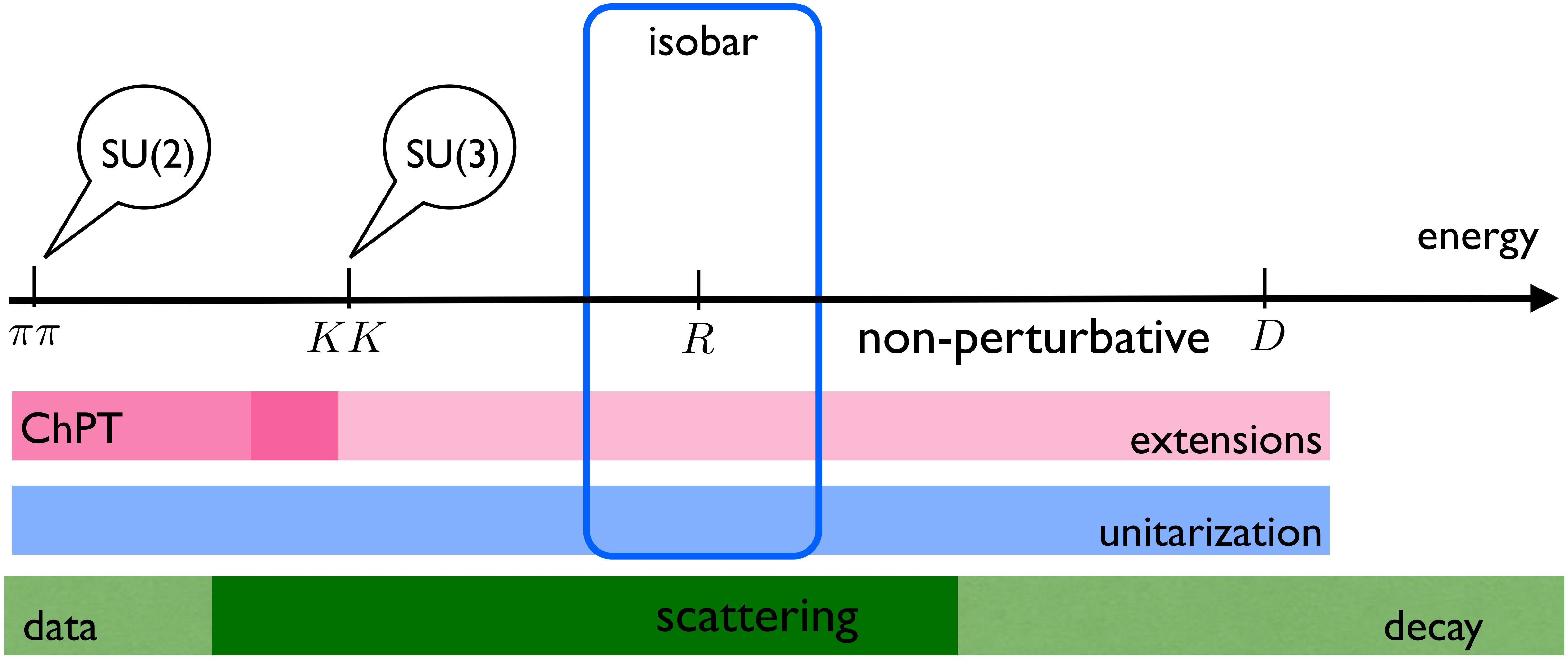
$< KK|0 >$ no data

Fit from 3-body data PCM, Robilotta + LHCb JHEP 1904 (2019) 063 will show how!

extrapolate from unitarity model Albaladejo and Moussallam EPJ C 75, 488 (2015).

quark model with isospin symmetry Bruch, Khodjamirian, and Kühn , EPJ C 39, 41 (2005)

scale issue



- we need non-perturbative meson-meson interactions up to…. 3 GeV
- extend 2-body amplitude theory validity

Ropertz, Kubis, Hanhart
EPJ Web Conf. 202 (2019) 06002

PCM, A.dos Reis, Robilotta
PRD 102, 076012 (2020)

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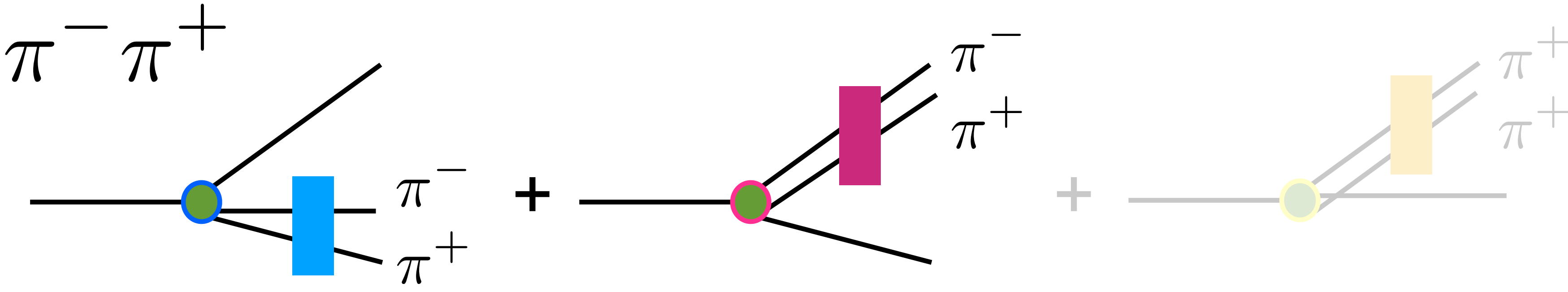
2-body x 3-body phases

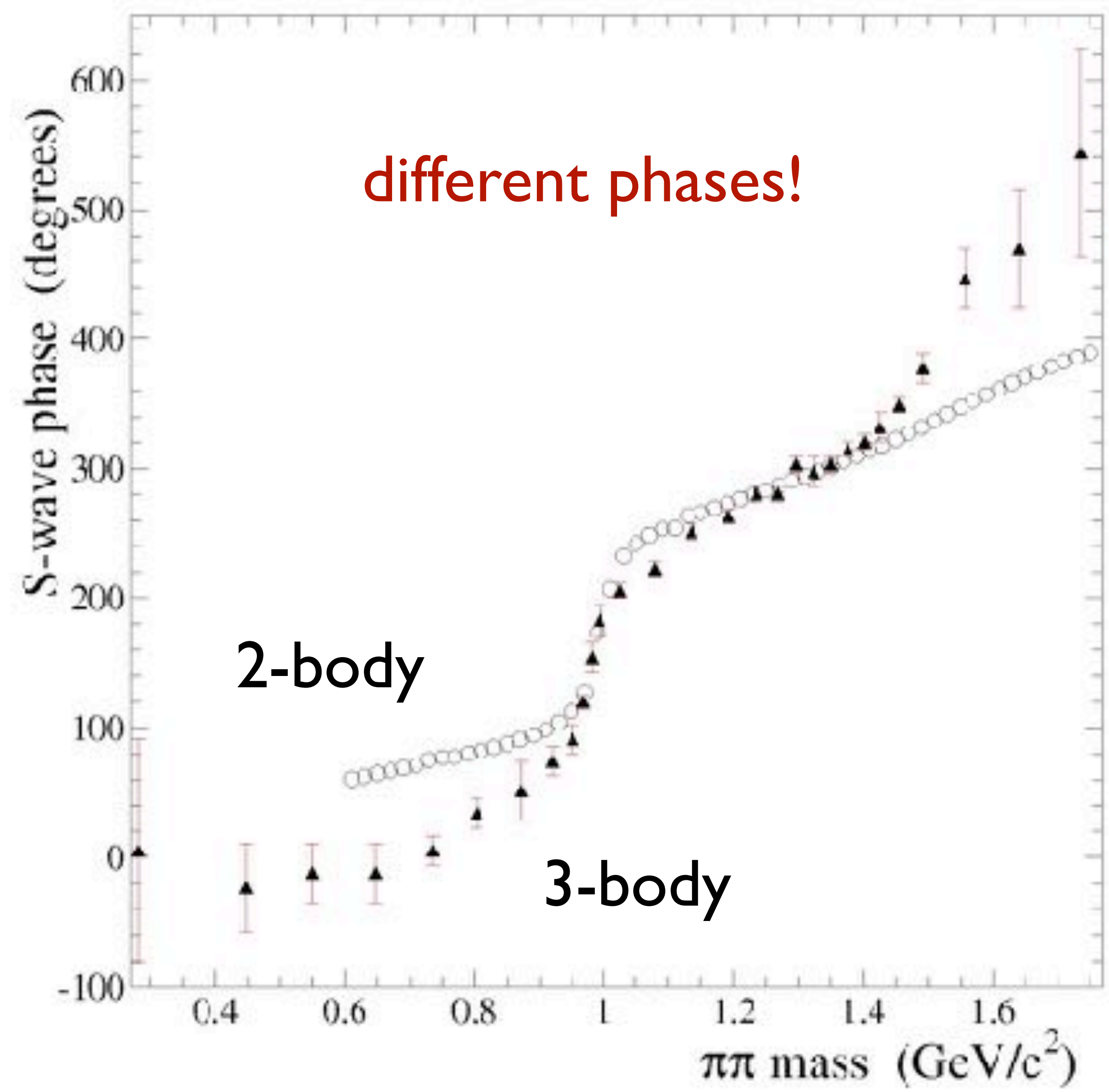
- $D_s^+ \to \pi^+\pi^-\pi^+$



(2+1)

  - If this is the "nature" picture → decay phase should be the same as 2-body

    → Watson's Theorem



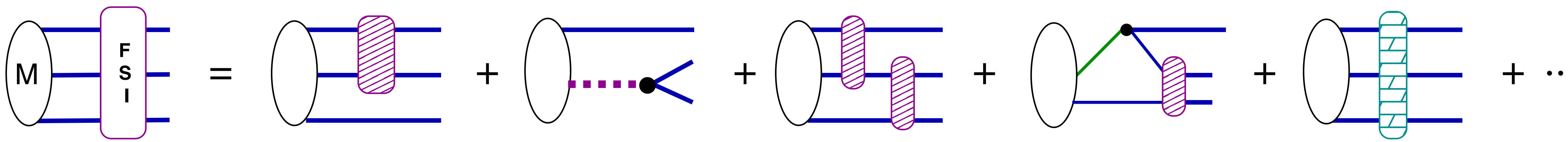
PRD 79 (2009) 032003

- Quantum numbers:
  - 2-body amplitude: spin and isospin well defined!
  - 3-body data: only spin! and $\neq$ dynamics

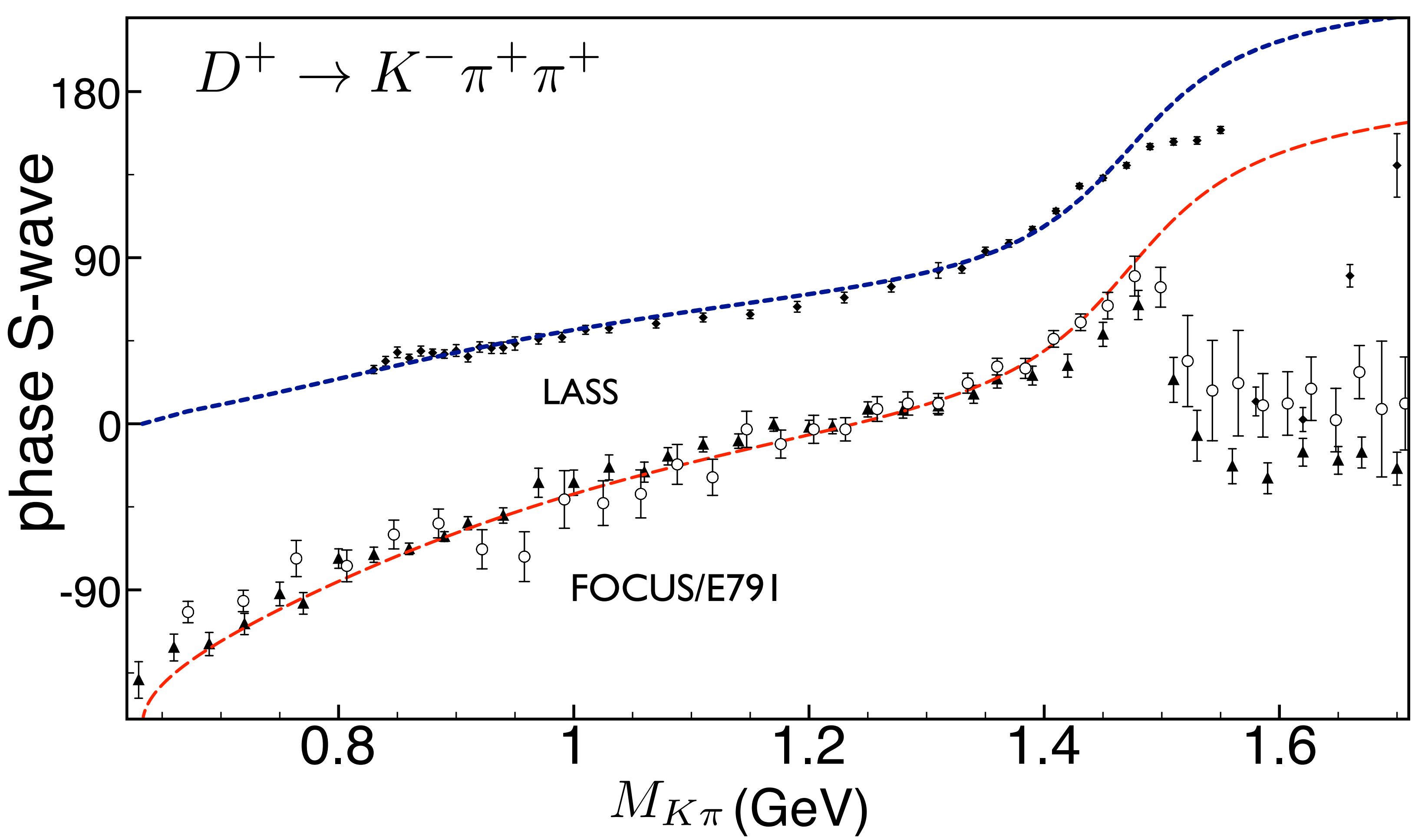
There is more than only 2-body

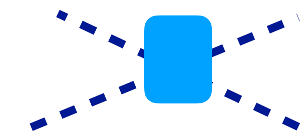
Three-body Models

- Three-body FSI (beyond 2+1)



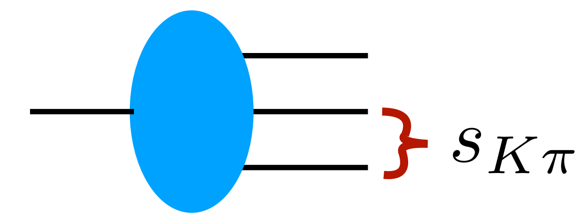
  - shown to be relevant on charm sector





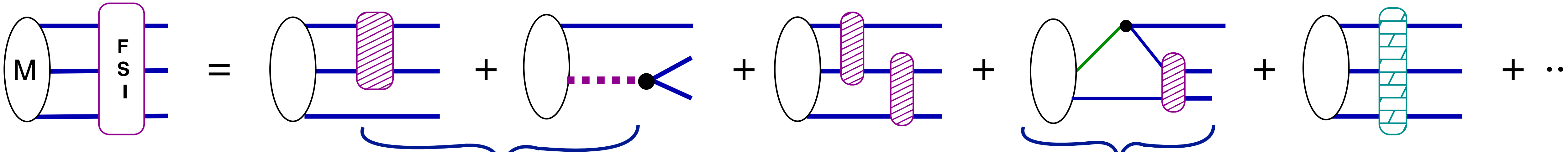
Scattering

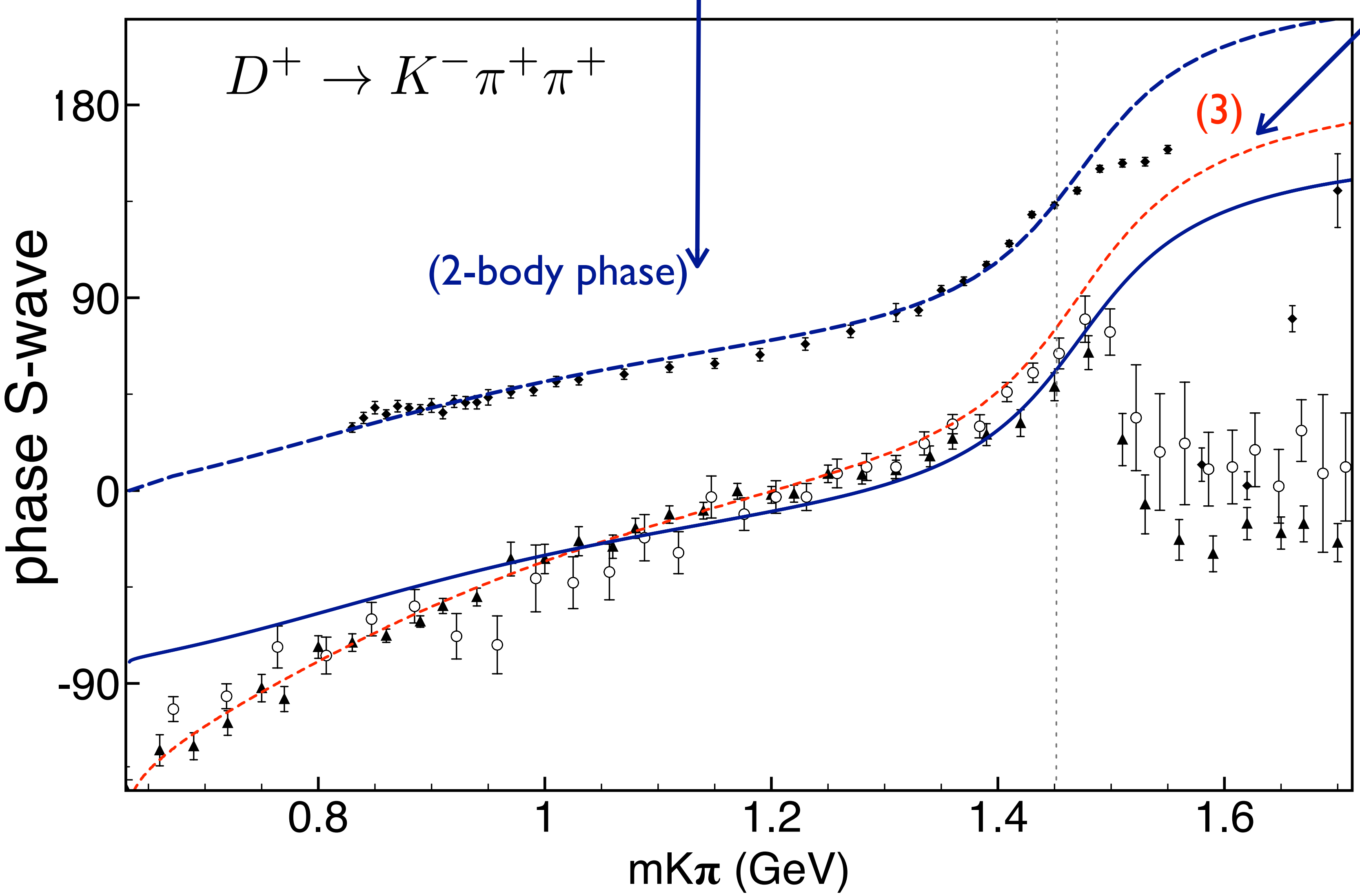
Decay projected in one pair mass



Models available

- Three-body FSI (beyond 2+1)





- 3-body approaches
  - Faddeev PCM et.al: PRD84 094001 (2011),
  - tri singularity S.Nakamura PRD93 014005 (2016)
  - Khuri-Treiman Niecknig, Kubis, JHEP10 142 (2015)

↳ 3-body FSI play a role

↳ will be important for precision

amplitude analysis @LHCb

$$D^+ \to K^- K^+ K^+$$

Theoretical model

PHYSICAL REVIEW D **98,** 056021 (2018)

arXiv:1805.11764 [hep-ph]

Multimeson model for the $D^+ \to K^+ K^- K^+$ decay amplitude

R. T. Aoude,[1,2] P. C. Magalhães,[1,3,*] A. C. dos Reis,[1] and M. R. Robilotta[4]

fitted to LHCb data

JHEP 1904 (2019) 063

KK scattering amplitude

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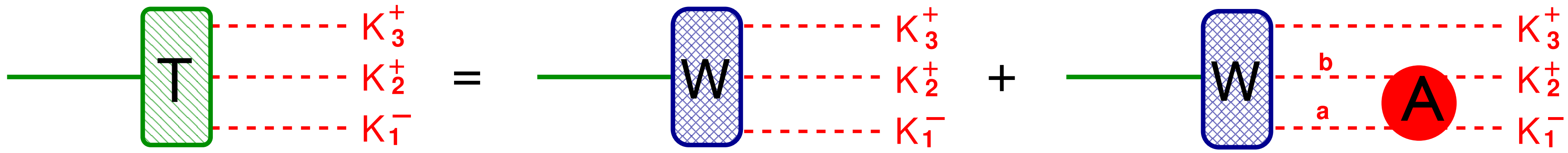
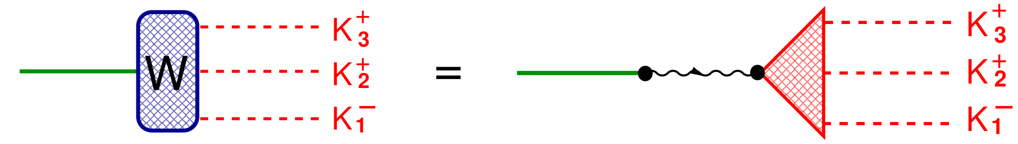
Triple-M



- hypothesis that annihilation is dominant

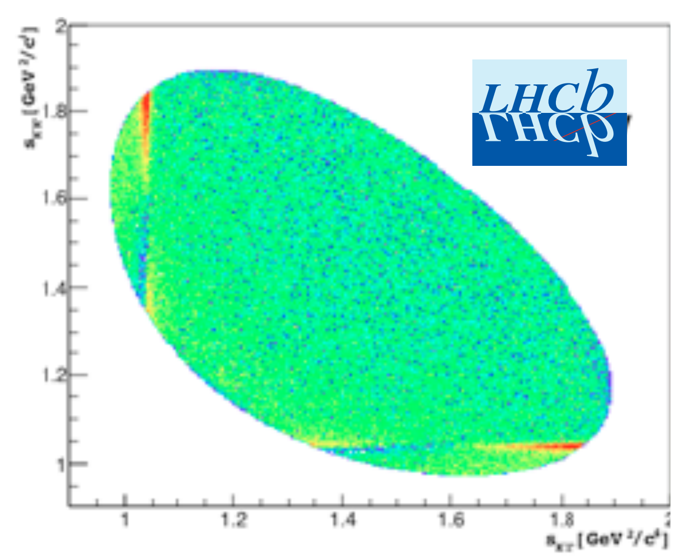


- depart from a fundamental theory → ChPT Lagrangian
  - track the ingredients we include in our model!
  - $A_{ab}^{JI} \longrightarrow$ unitary scattering amplitude for $ab \to K^+ K^-$

- fit the model to LHCb data

  run I (8 TeV CM) $2fb^{-1}$

  JHEP 1904 (2019) 063

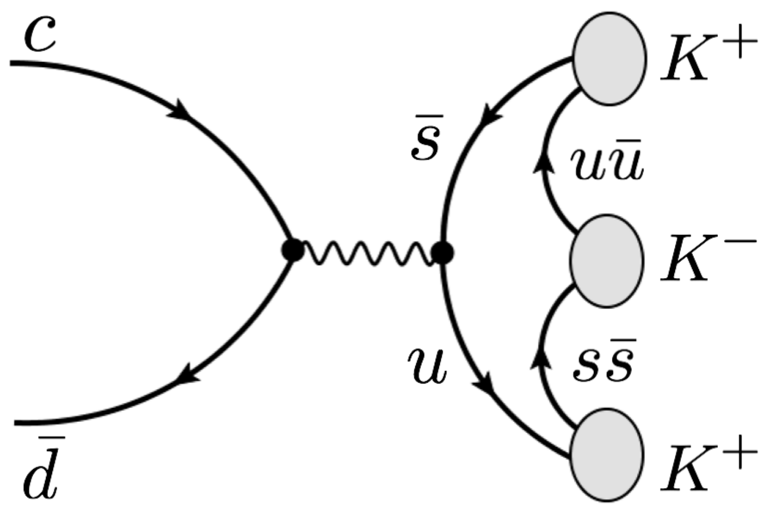


→ predict KK scattering amplitude

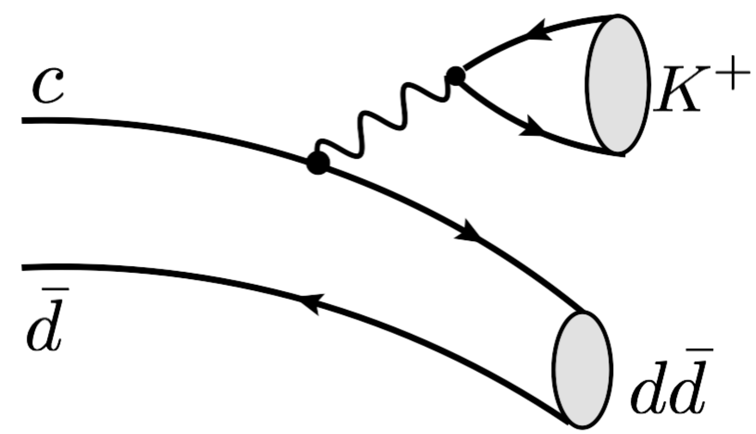
→ parameters have physical meaning: resonance masses and coupling constants

annihilation hypothesis

- annihilation

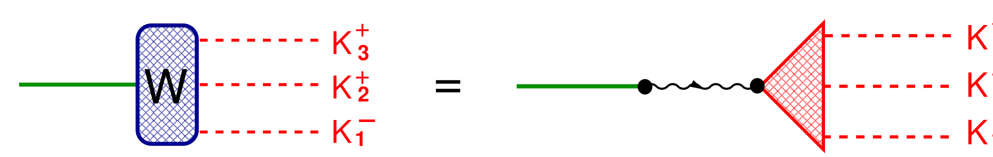


- color allowed



↪ need a rescattering!

- both are doubly Cabibbo-suppressed
- hypotheses that annihilation is dominant



↪ separate the different energy scales:

$$\mathcal{T} = \langle (KKK)^+|T|D^+\rangle = \underbrace{\langle (KKK)^+|A_\mu|0\rangle}_{\text{ChPT}} \; \langle 0|A^\mu|D^+\rangle.$$

↪ $-i\,G_F\,\sin^2\theta_C\,F_D\,P^\mu$

→ know how to calculate everything

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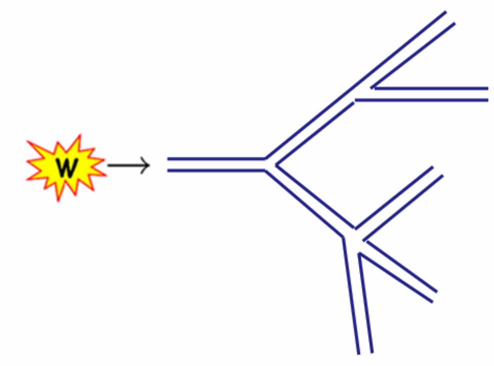
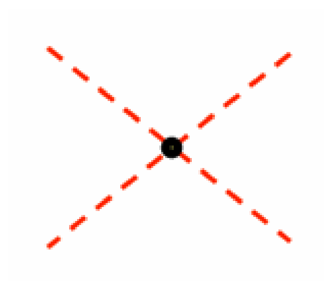
- solid theory to describe MM interactions at low energy

  - hadronization of Weak current



Gasser & Leutwyler
[Nucl. Phys. B250(1985)]

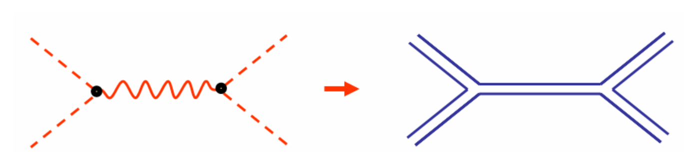
  - LO:



$$\mathcal{L}_M^{(2)} = -\frac{1}{6F^2} f_{ijs}\, f_{kls}\, \phi_i\, \partial_\mu \phi_j\, \phi_k\, \partial^\mu \phi_l + \frac{B}{24F^2}\left[\sigma_0 \left(\frac{4}{3}\delta_{ij}\,\delta_{kl} + 2\, d_{ijs}\, d_{kls}\right) + \sigma_8 \left(\frac{4}{3}\delta_{ij}\, d_{kl8} + \frac{4}{3} d_{ij8}\,\delta_{kl} + 2\, d_{ijm}\, d_{kln}\, d_{8mn}\right)\right] \phi_i\,\phi_j\,\phi_k\,\phi_l\,.$$

Gasser & Leutwyler
[Nucl. Phys. B250(1985)]

  - NLO: include resonances as a field



Ecker, Gasser, Pich and De Rafael
[Nucl. Phys. B321(1989)]

scalars

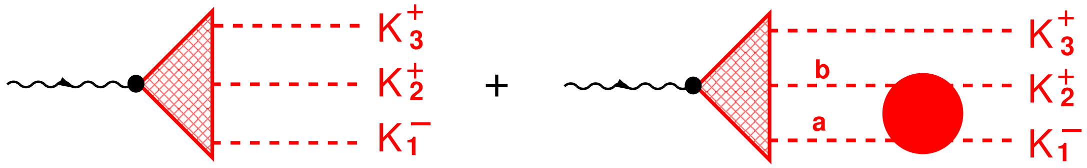
$$\mathcal{L}_S^{(2)} = \frac{2\,\tilde{c}_d}{F^2} R_0\, \partial_\mu \phi_i\, \partial^\mu \phi_i - \frac{4\,\tilde{c}_m}{F^2} B\, R_0 \left(\sigma_0\, \delta_{ij} + \sigma_8\, d_{8ij}\right) \phi_i\, \phi_j + \frac{2\, c_d}{\sqrt{2}F^2} d_{ijk}\, R_k\, \partial_\mu \phi_i\, \partial^\mu \phi_i - \frac{4Bc_m}{\sqrt{2}F^2}\left[\sigma_0\, d_{ijk} + \sigma_8 \left(\frac{2}{3}\delta_{ik}\,\delta_{j8} + d_{i8s}\, d_{jsk}\right)\right] \phi_i\, \phi_j R_k$$

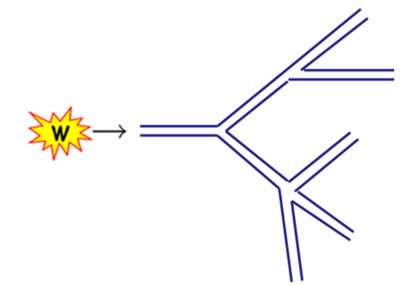
vectors

$$\mathcal{L}_V^{(2)} = \frac{iG_V}{\sqrt{2}} \langle V_{\mu\nu} u^\mu u^\nu \rangle$$

$$\langle V_{\mu\nu} u^\mu u^\nu \rangle = \frac{1}{F^2} V_a^{\mu\nu}\, \partial_\mu \phi_i\, \partial_\nu \phi_j\, \left(i f_{aij} + d_{aij}\right)$$

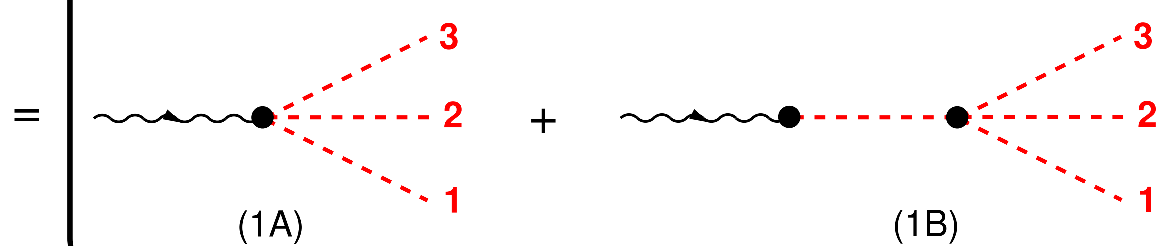
Triple - M





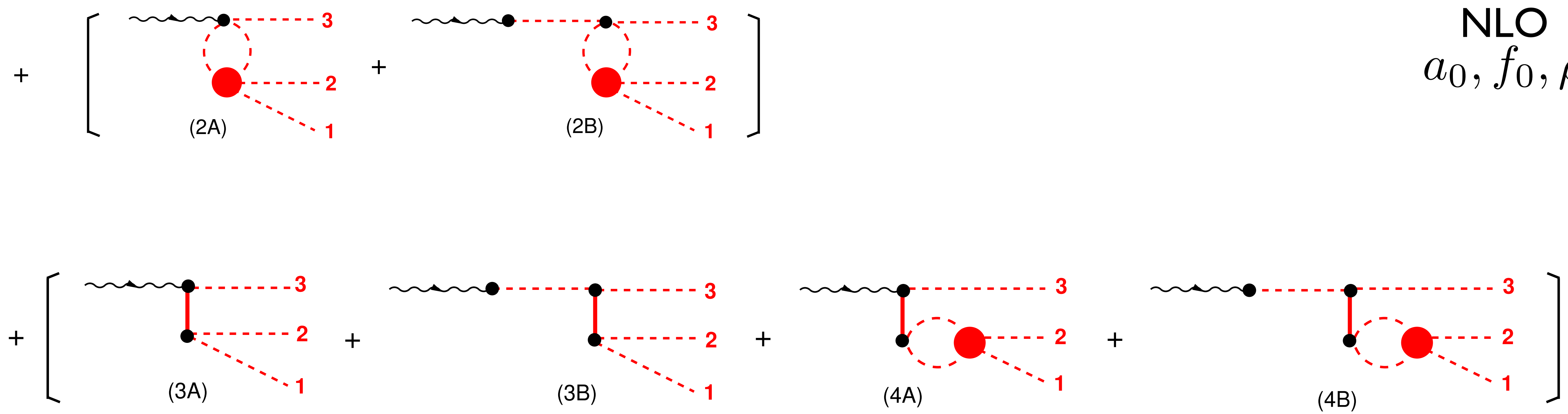
Chiral symmetry

LO: non-resonant

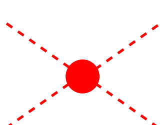


NLO

a_0, f_0, ρ, ϕ



width obtained through dynamics



$K\bar{K}$ coupled-channel unitary amplitude

$\pi\pi,\ \eta\eta,\ \pi\eta,\ \rho\pi$

- isospin decomposition $[J, I = (0,1), (0,1)]$

$\langle K^- K^+| = (i/2)\,\langle V_3^{KK} + V_8^{KK}| - (1/2)\,\langle U_3^{KK} + S^{KK}|$

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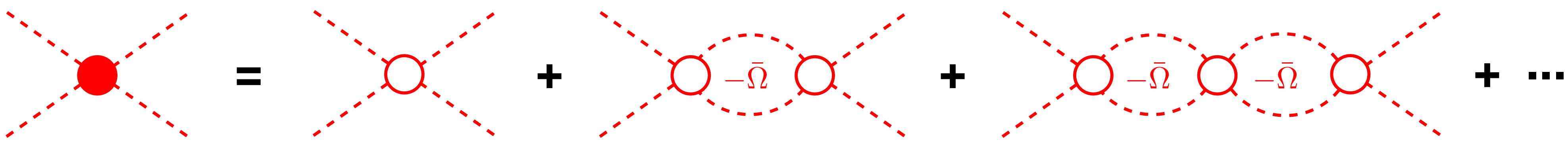
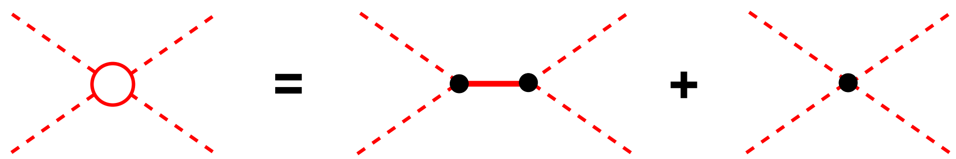
- unitarize amplitude by Bethe-Salpeter eq. [Oller and Oset PRD 60 (1999)]



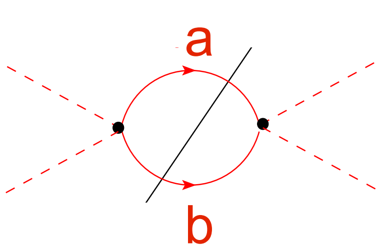
$$\boxed{\mathcal{A}^{JI}_{ab} = \frac{\mathcal{K}^{(JI)}_{ab\to cd}}{1+\bar{\Omega}_{ab}\,\mathcal{K}^{(JI)}_{ab\to cd}}}$$

- kernel $\mathcal{K}^{(J,I)}_{ab\to cd}$



resonance (NLO) + contact (LO)

- loops → K-matrix approximation: only on-shell



$$\{I_{ab};\, I^{\mu\nu}_{ab}\} = \int \frac{d^4\ell}{(2\pi)^4}\frac{\{1;\, \ell^\mu\, \ell^\nu\}}{D_a\, D_b}$$

$$D_a = (\ell+p/2)^2 - M_a^2 \qquad D_b = (\ell-p/2)^2 - M_b^2$$

→

$$\bar{\Omega}^S_{ab} = -\frac{i}{8\pi}\frac{Q_{ab}}{\sqrt{s}}\,\theta(s-(M_a+M_b)^2)$$

$$\bar{\Omega}^P_{aa} = -\frac{i}{6\pi}\frac{Q^3_{aa}}{\sqrt{s}}\,\theta(s-4\,M_a^2)$$

$$Q_{ab} = \frac{1}{2}\sqrt{s-2\,(M_a^2+M_b^2)+(M_a^2-M_b^2)^2/s}$$

- free parameters
  - masses:

    $m_\rho\,,\, m_{a_0}\,,\, \boxed{m_{s0}\,,\, m_{s1}}$ $SU(3)$ singlet and octet

  - coupling constants:

    $g_\rho\,,\, g_\phi$ vector

    $c_d\,,\, c_m\,,\, \tilde{c}_d\,,\, \tilde{c}_m$ scalar

→ physical f_0 states are linear combination of $m_{s0}\,,\, m_{s1}$

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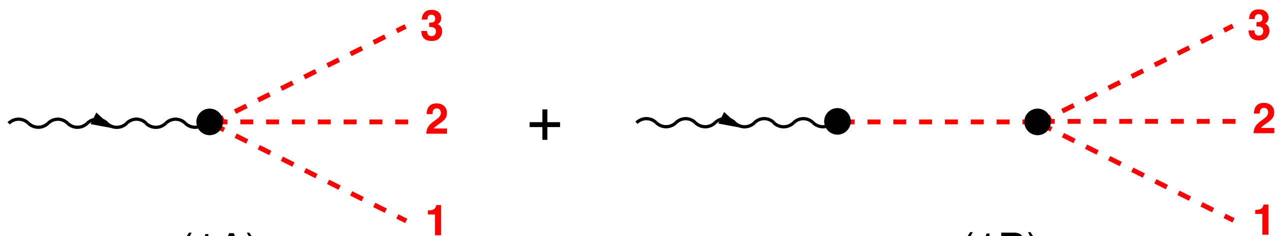
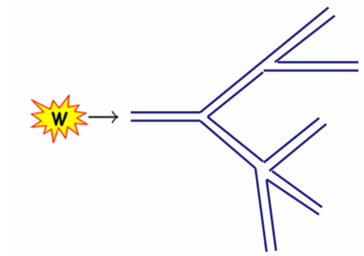
non-resonant



(1A) (1B)



Chiral symmetry

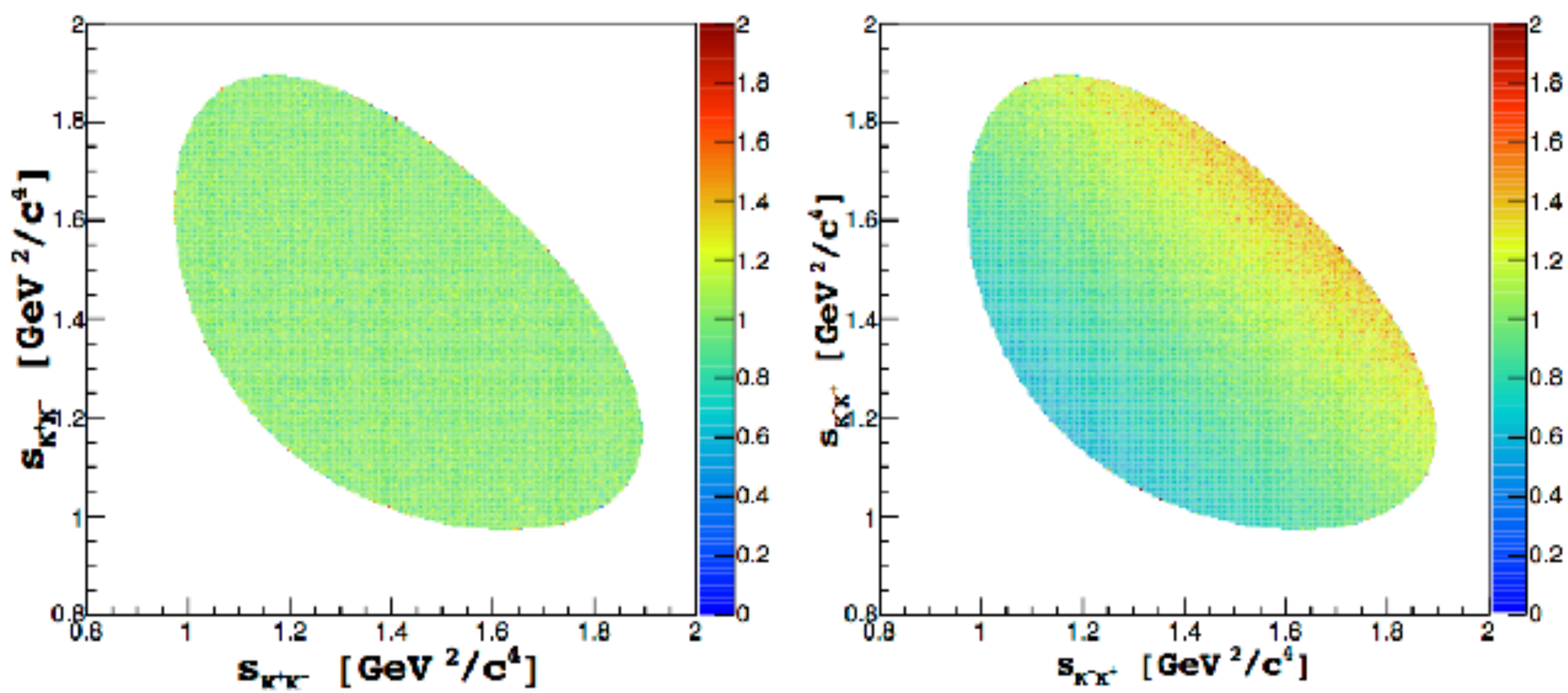
$$T_{NR} = \left[\frac{C}{4}\,(M^2 - M_K^2 + m_{12}^2) + \frac{C}{4}\,(m_{13}^2 - m_{23}^2) + (2 \leftrightarrow 3)\right]$$

$$C = \left\{\left[\frac{G_F}{\sqrt{2}}\,\sin^2\theta_C\right]\,\frac{2F_D}{F}\,\frac{M_K^2}{M_D^2 - M_K^2}\right\}$$

projected into S- and P- wave

3-body effect predicted by Chiral symmetry

- comparing with isobar (constant)



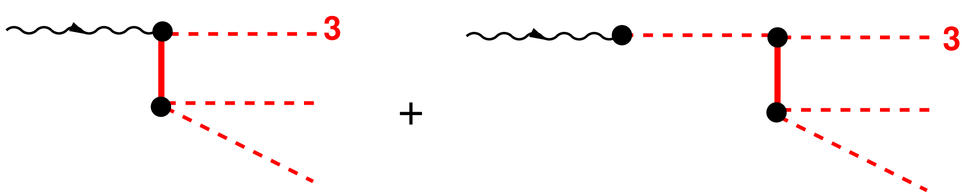
no free parameter

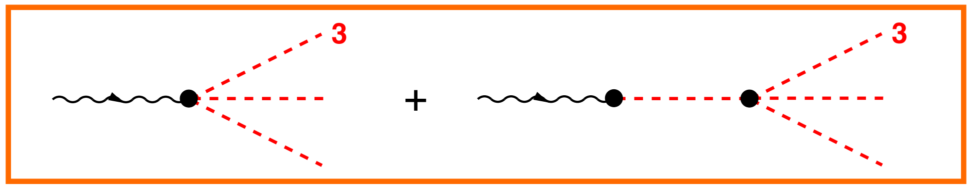
a_0 example

$[J, I = 0, 1] \rightarrow \eta\pi,\ KK$

- **tree** $D \to a\, b\, K^+$

$$\langle U_3\,(K^+)\,|\,T^{(0,1)}_{(0)}\,|\,D\,\rangle = \left\{ \Gamma^{(0,1)}_{(0)\,\pi 8}\,\langle\, U_3^{\pi 8}| + \Gamma^{(0,1)}_{(0)\,KK}\,\langle\, U_3^{KK}| \right\}$$



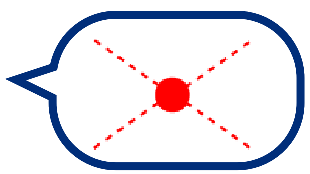


attention to double counting!

$$\Gamma^{(0,1)}_{(0)\,\pi 8} = C \left\{ \left[\frac{2\sqrt{2}}{\sqrt{3}\,F^2} \right] \frac{[-c_d\,P\cdot p_3 + c_m\,M_D^2]}{m_{12}^2 - m_{a_0}^2}\,[c_d\,(m_{12}^2 - M_\pi^2 - M_8^2) + 2\,c_m\,M_\pi^2] + \left[-\frac{\sqrt{3}}{\sqrt{2}}\,[M_D^2/3 - P\cdot p_3] \right]_c \right\}$$

$$\Gamma^{(0,1)}_{(0)\,KK} = C \left\{ \left[\frac{2}{F^2} \right] \frac{[-c_d\,P\cdot p_3 + c_m\,M_D^2]}{m_{12}^2 - m_{a_0}^2}\,[c_d\,(m_{12}^2 - 2M_K^2) + 2\,c_m\,M_K^2] + \left[-\frac{1}{2}\,[M_D^2 - P\cdot p_3] \right]_c \right\}$$

- **FSI**



one interaction

$$\Gamma^{(0,1)}_{(1)\,\pi 8} = -\mathcal{K}^{(0,1)}_{\pi 8|\pi 8}\,[\bar{\Omega}^S_{\pi 8}]\,\Gamma^{(0,1)}_{(0)\,\pi 8} - \mathcal{K}^{(0,1)}_{\pi 8|KK}\left[\frac{1}{2}\,\bar{\Omega}^S_{KK}\right]\Gamma^{(0,1)}_{(0)\,KK}$$

$$\Gamma^{(0,1)}_{(1)\,KK} = -\mathcal{K}^{(0,1)}_{\pi 8|KK}\,[\bar{\Omega}^S_{\pi 8}]\,\Gamma^{(0,1)}_{(0)\,\pi 8} - \mathcal{K}^{(0,1)}_{KK|KK}\left[\frac{1}{2}\,\bar{\Omega}^S_{KK}\right]\Gamma^{(0,1)}_{(0)\,KK}$$

$$\Gamma^{(0,1)}_{(1)} = \begin{bmatrix} \Gamma^{(0,1)}_{(1)\,\pi 8} \\ \Gamma^{(0,1)}_{(1)\,KK} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \Gamma^{(0,1)}_{(0)\,\pi 8} \\ \Gamma^{(0,1)}_{(0)\,KK} \end{bmatrix} = M^{(0,1)}\,\Gamma^{(0,1)}_{(0)}$$

infinity interactions

$$\Gamma^{(0,1)} = \{1 + M^{(0,1)} + [M^{(0,1)}]^2 + \cdots\}\,\Gamma^{(0,1)}_{(0)} \rightarrow \Gamma^{(0,1)} = \left[1 - M^{(0,1)}\right]^{-1}\Gamma^{(0,1)}_{(0)}$$

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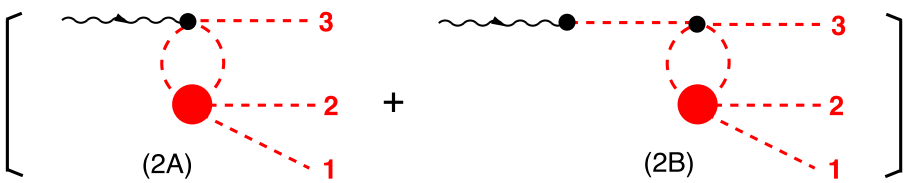
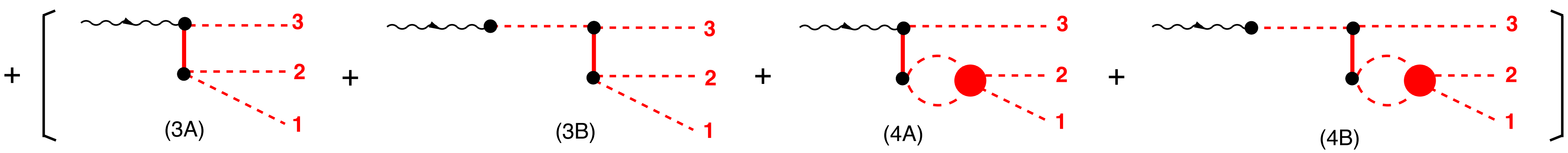
a_0 `example`

$[J, I = 0, 1] \rightarrow \eta\pi,\ KK$

- full FSI!





- $$T^{(0,1)} = -\frac{1}{2}\left[\bar{\Gamma}^{(0,1)}_{KK} - \Gamma^{(0,1)}_{c|KK}\right]$$

$$\rightarrow \quad \bar{\Gamma}^{(0,1)}_{KK} = \frac{(m_{12}^2 - m_{a_0}^2)}{D_{a_0}(m_{12}^2)}\left[M_{21}\,\Gamma^{(0,1)}_{(0)\,\pi 8} + (1 - M_{11})\,\Gamma^{(0,1)}_{(0)\,KK}\right]$$

$$\boxed{D_{a_0} = (m_{12}^2 - m_{a_0}^2)\,\left[(1 - M_{11})\,(1 - M_{22}) - M_{12}\,M_{21}\right]}$$

$$M_{11} = -\mathcal{K}^{(0,1)}_{\pi 8|\pi 8}\,\left[\bar{\Omega}^S_{\pi 8}\right]$$

$$M_{12} = -\mathcal{K}^{(0,1)}_{\pi 8|KK}\,\left[(1/2)\,\bar{\Omega}^S_{KK}\right]$$

$$M_{21} = -\mathcal{K}^{(0,1)}_{\pi 8|KK}\,\left[\bar{\Omega}^S_{\pi 8}\right]$$

$$M_{22} = -\mathcal{K}^{(0,1)}_{KK|KK}\,\left[(1/2)\,\bar{\Omega}^S_{KK}\right]$$

- only one channel in the scattering amplitude

$$\bar{\Gamma}^{(0,1)}_{KK} = \frac{(m_{12}^2 - m_{a_0}^2)}{D_{a_0}(m_{12}^2)}\,\Gamma^{(0,1)}_{(0)\,KK}$$

Flatté

$$D_{a_0}(s) = (s - m_{a_0}^2) + i\,m_{a_0}\,\Gamma_{a_0}(s)$$

$$m_{a_0}\Gamma_{a_0}(s) = \frac{1}{8\pi\sqrt{s}}\left\{\left[\frac{4}{3\,F^4}\right]\boxed{\left[c_d\,(s - M_\pi^2 - M_8^2) + 2\,c_m\,M_\pi^2\right]^2}\,Q_{\pi 8} + \left[\frac{1}{F^4}\right]\boxed{\left[c_d\,(s - 2\,M_K^2) + 2\,c_m\,M_K^2\right]^2}\,Q_{KK}\right\}$$

$\rightarrow$ parameter: $c_d,\ c_m\ m_{a_0}$

access two-body dynamics !

Triple M LHCb fit

- Theoretical sound model

$$T^S = T^S_{NR} + T^{00} + T^{01}$$

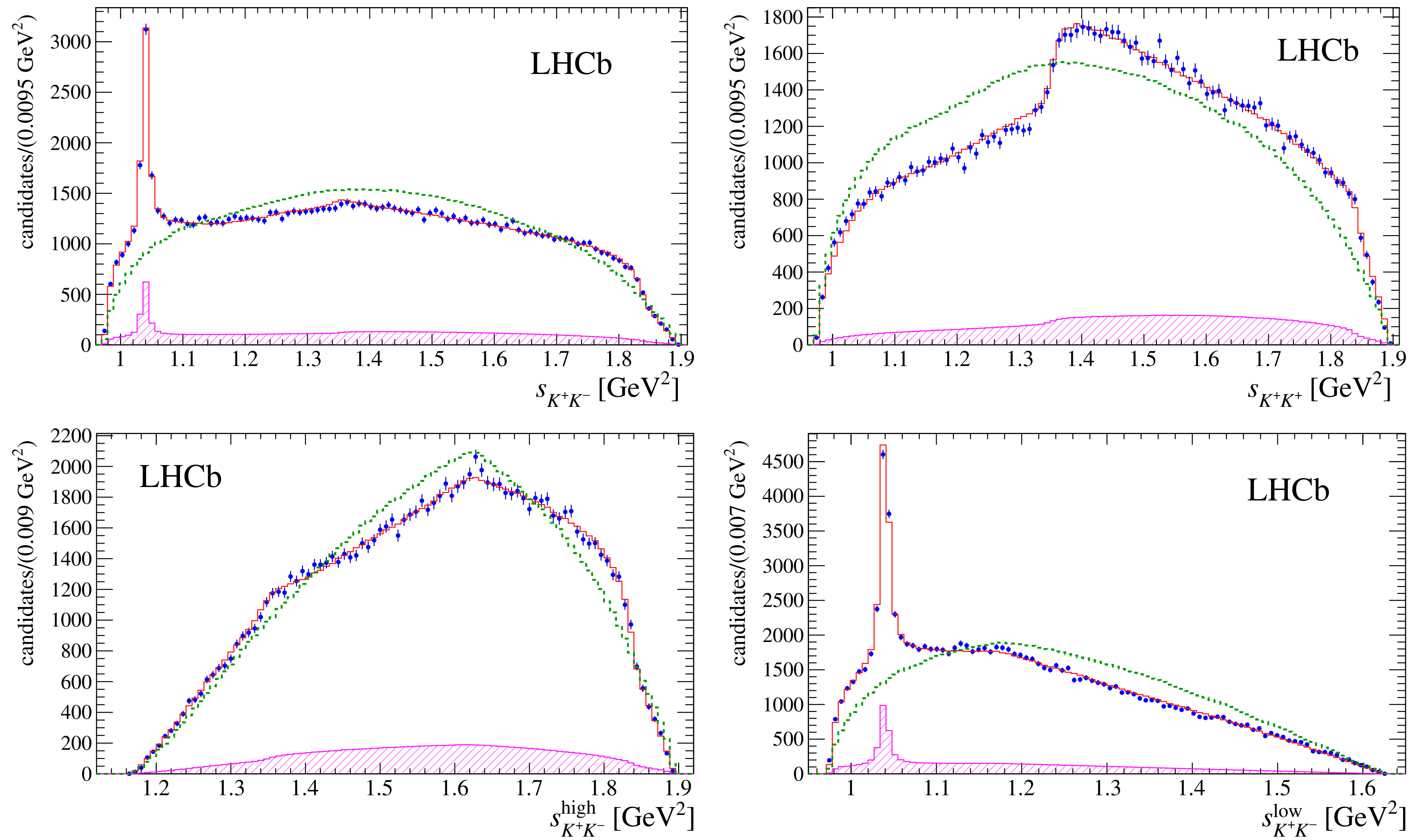
$$T^P = T^P_{NR} + T^{11} + T^{10}$$

- free parameters

parameter	value
F	$94.3^{+2.8}_{-1.7} \pm 1.5\,\mathrm{MeV}$
m_{a_0}	$947.7^{+5.5}_{-5.0} \pm 6.6\,\mathrm{MeV}$
m_{S_o}	$992.0^{+8.5}_{-7.5} \pm 8.6\,\mathrm{MeV}$
m_{S_1}	$1330.2^{+5.9}_{-6.5} \pm 5.1\,\mathrm{MeV}$
m_ϕ	$1019.54^{+0.10}_{-0.10} \pm 0.51\,\mathrm{MeV}$
G_ϕ	$0.464^{+0.013}_{-0.009} \pm 0.007$
c_d	$-78.9^{+4.2}_{-2.7} \pm 1.9\,\mathrm{MeV}$
c_m	$106.0^{+7.7}_{-4.6} \pm 3.3\,\mathrm{MeV}$
$\tilde{c}_d$	$-6.15^{+0.55}_{-0.54} \pm 0.19\,\mathrm{MeV}$
$\tilde{c}_m$	$-10.8^{+2.0}_{-1.5} \pm 0.4\,\mathrm{MeV}$

FF_{NR}	FF^{00}	FF^{01}	FF^{10}	FF^{11}	FF_{S-wave}
14 ± 1	29 ± 1	131 ± 2	7.1 ± 0.9	0.26 ± 0.01	94 ± 1

χ^2/ndof = 1.12 (Isobar 1.14-1.6)

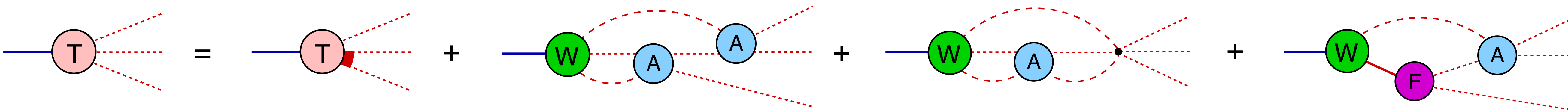


JHEP 1904 (2019) 063

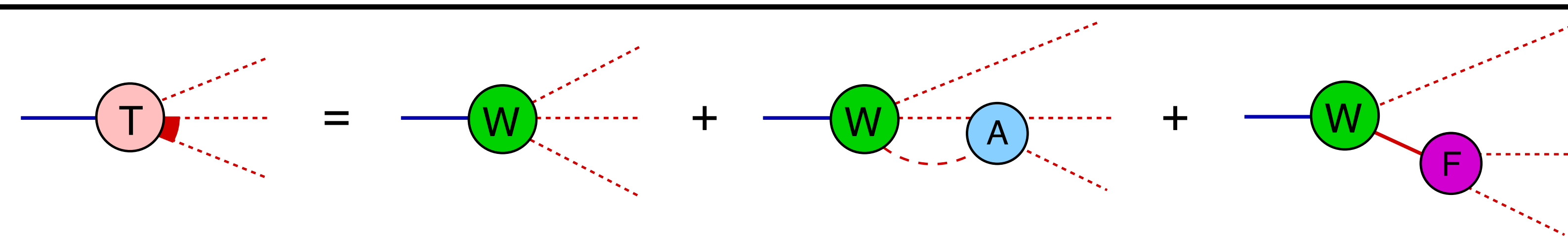
→ good fit with fewer parameters than the isobar

- Any 3-body decay amplitude

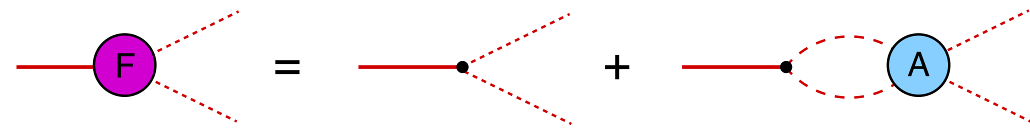
MAGALHAES, A.dos Reis, Robilotta
PRD 102, 076012 (2020)



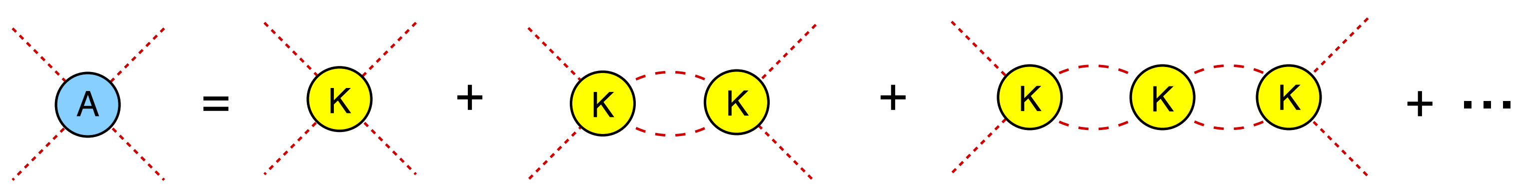
(2+1) approach

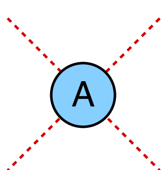


Form factor



meson-meson

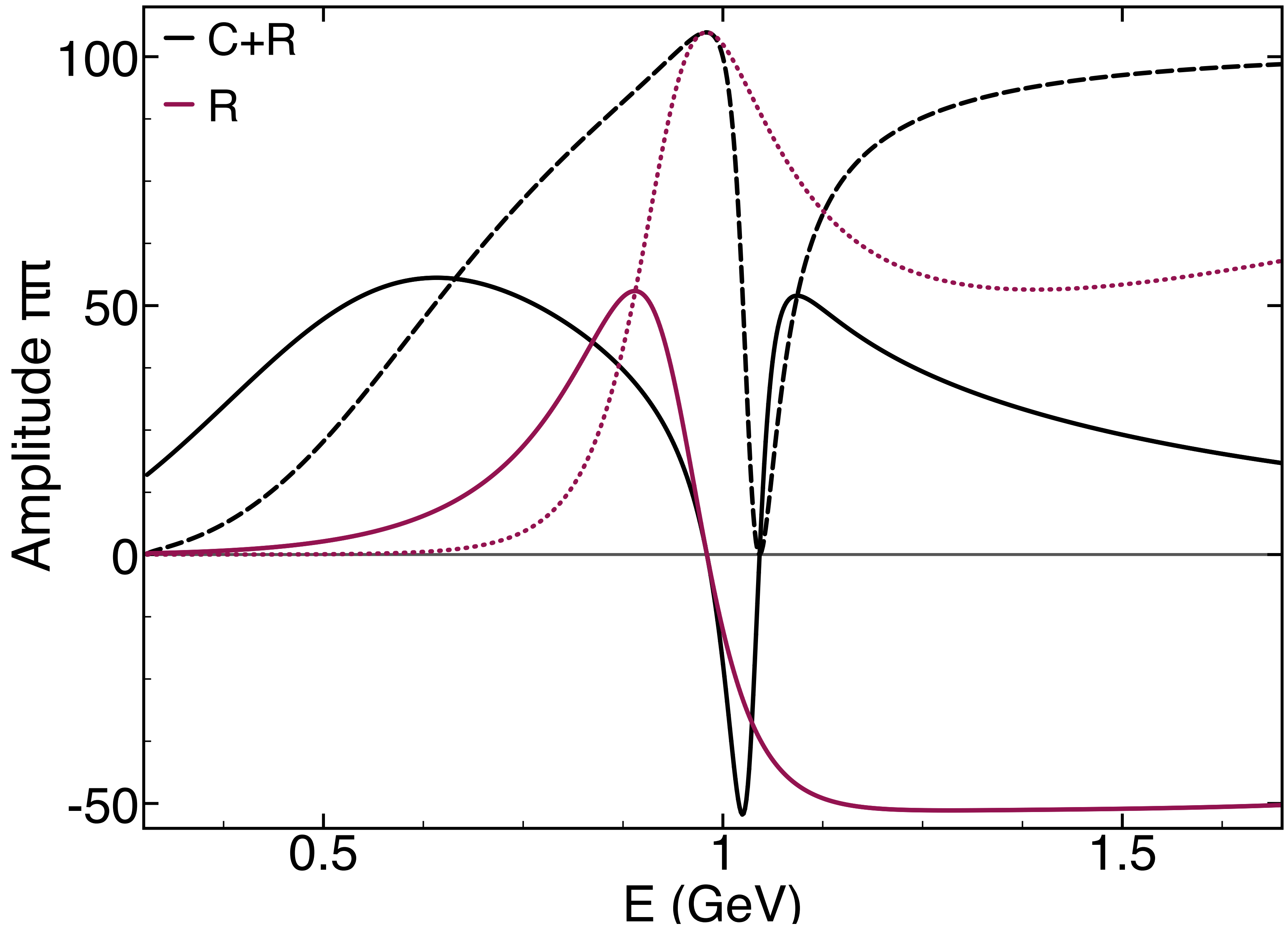


↳ provide the building block  in SU(3)

- includes multiple resonances in the same channel (as many as wanted)
- free parameter (massas and couplings) to be fitted to data.

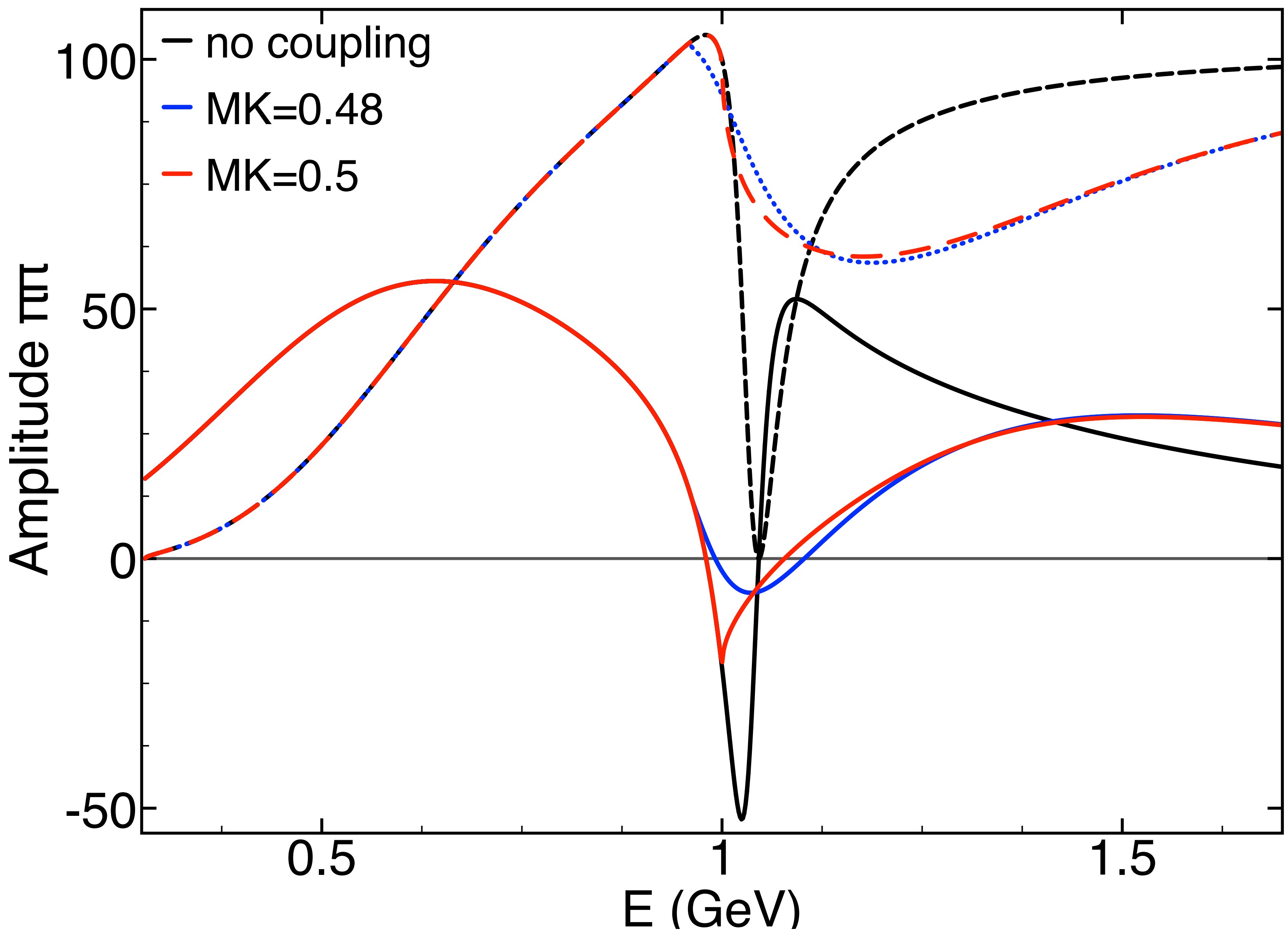
⟶ Available to be implement in data analysis!!

- beyond 1 resonance (BW description)



- ex: one resonance $f_0 = 980\,MeV$ one channel

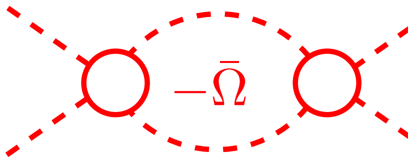
- Coupled-channel $\pi\pi \to KK$



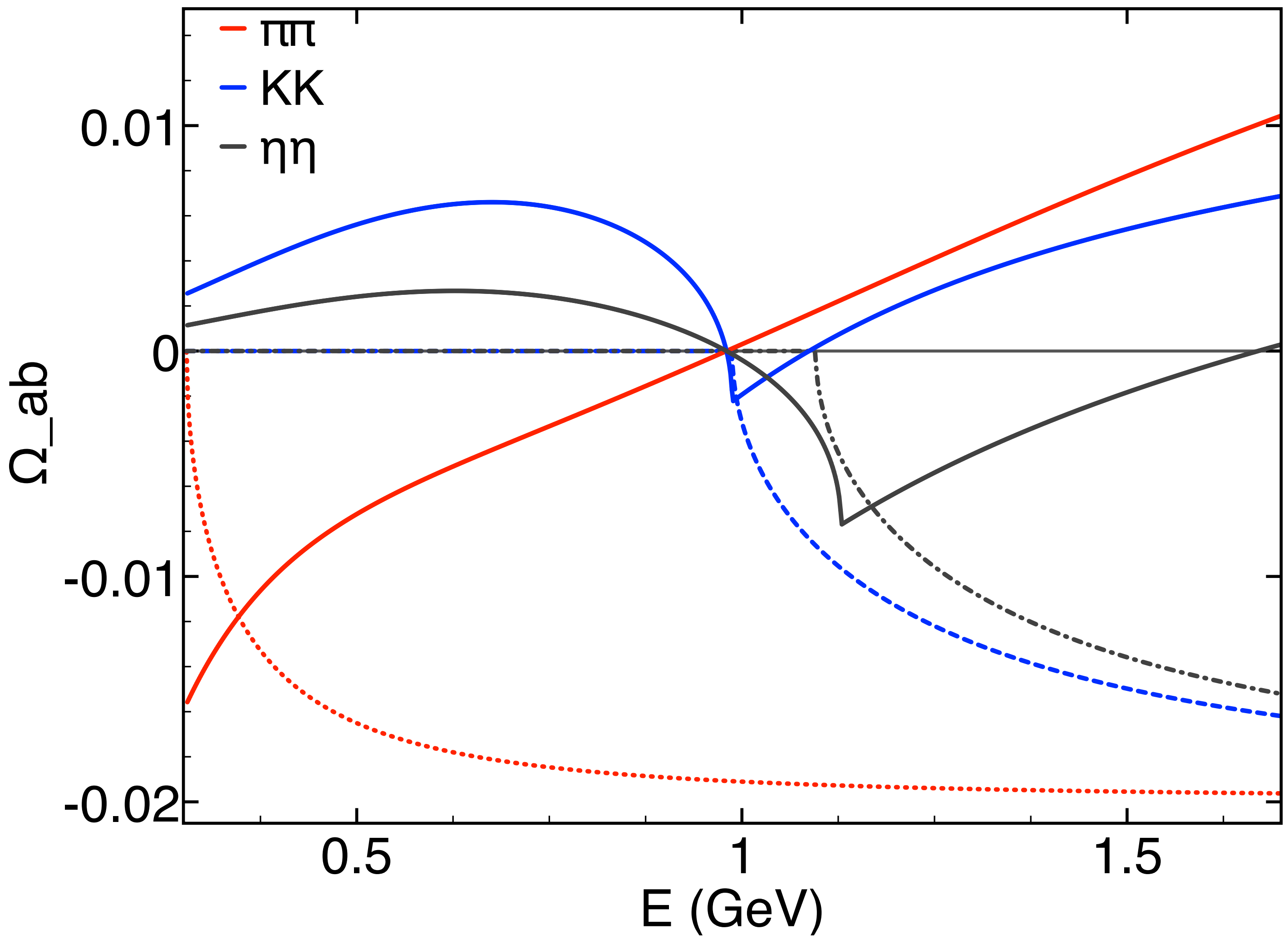
- all curves coincide below the thresholds
- cusp in the real parte for $m_{f_0} < 2M_K$ and a discontinuity in imaginary part for $m_{f_0} > 2M_K$

Unitarization with N resonances

- $$\Omega^S_{ab}(s) \to \frac{1}{16\pi^2}\left\{\left[F_x(s)\,\Pi^R_{ab}(m_x^2)\right] - \Pi_{ab}(s)\right\}\ ,$$



- beyond K-matrix approach → freedom to chose renormalization constant



- $\mathscr{Re}[\Omega(s = m_x^2)] = 0$
- $$F_x(s) = \frac{4\,m_x^2\,s}{(s+m_x^2)^2}$$

respect Chiral Symmetry is finite at $s \to \infty$

- extending it to 3 resonances

$$\Omega^S_{ab}(s) \to \frac{1}{16\pi^2}\left\{F_x(s)\,\frac{(s-m_y^2)\,(s-m_z^2)}{(m_x^2-m_y^2)\,(m_x^2-m_z^2)}\,\Pi^R_{ab}(m_x^2) + F_y(s)\,\frac{(m_x^2-s)\,(s-m_z^2)}{(m_x^2-m_y^2)\,(m_y^2-m_z^2)}\,\Pi^R_{ab}(m_y^2) + F_z(s)\,\frac{(m_x^2-s)\,(m_y^2-s)}{(m_x^2-m_z^2)\,(m_y^2-m_z^2)}\,\Pi^R_{ab}(m_z^2) - \Pi_{ab}(s)\right\}$$

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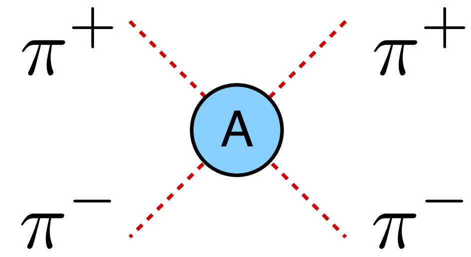
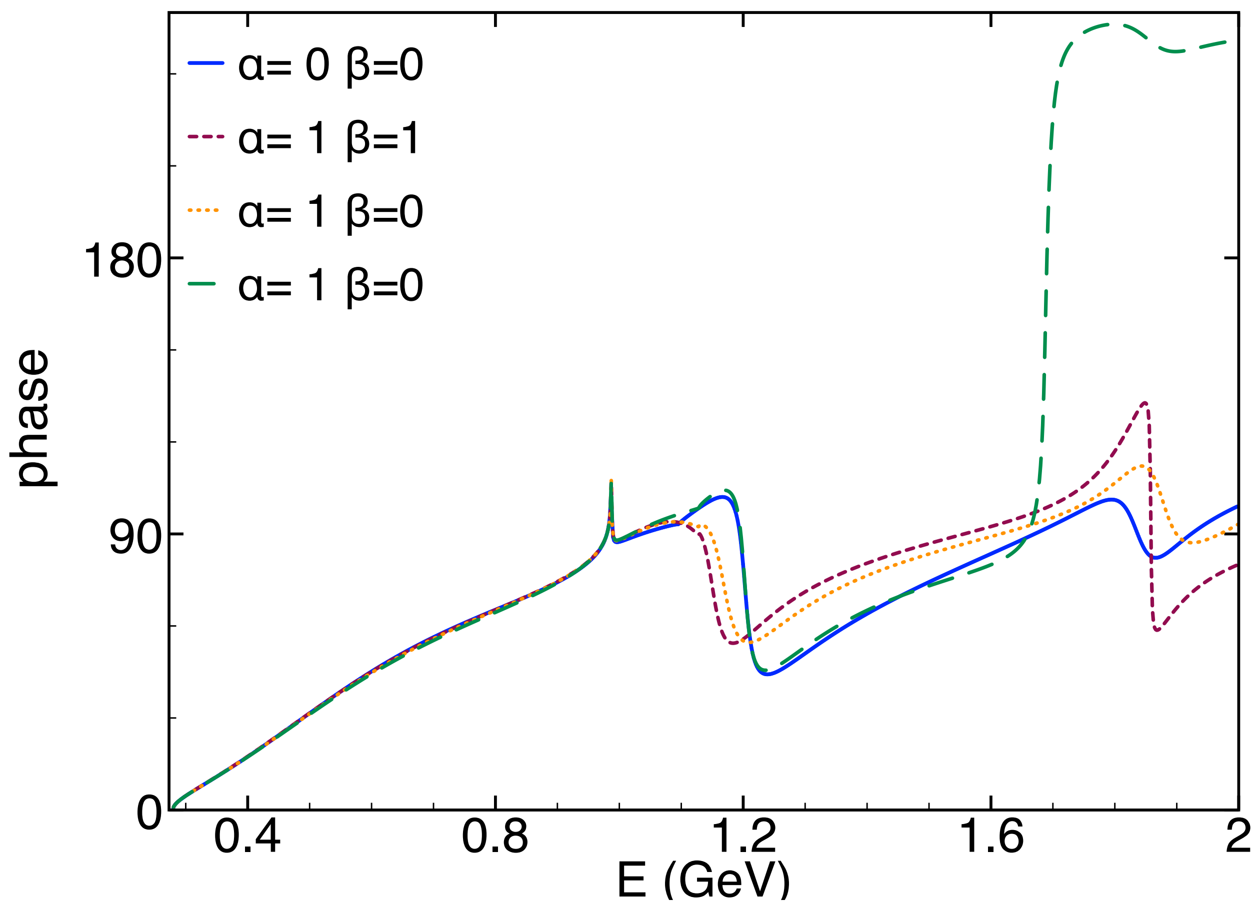
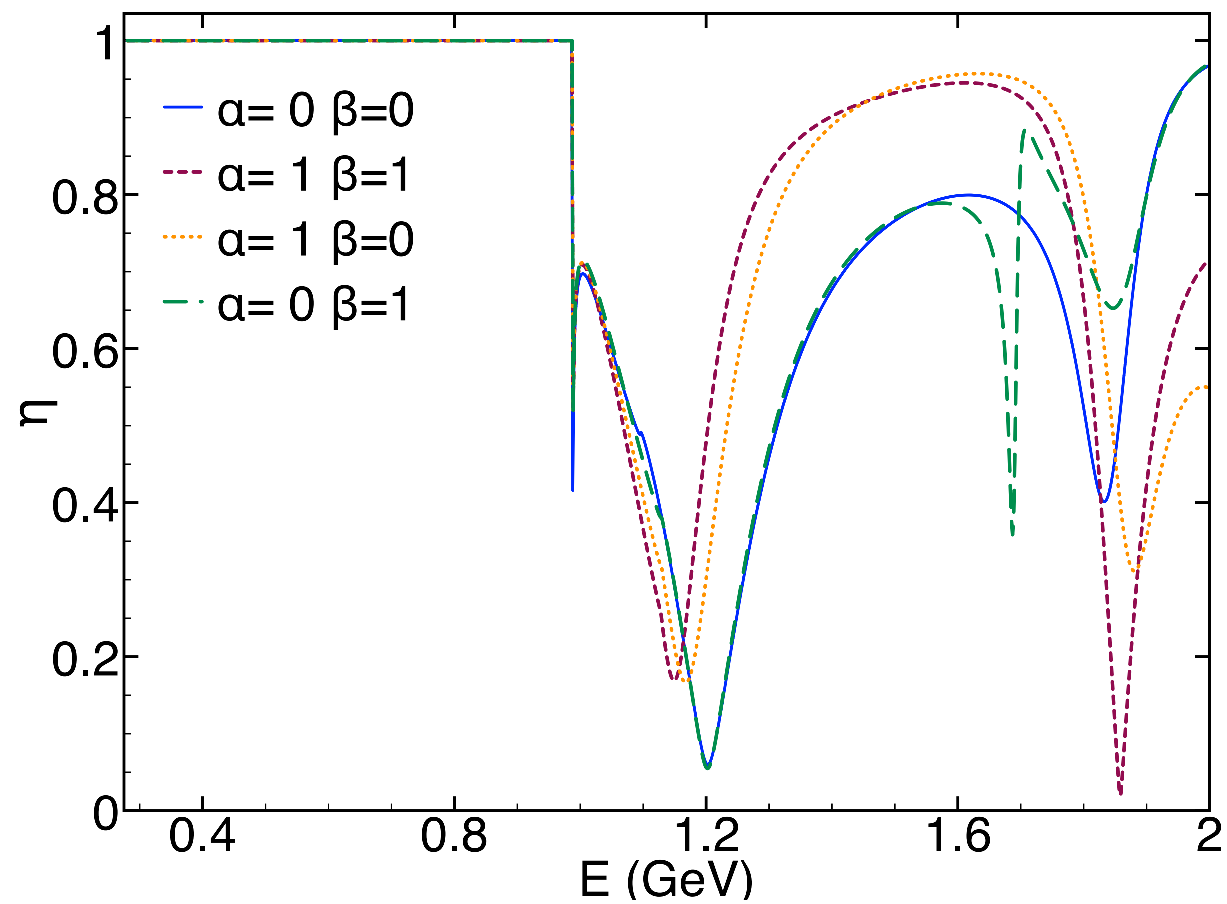
ππ amplitude 3 coupled-channels: $\pi\pi$, KK and $\eta\eta$

- 3 resonances: mx=0.98, my=1.37, mz=1.7 GeV

  ↳ α and β are couplings from mz







- extra res do not disturb the low-energy!
- parameter should be fixed by data

→ will apply this methodology in other $D \to hhh$

Final remarks

- A consistent treatment of FSI is crucial to reach precision in $D \to hhh$
  - → two-body coupled-channels description in mandatory
  - → a proper 2-body FSI have impact in both (2+1) and 3-body
  - → relevant for CPV search
- A full description of ANA need both weak and strong description
- $D^+ \to KKK$: example of theory/experimental join work
- tool kit for amplitude analysis with theoretically sound models to $D \to hhh$ ANA
- $D^+ \to h^+h^-h^+$ huge data samples on their way claiming for accurate models!

Backup slides !

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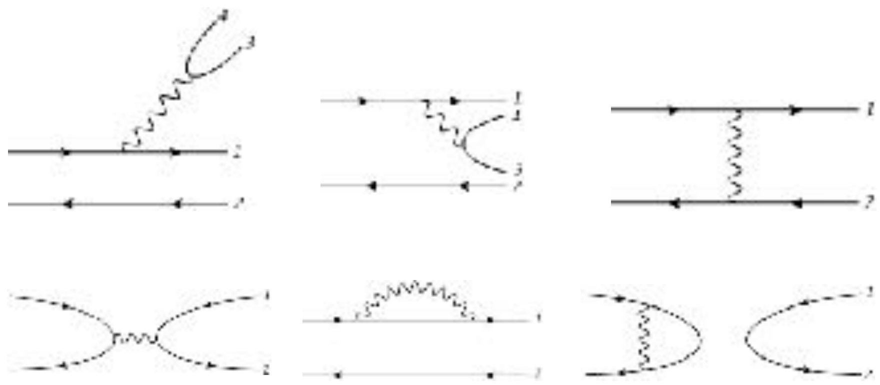
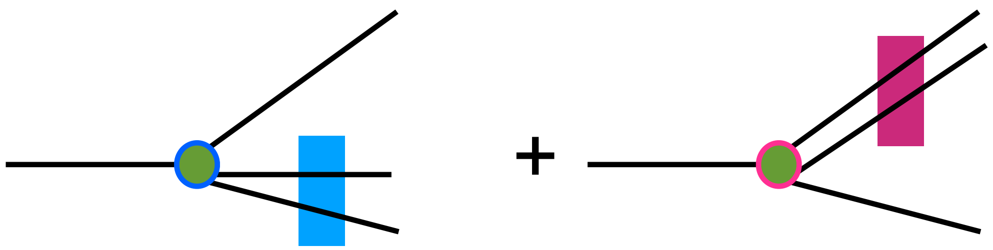
- QCD factorization approach → factorize the quark currents



Chau [Phys. Rep. 95,1(1983)]

challenging for 3-body
not all FSI and 3-body NR
scale issue with charm !



$$\mathcal{H}_{\rm eff}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pq}^* V_{pb} \left[C_1(\mu) O_1^p(\mu) + C_2(\mu) O_2^p(\mu) + \sum_{i=3}^{10} C_i(\mu) O_i(\mu) + C_{7\gamma}(\mu) O_{7\gamma}(\mu) + C_{8g}(\mu) O_{8g}(\mu) \right] + \text{ h.c.} \,,$$

⟶ ex: $B^+ \to \pi^+\pi^-\pi^+$

how to describe it?

$$A \sim \langle \underbrace{[\pi^+(p_2)\pi^-(p_3)]}_{R} \; |(\bar{u}b)_{V-A}|B^-\rangle \, \langle \pi^-(p_1)|(\bar{d}u)_{V-A}|0\rangle + \langle \pi^-(p_1)|(\bar{d}b)_{sc-ps}|B^-\rangle \, \underbrace{\langle [\pi^+(p_2)\pi^-(p_3)] \; |(\bar{d}d)_{sc+ps}|0\rangle}_{FF}$$

- naive factorization
  - intermediate by a resonance R;
  - FSI with scalar and vector form factors FF

↳ parametrizations for B and D→3h Boito et al. PRD96 113003 (2017)

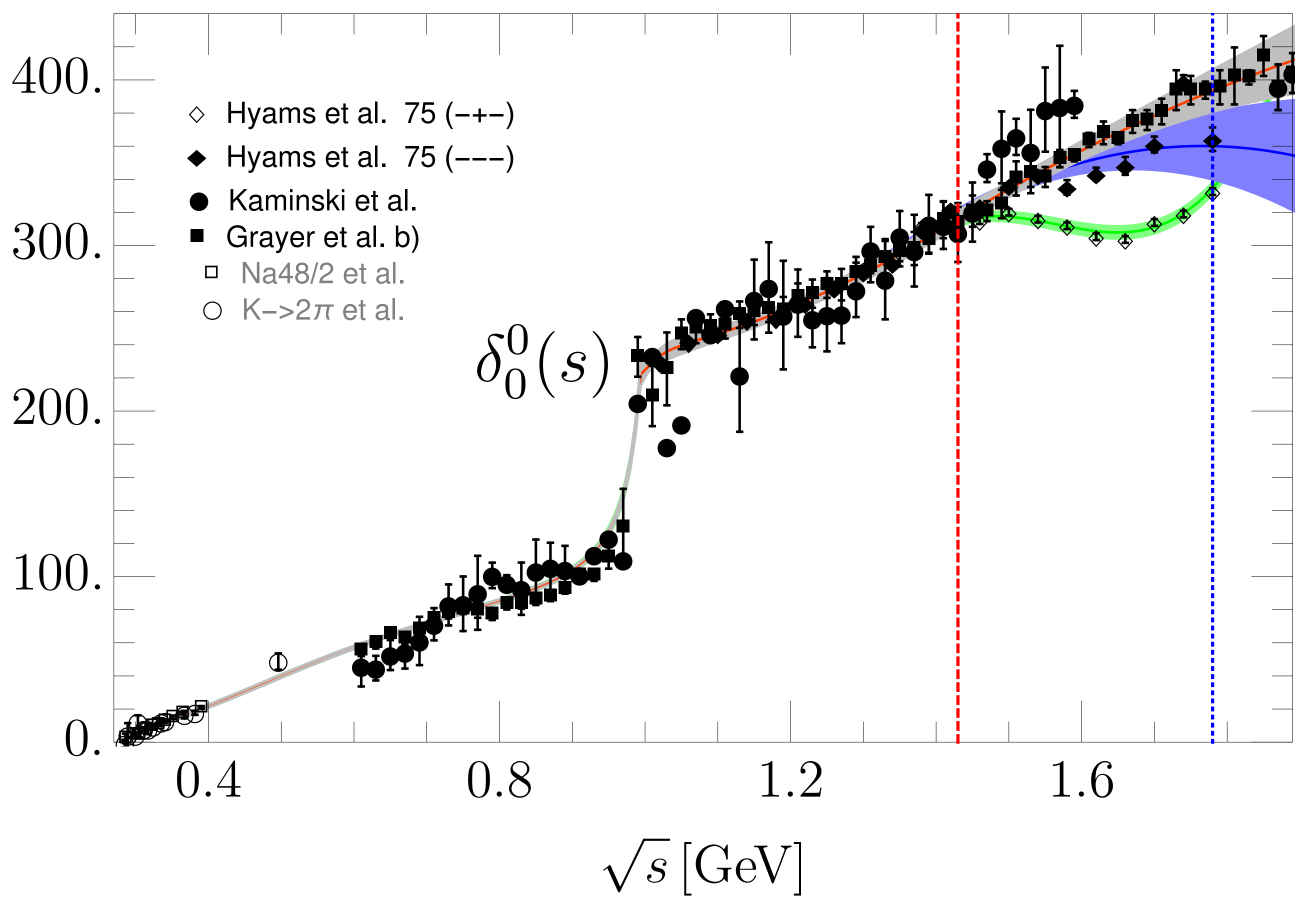
- modern QDC factorization: improvement to include "long distance"

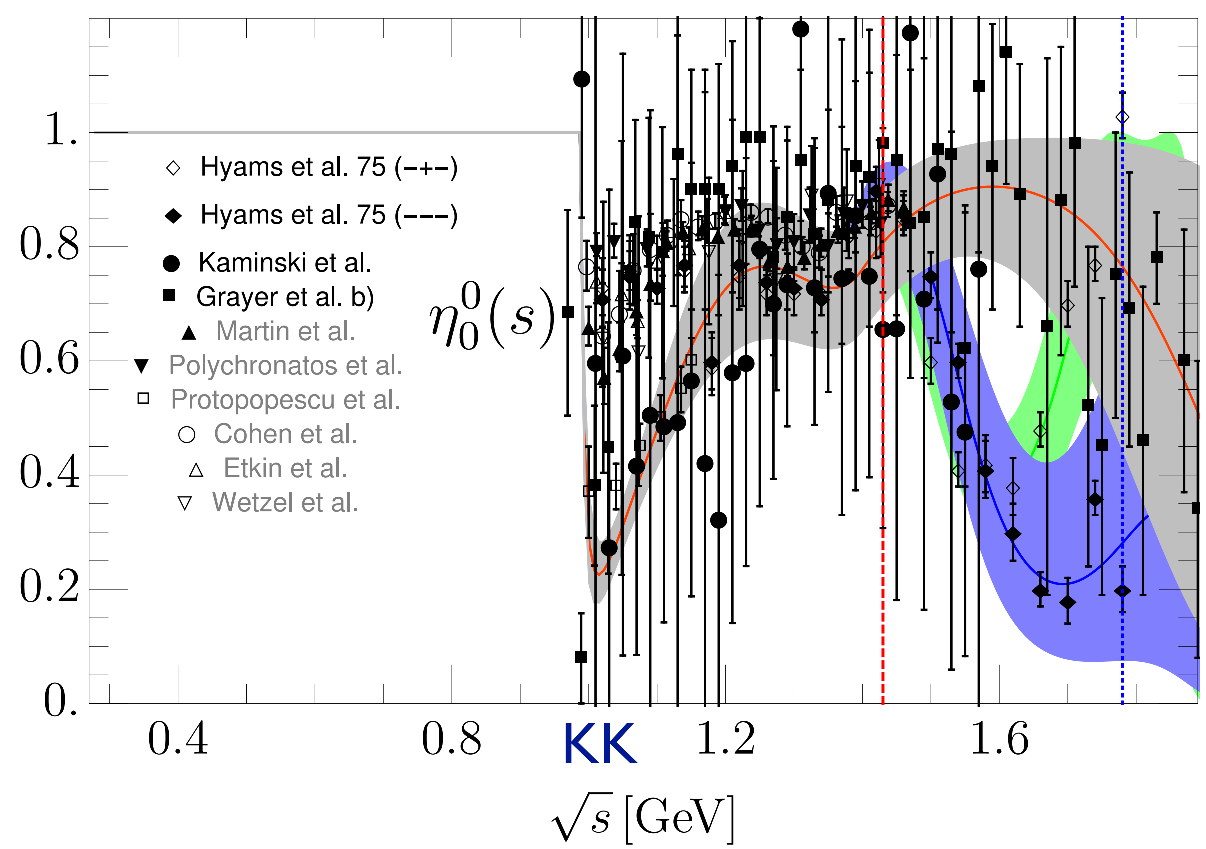
Klein, Mannel, Virto, Keri Vos JHEP10 117 (2017)

- $\pi\pi$ scattering data S-Wave

Pelaez, Rodas, Elvira *Eur.Phys.J.C* 79 (2019) 12, 1008

- amplitude $\hat{f}_l(s) = \left[\frac{\eta_l e^{2i\delta_l} - 1}{2i}\right].$
- elasticity





$$\sigma_l^{\mathrm{el}} = \frac{1}{2}\left\{\frac{1+\eta_l^2}{2} - \eta\cos 2\delta_l\right\},$$

Inelasticity: one minus the probability of losing signal (1=>elastic)

- mixing angle for singlet and octet resonances