

Higgsless simulations of gravitational wave in 1st-order phase transitions

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Before anything else, cool things always come with videos

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Notice two scales:

- Initially we see mainly the bubble size
- Later we see sound shells superposing

A message to take home...

New simulation scheme for sound-shell contribution in 1st order PT

Advantages:

- 1) fast (easier to explore parameter dependence);
- Don't need to include scalar field (solve particle physics scale in a cosmo simulation);
- 3) Incorporate shock front easily.
- 4) We can go non-linear!!!
- 5) Exploring alternative scenarios

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A user-friendly parametrization...

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$$q_l \simeq 1, \qquad q_h \simeq 1/\xi_{\text{shell}},$$



Outline of this talk

- Prerequisites of 1st order PT
- Gravitational waves
- Simulations
- Temperature fluctuations

Motivations for 1st Order PT

1) LISA is flying in the next decade



2) Electroweak Baryogenesis



Konstandin (13)

3) BSM physics



Breitbach+ (18)

1st Order PT



Scalar potential in thermal bath gets temperature corrections

$$V(\phi, T) = \frac{1}{2}M^{2}(T)\phi^{2} - \frac{1}{3}\delta(T)\phi^{3} + \frac{1}{4}\lambda\phi^{4}$$

At high temperatures we expect electroweak sym. to be restored. As temperature goes down higgs gets a vev and gives mass to particles

1st Order PT



Disjoint regions of space can make the transition, generating bubbles at different places

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The (three) GW frequency ranges



Low: We probe this region using Pulsar Timing Array (PTA). O(> 10^7 solar masses)

Intermediate: Supermassive BHs O(10^3-10^7 solar masses). One of the most interesting regions to probe the early Universe.

High: O(1-100) solar masses objects

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1st Order Phase Transition (PT)



3 sources:

Scalar field

Sound waves

Turbulence

1st Order Phase Transition (PT)





GW from 1st Order PT -- State of the art

Envelope approximation





Konstandin, Huber (08)

Energy contained in a thin non-collided yet shell (fluid or scalar)

See also Kamionkowski, Kosowsky, Turner (94) and Jinno, Takimoto (17a)

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Energy contained in a thin non-collided yet shell (fluid or scalar) Latter, it became clear that the sound shell contribution is larger than the envelope

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Enhanced by $\left(\frac{\beta}{H_*} \right)$

since they propagate for long time Hindmarsh, Huber, Rummukainen, Weir (13)

Difficult for bubbles to runaway, coupling to the plasma (Bodeker and Moore, 17)

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1st Order Phase Transition (PT)



(Caprini et al, 15)

3 sources:

Scalar field

Sound waves

Relevant when we talk about PTs in a plasma

Working flow



Working flow



How we can connect a Lagrangian to macroscopic properties of plasma+wall

- lpha PT strength
- β Bubble nucleation time scale (and bubble size)
- v_w Bubble wall velocity

$$T_{st}$$
 Temperature of PT

Working flow

Prediction of GW (analytics, simulations)



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For how long do sound waves propagate?

Turbulence

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Prediction of GW (analytics, simulations) Noise

Sensitivity

Foregrounds



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How we can connect a Lagrangian to macroscopic properties of plasma+wall

- lpha PT strength
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 v_w Bubble wall velocity

Non-linear dynamics are ultra-relevant!

For how long do sound

waves propagate?

- GW Templates

Turbulence

- Turbulence
- Strong PTs

 T_{st} Temperature of PT

Analytical prediction

Sound shell Hindmarsh (16) +Hijazi (19) model $\frac{d\Omega_{\rm GW}(k)}{d\ln(k)} \sim \begin{cases} (kR_*)^5, & k\Delta R_*, kR_* \ll 1, \\ (kR_*)^1, & k\Delta R_* \ll 1 \ll kR_*, \\ (kR_*)^{-3}, & 1 \ll k\Delta R_*, R_*. \end{cases}$

Assumes:

- Gaussianity of velocities
- Linear superposition of sound shells
- Hard to extrapolate to stronger PTs

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Simulations (two schemes)

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Scalar field + Fluid, Hindmarsh, Huber, Rummukainen, Weir (13,15,17)

Phenomenological friction term between between scalar and fluid

Model-dependence when assuming free energy for the scalar and T-dependence of friction

No separation of scales (scalar shell and bubble size)

Higgsless, Jinno, Konstandin, HR, Stromberg

Constant wall velocity + Bag equation of state

Model only the fluid part

Clear separation of scales

Lattice simulations: Higgsless

Bag Equation of State

Espinosa, Konstandin, No, Servant (10)



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Velocity profile is given by the self-similar equation in coordinates $\xi = r/t$

$$2\frac{v}{\xi} = \gamma^2 (1 - v\xi) \left[\frac{\mu^2}{c_s^2} - 1\right] \partial_{\xi} v$$

Lattice simulations: Higgsless

Bag Equation of State

Espinosa, Konstandin, No, Servant (10)



Henrique Rubira How our simulation works:

Lattice simulations: Higgsless

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 $\partial^{\mu}T_{\mu\nu} = \partial^{\mu}T^{\phi}_{\mu\nu} + \partial^{\mu}T^{\text{plasma}}_{\mu\nu} = 0$ Enters via a time dependent energy background $\epsilon(\vec{x}, t)$ You can spend all your computer energy solving the

fluid part!

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 $\partial^{\mu}T^{\mathrm{plasma}}_{\mu
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Lattice simulations: Higgsless

 $\partial^{\mu}T_{\mu
u}$ = Bag Equation of State Espinosa, Konstandin, No, Servant (10) Enters via a time dependent $\epsilon(ec{x},t)$ $p_{+} = \frac{1}{3}a_{+}T_{+}^{4} - \epsilon$ Higgs is $p_{-} = \frac{1}{3}a_{-}T_{-}^{4}$ You can spend all your computer energy solving the incorporated as a fluid part! boundary condition The scales of the problem: Bubble thickness: TeV Velocity profile is given by the self-similar equation in coordinates $\xi = r/t$ Sound Shell $2\frac{v}{\xi} = \gamma^2 (1 - v\xi) \left| \frac{\mu^2}{c^2} - 1 \right| \partial_{\xi} v$ Bubble size: Hubble size

Lattice simulations: Higgsless

Can we reproduce the correct result with one single bubble?

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Solving shocks

Solving shocks

$$\partial_t u + \partial_x f(u) = 0$$

Simples scheme, Lax Friedrichs (LF)

$$u_j^{n+1} = \frac{u_{j+1}^n + u_{j-1}^n}{2} - \frac{\lambda}{2} \left[f(u_{j+1}^n) - f(u_{j-1}^n) \right] - g_j^n \Delta t$$



Solving shocks

Henrique Rubira Gudunov's theorem: Linear scheme can only respect positivity at first order

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Solution 1: Riemann solvers or Godunov schemes Solve exact linearized scheme, diagonalize matrix locally

Solution 2: Hybridization Introduce non-linear terms via 'flux limiters'



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Kurganov - Tadmor (KT) scheme $~~\partial_\mu T^{\mu
u}=0$ for conservative schemes







 q/β



Henrique Rubira









A double-broken power law

$$S_f(q) = S_0 \times \frac{(q/q_0)^3}{1 + (q/q_0)^2 [1 + (q/q_1)^4]} \times e^{-(q/q_e)^2}$$

Position of the peaks





A double-broken power law

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Position of the peaks

Amplitude of the spectrum





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Position of the peaks

Amplitude of the spectrum



Comparison to other works

We find very similar GW spectra when comparing weak and intermediate transitions

1000x faster! (Fluid only + second-order discretization scheme)

More bubbles -- O(2500)

Realistic nucleation (not simultaneous)

Now we can move to non-linear fluid dynamics! Easy to go to strong PTs and to explore turbulence

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Is it the end of the story connecting observed spectra to sourced spectra?

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Domcke, Jinno, HR; 19

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We know that temperature fluctuations in the line-of-sight can affect the GW spectrum



More videos...





Can temperature fluctuations also affect the sourcing of GWs from 1st order PTs?

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Nucleation rate proportional to the 3d bounce action:

.

$$\Gamma \propto e^{-S_3/T}$$

After expanding it around
$$t = t_*$$

$$\Gamma = \Gamma_* \exp\left[\beta(t - t_*) - \frac{\beta}{H_*} \frac{\delta T}{\overline{T}}\right]$$
This next term in the expansion may be large if $\frac{\delta T}{T} \sim \left(\frac{\beta}{H_*}\right)^{-1}$

expansion may be large if

$$\sim \left(\frac{\beta}{H_*}\right)$$

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Nucleation rate proportional to the 3d bounce action:

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What is the effect in the GW spectra?

We simulate bubble nucleation under temperature fluctuations and plug it into the hybrid simulation



Result: increase the bubbles size and therefore enhance GW spectra

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We also parametrized how spectra depends on temperature fluctuations



Conclusions

- We have a fast and precise scheme to calculate GWs from PT that can explore non-linear PTs also alternative scenarios (PBHs, topological defects, ...)

What comes next?

- Deep into non-linear PTs
- Topological defects
- Turbulence
- Parameter extraction
- Template building



Extra slide: parametrizing the spectra

$$k_{\text{ave}} \equiv \int d\ln k \, k \, Q'(k) \Big/ \int d\ln k \, Q'(k),$$
$$Q'_{\text{int}} \equiv \int d\ln k \, Q'(k).$$



Extra slide: deep IR, IR and UV

Deep IR: temperature is uniform, no difference



Only IR affects the spectra



In the limit in which $k_* \to \infty$, it does not affect the spectra since any volume has many hot and cold spots

Extra slide: the algorithm

Instead of letting bubbles nucleate linearly distributed in space and (exp) time, we consider the cumulative probability calculated from the Temperature grid

