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Maria Zambrano grants to attract international talent

GOBIERNO DE ESPAÑA

CP violation in heavy hadrons decay

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seminar at IFAE Institut de Física d'Altes Energies 20/12/2022

Outline

- CPV introduction/motivation

- hadronic heavy meson decay and FSI

- FSI as source of CPV in B decays

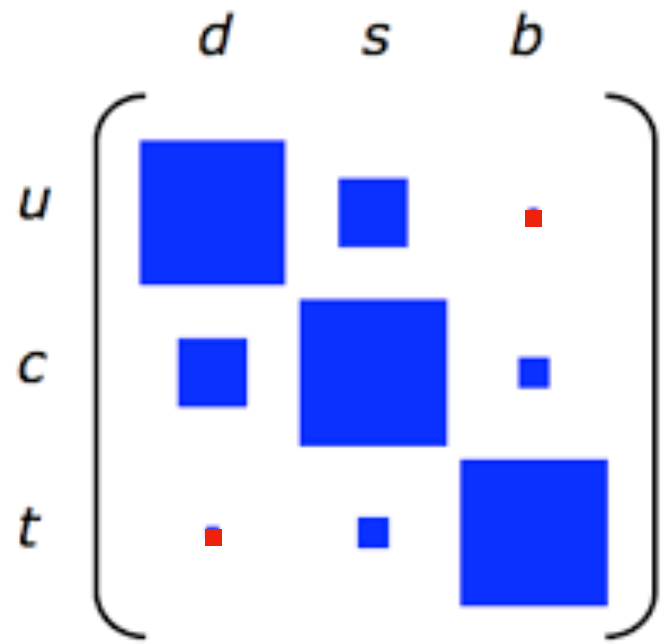
- understanding CPV in D decays

- final remarks

Many open questions in Particle physics

- Standard Model for all its success can not explain everything

Quark mixing matrix



→ No Dark matter candidate

→ Matter anti-matter asymmetry in the Universe

(need new CP violation source)

- development of powerful tools

→ theory and experiment

CP asymmetry measurements

1956
Parity violation
T.D. Lee, C.N. Yang, C.S. Wu et al.

1963
Cabibbo Mixing
N. Cabibbo

1964
Strange particles: CPV in K meson decays
J.W. Cronin, V.L Fitch et al.

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CKM matrix
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2001
Beauty particles: CPV in B^0 meson decays
Babar and Belle

2019
Charm particle: CPV in D^0 meson decays
LHCb

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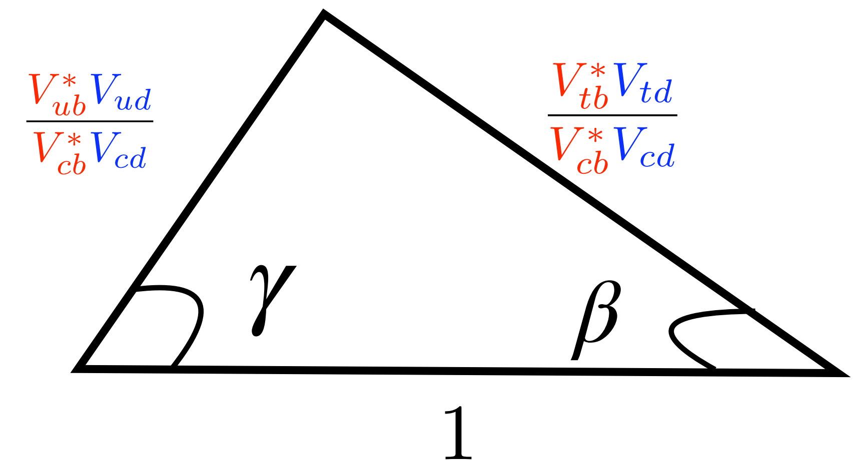
LHCb

2022
1st obs of CPV in $D^0 \to \pi\pi$
LHCb

today

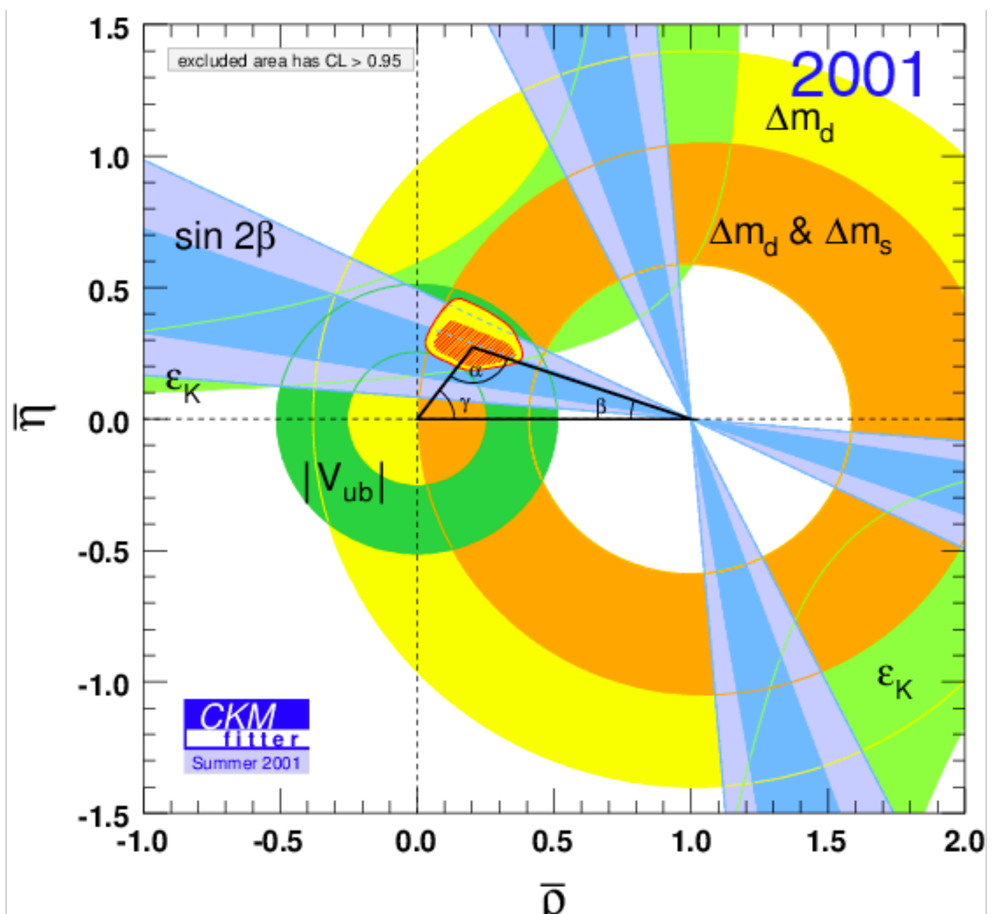
Unitary CKM matrix

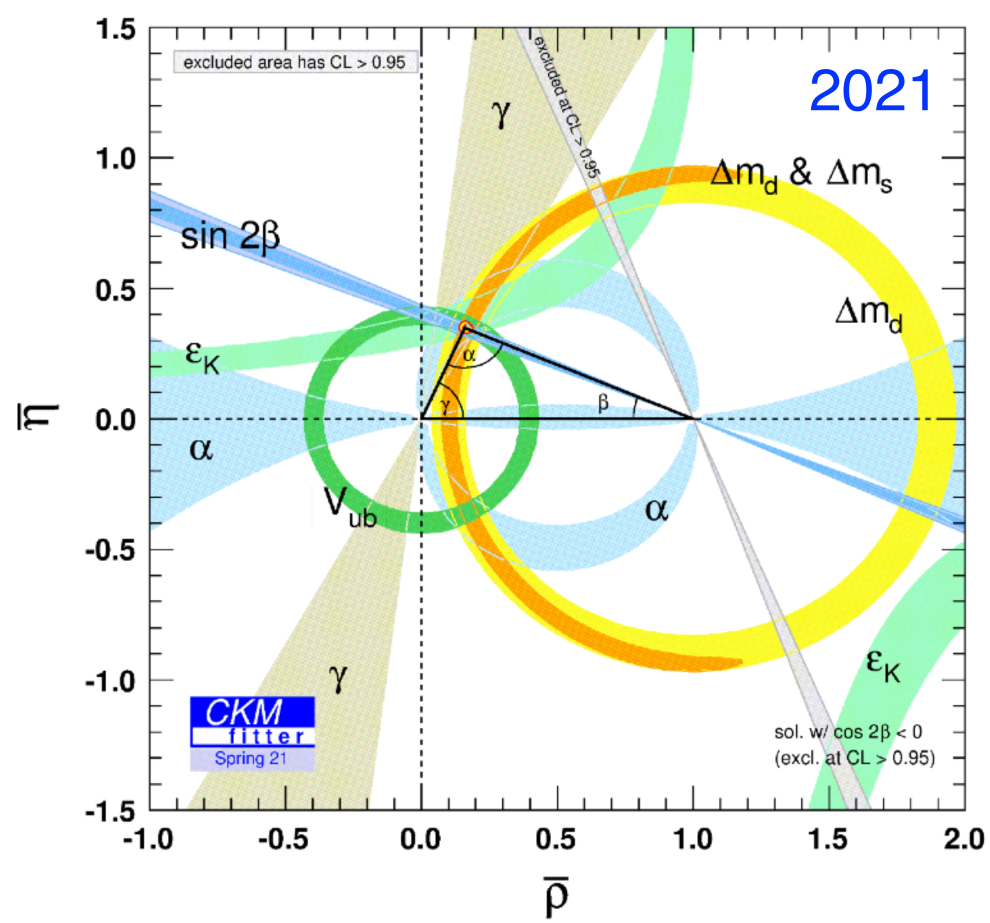
- $$V_{\rm CKM} = \begin{array}{c c} & \begin{array}{ccc} d & s & b \end{array} \\ \begin{array}{c} u \\ c \\ t \end{array} & \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \end{array} + \mathcal{O}(\lambda^4)$$



  - loads of CPV expected/found in B decays and not much in Kaon or D

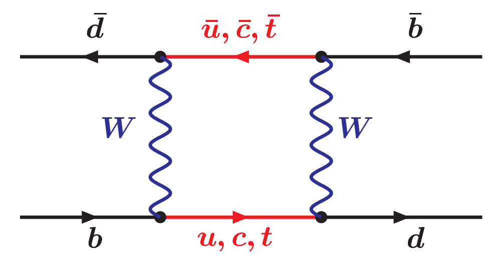
- test Standard model limits

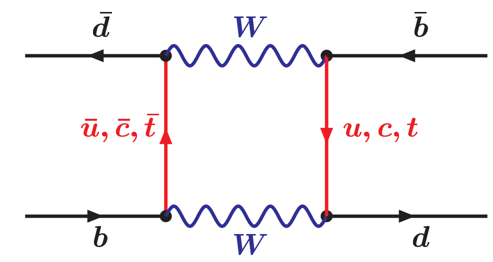


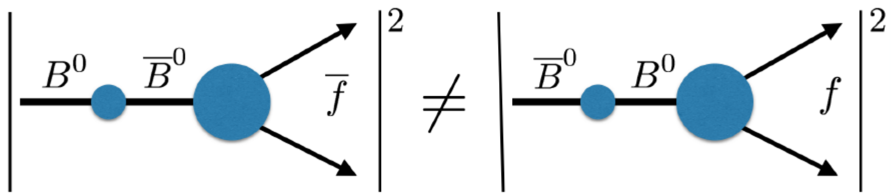


CPV in neutral particles

- mixing





  - 

$$B^0 \to f \nleftarrow \overline{B}^0$$

$$B^0 \nrightarrow \bar{f} \leftarrow \overline{B}^0$$

→ mass eigenstates: $B_1 = p|B^0\rangle + q|\overline{B}^0\rangle$

$B_2 = p|B^0\rangle - q|\overline{B}^0\rangle$

$$\left|\frac{q}{p}\right| \neq 1$$

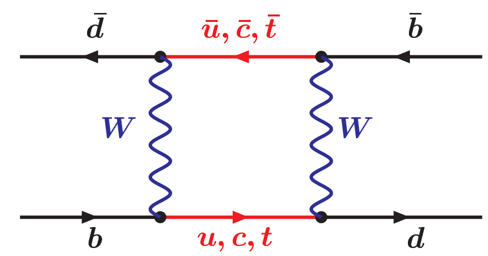
CP

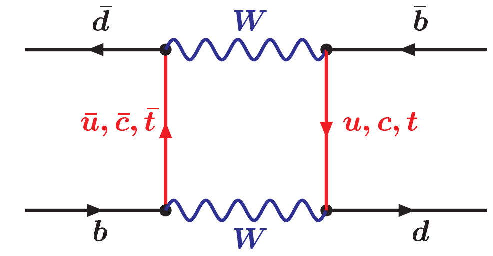
↳ very tiny effect

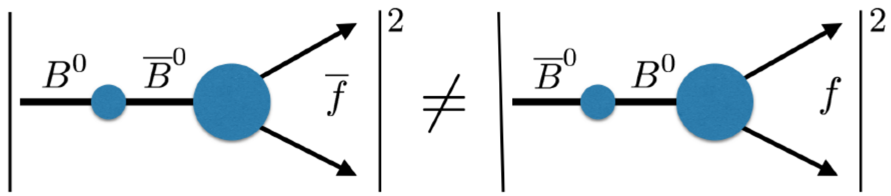
$$\mathcal{A}_{\rm SL}(B^0) = (0.7 \pm 2.7) \times 10^{-3}$$

CPV in neutral particles

- mixing





- 

$B^0 \to f \not\leftarrow \overline{B}^0$

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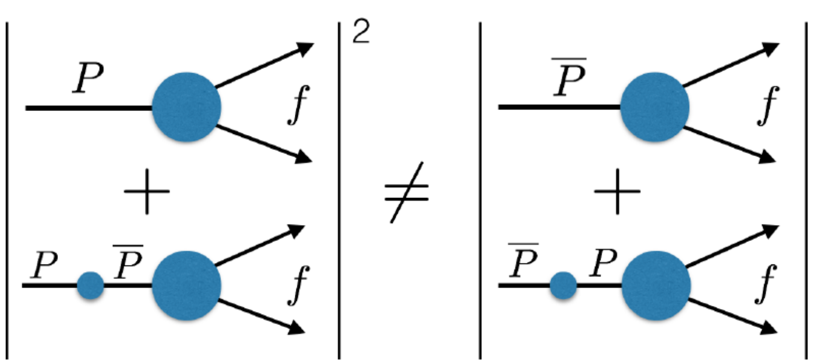
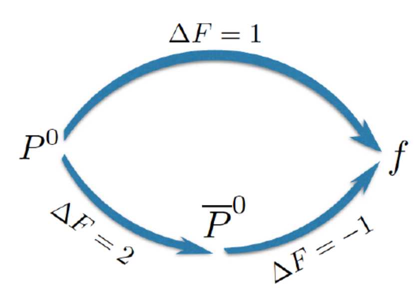
- mixing + decay

- 

→ 

→ 3 possibilities of CP

↳ mixing: $\frac{q}{p} = \left|\frac{q}{p}\right| e^{-i\phi_M}$

↳ decay: $\frac{\overline{A}_f}{A_f} = \left|\frac{\overline{A}_f}{A_f}\right| e^{-i(\Delta\phi_f + \Delta\delta_f)}$

↳ interference between them

direct CP violation

- 2 amplitudes: SAME final state, $\neq$ strong (δ_i) and weak (ϕ_i) phases

$$\langle f \,|\, T \,|\, M\rangle = A_1\, e^{i(\delta_1+\phi_1)} + A_2\, e^{i(\delta_2+\phi_2)}$$

$$\langle \bar{f} \,|\, T \,|\, \bar{M}\rangle = A_1\, e^{i(\delta_1-\phi_1)} + A_2\, e^{i(\delta_2-\phi_2)}$$

- weak phase → CKM
- strong phase → QCD

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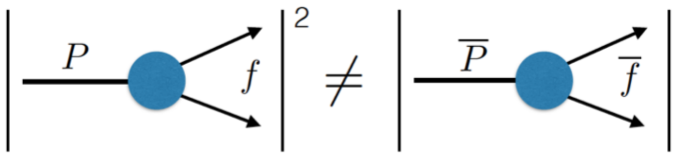
- $$\boxed{\Gamma(M\to f)-\Gamma(\bar M\to \bar f) = |\langle f\,|\,T\,|\,M\rangle|^2 - |\langle \bar f\,|\,T\,|\,\bar M\rangle|^2 = -4A_1A_2\sin(\delta_1-\delta_2)\,\sin(\phi_1-\phi_2)}$$



- $$A_{CP} = \frac{\Gamma(M\to f)-\Gamma(\bar M\to \bar f)}{\Gamma(M\to f)+\Gamma(\bar M\to \bar f)}$$

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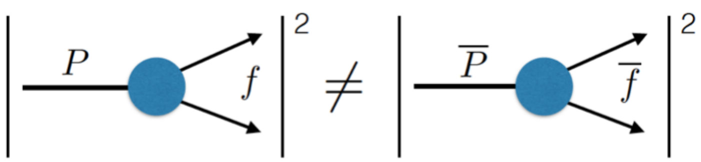

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- CPV at quark level: BSS model Bander Silverman & Soni PRL 43 (1979) 242

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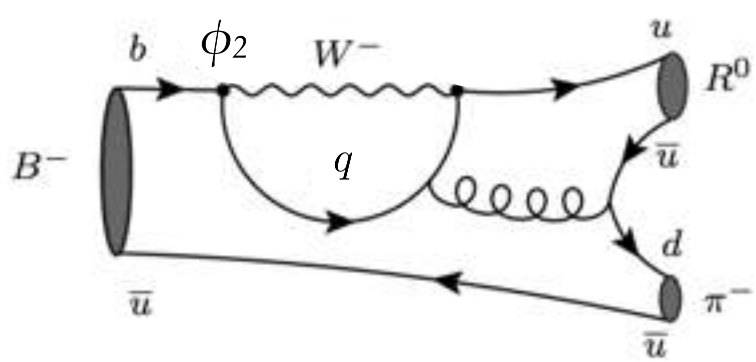

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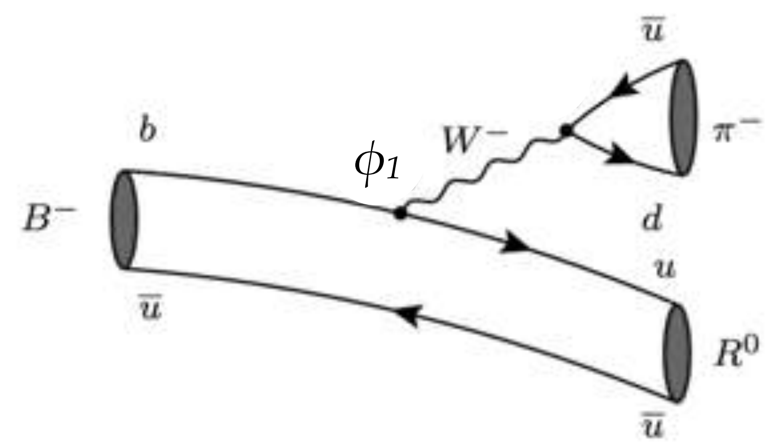

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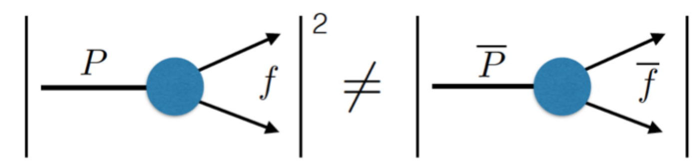

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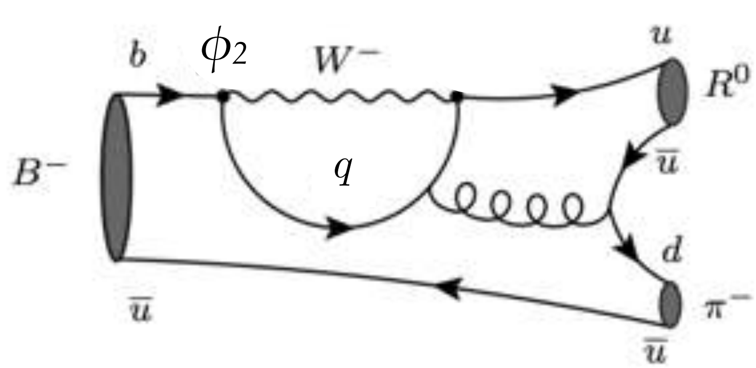

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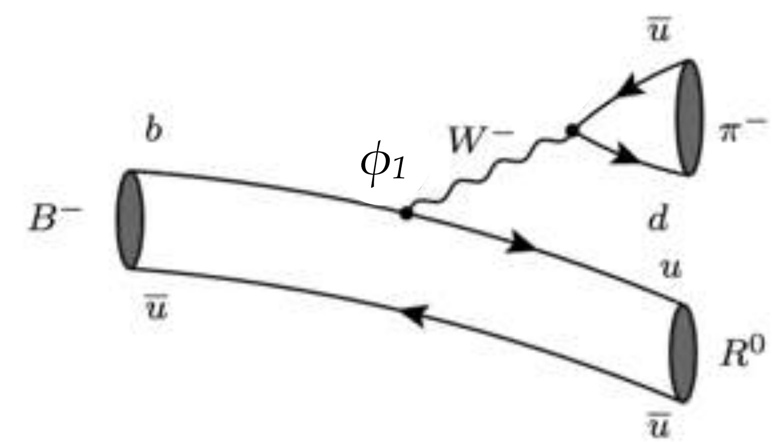


- not enough for CPV

- hadronic interactions are natural sources of strong phase!

CPV on heavy meson decays

- CPV in $B^{\pm} \to h^{\pm}h^{-}h^{+}$

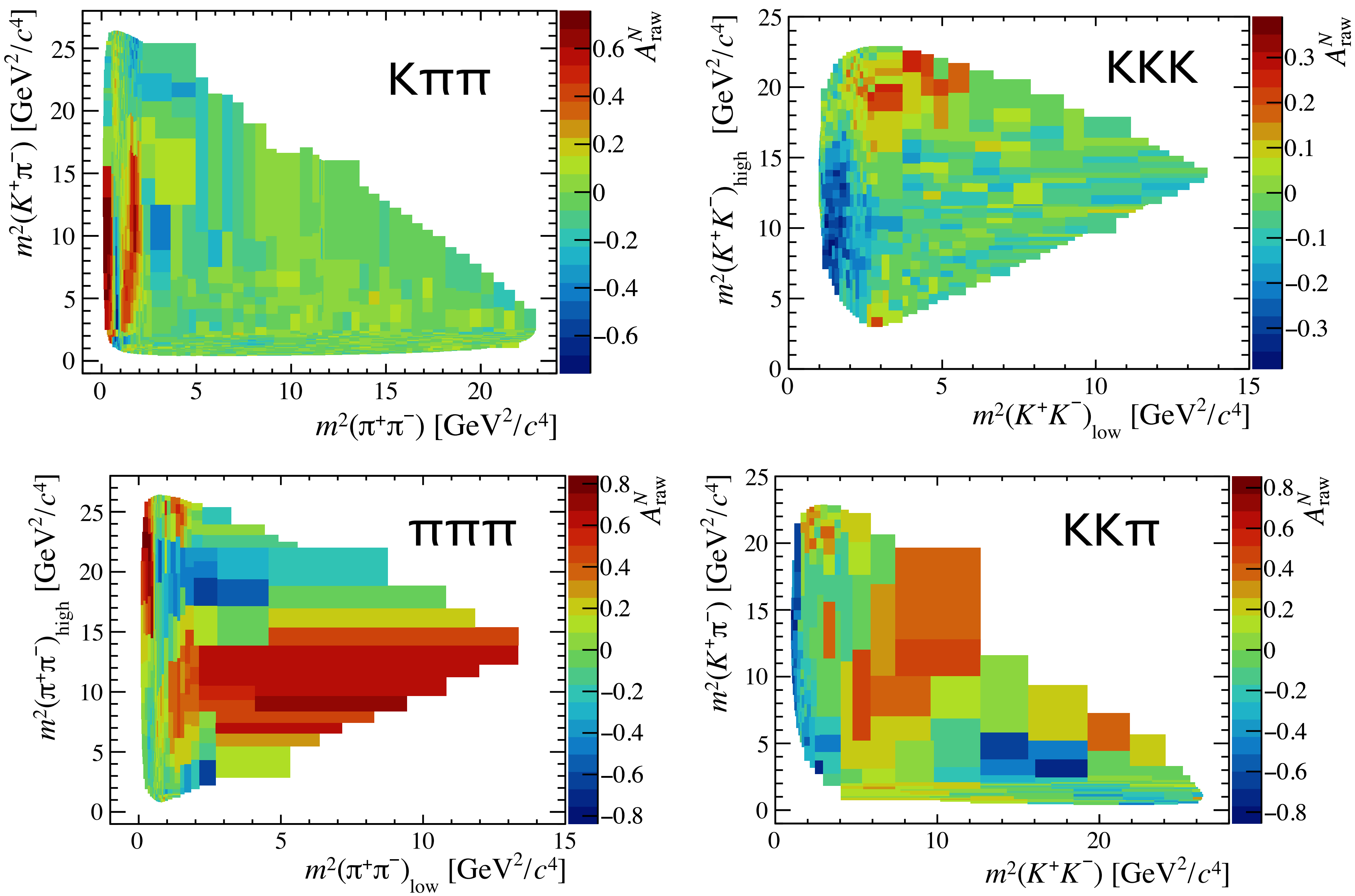
LHCb Run II 5.9 fb^{-1}

arXiv:2206.07622 PRD 2022 XX

- integrated

$$A_{CP}(B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}) = +0.011 \pm 0.002,$$
$$A_{CP}(B^{\pm} \to K^{\pm}K^{+}K^{-}) = -0.037 \pm 0.002,$$
$$A_{CP}(B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}) = +0.080 \pm 0.004,$$
$$A_{CP}(B^{\pm} \to \pi^{\pm}K^{+}K^{-}) = -0.114 \pm 0.007,$$

- massive localized Acp



CPV on heavy meson decays

- $\Delta A_{CP}^{\mathrm{LHCb}} = A_{cp}(D^0 \to K^+K^-) - A_{cp}(D^0 \to \pi^+\pi^-) = -(1.54 \pm 0.29) \times 10^{-3}$

Phys. Rev. Lett. 122, 211803 (2019)

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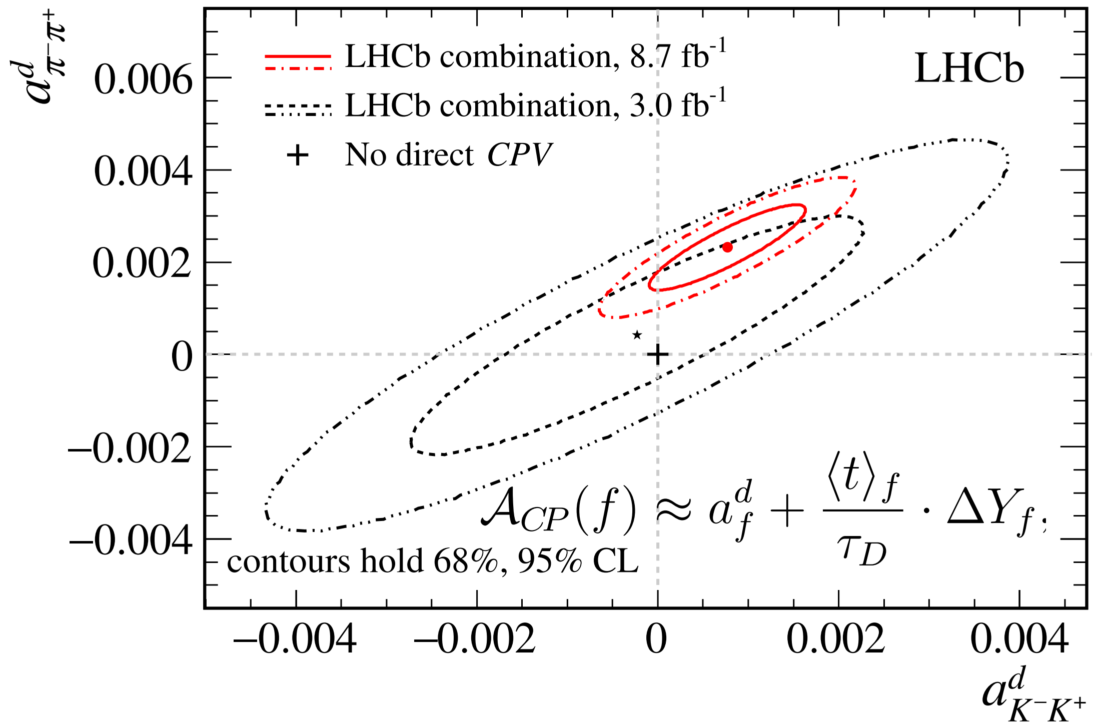
➔ direct CP asymmetry observation

- $A_{CP}^{LHCb}(KK) = (0.77 \pm 0.57) \times 10^{-3}$

↳ $A_{CP}^{LHCb}(\pi\pi) = (2.32 \pm 0.61) \times 10^{-3}$

arXiv:2209.03179



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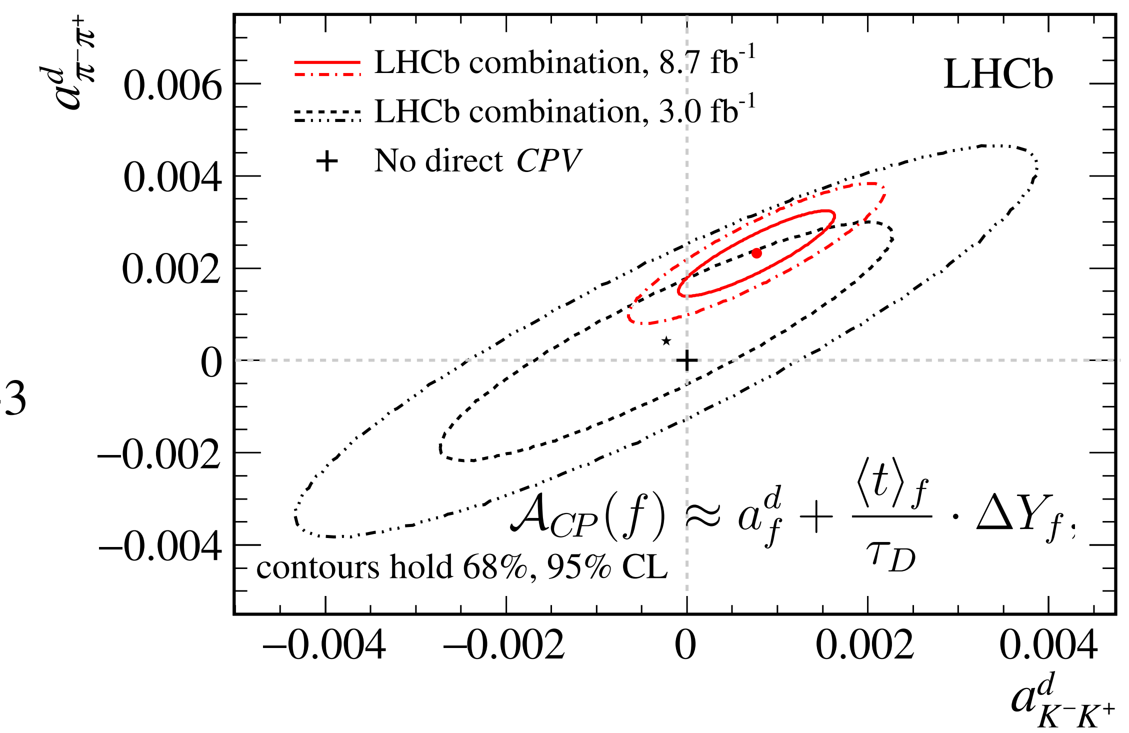

- QCD → LCSR predictions $A_{CP} \approx 10^{-4}$ (1 order magnitude bellow)

↳ new physics? nonperturbative effects?!

Khodjamirian, Petrov, Phys. Lett. B 774, 235 (2017)

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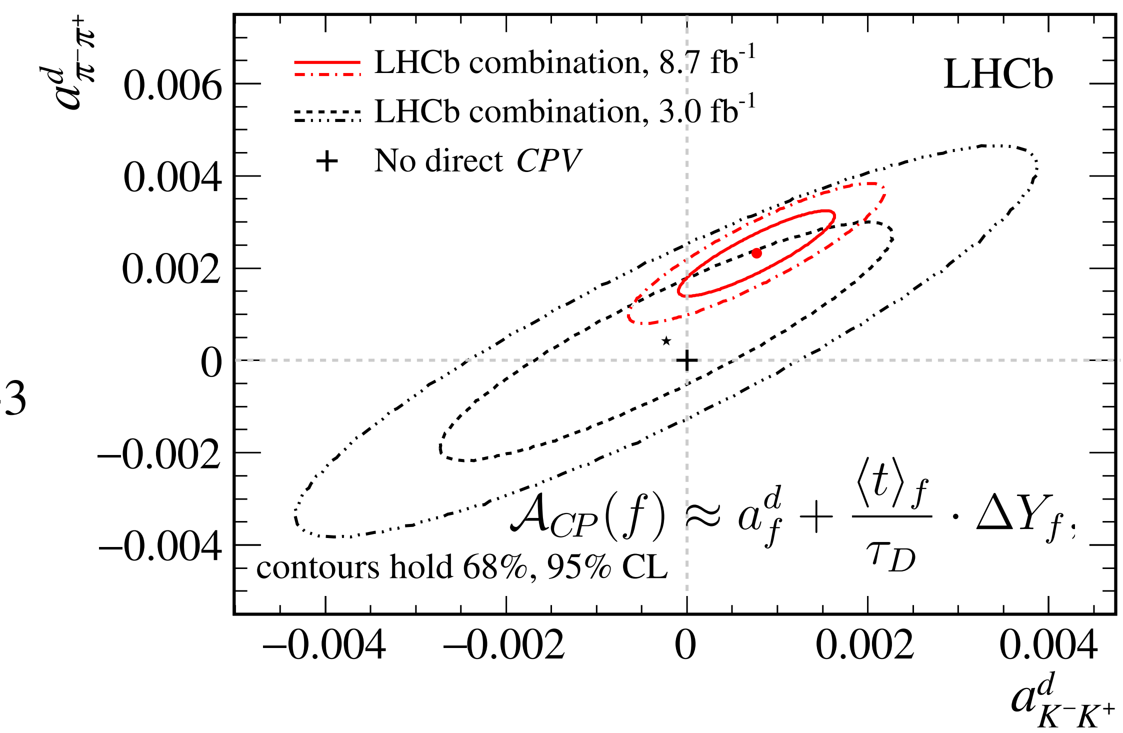

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→ what about CPV on $D \to hhh$?

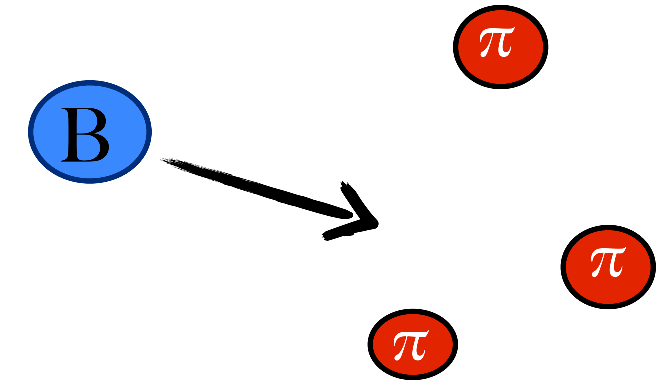
→ searches in many process at LHCb, BESIII, BeleII

↳ is expected soon with LHCb run II


heavy meson decays and FSI
LHCb

- Multi-body hadronic decays of B and D mesons

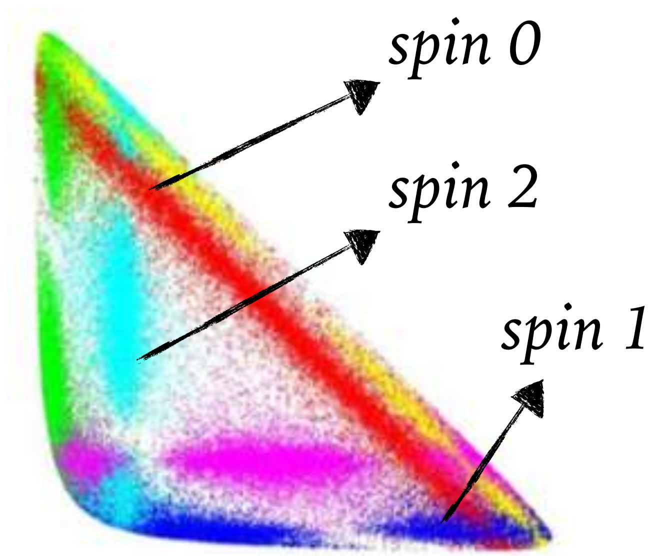
  ↳ are sensitive to strong phases



- signature of resonances on data

- spectroscopy → exotics, tetra-quarks, pentaquark,…

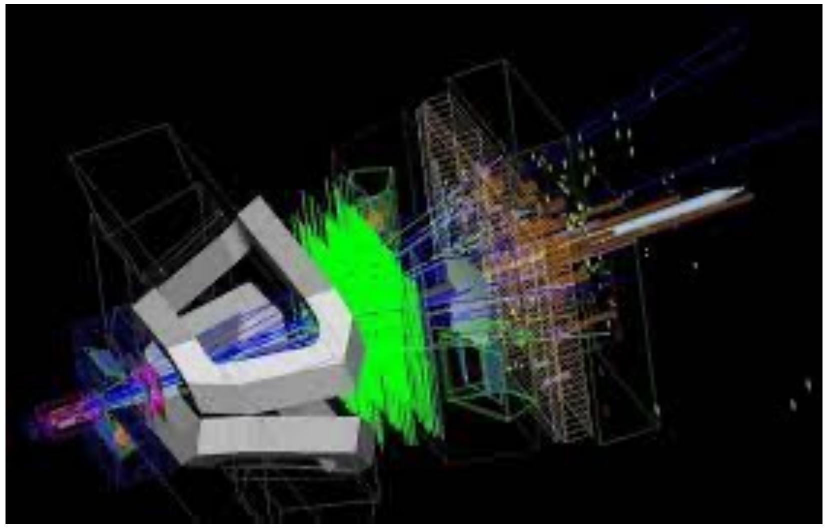
- study CP violation



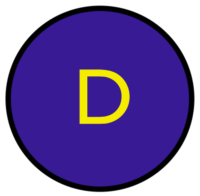
- new high data sample from LHCb

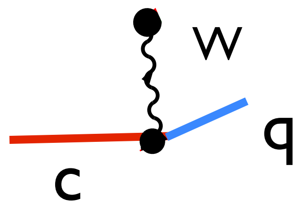
  ⟶ more to come from LHCb, BelleII, BESIII

  ⟶ better models are needed (challenge)

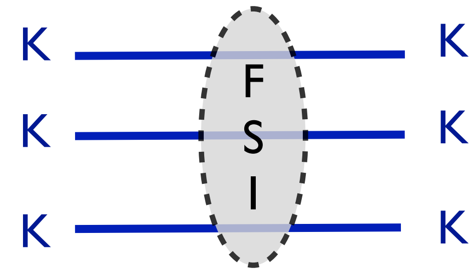


- ex: $D^+ \to K^- K^+ K^-$

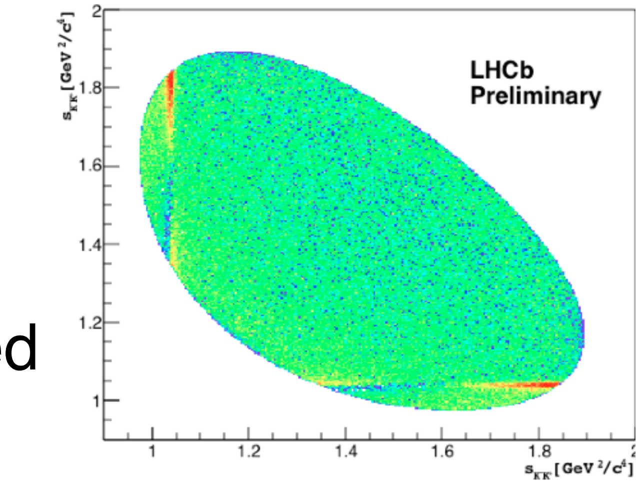




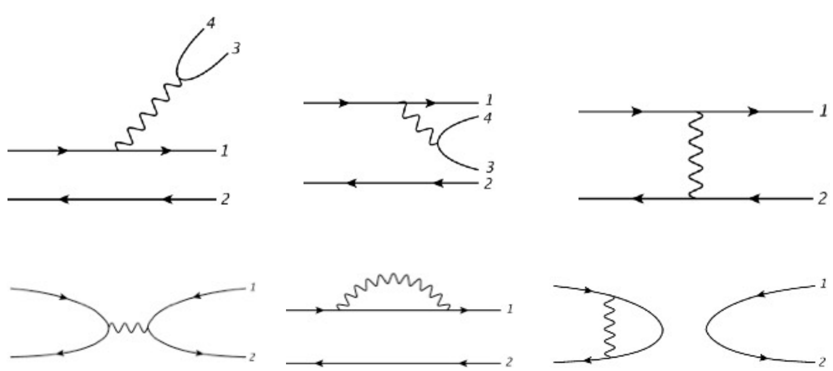
hadronize



observed

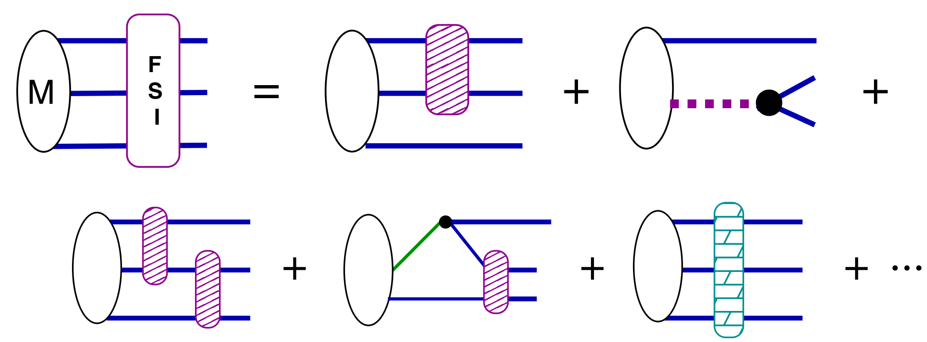


primary vertex
- weak -



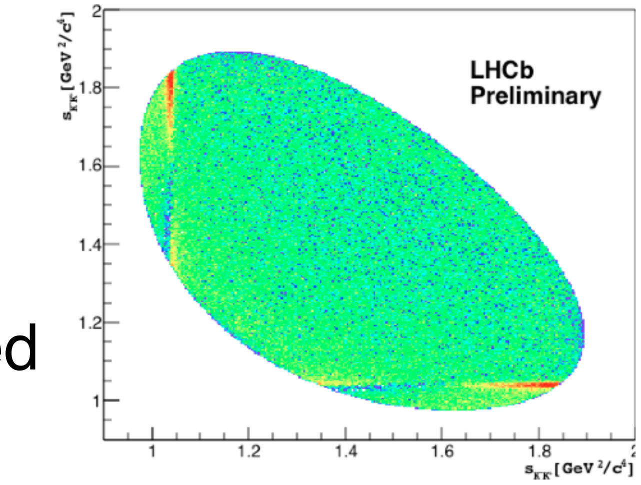
QCD, CKM coupling and phase

Final State Interactions
- strong -



(2+1) + 3-body interactions

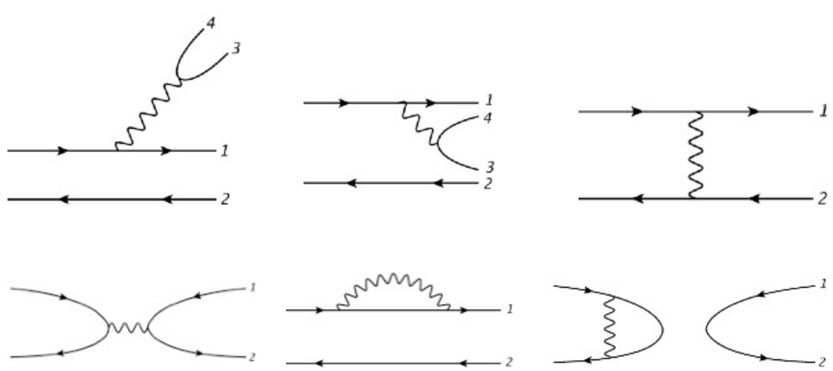
- ex: $D^+ \to K^- K^+ K^-$

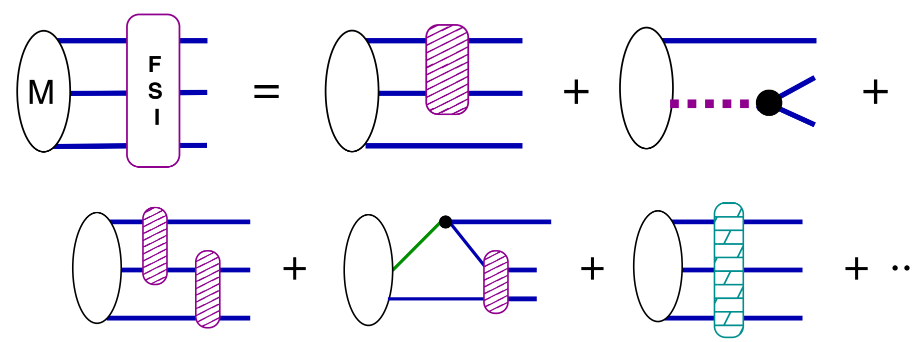


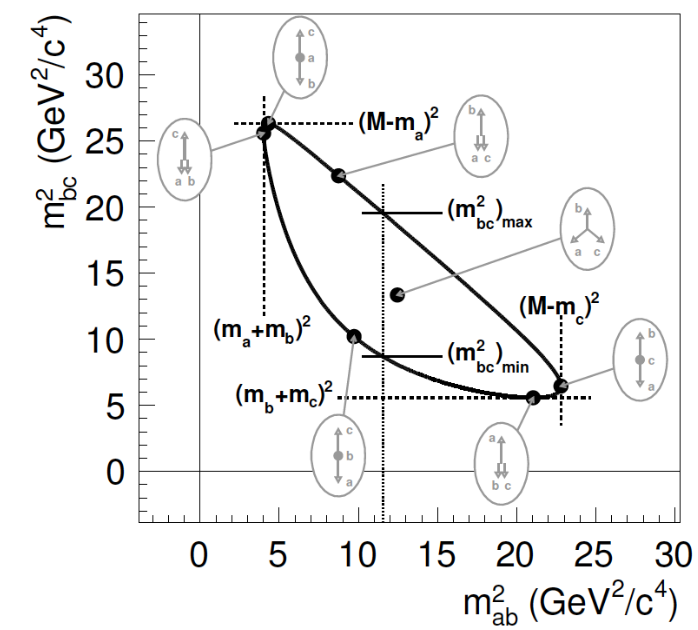
primary vertex
- weak -

Final State Interactions
- strong -

Dalitz plot







QCD, CKM coupling and phase

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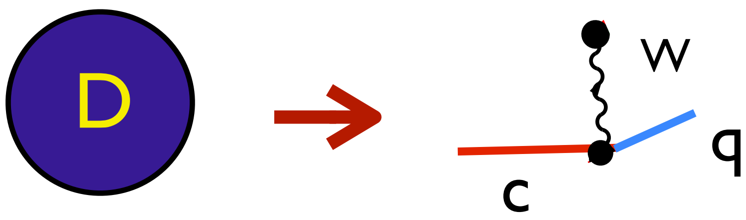
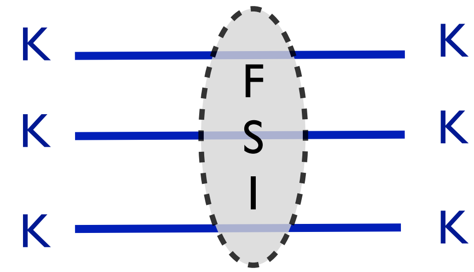
heavy meson decay Dynamics

- ex: $D^+ \to K^- K^+ K^-$



hadronize



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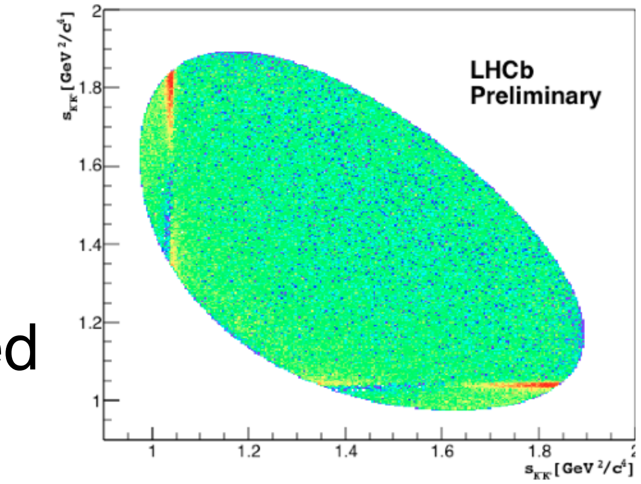

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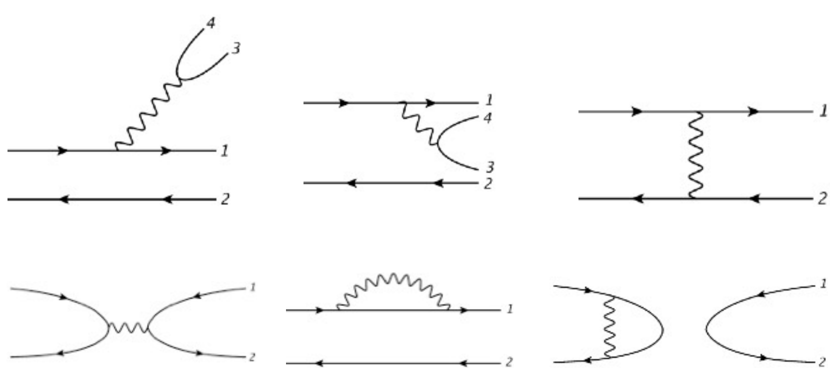

QCD, CKM coupling and phase

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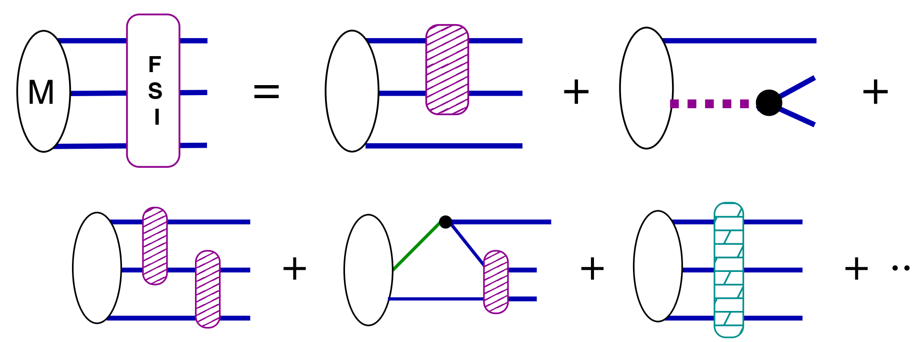

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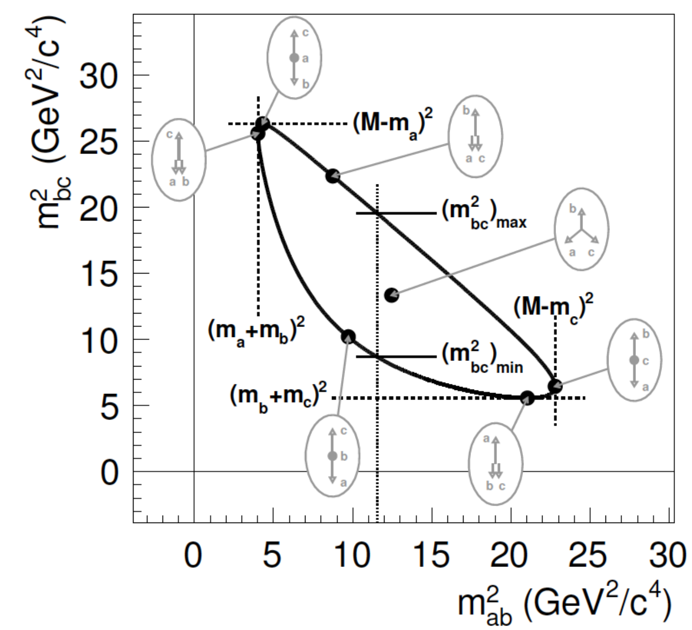


- To extract information from data we need an amplitude MODEL

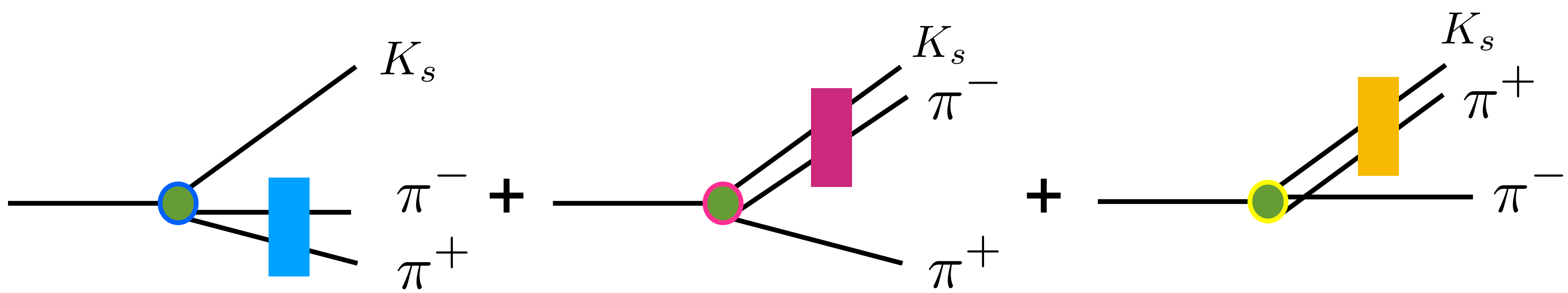
$$\frac{d\Gamma}{ds_{12}ds_{23}} = \frac{1}{(2\pi)^3}\frac{1}{32M^3}|\mathcal{A}(s_{12},s_{23})|^2$$

dynamics

standard experimental approach

- ex $D^0 \to K_s \pi^- \pi^+$

$\mathcal{A}(s_{12}, s_{23})$ =

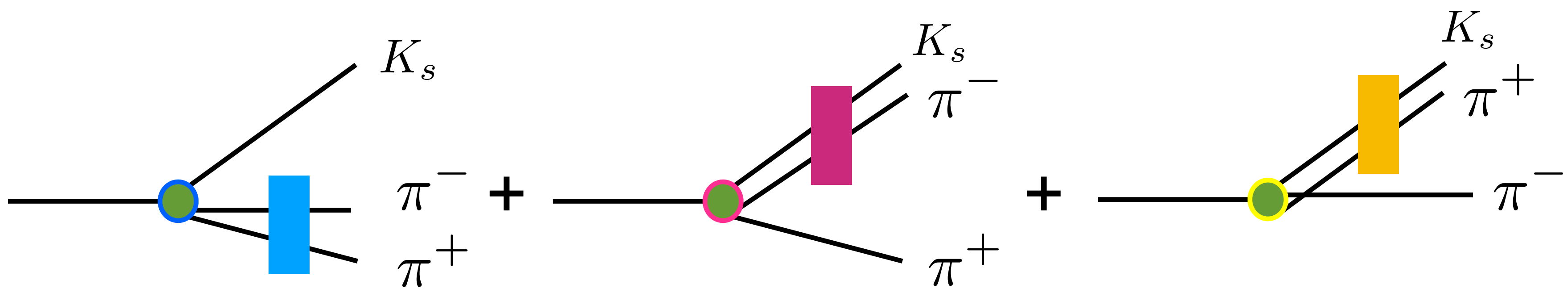


- isobar model widely used by experimentalists:
  - (2+1) approximation → ignore the interaction with 3rd particle (bachelor)
  - $A = \sum c_k A_k$; + NR coherent sum of amplitude's in different parcial waves

standard experimental approach

- ex $D^0 \to K_s \pi^- \pi^+$

$\mathcal{A}(s_{12}, s_{23})$ =



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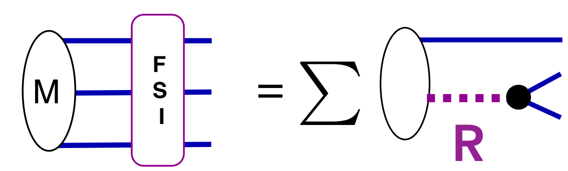
! Warning: when A_k are single resonances

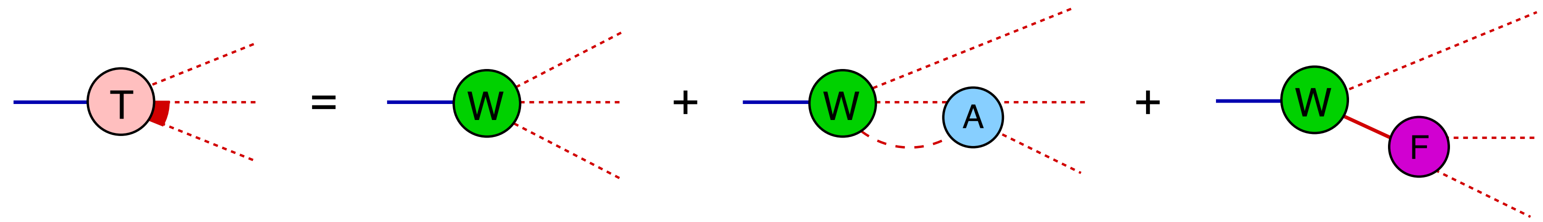


!

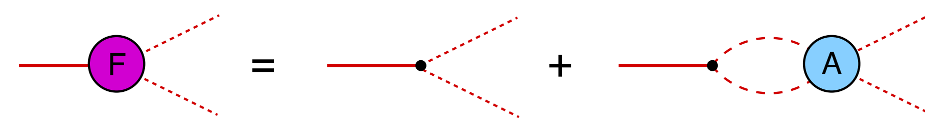
↪ defined as Breit-Wigner $\mathrm{BW}(s_{12}) = \dfrac{1}{m_R^2 - s_{12} - i m_R \Gamma(s_{12})},$

FSI is a key ingredient

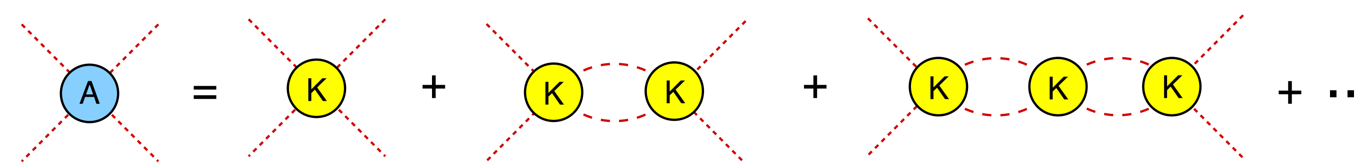
- (2+1) approach



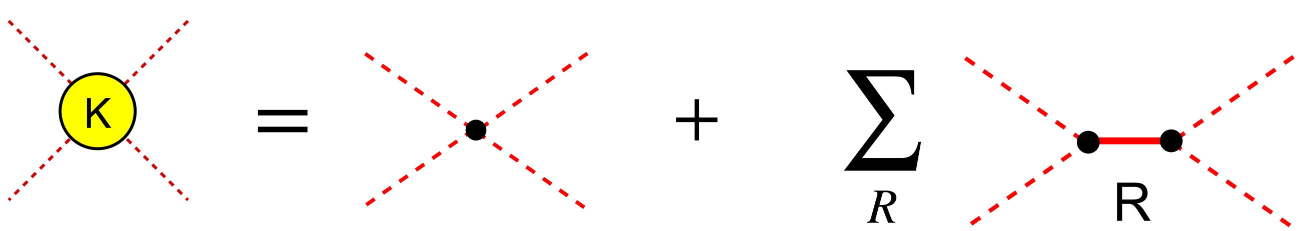
Form factor



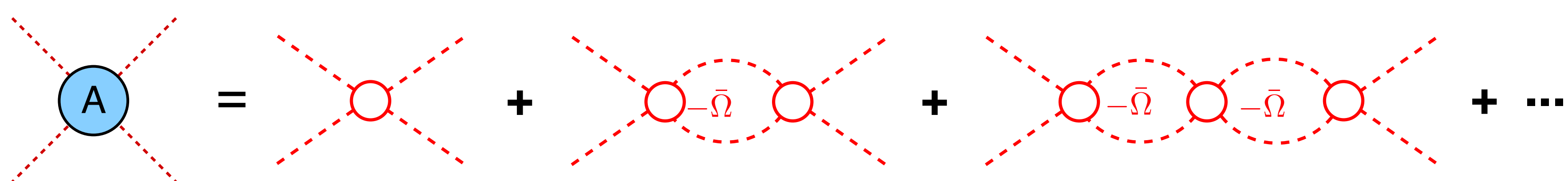
meson-meson



- kernel should includes all the mm dynamics



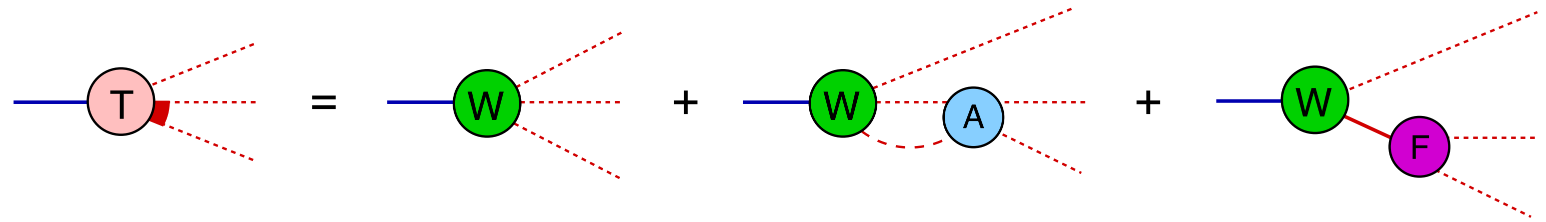
- unitarize amplitude (ie by Bethe-Salpeter eq.) Oller and Oset PRD 60 (1999)



should includes all channels with the same (J,I)

FSI is a key ingredient

- (2+1) approach



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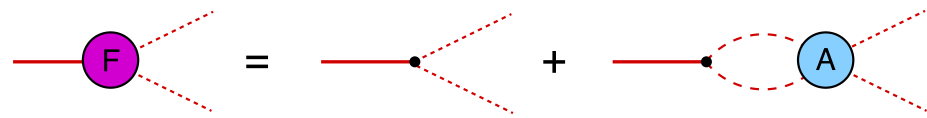

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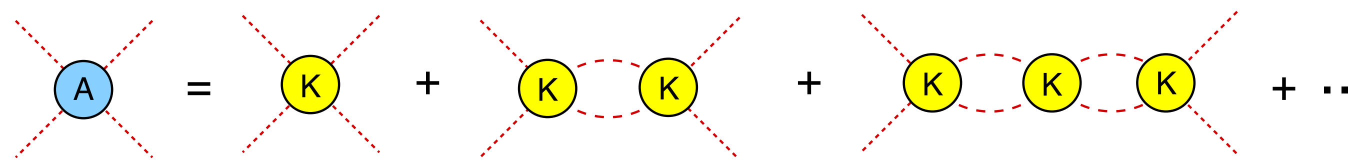

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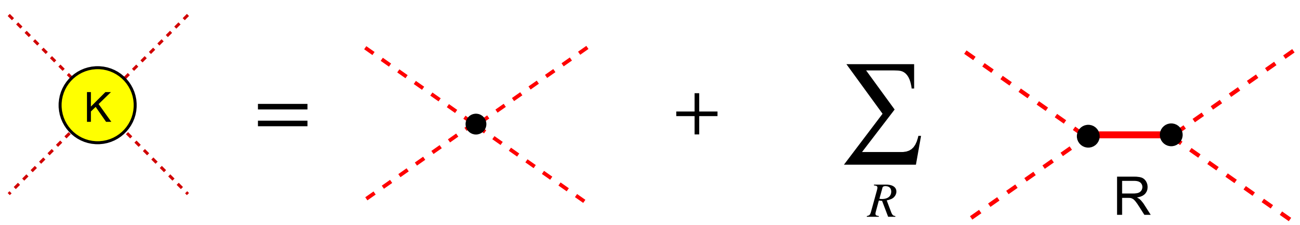

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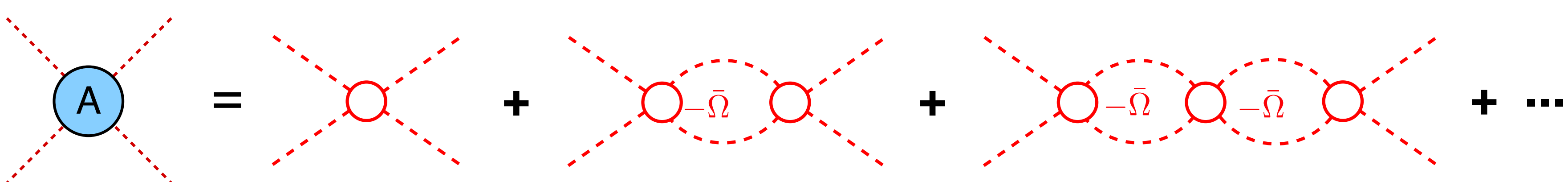

$$\boxed{\mathcal{A}_{ab}^{JI} = \frac{\mathcal{K}_{ab\to cd}^{(JI)}}{1+\bar{\Omega}_{ab}\,\mathcal{K}_{ab\to cd}^{(JI)}}}$$

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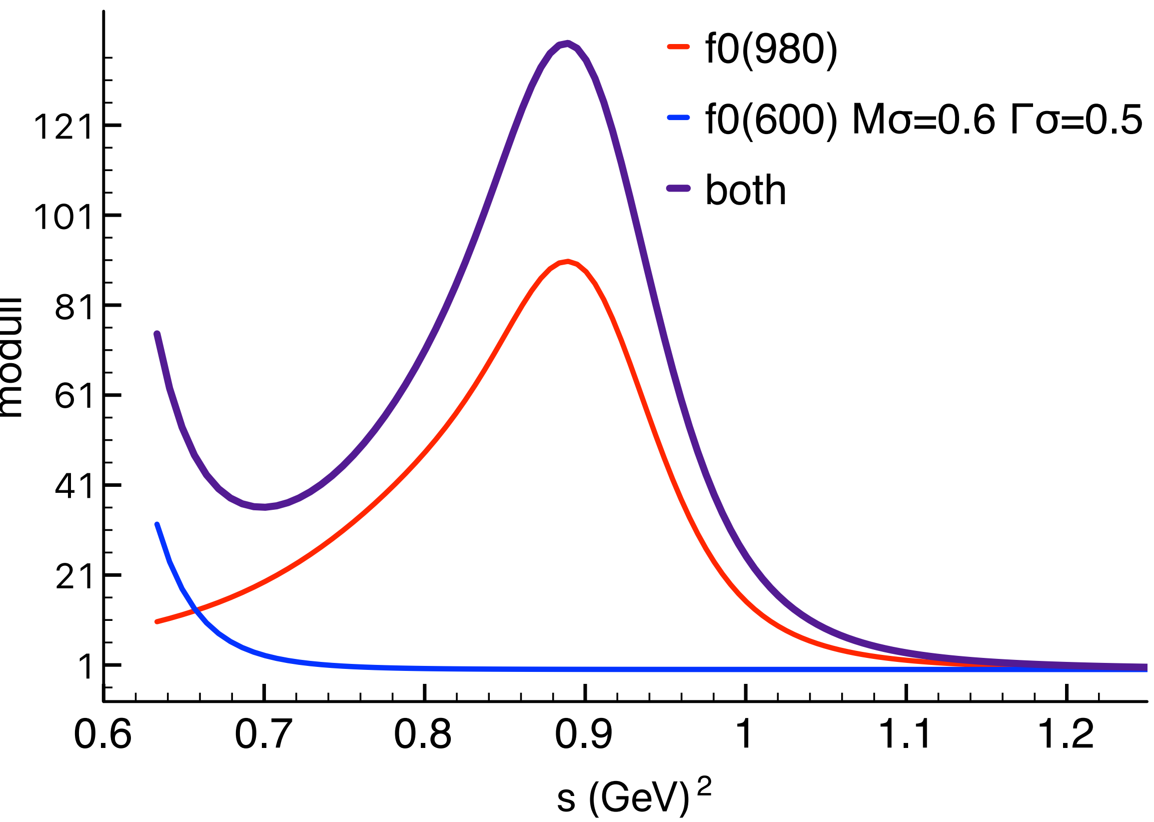
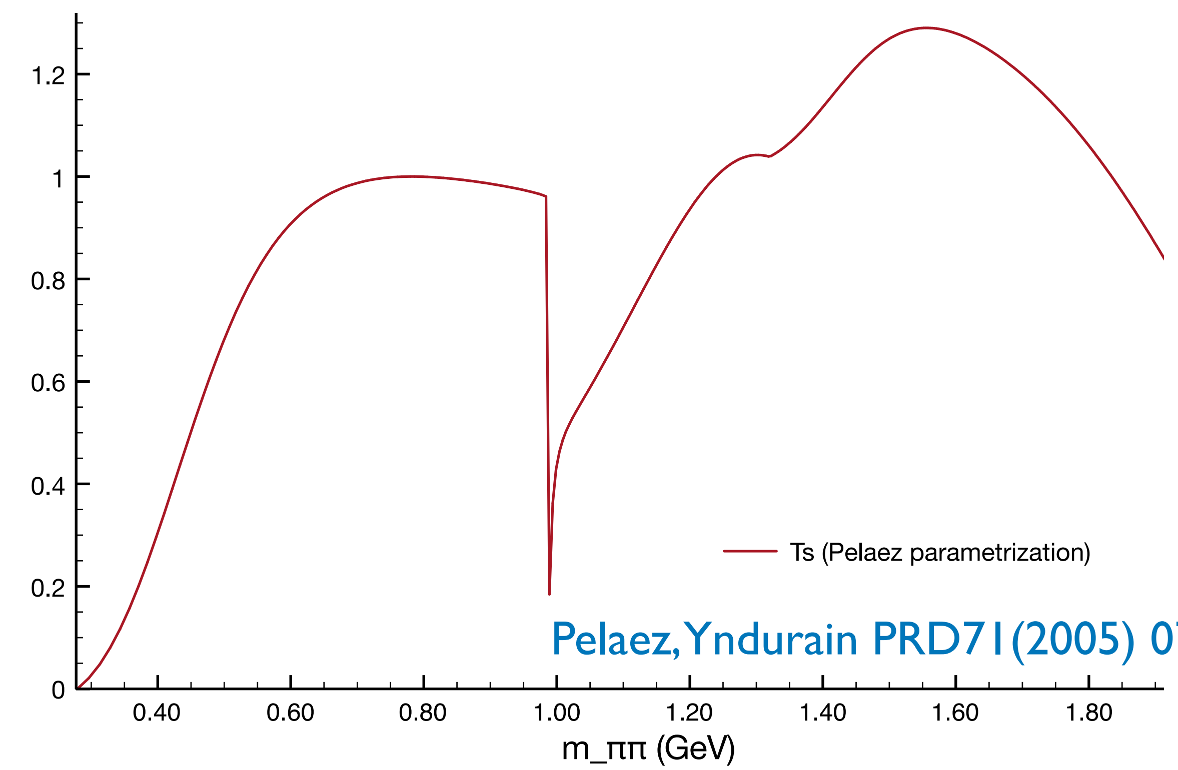
FSI is a key ingredient

- examples of different ππ S-wave amplitudes

isobar BW



unitary, analytic, coupled-channels…



Pelaez, Yndurain PRD71(2005) 074016

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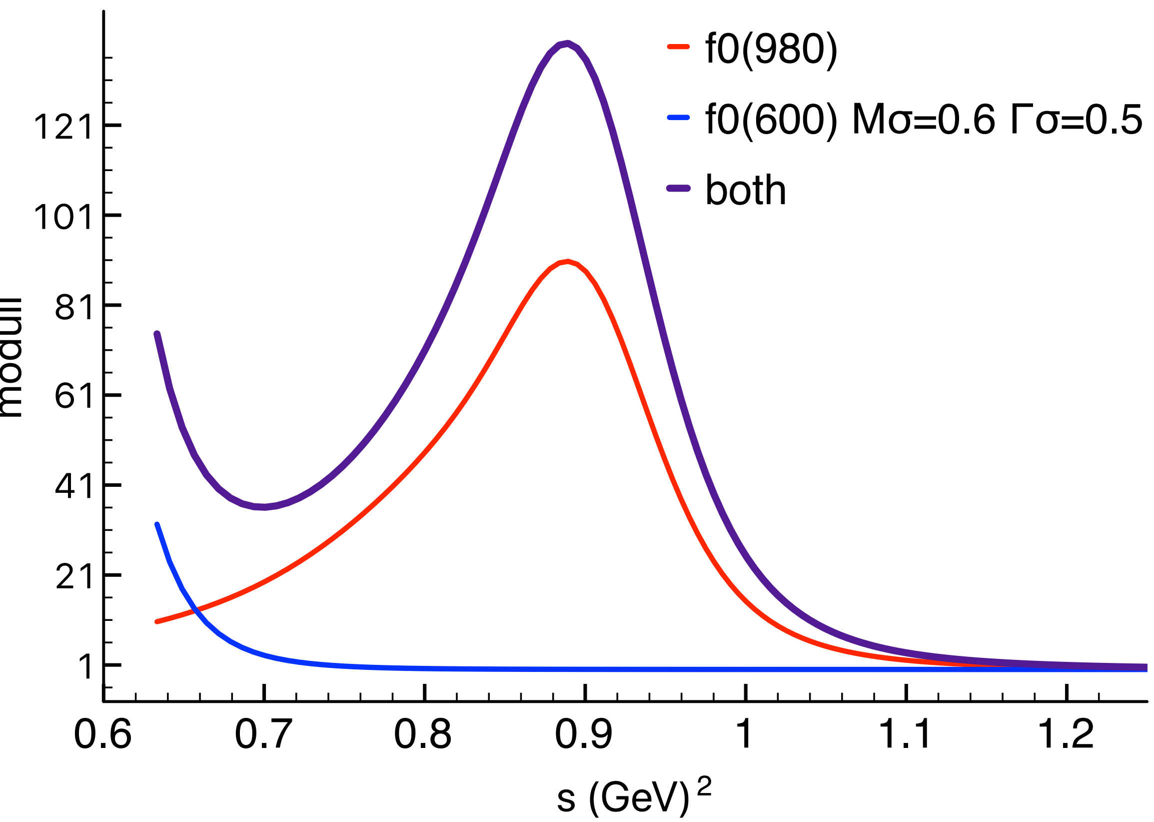

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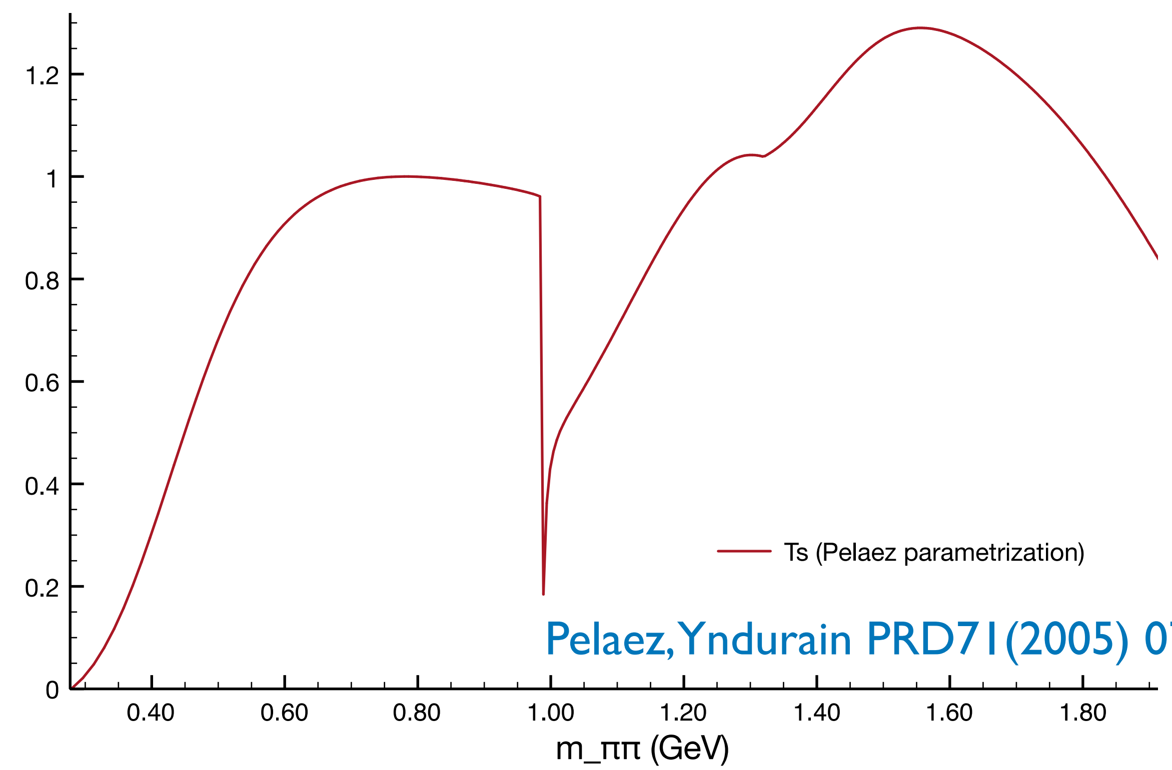


Pelaez, Yndurain PRD71(2005) 074016

- sum of BW violates two-body unitarity (close Rs in the same channel - scalars)
- resonance's mass and width are processes dependent

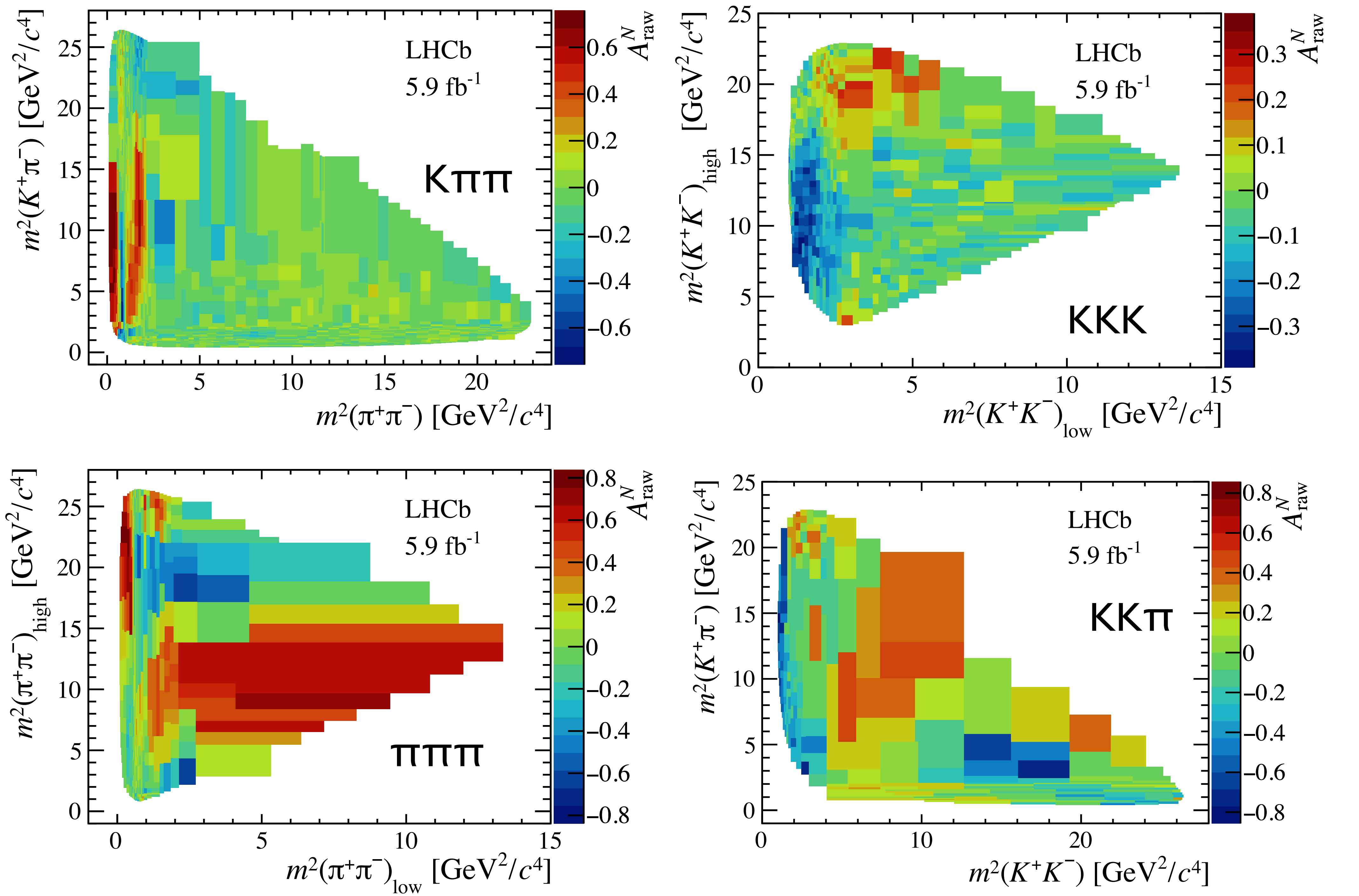
CP violation and FSI

FSI as source of CP asymmetry in B decays

- $B^{\pm} \to h^{\pm}h^{-}h^{+}$ massive localized Acp

$$A_{CP} = \frac{\Gamma(M \to f) - \Gamma(\bar{M} \to \bar{f})}{\Gamma(M \to f) + \Gamma(\bar{M} \to \bar{f})}$$

- suggest dynamic effect



- CPT must be preserved

Lifetime $\tau = 1 / \Gamma_{total} = 1 / \overline{\Gamma}_{total}$

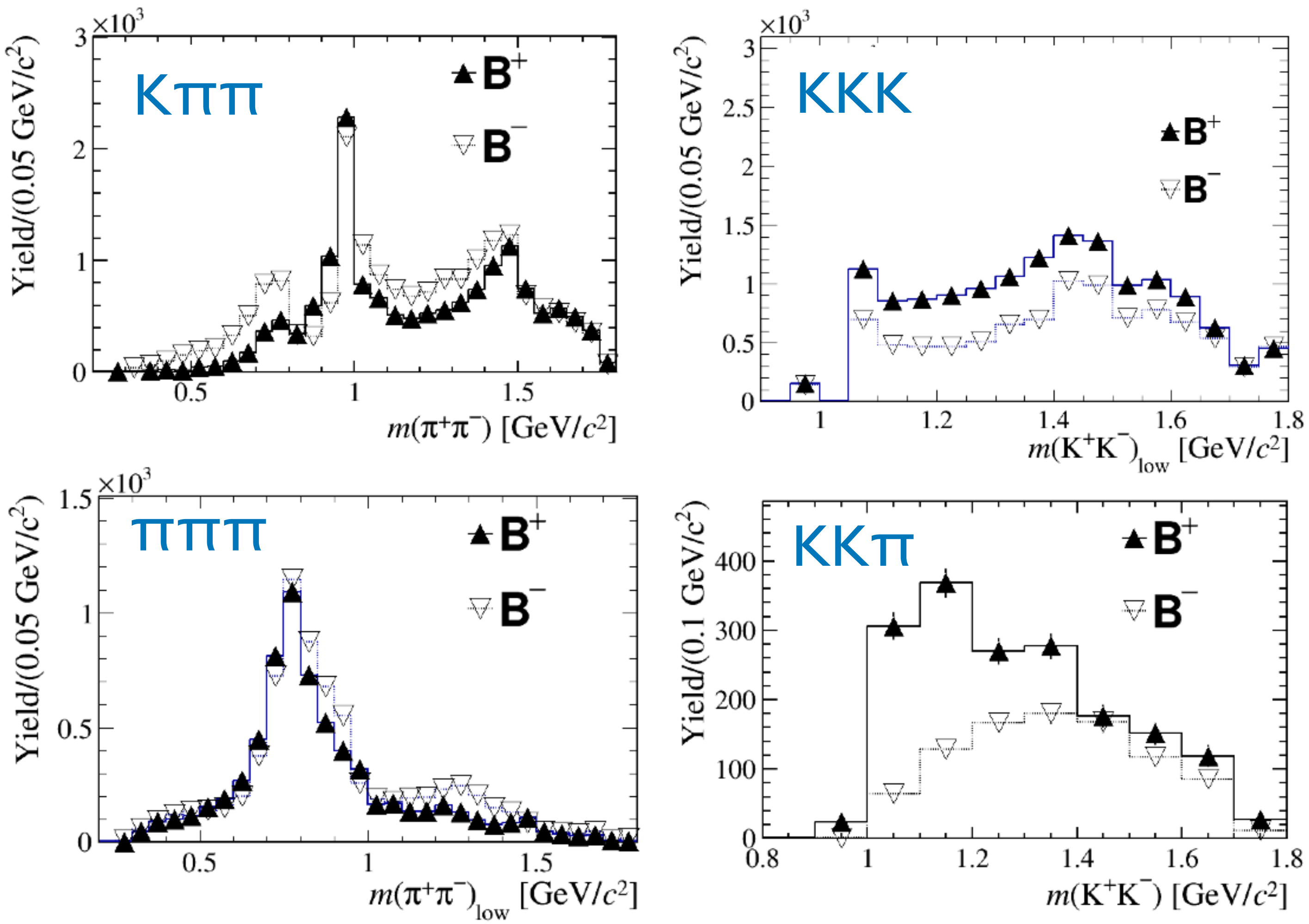
$\Gamma_{total} = \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4 + \Gamma_5 + \Gamma_6 + \ldots$

$\overline{\Gamma}_{total} = \overline{\Gamma}_1 + \overline{\Gamma}_2 + \overline{\Gamma}_3 + \overline{\Gamma}_4 + \overline{\Gamma}_5 + \overline{\Gamma}_6 + \ldots$

$$\sum \Delta\Gamma_{CP} = 0$$

CPV in one channel should be compensated by another, same quantum #, with opposite sign

- LHCb run 1 projections



rescattering as a CPV mechanism

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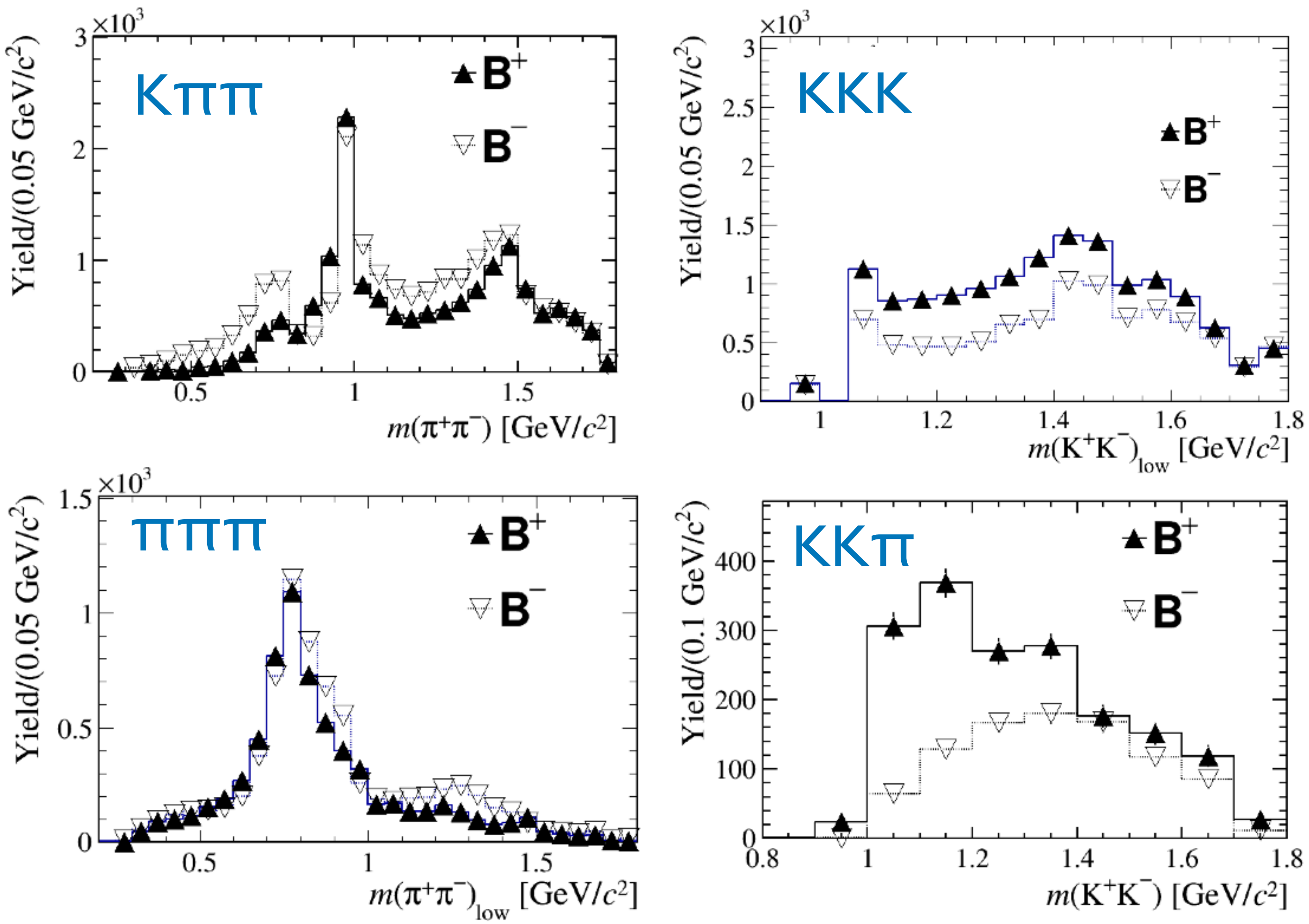

- rescattering $\boxed{\pi\pi \rightarrow KK}$

CPV at [1 -1.6] GeV

Frederico, Bediaga, Lourenço
PRD89(2014)094013

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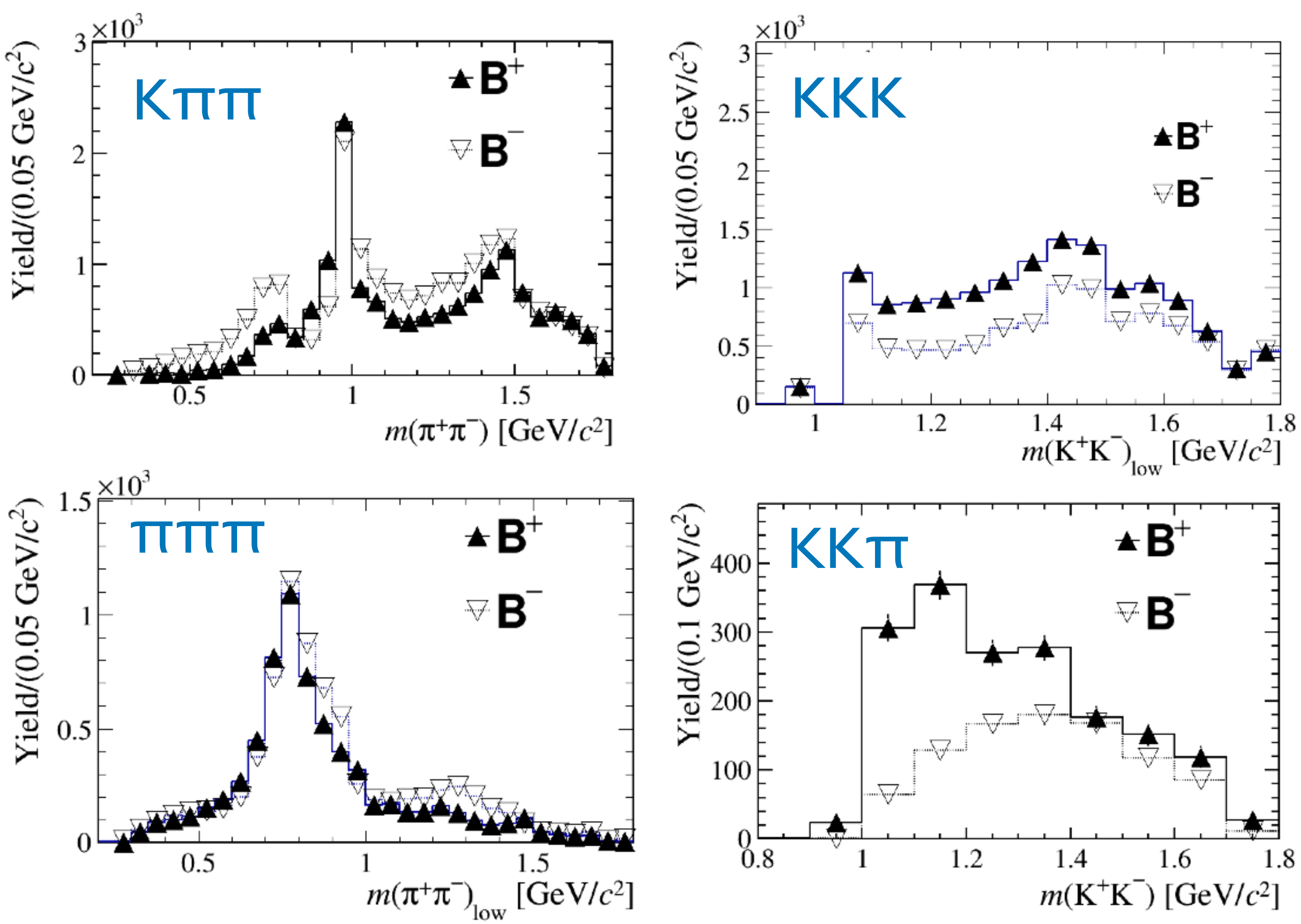

- rescattering $\boxed{\pi\pi \rightarrow KK}$
  - CPV at [1 -1.6] GeV

  Frederico, Bediaga, Lourenço PRD89(2014)094013

- implemented in LHCb amplitude analysis:
  - $B^\pm \rightarrow \pi^-\pi^+\pi^\pm$ PRD101 (2020) 012006; PRL 124 (2020) 031801
  - $B^\pm \rightarrow \pi^\pm K^- K^+$ PRL 123 (2019) 231802

CPV: amplitude analysis $B^\pm \to \pi^-\pi^+\pi^\pm$

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  - $(\pi^-\pi^+)_{S-Wave}$ 3 different model:
    - ↪ σ as BW (!) + rescattering;
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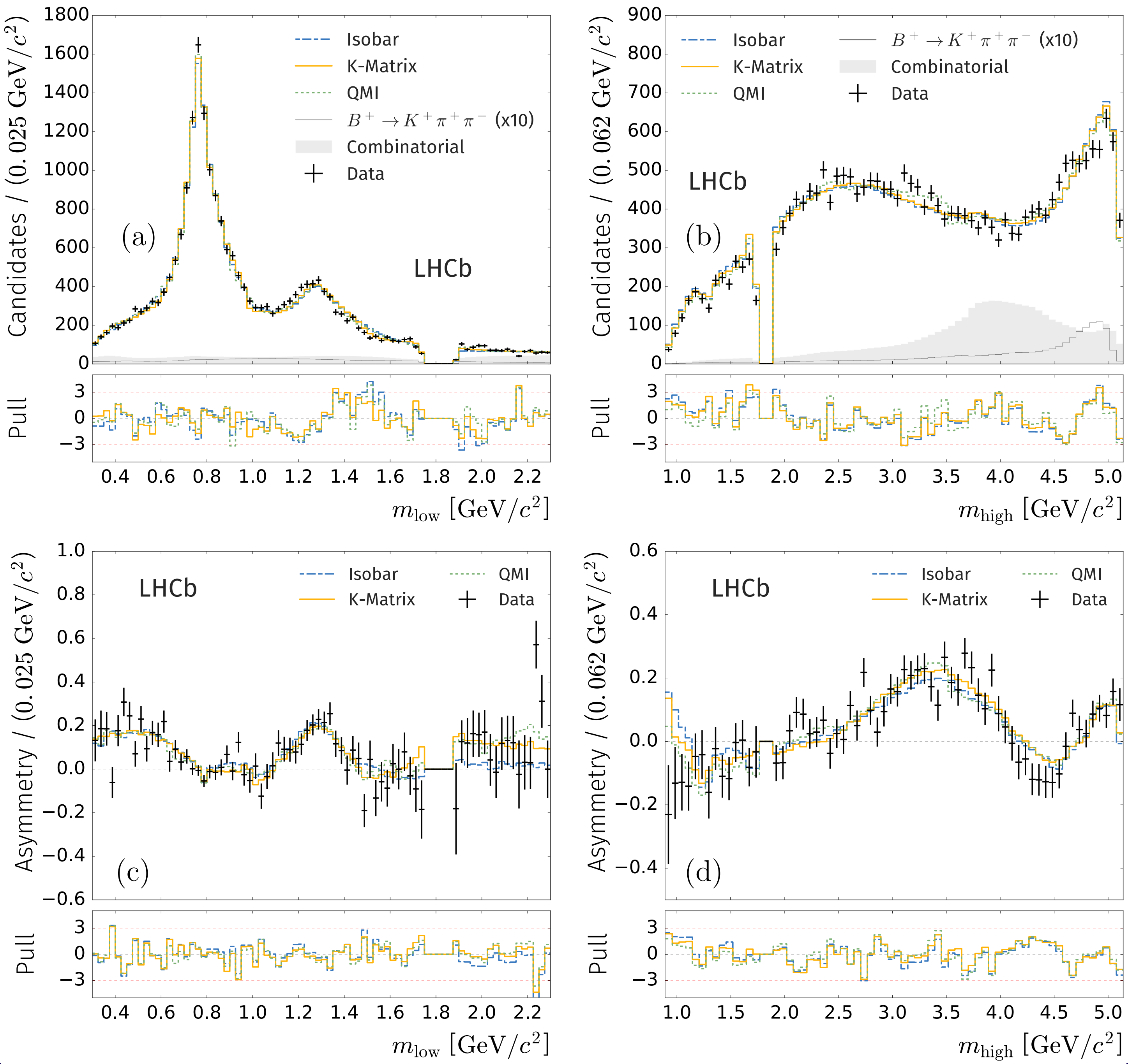

Contribution	Fit fraction (10^{-2})	A_{CP} (10^{-2})	B^+ phase (°)	B^- phase (°)
Isobar model				
$\rho(770)^0$	$55.5 \pm 0.6 \pm 2.5$	$+0.7 \pm 1.1 \pm 1.6$	—	—
$\omega(782)$	$0.50 \pm 0.03 \pm 0.05$	$-4.8 \pm 6.5 \pm 3.8$	$-19 \pm 6 \pm 1$	$+8 \pm 6 \pm 1$
$f_2(1270)$	$9.0 \pm 0.3 \pm 1.5$	$+46.8 \pm 6.1 \pm 4.7$	$+5 \pm 3 \pm 12$	$+53 \pm 2 \pm 12$
$\rho(1450)^0$	$5.2 \pm 0.3 \pm 1.9$	$-12.9 \pm 3.3 \pm 35.9$	$+127 \pm 4 \pm 21$	$+154 \pm 4 \pm 6$
$\rho_3(1690)^0$	$0.5 \pm 0.1 \pm 0.3$	$-80.1 \pm 11.4 \pm 25.3$	$-26 \pm 7 \pm 14$	$-47 \pm 18 \pm 25$
S-wave	$25.4 \pm 0.5 \pm 3.6$	$+14.4 \pm 1.8 \pm 2.1$	—	—
Rescattering	$1.4 \pm 0.1 \pm 0.5$	$+44.7 \pm 8.6 \pm 17.3$	$-35 \pm 6 \pm 10$	$-4 \pm 4 \pm 25$
σ	$25.2 \pm 0.5 \pm 5.0$	$+16.0 \pm 1.7 \pm 2.2$	$+115 \pm 2 \pm 14$	$+179 \pm 1 \pm 95$
K-matrix				
$\rho(770)^0$	$56.5 \pm 0.7 \pm 3.4$	$+4.2 \pm 1.5 \pm 6.4$	—	—
$\omega(782)$	$0.47 \pm 0.04 \pm 0.03$	$-6.2 \pm 8.4 \pm 9.8$	$-15 \pm 6 \pm 4$	$+8 \pm 7 \pm 4$
$f_2(1270)$	$9.3 \pm 0.4 \pm 2.5$	$+42.8 \pm 4.1 \pm 9.1$	$+19 \pm 4 \pm 18$	$+80 \pm 3 \pm 17$
$\rho(1450)^0$	$10.5 \pm 0.7 \pm 4.6$	$+9.0 \pm 6.0 \pm 47.0$	$+155 \pm 5 \pm 29$	$-166 \pm 4 \pm 51$
$\rho_3(1690)^0$	$1.5 \pm 0.1 \pm 0.4$	$-35.7 \pm 10.8 \pm 36.9$	$+19 \pm 8 \pm 34$	$+5 \pm 8 \pm 46$
S-wave	$25.7 \pm 0.6 \pm 3.0$	$+15.8 \pm 2.6 \pm 7.2$	—	—
QMI				
$\rho(770)^0$	$54.8 \pm 1.0 \pm 2.2$	$+4.4 \pm 1.7 \pm 2.8$	—	—
$\omega(782)$	$0.57 \pm 0.10 \pm 0.17$	$-7.9 \pm 16.5 \pm 15.8$	$-25 \pm 6 \pm 27$	$-2 \pm 7 \pm 11$
$f_2(1270)$	$9.6 \pm 0.4 \pm 4.0$	$+37.6 \pm 4.4 \pm 8.0$	$+13 \pm 5 \pm 21$	$+68 \pm 3 \pm 66$
$\rho(1450)^0$	$7.4 \pm 0.5 \pm 4.0$	$-15.5 \pm 7.3 \pm 35.2$	$+147 \pm 7 \pm 152$	$-175 \pm 5 \pm 171$
$\rho_3(1690)^0$	$1.0 \pm 0.1 \pm 0.5$	$-93.2 \pm 6.8 \pm 38.9$	$+8 \pm 10 \pm 24$	$+36 \pm 26 \pm 46$
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- ANA for $B^\pm \to \pi^\pm K^- K^+$ PRL 123 (2019) 231802

Contribution	Fit Fraction(%)	A_{CP}(%)	Magnitude (B^+/B^-)	Phase[°] (B^+/B^-)
$K^*(892)^0$	$7.5 \pm 0.6 \pm 0.5$	$+12.3 \pm 8.7 \pm 4.5$	$0.94 \pm 0.04 \pm 0.02$	0 (fixed)
			$1.06 \pm 0.04 \pm 0.02$	0 (fixed)
$K_0^*(1430)^0$	$4.5 \pm 0.7 \pm 1.2$	$+10.4 \pm 14.9 \pm 8.8$	$0.74 \pm 0.09 \pm 0.09$	$-176 \pm 10 \pm 16$
			$0.82 \pm 0.09 \pm 0.10$	$136 \pm 11 \pm 21$
Single pole	$32.3 \pm 1.5 \pm 4.1$	$-10.7 \pm 5.3 \pm 3.5$	$2.19 \pm 0.13 \pm 0.17$	$-138 \pm 7 \pm 5$
			$1.97 \pm 0.12 \pm 0.20$	$166 \pm 6 \pm 5$
$\rho(1450)^0$	$30.7 \pm 1.2 \pm 0.9$	$-10.9 \pm 4.4 \pm 2.4$	$2.14 \pm 0.11 \pm 0.07$	$-175 \pm 10 \pm 15$
			$1.92 \pm 0.10 \pm 0.07$	$140 \pm 13 \pm 20$
$f_2(1270)$	$7.5 \pm 0.8 \pm 0.7$	$+26.7 \pm 10.2 \pm 4.8$	$0.86 \pm 0.09 \pm 0.07$	$-106 \pm 11 \pm 10$
			$1.13 \pm 0.08 \pm 0.05$	$-128 \pm 11 \pm 14$
Rescattering	$16.4 \pm 0.8 \pm 1.0$	$-66.4 \pm 3.8 \pm 1.9$	$1.91 \pm 0.09 \pm 0.06$	$-56 \pm 12 \pm 18$
			$0.86 \pm 0.07 \pm 0.04$	$-81 \pm 14 \pm 15$
$\phi(1020)$	$0.3 \pm 0.1 \pm 0.1$	$+9.8 \pm 43.6 \pm 26.6$	$0.20 \pm 0.07 \pm 0.02$	$-52 \pm 23 \pm 32$
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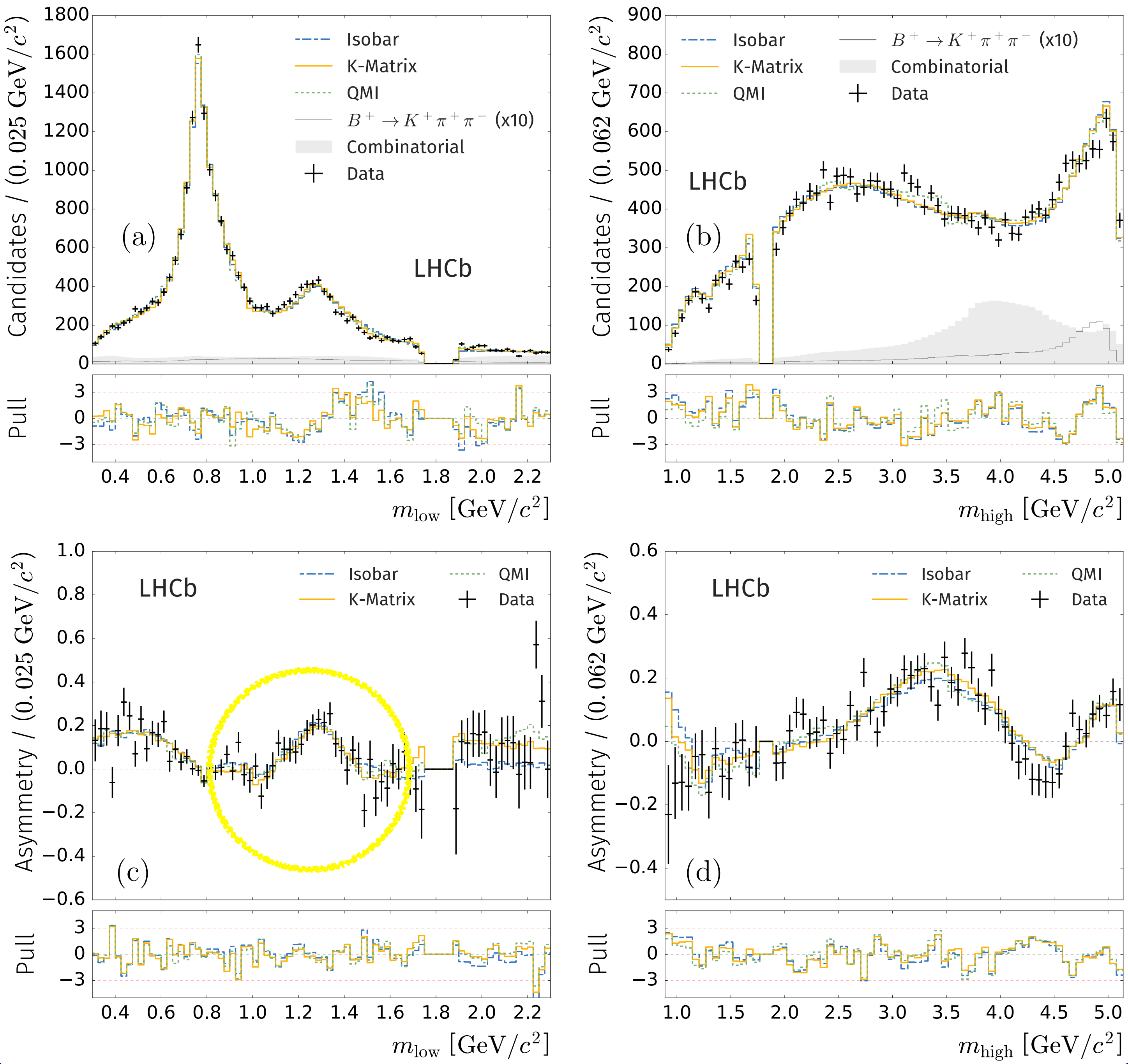

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FSI as source of CP asymmetry in D decays

- how to explain the CPV in charm?

$$\Delta A_{CP}^{\mathrm{LHCb}} = A_{cp}(D^0 \to K^+K^-) - A_{cp}(D^0 \to \pi^+\pi^-) = -(1.54 \pm 0.29) \times 10^{-3}$$

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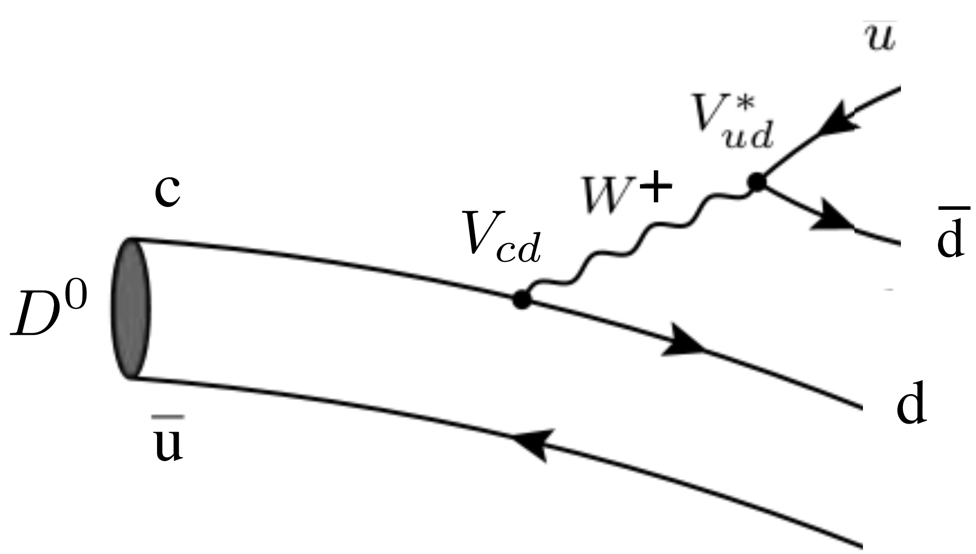
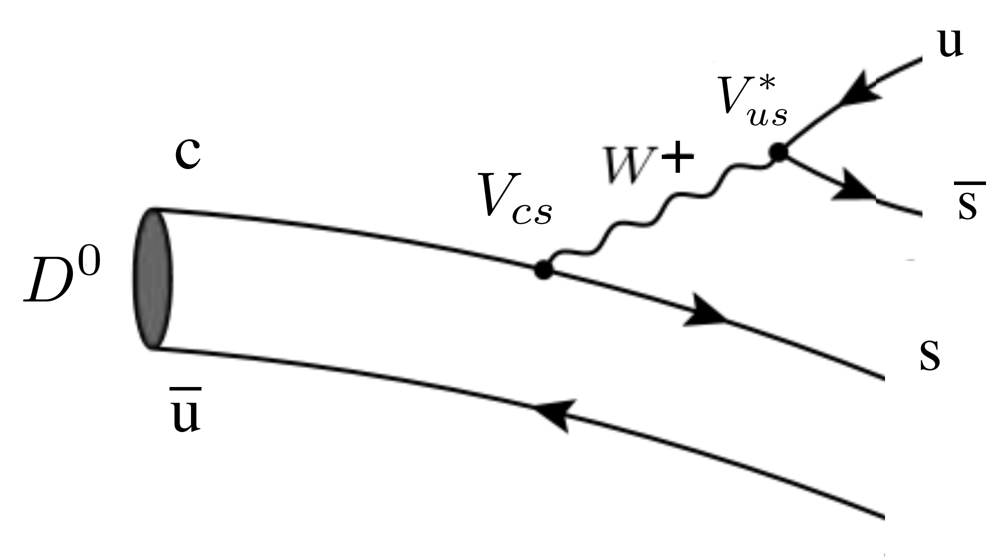
  - single cabibbo suppressed decays

$D^0 \to \pi^+\pi^-$



$D^0 \to K^+K^-$



$V_{cd}V_{ud}^* \approx \lambda(1 - \lambda^4\, e^{i\delta})$

$V_{cs}V_{us}^* \approx \lambda(1 - \lambda^2)$

  - weak phase in KK is 20 times smaller

Lenz, Wilkinson, Annu. Rev. Nucl. Part. Sci. 71, 59 (2021)

FSI as source of CP asymmetry in D decays

- how to explain the CPV in charm?

$$\Delta A_{CP}^{\mathrm{LHCb}} = A_{cp}(D^0 \to K^+K^-) - A_{cp}(D^0 \to \pi^+\pi^-) \; = -(1.54 \pm 0.29) \times 10^{-3}$$

  - single cabibbo suppressed decays

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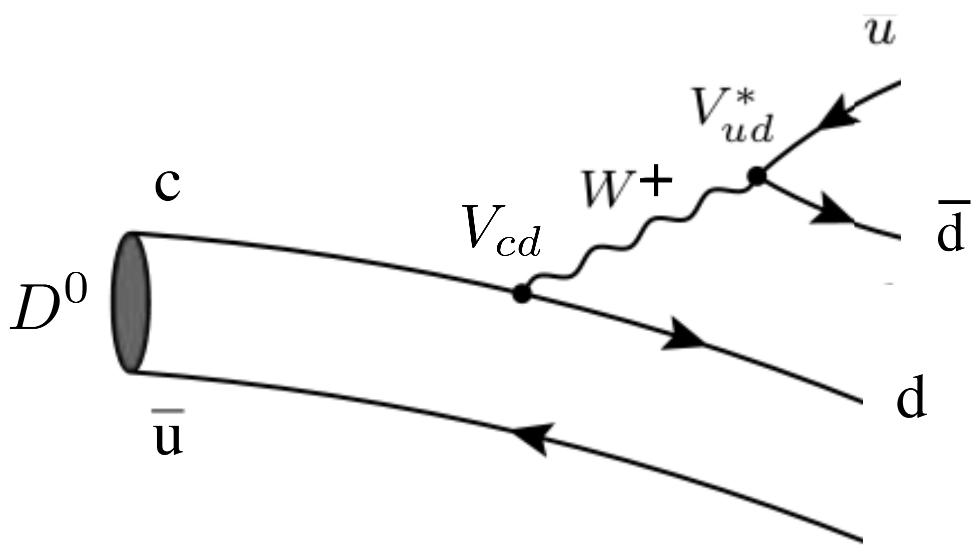

$$V_{cd}V_{ud}^* \approx \lambda(1 - \lambda^4\, e^{i\delta})$$

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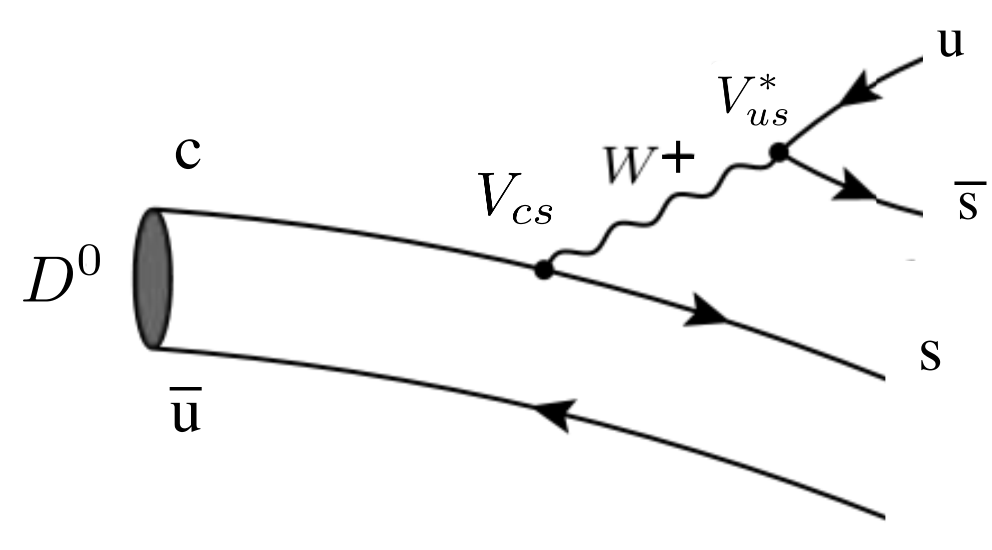


$$V_{cs}V_{us}^* \approx \lambda(1 - \lambda^2)$$

  - weak phase in KK is 20 times smaller Lenz ,Wilkinson, Annu. Rev. Nucl. Part. Sci. 71, 59 (2021)

→ what about strong phases if not from penguin? hadronic FSI

Grossman, Schacht JHEP07 20 (2019); Schacht, Soni PLB825 136855 (2022).

Theoretical approaches to CPV on charm

QCD short-distance

- QCDF → how to calculate penguin contributions? call BSM effects

Chala, Lenz, Rusov, Scholtz, JHEP 07, 161 (2019).

- LCSR → QCD, model independent but predictions are 1 order magnitude bellow

Khodjamirian, Petrov, Phys. Lett. B 774, 235 (2017)

long distance effect:

- topological and group symmetry approach
  - with SU(3) breaking through FSI (fit agrees)

    H.-Y. Cheng and C.-W. Chiang, PRD 100, 093002 (2019).
    F. Buccella, A. Paul and P. Santorelli, PRD 99, 113001 (2019)
    Franco, Mishima, Silvestrini JHEP05, 140 (2012)

  - with resonances (fit agrees)

    Schacht and A. Soni, Phys. Lett. B 825, 136855 (2022).
    Y. Grossman and S. Schacht, JHEP 07, 20 (2019)

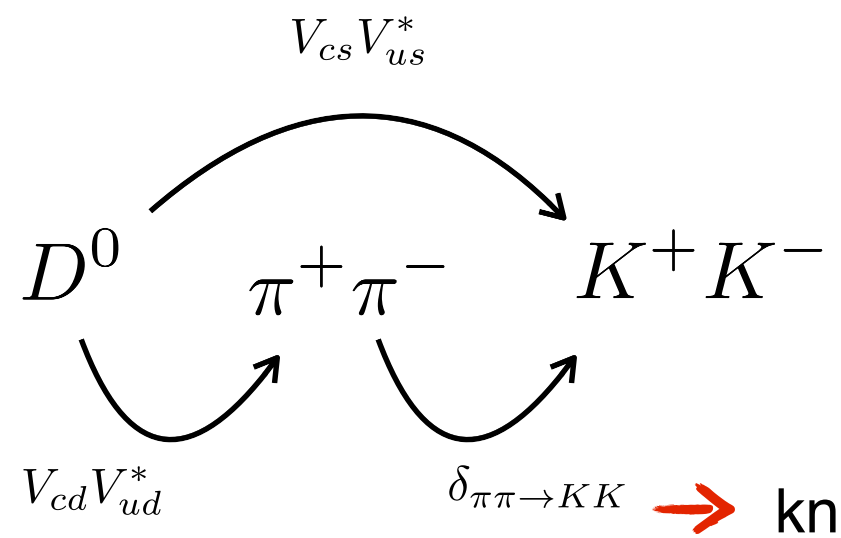
- FSI with CPT (prediction agrees)

bediaga, Frederico, PCM
arxiv 2203.04056v2

FSI as the source of CPV

- D and $\bar{D}$ can decay to $\pi\pi$ and KK

arxiv 2203.04056v2
Bediaga, Frederico, PCM



known from 80's experiment

FSI as the source of CPV

- D and $\bar{D}$ can decay to $\pi\pi$ and KK

arxiv 2203.04056v2

Bediaga, Frederico, PCM

$$\begin{tikzcd}
D^0 \arrow[r, bend right, swap, "V_{cd}V^*_{ud}"] \arrow[rr, bend left, "V_{cs}V^*_{us}"] & \pi^+\pi^- \arrow[r, bend right, swap, "\delta_{\pi\pi\to KK}"] & K^+K^-
\end{tikzcd}$$

→ known from 80's experiment

- build amplitudes decays implying three constraints:
  - CPT invariance relates channels with same quantum numbers

    $$\rightarrow \quad \sum \Delta\Gamma_{CP} = 0$$

  - Watson theorem relates the strong phase from the rescattering process to the decay amplitudes
  - the unitarity of the strong S-matrix.

FSI and CPT in D decays

- FSI in $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$ can include multiple mesons
  - general S-matrix can mix many FSI states $S = \begin{pmatrix} S_{2M,2M} & S_{2M,3M} & S_{2M,4M} & \cdots \\ S_{3M,2M} & S_{3M,3M} & S_{3M,4M} & \cdots \\ S_{4M,2M} & S_{4M,3M} & S_{4M,4M} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}$

- assume only 2 couple-channels to FSI, ie the dominant ones $\pi\pi, K\bar{K}$

$$\to \quad S_{2M,2M} = \begin{pmatrix} S_{\pi\pi,\pi\pi} & S_{\pi\pi,KK} \\ S_{KK,\pi\pi} & S_{KK,KK} \end{pmatrix}$$

$$\boxed{\begin{array}{ll} S_{\pi\pi,\pi\pi} = \eta\, \mathrm{e}^{2i\delta_{\pi\pi}} & S_{KK,KK} = \eta\, \mathrm{e}^{2i\delta_{KK}} \\ S_{\pi\pi,KK} = S_{KK,\pi\pi} = \imath\sqrt{1-\eta^2}\, \mathrm{e}^{\imath(\delta_{\pi\pi}+\delta_{KK})} & \end{array}}$$

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- two pions cannot go to three pions due to G-parity
- ignore four pion coupling to the 2M channel based on 1/Nc counting
- ignore $\eta\eta$ channel once their coupling to the $\pi\pi$ channel are suppressed with respect to $K\bar{K}$.

FSI and CPT in D decays

- FSI in $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$ can include multiple mesons
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  - two pions cannot go to three pions due to G-parity
  - ignore four pion coupling to the 2M channel based on 1/Nc counting
  - ignore $\eta\eta$ channel once their coupling to the $\pi\pi$ channel are suppressed with respect to $K\bar{K}$.
- CPT constraint restricted to the two-channels: $\sum_{f=(\pi\pi,KK)} (|\mathcal{A}_{D^0\to f}|^2 - |\mathcal{A}_{\bar{D}^0\to f}|^2) = 0$

Decay amplitudes

- dressing the weak tree topology with FSI
  - → penguin are suppressed

$$\begin{tikzcd} D^0 \arrow[r, bend right, "V_{cd}V_{ud}^*"'] \arrow[rr, bend left, "V_{cs}V_{us}^*"] & \pi^+\pi^- \arrow[r, bend right, "\delta_{\pi\pi\to KK}"'] & K^+K^- \end{tikzcd}$$

Decay amplitudes

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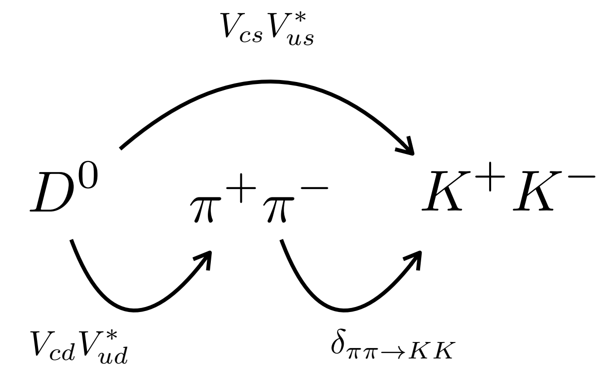
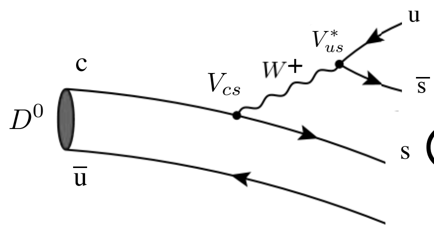
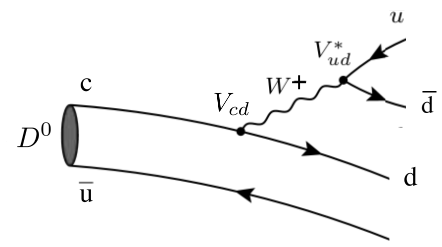


- $D^0 \to KK$



$\otimes$ $KK \to KK$ +



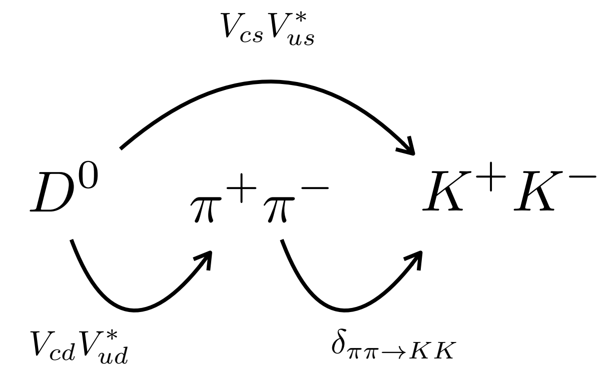
$\otimes$ $\pi\pi \to KK$

→ $$\mathcal{A}_{D^0\to KK} = \eta\, e^{2i\delta_{KK}}\, V_{cs}^* V_{us}\, a_{KK} + i\sqrt{1-\eta^2}\, e^{i(\delta_{\pi\pi}+\delta_{KK})}\, V_{cd}^* V_{ud}\, a_{\pi\pi}$$

→ $\mathcal{A}_{\bar{D}^0\to f}$ same with CKM cc.

Decay amplitudes

- dressing the weak tree topology with FSI
  - → penguin are suppressed



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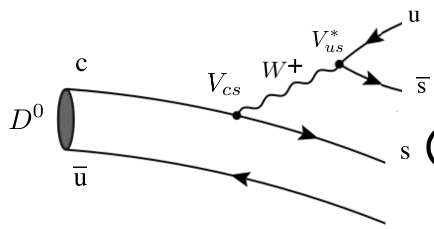

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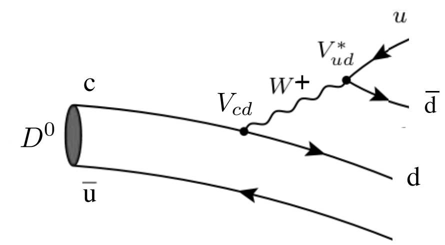
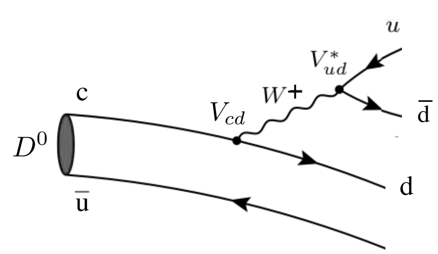


$\otimes$ $\pi\pi \to KK$

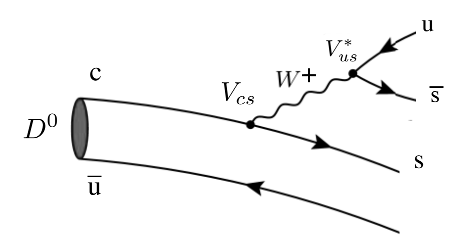
→ $\mathcal{A}_{D^0\to KK} = \eta\, \mathrm{e}^{2i\delta_{KK}}\, V_{cs}^* V_{us}\, a_{KK} + i\sqrt{1-\eta^2}\, \mathrm{e}^{i(\delta_{\pi\pi}+\delta_{KK})}\, V_{cd}^* V_{ud}\, a_{\pi\pi}$

→ $\mathcal{A}_{\bar{D}^0\to f}$ same with CKM cc.

- $D^0 \to \pi\pi$



$\otimes$ $\pi\pi \to \pi\pi$ +



$\otimes$ $KK \to \pi\pi$

→ $\mathcal{A}_{D^0\to \pi\pi} = \eta\, \mathrm{e}^{2i\delta_{\pi\pi}}\, V_{cd}^* V_{ud}\, a_{\pi\pi} + i\sqrt{1-\eta^2}\, \mathrm{e}^{i(\delta_{\pi\pi}+\delta_{KK})}\, V_{cs}^* V_{us}\, a_{KK}$

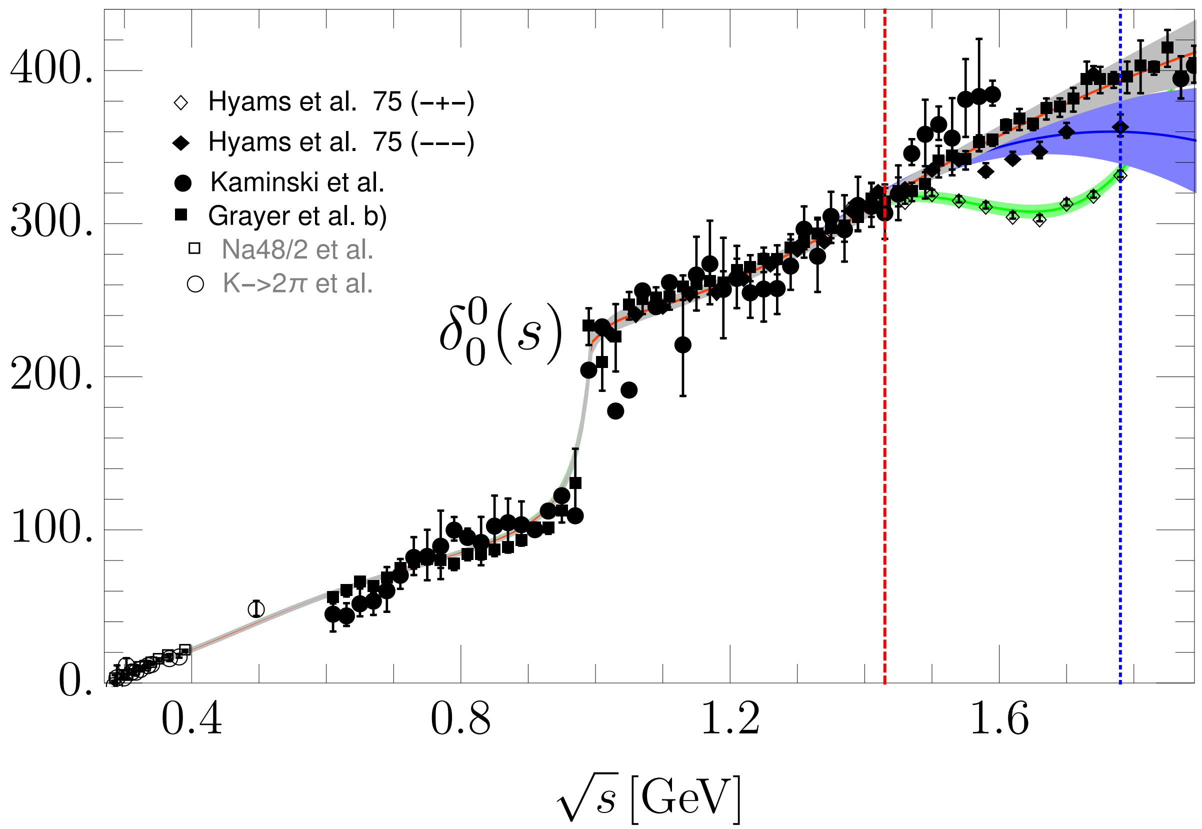
- a_{KK} and $a_{\pi\pi}$ do not carry any strong phases (real)

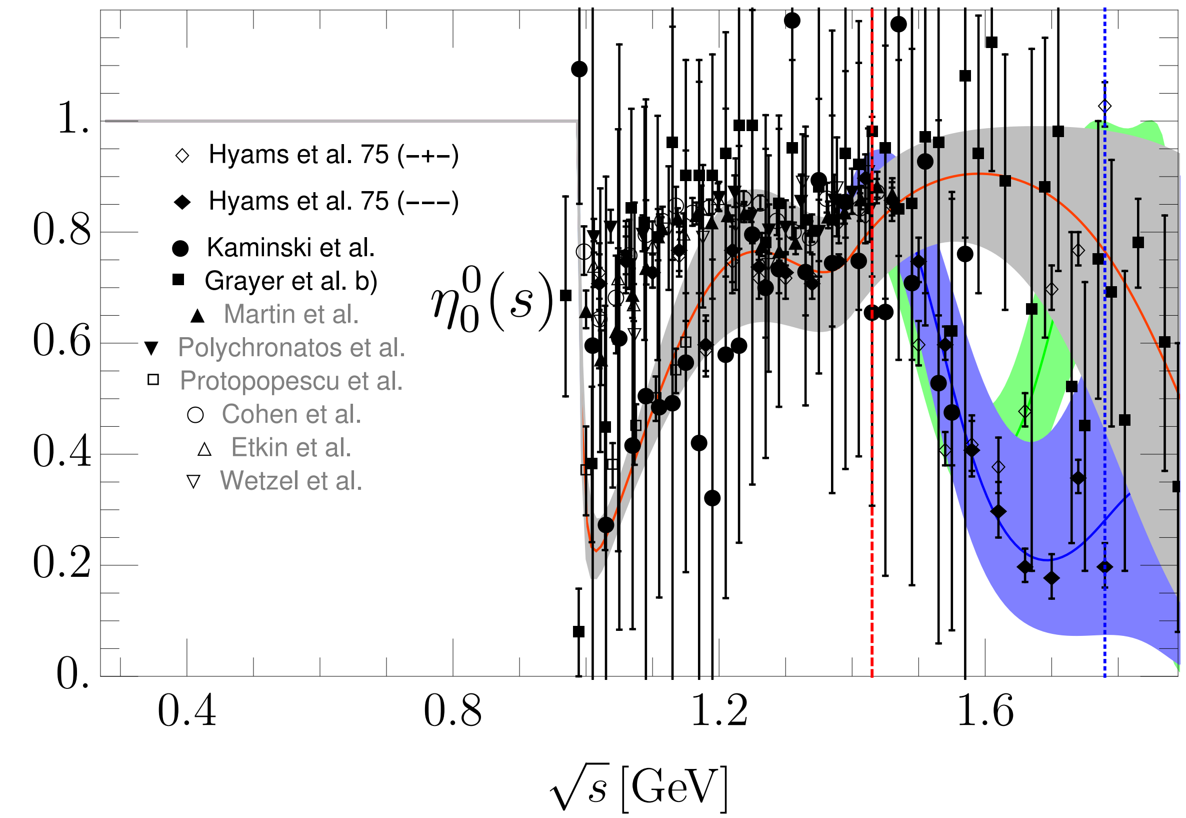
- $\delta_{\pi\pi}$, δ_{KK} and $\delta_{\pi\pi\to KK}$ are the same independent of the initial process

→ we can use CERN-Munich data from 80's

Longacre et al., Phys. Lett. B 177, 223 (1986)
Hyams et al., Nucl. Phys. B 100, 205 (1975)
Ochs, J. Phys. G 40, 043001 (2013)

- $\pi\pi \to \pi\pi$





Pelaez, Rodas, Elvira *Eur.Phys.J.C* 79 (2019) 12, 1008

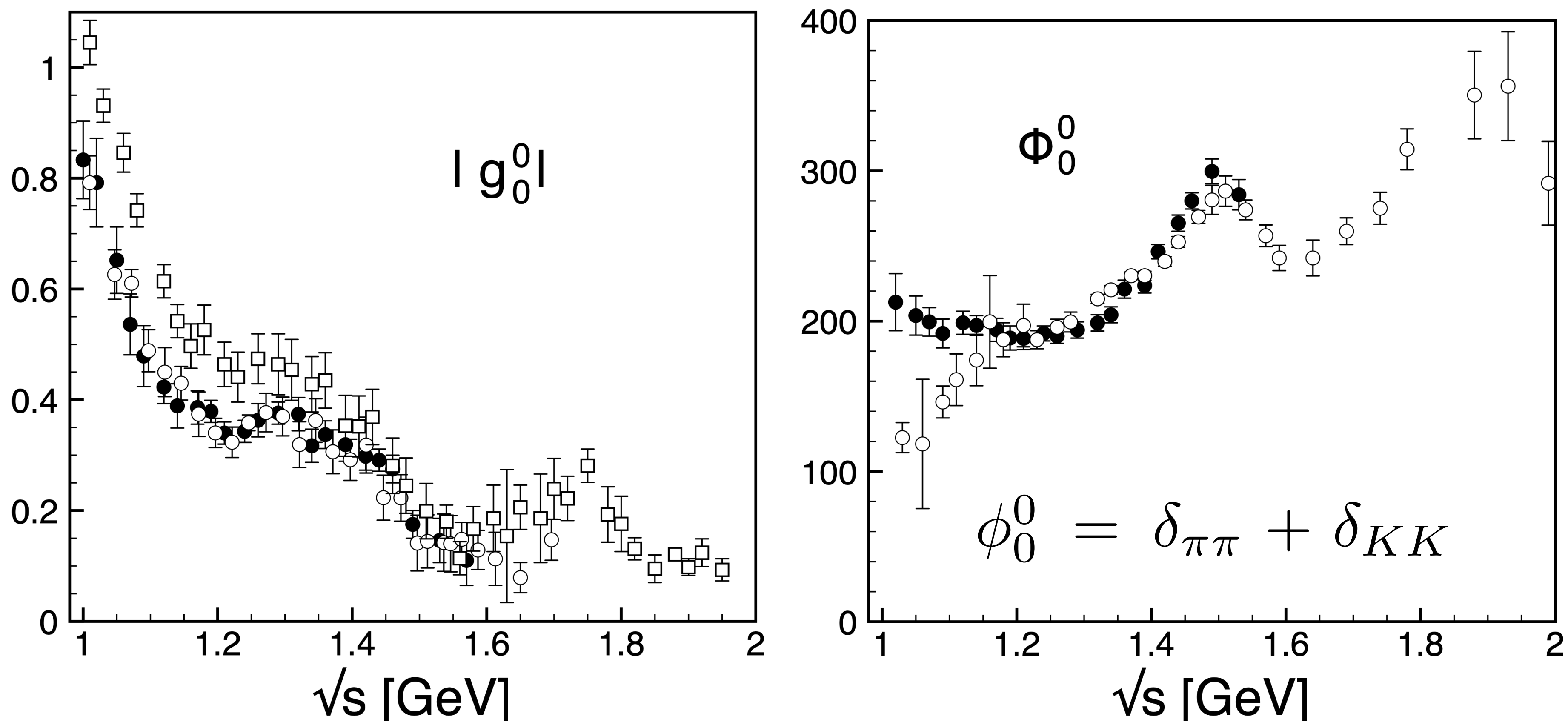
amplitude $\hat{f}_l(s) = \left[\frac{\eta_l e^{2i\delta_l} - 1}{2i}\right].$

→ elasticity drops dramatically near $K\bar{K}$ → strongly couple

- $\pi\pi \to KK$

$$\rightarrow \quad S_{\pi\pi,KK}(s) = \imath\sqrt{1-\eta^2}\,\mathrm{e}^{\imath(\delta_{\pi\pi}+\delta_{KK})} = i\,4\sqrt{\frac{q_\pi q_K}{s}}|g_0^0(s)|\,e^{i\phi_0^0(s)}\,\Theta(s-4m_K^2)$$

Pelaez and Rodas, Eur. Phys. J. C 78, 897 (2018)



Cohen et al., Phys. Rev. D 22, 2595 (1980)
Etkin et al., Phys. Rev. D 25, 1786 (1982)

- Pelaez parametrization @ M_D^2:

$$|g_0^0(M_D^2)| \approx 0.125 \pm 0.025 \quad \rightarrow \quad \sqrt{1-\eta^2} \approx 0.229 \pm 0.046 \quad \rightarrow \quad \eta \approx 0.973$$

$$\phi_0^0 = \delta_{\pi\pi} + \delta_{KK} \approx 343^o \pm 8^o$$

Partial decay widths

- $\Delta\Gamma_f = \Gamma\left(D^0 \to f\right) - \Gamma(\bar{D}^0 \to f)$

$$\mathcal{A}_{D^0\to\pi\pi} = \eta\, \mathrm{e}^{2i\delta_{\pi\pi}}\, V_{cd}^* V_{ud}\, a_{\pi\pi} + i\sqrt{1-\eta^2}\, \mathrm{e}^{i(\delta_{\pi\pi}+\delta_{KK})}\, V_{cs}^* V_{us}\, a_{KK}$$

$$\mathcal{A}_{D^0\to KK} = \eta\, \mathrm{e}^{2i\delta_{KK}}\, V_{cs}^* V_{us}\, a_{KK} + i\sqrt{1-\eta^2}\, \mathrm{e}^{i(\delta_{\pi\pi}+\delta_{KK})}\, V_{cd}^* V_{ud}\, a_{\pi\pi}$$

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$\rightarrow$ $\Delta\Gamma_{\pi\pi} = -\Delta\Gamma_{KK} = 4\,\mathrm{Im}[V_{cs} V_{us}^* V_{cd}^* V_{ud}]\, a_{\pi\pi}\, a_{KK}\, \eta\sqrt{1-\eta^2}\cos\phi$

  - $\phi = \delta_{KK} - \delta_{\pi\pi}$
  - the sign of $\Delta\Gamma_f$ is determined by the CKM elements and the S-wave phase-shifts

Partial decay widths

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$$\rightarrow \Delta\Gamma_{\pi\pi} = -\Delta\Gamma_{KK} = 4\,\mathrm{Im}[V_{cs}V_{us}^* V_{cd}^* V_{ud}]\, \underline{a_{\pi\pi}\, a_{KK}}\, \eta\sqrt{1-\eta^2}\cos\phi$$

- $\phi = \delta_{KK} - \delta_{\pi\pi}$
- the sign of $\Delta\Gamma_f$ is determined by the CKM elements and the S-wave phase-shifts
- need to quantify $a_{\pi\pi}$ and a_{KK}:

at D^0 mass $\sqrt{1-\eta^2} << 1$ $\rightarrow$

$$\Gamma_{\pi\pi} \approx \eta^2 |V_{cd}^* V_{ud}|^2\, a_{\pi\pi}^2$$
$$\Gamma_{KK} \approx \eta^2 |V_{cs}^* V_{us}|^2 a_{KK}^2$$

$Br[D \to f] = \Gamma_f / \Gamma_{total}$

we can use experimental input

Partial decay widths

- $\Delta\Gamma_f = \Gamma\left(D^0 \to f\right) - \Gamma(\bar{D}^0 \to f)$

$$\mathcal{A}_{D^0\to\pi\pi} = \eta\, \mathrm{e}^{2i\delta_{\pi\pi}}\, V_{cd}^* V_{ud}\, a_{\pi\pi} + i\sqrt{1-\eta^2}\, \mathrm{e}^{i(\delta_{\pi\pi}+\delta_{KK})}\, V_{cs}^* V_{us}\, a_{KK}$$

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  - $\phi = \delta_{KK} - \delta_{\pi\pi}$
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at D^0 mass $\sqrt{1-\eta^2} << 1$ $\rightarrow$ $\Gamma_{\pi\pi} \approx \eta^2 |V_{cd}^* V_{ud}|^2\, a_{\pi\pi}^2$

$\Gamma_{KK} \approx \eta^2 |V_{cs}^* V_{us}|^2 a_{KK}^2$

$Br[D \to f] = \Gamma_f / \Gamma_{total}$

we can use experimental input

- $A_{CP}(f) = \dfrac{\Gamma\left(D^0 \to f\right) - \Gamma(\bar{D}^0 \to f)}{\Gamma\left(D^0 \to f\right) + \Gamma(\bar{D}^0 \to f)}$ $= \Delta\Gamma_f / 2\Gamma_f$

Final values for A_{CP}

- $$A_{CP}(f) \approx \pm 2 \frac{\mathrm{Im}[V_{cs}V_{us}^* V_{cd}^* V_{ud}]}{|V_{cs}V_{us}^* V_{cd}^* V_{ud}|} \eta^{-1} \sqrt{1-\eta^2} \cos\phi \left[\frac{\mathrm{Br}(D^0 \to K^+K^-)}{\mathrm{Br}(D^0 \to \pi^+\pi^-)} \right]^{\pm\frac{1}{2}}$$

$+\ \pi\pi$
$-\ KK$

  - $\mathrm{Br}(D^0 \to \pi^+\pi^-) = (1.455 \pm 0.024) \times 10^{-3}$ PDG
    $\mathrm{Br}(D^0 \to K^+K^-) = (4.08 \pm 0.06) \times 10^{-3}$

  - $$\frac{\mathrm{Im}[V_{cs}V_{us}^* V_{cd}^* V_{ud}]}{|V_{cs}V_{us}^* V_{cd}^* V_{ud}|} = (6.02 \pm 0.32) \times 10^{-4}$$ PDG

  - $\cos\phi$: $\phi = \delta_{KK} - \delta_{\pi\pi} = (\delta_{KK} + \delta_{\pi\pi}) - 2\delta_{\pi\pi}$

Final values for A_{CP}

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  - $\cos\phi$: $$\phi = \delta_{KK} - \delta_{\pi\pi} = \underbrace{(\delta_{KK}+\delta_{\pi\pi})}_{\phi_0^0} - 2\delta_{\pi\pi}$$

Final values for A_{CP}

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  $+ \ \pi\pi$
  $- \ KK$

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  - $\dfrac{\mathrm{Im}[V_{cs}V_{us}^*V_{cd}^*V_{ud}]}{|V_{cs}V_{us}^*V_{cd}^*V_{ud}|} = (6.02\pm 0.32)\times 10^{-4}$ PDG

  - $\cos\phi$: $\quad \phi = \delta_{KK} - \delta_{\pi\pi} = (\delta_{KK}+\delta_{\pi\pi}) - 2\delta_{\pi\pi}$

    ϕ_0^0

  from $\pi\pi$ and $\pi\pi \to KK$ data: $\cos\phi = 0.99 \pm 0.18$.

Final values for A_{CP}

- $$A_{CP}(f) \approx \pm 2 \, \frac{\mathrm{Im}[V_{cs}V_{us}^{*}V_{cd}^{*}V_{ud}]}{|V_{cs}V_{us}^{*}V_{cd}^{*}V_{ud}|} \, \eta^{-1}\sqrt{1-\eta^2}\,\cos\phi \left[\frac{\mathrm{Br}(D^0\to K^+K^-)}{\mathrm{Br}(D^0\to\pi^+\pi^-)}\right]^{\pm\frac{1}{2}}$$

  $+$ $\pi\pi$
  $-$ KK

  - $\mathrm{Br}(D^0 \to \pi^+\pi^-) = (1.455 \pm 0.024) \times 10^{-3}$
    $\mathrm{Br}(D^0 \to K^+K^-) = (4.08 \pm 0.06) \times 10^{-3}$ PDG

  - $$\frac{\mathrm{Im}[V_{cs}V_{us}^{*}V_{cd}^{*}V_{ud}]}{|V_{cs}V_{us}^{*}V_{cd}^{*}V_{ud}|} = (6.02 \pm 0.32) \times 10^{-4}$$ PDG

  - $\cos\phi$: $\phi = \delta_{KK} - \delta_{\pi\pi} = (\delta_{KK} + \delta_{\pi\pi}) - 2\delta_{\pi\pi}$ $\quad \phi_0^0$

  from $\pi\pi$ and $\pi\pi \to KK$ data: $\cos\phi = 0.99 \pm 0.18$

- $$\boxed{\begin{aligned} A_{CP}(\pi\pi) &= (1.99 \pm 0.37) \times 10^{-3}\sqrt{\eta^{-2}-1} \\ A_{CP}(KK) &= -(0.71 \pm 0.13) \times 10^{-3}\sqrt{\eta^{-2}-1} \end{aligned}}$$

as a function of inelasticity

Predictions for ΔA_{CP}

- $\Delta A_{CP}^{th} = -(2.70 \pm 0.50) \times 10^{-3} \sqrt{\eta^{-2} - 1}$

$*$

$* \sqrt{1 - \eta^2} \approx 1$ not valid

Predictions for ΔA_{CP}

$$\Delta A_{CP}^{\mathrm{LHCb}} = -(1.54 \pm 0.29) \times 10^{-3}$$

- $\Delta A_{CP}^{th} = -(2.70 \pm 0.50) \times 10^{-3} \sqrt{\eta^{-2} - 1}$

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$$\boxed{\Delta A_{CP}^{th} = -(0.64 \pm 0.18) \times 10^{-3}}$$ 3σ

→ largest theoretical prediction within SM without relying on fitting parameters

→ systematic uncertainties are unknown in η → error is underestimated

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- Alternatively one can assume all inelasticity in $\pi\pi \rightarrow \pi\pi$ is due to KK

→ more precise data (Grayer) → $\eta = 0.78 \pm 0.08$ *

$$\boxed{\Delta A_{CP}^{th} = -(1.31 \pm 0.20) \times 10^{-3}}$$

1σ

* $\sqrt{1 - \eta^2} \approx 1$ not valid

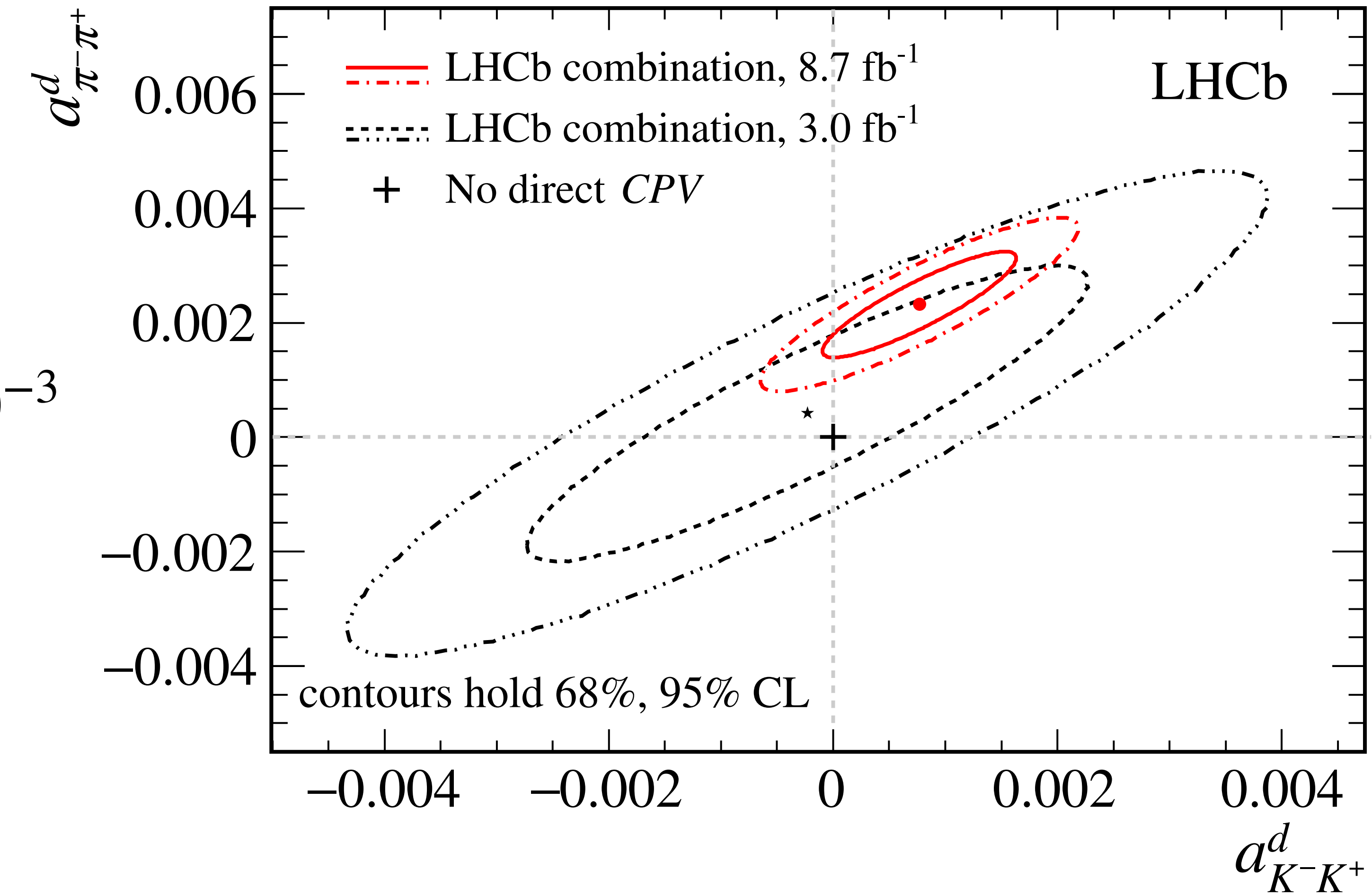
Predictions for $A_{CP}(hh)$

- direct CP asymmetry observation

$$A_{CP}^{LHCb}(KK) = (0.77 \pm 0.57) \times 10^{-3}$$

$$\hookrightarrow A_{CP}^{LHCb}(\pi\pi) = (2.32 \pm 0.61) \times 10^{-3}$$

arXiv:2209.03179



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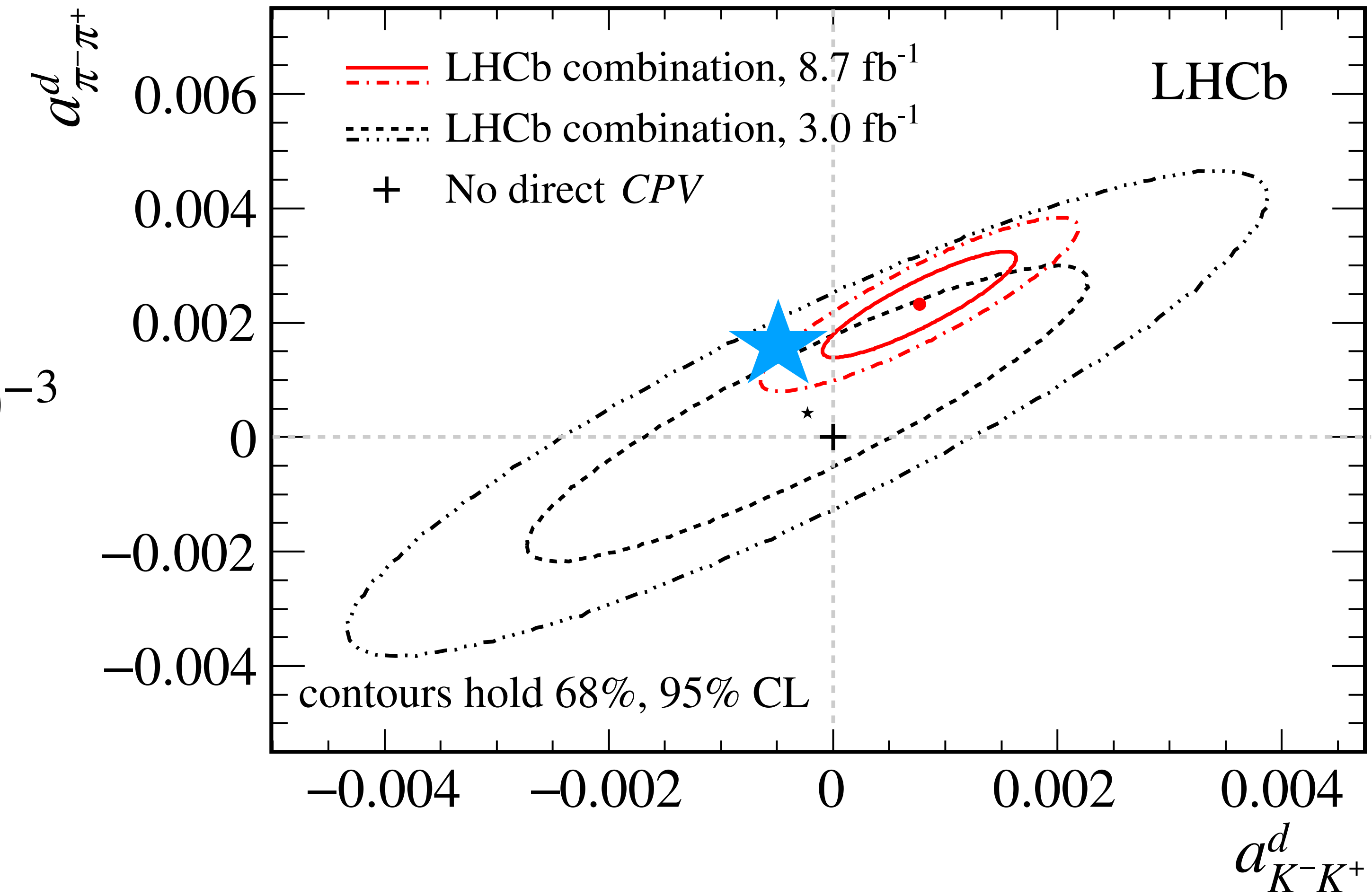


- with $\eta = 0.78 \pm 0.08$

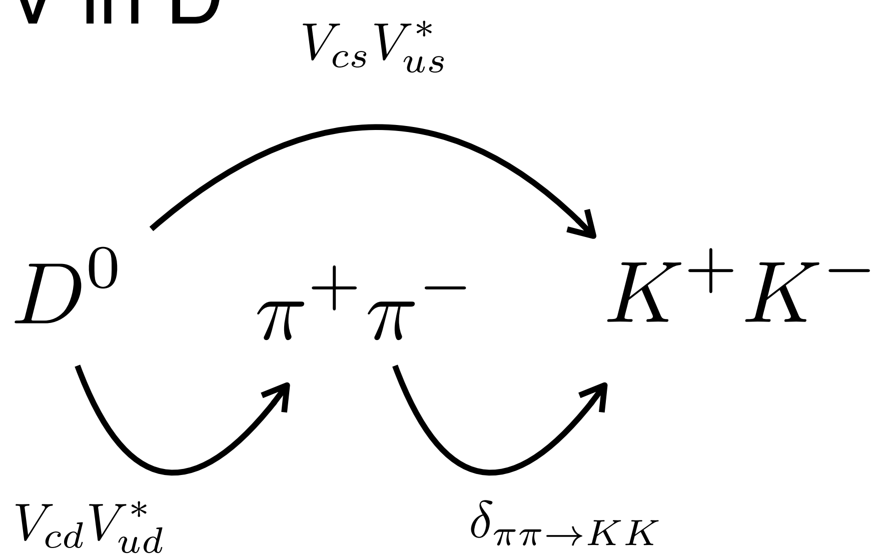
$$\boxed{\begin{aligned} A_{CP}(KK) &= -(0.34 \pm 0.15) \times 10^{-3} \\ A_{CP}(\pi\pi) &= (0.97 \pm 0.05) \times 10^{-3} \end{aligned}}$$

2σ

Final remarks

- hadronic FSI (and their strong phases) are crucial to explain CP violation in B and D decays

- we proposed a mechanism that can explain CPV in D
  - coupling $\pi\pi \leftrightarrow K\bar{K}$ in a CPT invariant framework
  - still room to add 2nd order effects



→ predicted ΔA_{CP} which is compatible with LHCb

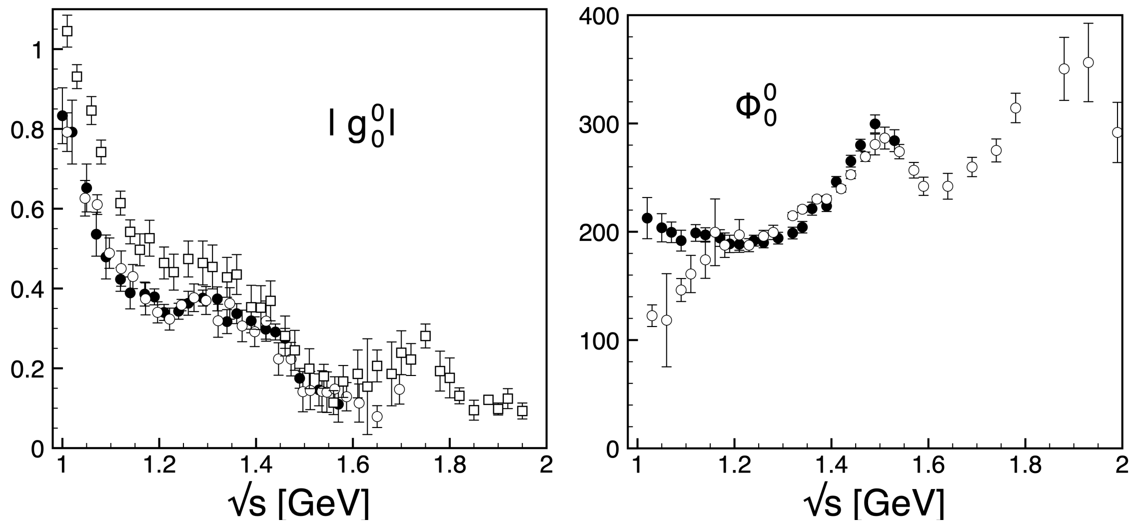
- new measurement for $A_{CP}(hh)$ from LHCb
  - agrees with our predictions with 2σ

  ↳ we still need more data to fully understood it

What's next?

- In 3-body decays this effect will be bigger and phase-space distributed

  ↪ $D^+ \to \pi^+\pi^-\pi^+$ and $D^+ \to \pi^+K^-K^+$ have exactly the same Weak vertex



- expected CPV in run II analysis

50 million events…

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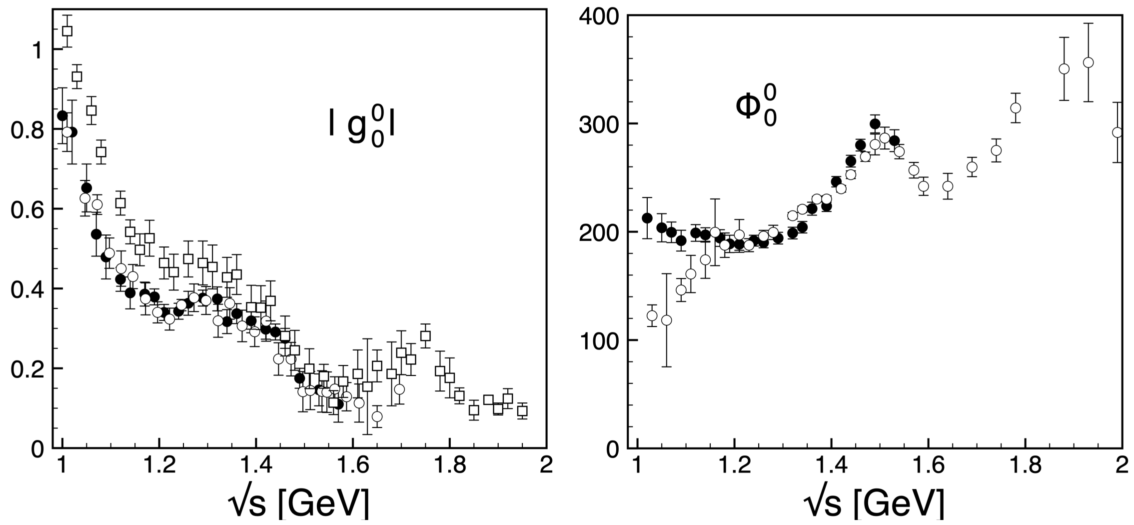


- expected CPV in run II analysis

50 million events…

thank you!

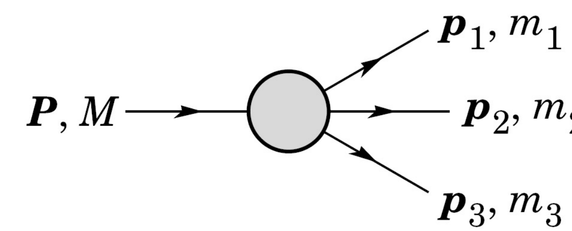
Gracias!

Obrigada!

#forabolsonaro

Backup slides

- In three-body decay phase-space is NOT one-dimension!
  - bi-dimension phase-space information



- DALITZ PLOT : proposed by Richard Dalitz (1925-2006) in 1953

Mandelstam variables for 3-body

$$s_{12} = (p_1 + p_2)^2 = m_{12}^2$$

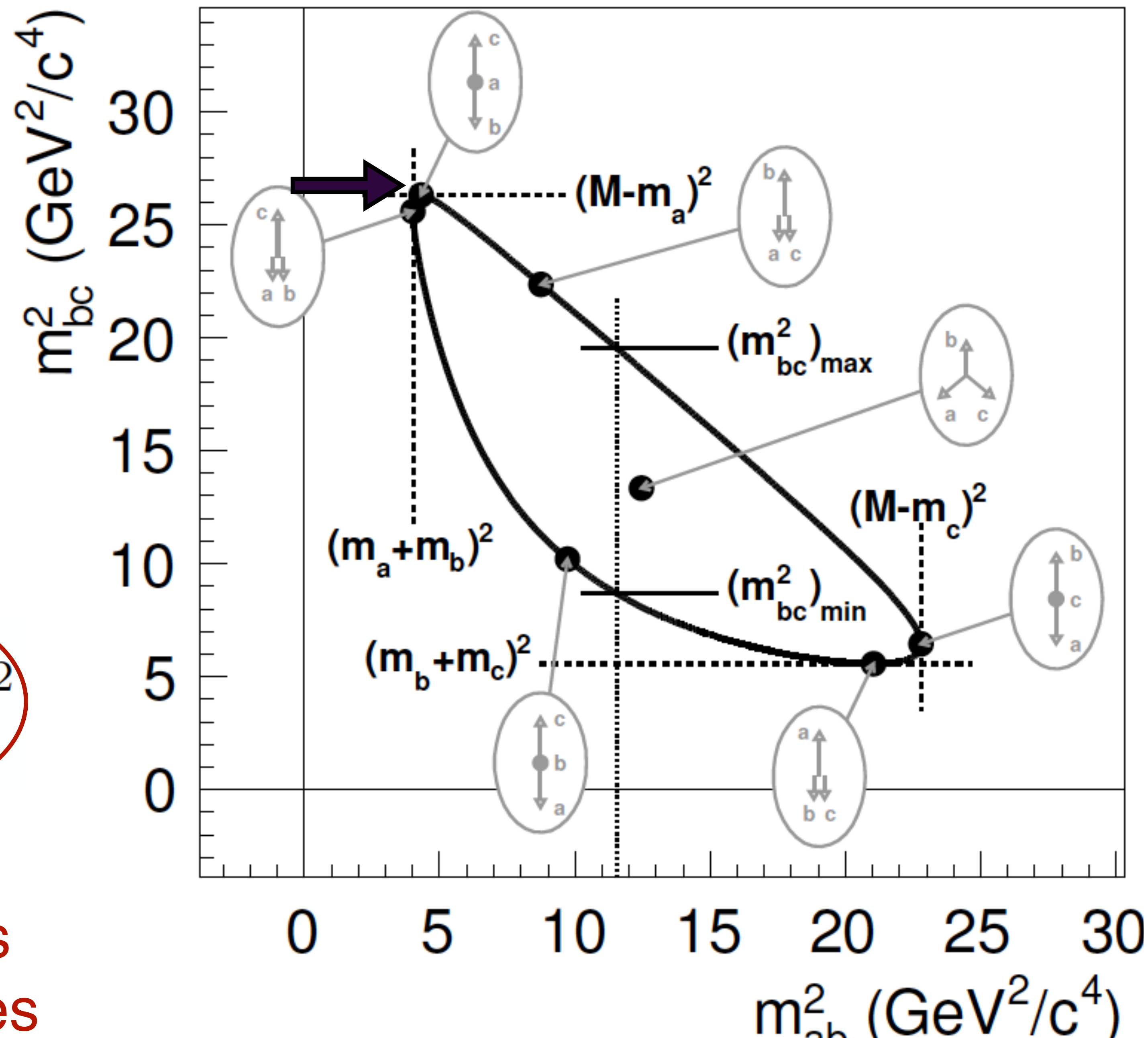
$$s_{13} = (p_1 + p_3)^2 = m_{13}^2$$

$$s_{23} = (p_2 + p_3)^2 = m_{23}^2$$

$$s_{12} + s_{13} + s_{12} = M^2 + m_1^2 + m_2^2 + m_3^2$$

$$\frac{d\Gamma}{ds_{12}ds_{23}} = \frac{1}{(2\pi)^3}\frac{1}{32M^3}|\mathcal{A}(s_{12}, s_{23})|^2$$

dynamics
resonances



- we don't have data from KK scattering !
  - we can use $\pi\pi$ and $KK \to \pi\pi$ data: $\delta_{KK} - \delta_{\pi\pi} = \phi_0^0 - 2\delta_{\pi\pi} = (\delta_{KK} + \delta_{\pi\pi}) - 2\delta_{\pi\pi}$
  - CERN-Munich data (revised Ochs)

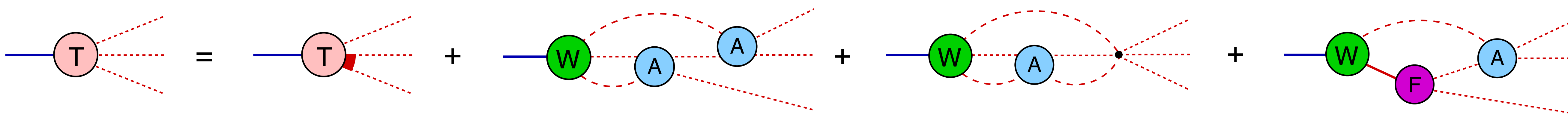
$\sqrt{s}$ [GeV]	$\cos\phi$
1.58	0.989 ± 0.149
1.62	0.994 ± 0.105
1.66	0.999 ± 0.040
1.70	0.987 ± 0.160
1.74	0.999 ± 0.048
1.78	0.999 ± 0.037
$m_D^2 \rightarrow$ 1.846	0.987 ± 0.175

$\rightarrow \cos(\delta_{KK} - \delta_{\pi\pi}) \lesssim 1$

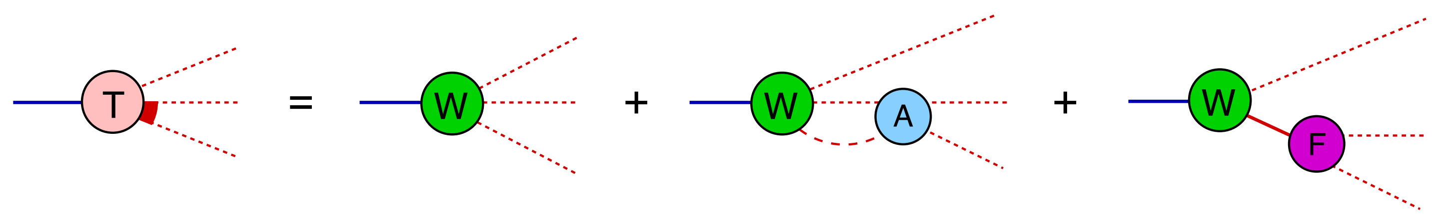
$\rightarrow$ Pelaez parametrization

Full story in 3-body decay

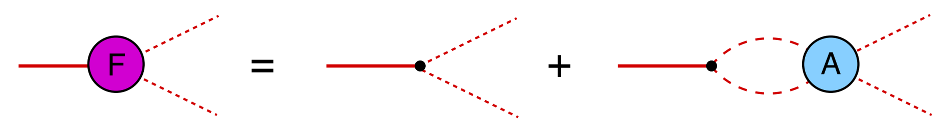
- Any 3-body decay amplitude



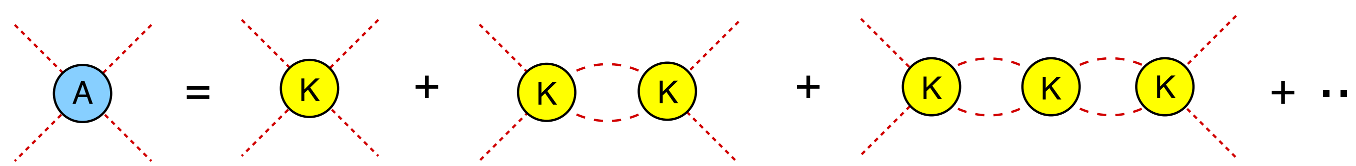
(2+1) approach



Form factor

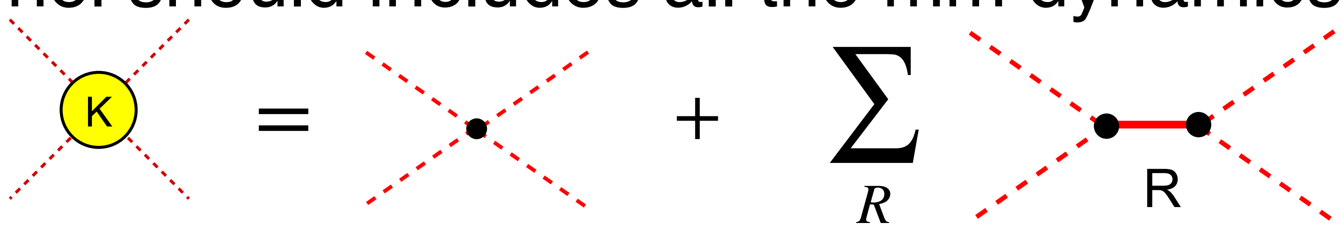


meson-meson

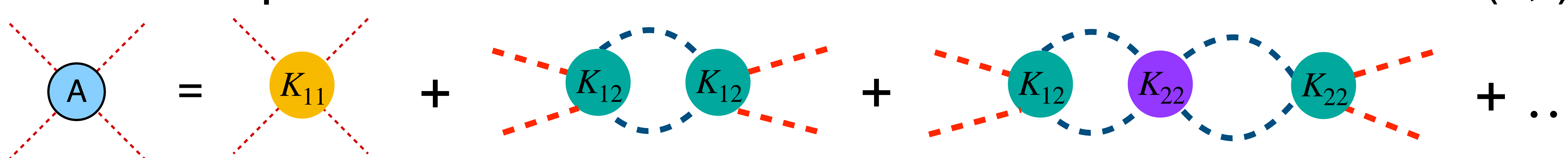


MAGALHAES, A.dos Reis, Robilotta
PRD 102, 076012 (2020)

- kernel should includes all the mm dynamics

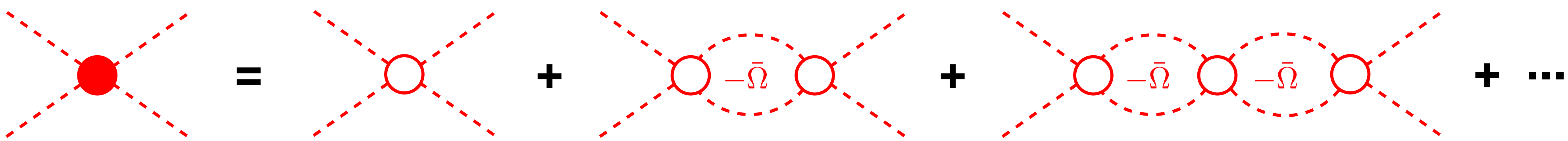


- Unitarized amplitude should includes all channels with the same (J,I)

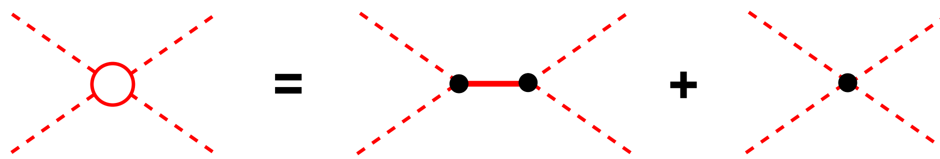


unitarized amplitude $P^a P^b \to P^c P^d$

- unitarize amplitude by Bethe-Salpeter eq. [Oller and Oset PRD 60 (1999)]

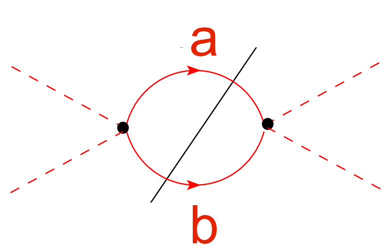


- kernel $\mathcal{K}^{(J,I)}_{ab\to cd}$



resonance (NLO) + contact (LO)

- loops $\longrightarrow$ K-matrix approximation: only on-shell



$$\{I_{ab};\, I^{\mu\nu}_{ab}\} = \int \frac{d^4\ell}{(2\pi)^4}\frac{\{1;\, \ell^\mu\,\ell^\nu\}}{D_a\, D_b}$$

$$D_a = (\ell+p/2)^2 - M_a^2 \qquad D_b = (\ell-p/2)^2 - M_b^2$$

$\longrightarrow$

$$\bar{\Omega}^S_{ab} = -\frac{i}{8\pi}\frac{Q_{ab}}{\sqrt{s}}\,\theta(s-(M_a+M_b)^2)$$

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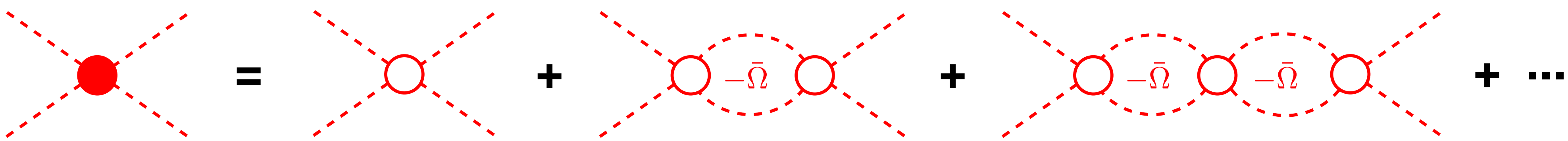

$$\boxed{\mathcal{A}^{JI}_{ab} = \frac{\mathcal{K}^{(JI)}_{ab\to cd}}{1+\bar{\Omega}_{ab}\,\mathcal{K}^{(JI)}_{ab\to cd}}}$$

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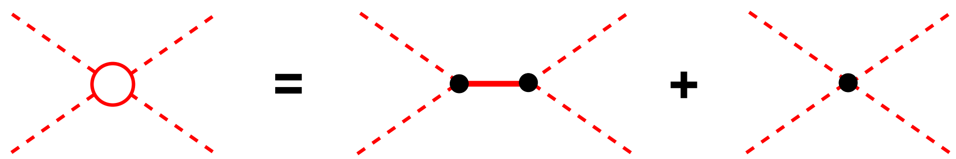

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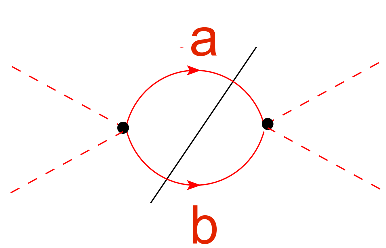

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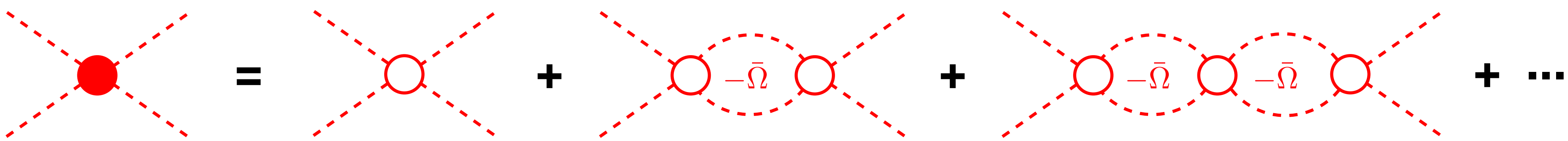

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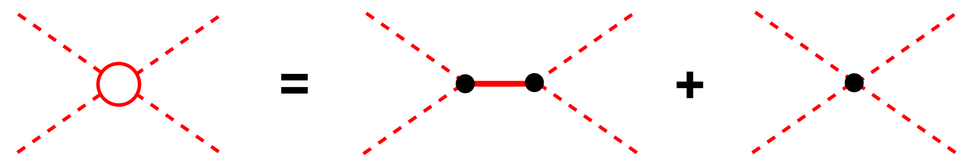

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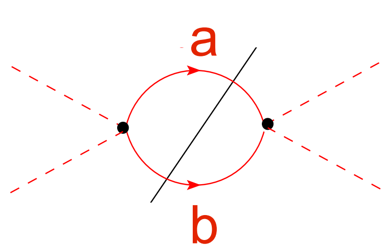

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- free parameters
  - masses:

$m_\rho\,,\ m_{a_0}\,,\ \boxed{m_{s0}\,,\ m_{s1}}$ — $SU(3)$ singlet and octet

  - coupling constants:

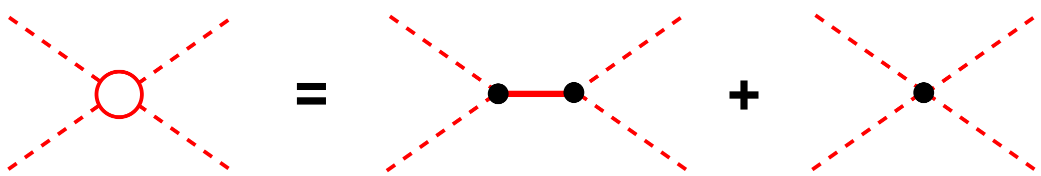
$g_\rho\,,\ g_\phi$ vector

$c_d\,,\ c_m\,,\ \tilde{c}_d\,,\ \tilde{c}_m$ scalar

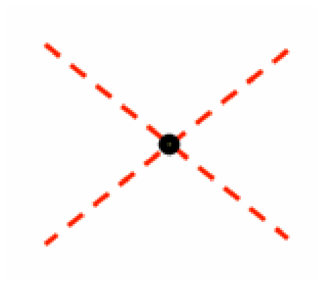
$\rightarrow$ physical f_0 states are linear combination of $m_{s0}\,,\ m_{s1}$

meson-meson interactions at low E

- solid theory to describe MM interactions at low energy == ChPT



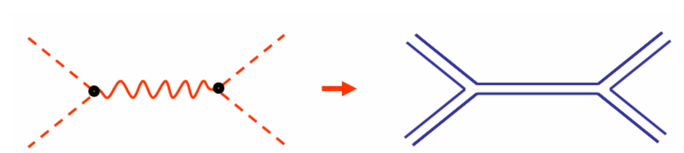
- LO:



$$\mathcal{L}_M^{(2)} = -\frac{1}{6F^2} f_{ijs}\, f_{kls}\, \phi_i\, \partial_\mu \phi_j\, \phi_k\, \partial^\mu \phi_l + \frac{B}{24F^2}\Big[\sigma_0 \left(\frac{4}{3}\delta_{ij}\,\delta_{kl} + 2\, d_{ijs}\, d_{kls}\right) + \sigma_8 \left(\frac{4}{3}\delta_{ij}\, d_{kl8} + \frac{4}{3} d_{ij8}\,\delta_{kl} + 2\, d_{ijm}\, d_{kln}\, d_{8mn}\right)\Big]\, \phi_i\, \phi_j\, \phi_k\, \phi_l$$

Gasser & Leutwyler [Nucl. Phys. B250(1985)]

- NLO: include resonances as a field



Ecker, Gasser, Pich and De Rafael [Nucl. Phys. B321(1989)]

scalars

$$\mathcal{L}_S^{(2)} = \frac{2\,\tilde{c}_d}{F^2} R_0\, \partial_\mu \phi_i\, \partial^\mu \phi_i - \frac{4\,\tilde{c}_m}{F^2} B\, R_0 \left(\sigma_0\, \delta_{ij} + \sigma_8\, d_{8ij}\right) \phi_i\, \phi_j$$
$$+ \frac{2\, c_d}{\sqrt{2}F^2} d_{ijk}\, R_k\, \partial_\mu \phi_i\, \partial^\mu \phi_i - \frac{4Bc_m}{\sqrt{2}F^2}\left[\sigma_0\, d_{ijk} + \sigma_8 \left(\frac{2}{3}\delta_{ik}\,\delta_{j8} + d_{i8s}\, d_{jsk}\right)\right] \phi_i\, \phi_j R_k$$

vectors

$$\mathcal{L}_V^{(2)} = \frac{iG_V}{\sqrt{2}} \langle V_{\mu\nu} u^\mu u^\nu \rangle$$
$$\langle V_{\mu\nu} u^\mu u^\nu \rangle = \frac{1}{F^2} V_a^{\mu\nu}\, \partial_\mu \phi_i\, \partial_\nu \phi_j\, (i f_{aij} + d_{aij})$$

→ because we want to extend this to high E the parameters change meaning and can be free to fit!