Dilaton chiral perturbation theory and applications

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arXiv:1603.04575 PRD94 (2016) 025020

arXiv:1611.04275 PRD95 (2017) 016003

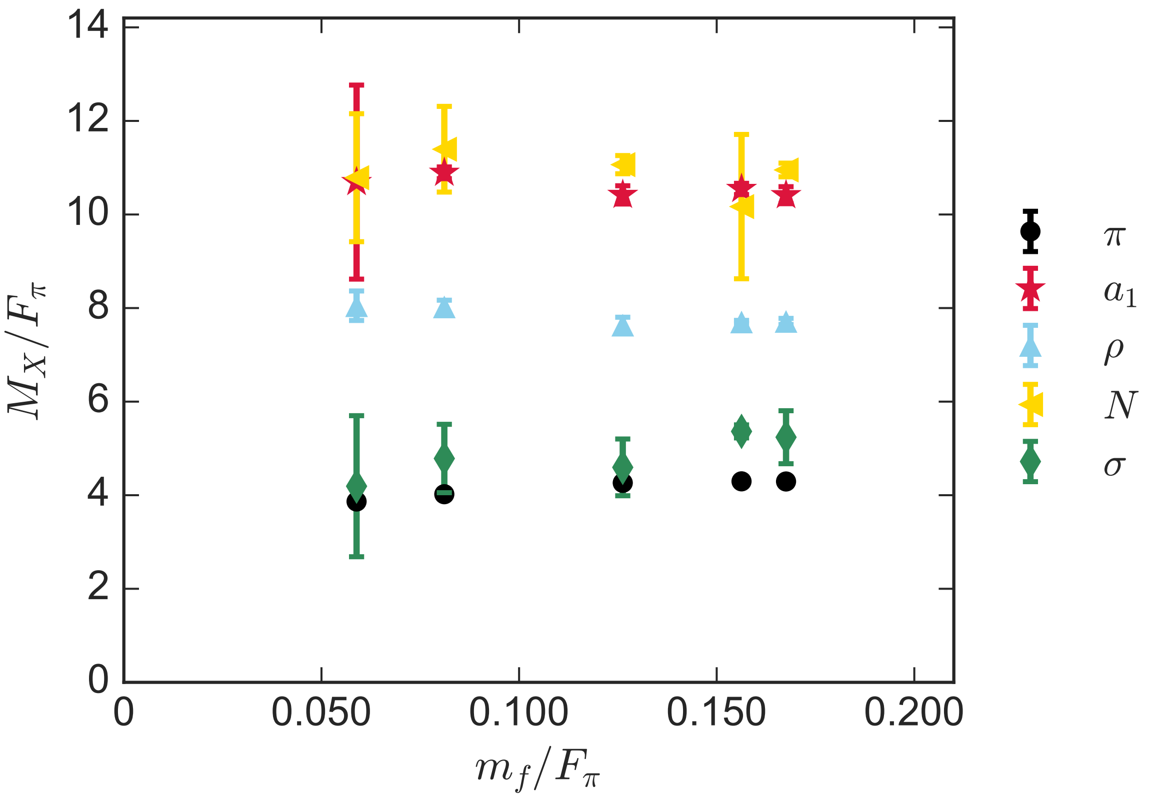
arXiv:1805.00198 PRD98 (2018) 056025

arXiv:1909.10796 PRD100 (2019) 114515
(with Taro Brown, Svend Krøjer, Kim Splittorff)

arXiv:2003.00114 PRD102 (2020) 034515 (with Ethan Neil)

arXiv:2009.13846 PRD102 (2020) 114507

Spectrum of SU(3) with $N_f = 8$ fundamental flavors



LSD collaboration 2018

similar: SU(3), $N_f = 2$ sextets [LatHC]

⇒ Approx. scale symmetry!

(1) Light scalar

$$M_{\text{scalar}} \approx M_\pi$$

(2) Approx. hyperscaling:

$$\frac{M_X}{F_\pi} \sim \text{const.}$$

$$vs.\ \text{QCD}: \quad \frac{M_\pi}{F_\pi} \sim \sqrt{\frac{m}{F_\pi}}$$

(3) Staggered fermion
"taste" splittings

Outline

- SU(N_c) with N_f fundamental flavors has two phases:

  $0 < N_f < N_f^*$ $\cdots\cdots\cdots\cdots\cdots$ Confining & chirally broken

  $N_f^* \le N_f < (11/2)N_c$ $\cdots\cdots$ IR conformal

  "Walking": approximate scale symmetry for $N_f \nearrow N_*$

- Construct effective field theory with pions and a dilaton

  Chiral perturbation theory (pions) + dilaton = dChPT

  Power counting & systematic expansion

- Tree-level lagrangian

  Large-mass regime and hyperscaling

- Fits of dChPT to $N_f = 8$ lattice data (LSD, LatKMI)

- Open questions, directions for future work

Effective Field Theory for pions and a dilatonic meson

∘ Theory contains pions = pseudo Nambu-Goldstone (NG) bosons associated with spontaneous chiral symmetry breaking; become massless for $m \to 0$.

Assumptions:

- Scale invariance gets restored in the infrared for $N_f \nearrow N_f^*$.
- But N_f takes discrete values, unlike m... [Similar problem: η' in large N_c.]

$\Rightarrow$ Make N_f a continuous parameter (Veneziano limit): $N_c, N_f \to \infty$, with $n_f = N_f/N_c$ fixed; also let $n_f^* = \lim_{N_c \to \infty} N_f^*(N_c)/N_c$.

$\Rightarrow$ Theory contains a "dilaton" = pseudo NG boson associated with spontaneous breaking of approx. scale symmetry, which becomes massless in the combined limit $n_f \nearrow n_f^*$ and $m \to 0$.

- Also need some technical assumptions on the dilaton potential

Approximate scale symmetry and power counting

- ◦ Chiral symmetry: $M_\pi^2 = O(m)$ vanishes for $m \to 0$
- Dilatation current $S_\mu = x_\nu T_{\mu\nu}$ satisfies

$$\begin{aligned} \partial_\mu S_\mu &= T_{\mu\mu} = -T_{\rm cl} - T_{\rm an} \\ T_{\rm cl} &= m\,\overline{\psi}\psi \\ T_{\rm an} &= \beta(g^2)/(4g^2)\,G^2 + \gamma_m\,m\,\overline{\psi}\psi \end{aligned}$$

- $T_{\rm an}$ is the trace anomaly [Collins, Duncan & Joglekar, '77]
- We assume: $T_{\rm an}$(ChSB scale) $\sim$ $O(n_f - n_f^*) + O(1/N_c) + O(m)$

$\Rightarrow$ *Systematic* expansion in m, $|n_f - n_f^*|$, $1/N_c$, and p^2

$\Rightarrow$ $M_{\rm dilaton}^2 = O(n_f - n_f^*) + O(m)$ (in Veneziano limit)

Spurions fields, effective fields

- Augment underlying *bare* lagrangian with spurions, using dim. reg.:

$$\mathcal{L}(\sigma,\chi) = e^{(d-4)\sigma}\left(\frac{1}{4}G^2 + \overline{\psi}\not{D}\psi + \overline{\psi}_R\chi^\dagger\psi_L + \overline{\psi}_L\chi\psi_R\right)$$

transforming as

$$\begin{aligned} \chi(x) &\rightarrow \lambda\, g_L\, \chi(\lambda x)\, g_R^\dagger \\ \sigma(x) &\rightarrow \sigma(\lambda x) + \log\lambda \qquad \Leftarrow \end{aligned}$$

(all *bare* fields transform canonically as in 4-dim) so that

$$\mathcal{L}(\sigma(x),\chi(x),\ldots) \rightarrow \lambda^d\, \mathcal{L}(\sigma(\lambda x),\chi(\lambda x),\ldots)$$

Then fix spurions: $\chi(x) = m, \quad \sigma(x) = 0$ and recover explicit breaking.

- Effective field theory: transformations of the dynamical fields

pions: $\Sigma(x) = e^{2i\pi(x)/f_\pi} \rightarrow g_L\,\Sigma(\lambda x)\, g_R^\dagger$

dilaton: $\tau(x) \rightarrow \tau(\lambda x) + \log\lambda \qquad \Leftarrow$

Effective field theory

- Use (*renorm.*) spurions and effective fields to build leading-order EFT

$$
\begin{aligned}
\mathcal{L}^{\mathrm{EFT}} &= \tfrac{1}{4} V_\pi(\tau-\sigma)\, f_\pi^2\, e^{2\tau}\, \mathrm{tr}\big(\partial_\mu \Sigma^\dagger \partial_\mu \Sigma\big) \\
&\quad + \tfrac{1}{2} V_\tau(\tau-\sigma)\, f_\tau^2\, e^{2\tau} (\partial_\mu \tau)^2 \\
&\quad - \tfrac{1}{2} V_m(\tau-\sigma)\, f_\pi^2 B_\pi\, e^{(3-\gamma_*)\tau}\, \mathrm{tr}\big(\chi^\dagger \Sigma + \Sigma^\dagger \chi\big) \\
&\quad + V_d(\tau-\sigma)\, f_\tau^2 B_\tau\, e^{4\tau}
\end{aligned}
$$

- γ_* = mass anomalous dimension at IRFP for $n_f = n_f^*$ $\qquad (m \to \lambda^{1+\gamma_*} m)$
- Invariant potentials: $V(\tau(x) - \sigma(x)) \;\to\; V(\tau(\lambda x) - \sigma(\lambda x))$

The $V(\tau - \sigma)$ potentials are arbitrary functions of their argument

hence <u>no predictability</u> without a power counting for them!

[Unlike chiral limit $\chi = m = 0$, scale symmetry not restored for $\sigma = 0$.]

[Similar: in large-N_c ChPT encounter potentials $V(\eta' - \theta)$.]

Power counting hierarchy from matching

- *Renorm.* mic. theory $\mathcal{L}^{\rm MIC}(\sigma, \chi) = \mathcal{L}^{\rm MIC}(0, \chi) - \sigma T_{\rm an}(\chi) + O(\sigma^2)$

$$\left.\frac{\partial \mathcal{L}^{\rm MIC}}{\partial \sigma(x)}\right|_{\sigma=\chi=0} = \left. T_{\rm an}(x)\right|_{\chi=0} = \frac{\beta(g^2)}{4g^2}\left[G^2(x)\right] \sim |n_f - n_f^*|$$

$\Rightarrow$ $\partial^n/\partial\sigma^n \sim |n_f - n_f^*|^n$

- EFT: $$\left.\frac{(-1)^n}{n!}\,\frac{\partial^n V(\tau - \sigma)}{\partial \sigma^n}\right|_{\sigma=0} = \boxed{c_n} + (n+1)c_{n+1}\tau + \cdots$$

Hence $$V(\tau - \sigma) = \sum_{n=0}^{\infty} c_n (\tau - \sigma)^n \qquad \text{with} \quad c_n = O\big(|n_f - n_f^*|^n\big)$$

$\Rightarrow$ Only a finite number of low-energy constants at each order!

Leading-order dilaton chiral perturbation theory (dilaton-ChPT)

$$
\begin{aligned}
\mathcal{L}^{\rm EFT} \;=\; & \tfrac{1}{4} f_\pi^2\, e^{2\tau}\, \boxed{\mathrm{tr}\big(\partial_\mu\Sigma^\dagger\partial_\mu\Sigma\big)} \\
& + \tfrac{1}{2} f_\tau^2\, e^{2\tau} (\partial_\mu\tau)^2 \\
& - \tfrac{1}{2} f_\pi^2 B_\pi m\, e^{(3-\gamma_*)\tau}\, \boxed{\mathrm{tr}\big(\Sigma+\Sigma^\dagger\big)} \\
& + f_\tau^2 B_\tau\, e^{4\tau}\, c_1(\tau - 1/4)
\end{aligned}
$$

- Spurions: set $\sigma = 0$, fermion mass $\langle\chi\rangle = m$
- Assumed small: $p^2 \sim m \sim c_1 \propto |n_f - n_f^*|$
- Can set $V_\pi = V_\tau = V_m = 1$
- Shift τ so that $V_d = c_0 + c_1\tau = c_1(\tau - 1/4)$ [redefine LECs $f_{\pi,\tau}, B_{\pi,\tau}$]

$\Rightarrow$ classical vacuum $v = \langle\tau\rangle$ is $v(m) = 0$ for $m = 0$

Leading order predictions

- Minimize potential: $\frac{m}{c_1 \mathcal{M}} = v e^{(1+\gamma_*)v}$ $\mathcal{M} = \frac{4 f_\tau^2 B_\tau}{f_\pi^2 B_\pi N_f (3-\gamma_*)}$
- Pion mass: $M_\pi^2 = 2B_\pi m e^{(1-\gamma_*)v} = 2B_\pi \mathcal{M}\, c_1 v\, e^{2v}$
- Dilaton mass: $M_\tau^2 = 4B_\tau c_1 e^{2v}\big(1 + (1+\gamma_*)v\big)$
- Decay constants: $F_{\pi,\tau} = f_{\pi,\tau}\, e^{v}$
- Other hadron masses: $M_{\rm h} = M_0\, e^{v}$

Ratio $\frac{m}{c_1} = O(1)$ parametrically, but can be large or small

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Chiral (small-mass) regime: $\frac{m}{c_1\mathcal{M}} \ll 1 \quad \Rightarrow \quad v \propto m$ small, $e^v \approx 1$

Pion mass: $M_\pi^2 = 2B_\pi m$

Dilaton mass: $M_\tau^2 = 4B_\tau c_1 \qquad \propto |n_f - n_f^*|$

Large-mass regime: $\frac{m}{c_1\mathcal{M}} \gg 1$

- Neglect v compared to e^v

$$\frac{m}{c_1\mathcal{M}} = v e^{(1+\gamma_*)v} \quad \Rightarrow \quad e^{v(m)} \sim \left(\frac{m}{c_1\mathcal{M}}\right)^{\frac{1}{1+\gamma_*}}$$

- Approx. hyperscaling: $M_\pi \sim M_\tau \sim F_\pi \sim F_\tau \sim M_{\rm h} \sim \cdots \sim m^{1/(1+\gamma_*)}$

  Mass dominates (slow!) running — like a mass deformed conformal theory!

- pNG bosons still lighter: $\frac{M_\pi}{M_{\rm h}} \sim \frac{M_\tau}{M_{\rm h}} \sim c_1 v(m) \propto |n_f - n_f^*| v(m)$

- Loop-expansion parameter: $\frac{M_\pi^2}{(4\pi F_\pi)^2} \sim c_1 v(m) \sim c_1 \log\left(\frac{m}{c_1\mathcal{M}}\right)$

$\Rightarrow$ Still systematic expansion provided $c_1 \log\left(m/(c_1\mathcal{M})\right) \ll 1$

By contrast: $m/\mathcal{M} \ll 1$ required in ordinary ChPT!

Fitting data

- Use exact tree-level expressions (hyperscaling + corrections)
- Single bare coupling $\Rightarrow$ single lattice spacing a (independent of m)
- Basic fit ($W_0 =$ Lambert function):

$$\frac{M_\pi^2}{F_\pi^2} = \frac{v(m)}{d_1} \equiv \frac{1}{(1+\gamma_*)d_1} W_0\left(\frac{(1+\gamma_*)d_1}{d_2}\, m\right)$$

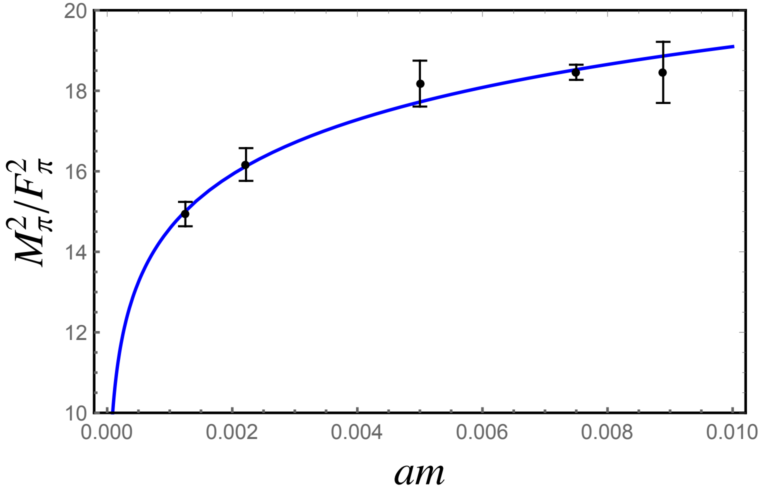
$$aF_\pi = af_\pi\, e^{v(m)}$$

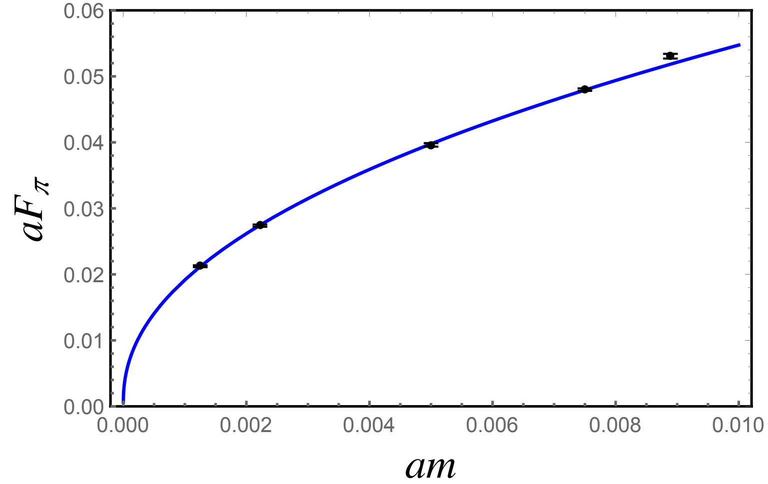
$$\frac{M_\tau^2}{F_\pi^2} = d_3\big(1 + (1+\gamma_*)\, v(m)\big)$$

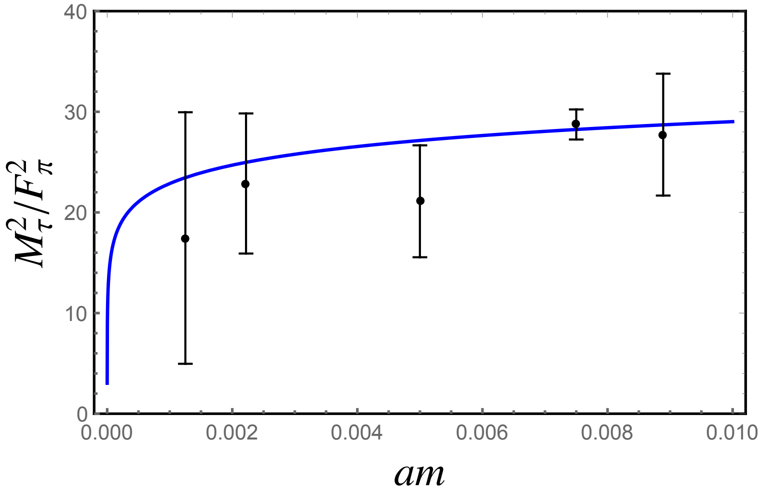
- combinations of tree level parameters: d_1, d_2, d_3

Fits to $N_f = 8$ data from LSD collaboration

PRD99 (2019) 014509







5 Mass values:

$0.00125 \leq am \leq 0.00889$

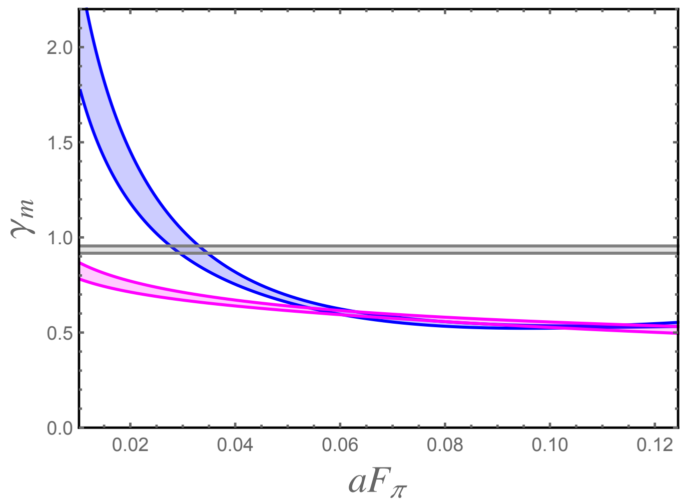
Shown: 4-ensemble fit

Main result: $\gamma_* = 0.94(2)$

Fits to $N_f = 8$ data from LatKMI collaboration PRD96 (2017) 014508

- Same theory, different lattice action, different (bare) coupling
  different mass range: $0.012 \leq am \leq 0.1$

- Need N(N?)LO dChPT — too many parameters for limited data!
  Instead: model an m-dependent mass anomalous dimension:
  $\gamma(m) = \gamma_0 - bv(m) + cv(m)^2$
  Still satisfies anomalous Ward–Takahashi identity for scale invariance



Good description of data

Gray band: LSD value $\gamma_* = 0.94(2)$

Magenta band: $c = 0$, eight masses

Blue band: $c \neq 0$, nine masses

[KMI data: $0.045 \leq aF_\pi \leq 0.12$]

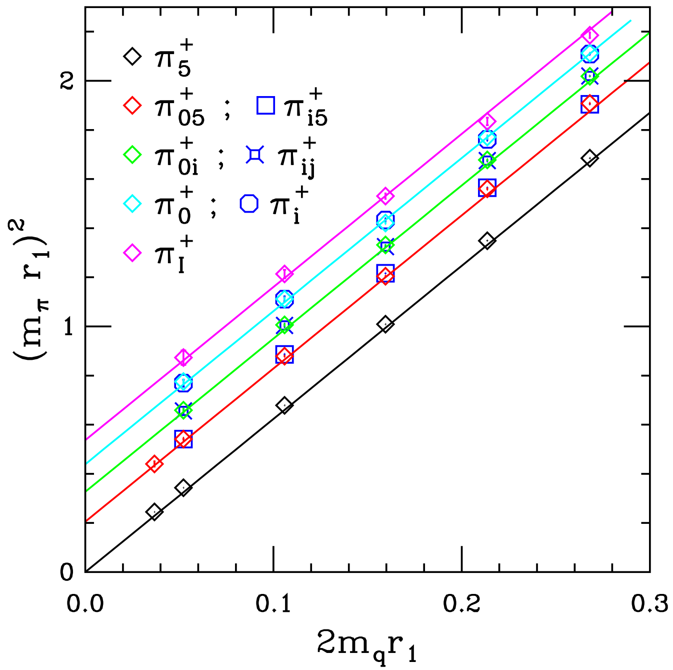
Staggered fermions "taste" splittings (LatKMI data)

Staggered fermion = 4 quarks with remnant of flavor ("taste") symmetry

Pions: $\pi_\Gamma = \bar{q}\gamma_5\Gamma\, q$ where $\Gamma \Rightarrow P$ (exact NGB), A, T, V, S

Taste splittings from 4-fermi operators $\sim a^2(\bar{q}\,\Gamma\, q)(\bar{q}\,\Gamma\, q)$

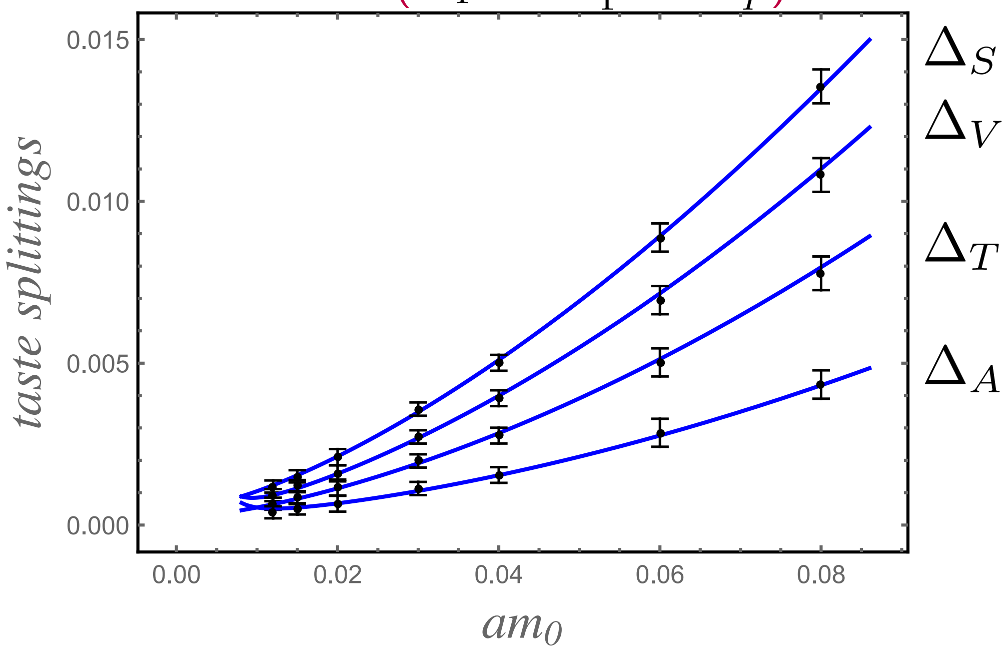
MILC 0903.3598



QCD: $M_\Gamma^2 = Bm + c_\Gamma a^2$

($c_\Gamma = 0$ for NG pion)

LatKMI ($\Delta_\Gamma = M_\Gamma^2 - M_P^2$)



$N_f = 8$: tree-level splittings depend on m through $e^{v(m)}$

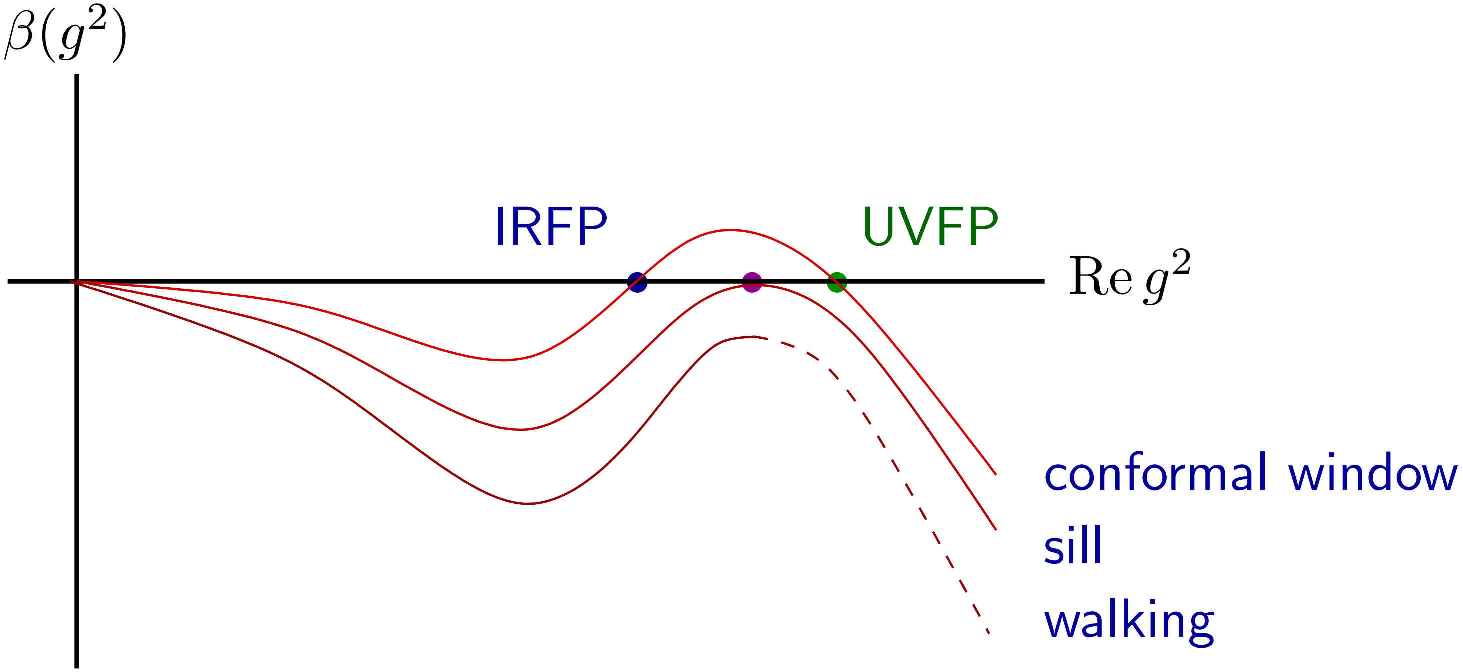
Open questions

The chiral limit

- Chiral limit pion decay constant: $af_\pi = 0.0006(3)$
  vs. values in LSD simulation: $0.02 \lesssim aF_\pi(m) \lesssim 0.06$.
- $F_\pi(m)L \gtrsim 1$ in simulation. But having $f_\pi L \gtrsim 1$ in chiral limit requires unrealistically large lattices. (Recall: $F_\pi(m) = f_\pi e^{v(m)}$.)
- Present day $N_f = 8$ simulations are deep in the large-mass regime.
  Very long extrapolation to the chiral limit.
- Explains why it is so hard to distinguish an infrared conformal theory from a chirally broken (confining) theory with "walking" coupling.
- Result may depend on higher orders in dChPT and mass range in the fit.
- Actually, it's hard to settle whether af_π and c_1 are non-zero!
- Does dChPT also–effectively–apply inside the conformal window?

What sets the sill of CW? Colliding FP scenario

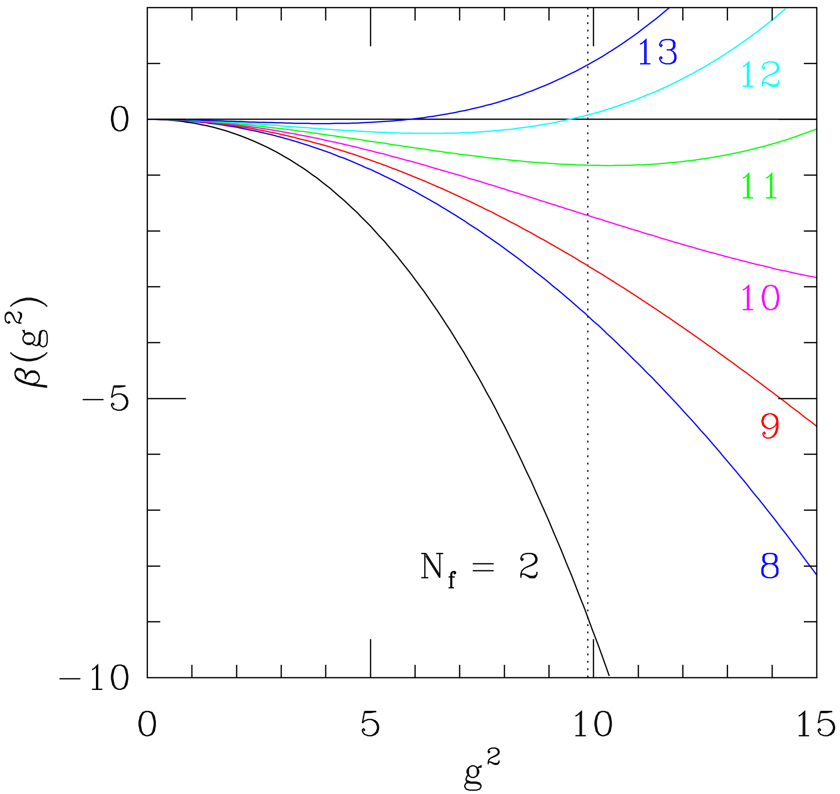
Inside the conformal window: IRFP followed by UVFP.
At the sill the FPs collide and move off into the complex (coupling) plane.



Q: Once hadrons form, can we make sense of the β function at scales $\mu \ll m_{\text{constituent}}$?

Kaplan, Lee, Son & Stephanov, '09
Gorbenko, Rychkov & Zan, '18
Pomarol, Pujolas & Salas, '19

What sets the sill of CW? Chiral symmetry breaking scenario



- Running slows down for increasing N_f

$$\frac{\partial g^2}{\partial \log \mu} = -\frac{b_1}{16\pi^2} g^4 - \frac{b_2}{(16\pi^2)^2} g^6$$

When $b_1 > 0 > b_2$, 2-loop IRFP g_*

- Walking gap equation: ChSB when $g^2(\mu)$ reaches the critical coupling

$$g_c^2 = \frac{4\pi^2}{3C_2} = \pi^2 \quad \text{for SU(3)}$$

- $g_*(N_f) > g_c$: chirally broken
  $g_*(N_f) < g_c$: conformal window

- $\beta(g_c) \to 0$ for $N_f \to N_f^*$
  sill of conformal window: $g_*(N_f^*) = g_c$

Is QCD near conformal?

- Is the σ resonance a Dilaton?

Caprini, Colangelo, Leutwyler
PRL 96 (2006) 132001

$$M_\sigma = 441^{+16}_{-8}\ \mathrm{MeV}, \quad \Gamma_\sigma = 544^{+18}_{-25}\ \mathrm{MeV}$$

Let's assume that QCD is close to the conformal sill $\Rightarrow$

- $\tau \to \pi\pi$ in tree-level dChPT

$$\Gamma_{\tau\to\pi\pi} \; = \; \frac{1}{32\pi}\frac{N_f^2-1}{M_\tau F_\tau^2}\sqrt{1-\frac{4M_\pi^2}{M_\tau^2}}\left(M_\tau^2+(1-\gamma_*)M_\pi^2\right)^2$$

- Use M_π, F_π and $M_\tau = M_\sigma$ from QCD. Use $N_f = 2$, estimate γ_* and F_π/F_τ from $N_f = 8$ (we're assuming $N_f - N_f^*$ corrections are small!)

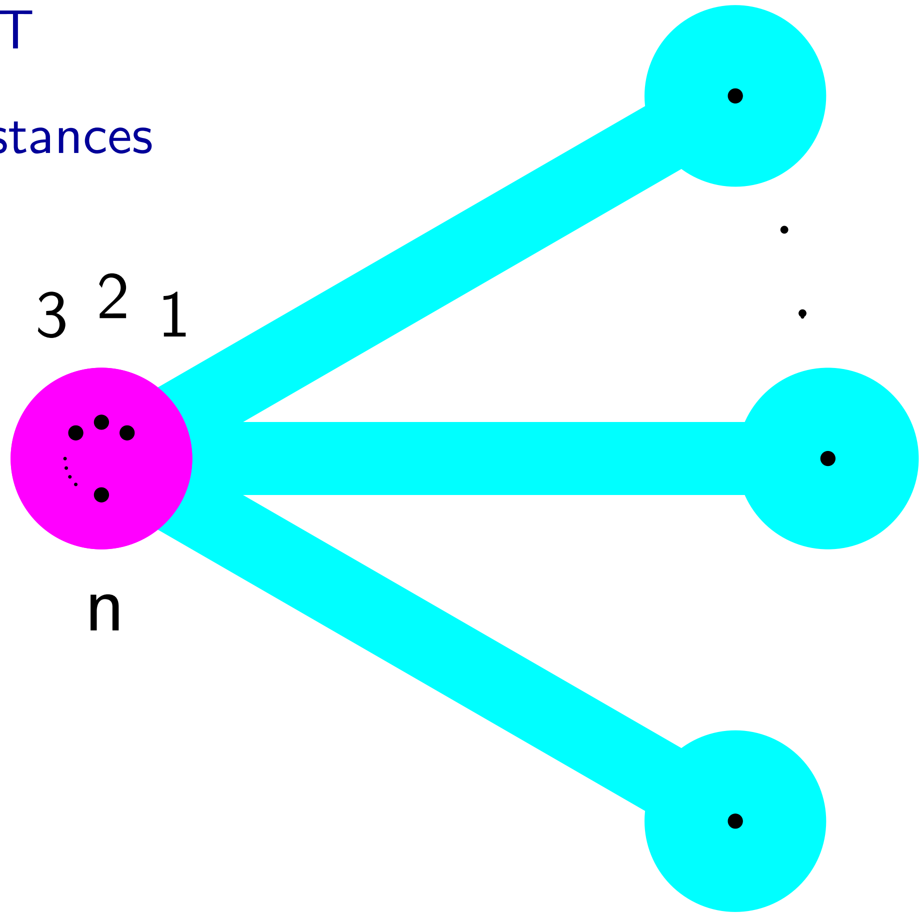
$$\Gamma_{\sigma\to\pi\pi} \; \simeq \; 240\left(\frac{F_\pi}{F_\tau}\right)^2 \ \mathrm{MeV} \; < \; 60\ \mathrm{MeV}$$

$\Rightarrow$ About a factor 10 too small!

Thank you

Matching – role of non-coinciding points

- Recall $p^2 \ll$ meson size
- Magenta: points at distances $\ll$ meson size collapse to a single point in the EFT
- Cyan: points at asympt. large distances



$\Rightarrow \quad c_n = O(|n_f - n_f^*|^n)$

$$V(\tau - \sigma) \;=\; \sum_{n=0}^{\infty} c_n (\tau - \sigma)^n$$

Other approaches

- Let $\Phi = e^{\tau}$, so that $V_d \propto \Phi^4(\log\Phi - 1/4)$
  Recall this is the tree-level potential dictated by dChPT power counting.

- Appelquist, Ingoldby & Piai: Consider instead $V_\Delta \propto \frac{\Phi^4}{4-\Delta}\left(1 - \frac{4}{\Delta}\,\Phi^{(\Delta-4)}\right)$.
  $\Rightarrow$ No power counting for Δ not close to 4!
  ($\Delta \to 4$ is dChPT, $\Delta = 2$ is σ-model)

- p-regime fits ignoring taste splittings work for both $V_d = V_{\Delta\to4}$ and $V_{\Delta=2}$,
  in both $N_f = 8$ and sextet theories.

- ϵ-regime study of sextet model (LatHC coll., PoS LATTICE2019)
  finds chiral-limit condensate in agreement with $V_{\Delta=2}$.
  Note, however, that dChPT makes no predictions for this study.