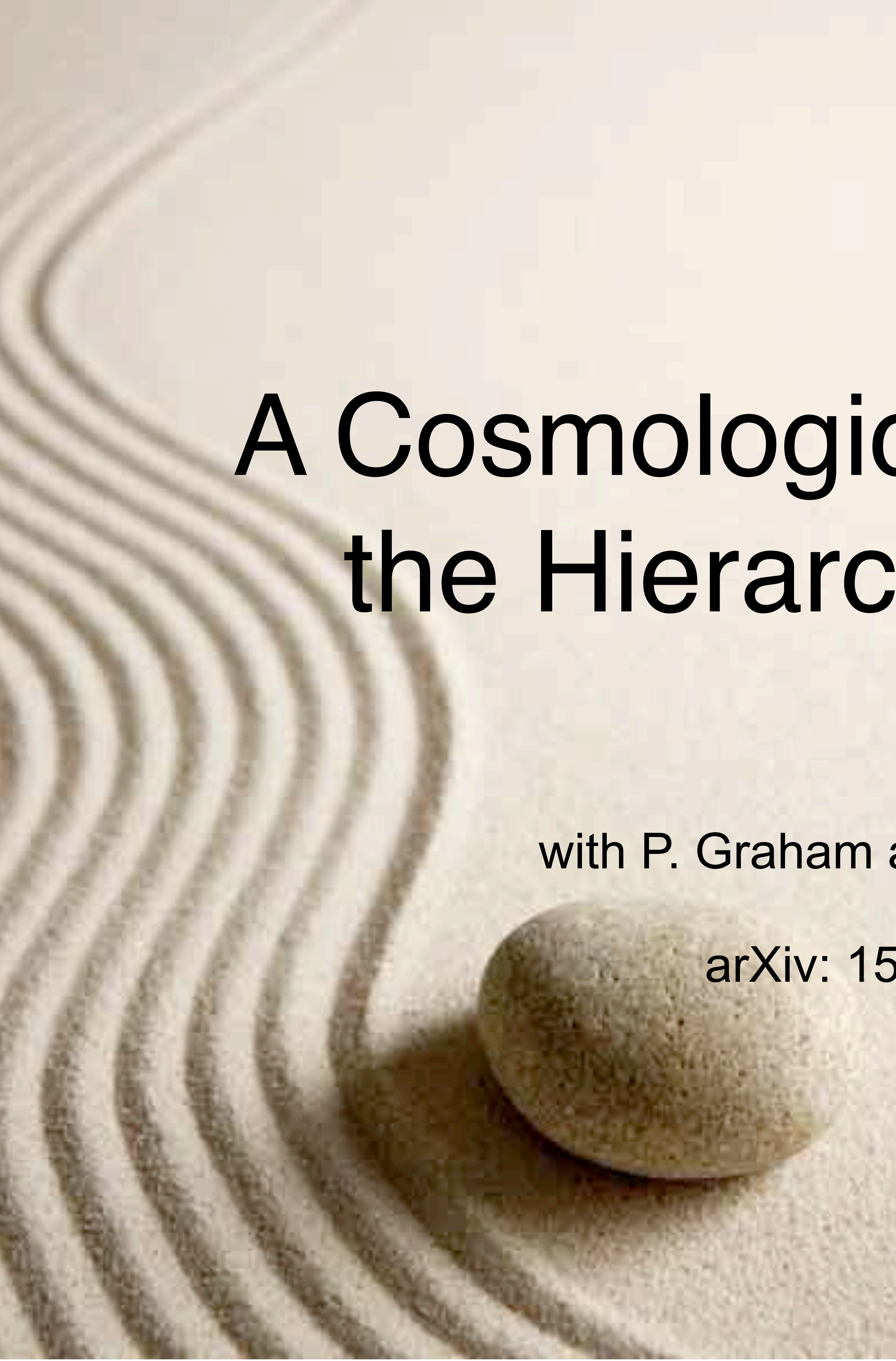


A Cosmological Solution to the Hierarchy Problem

with P. Graham and S. Rajendran

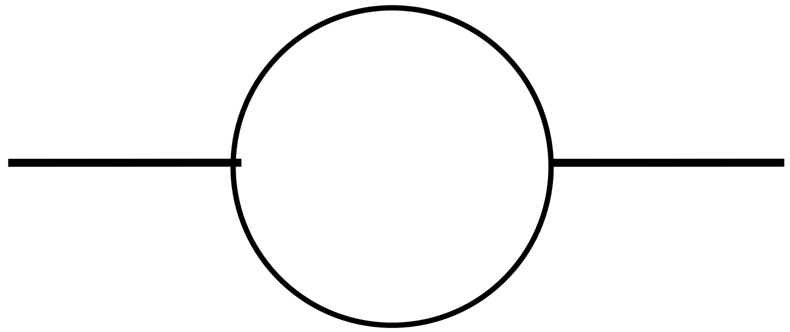
arXiv: 1504.07551

the Relaxion

The Hierarchy Problem

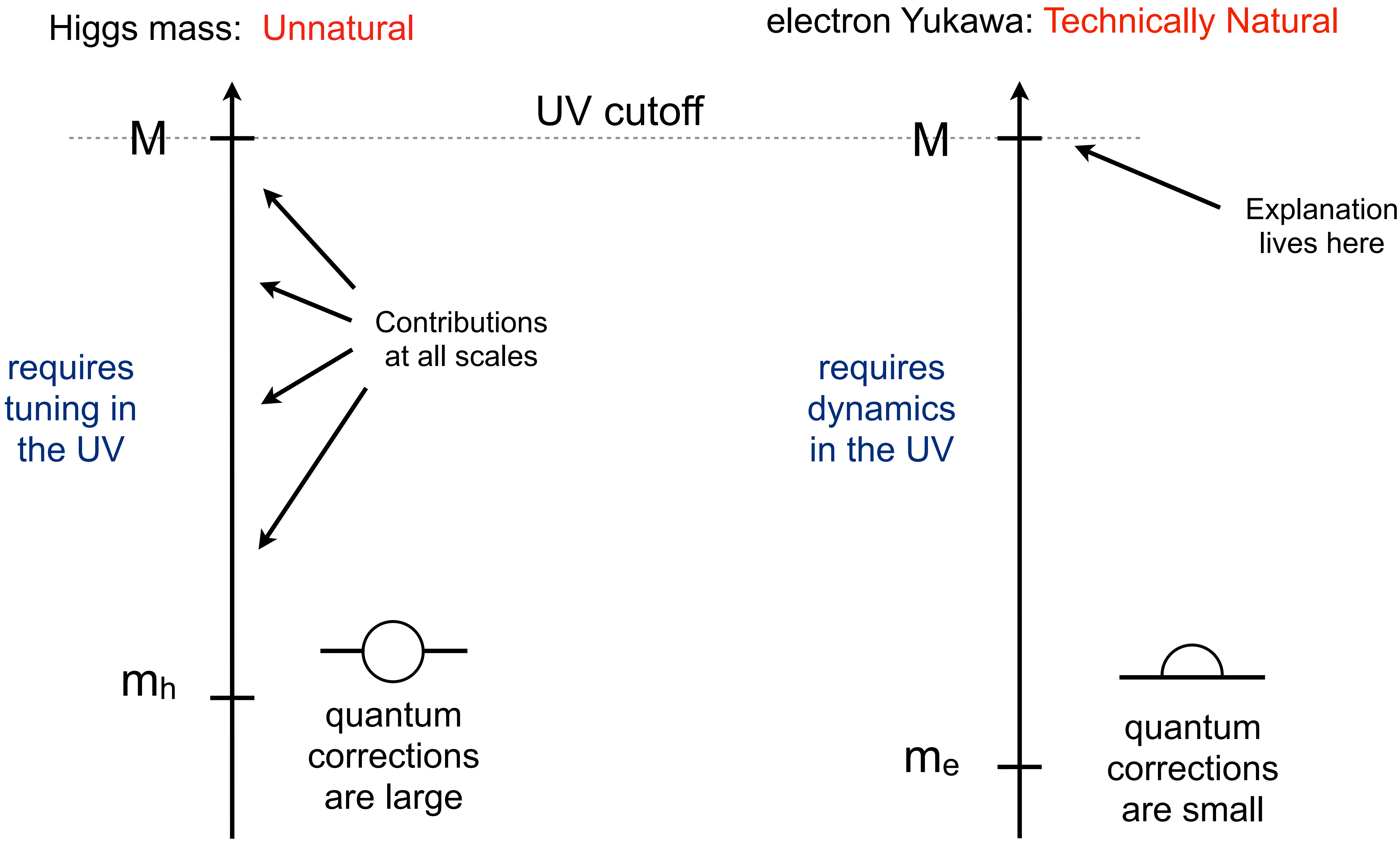
The Higgs mass in the standard model is sensitive to the ultraviolet.

$$m_{h_{\rm phys}}^2 = m_0^2 + \delta m_h^2$$



This term gets contributions all the way to the Planck scale

Unnatural vs. Technically Natural in the SM



The Hierarchy Problem

The Higgs mass in the standard model is sensitive to the ultraviolet.

Two approaches to explain:

- New symmetry or new dynamics realized at the electroweak scale. (SUSY, composite Higgs, EOFT)
- An anthropic explanation for fine tuning of ultraviolet parameters. (Multiverse)

We Propose:
A Dynamical Solution

- Higgs mass-squared promoted to a field.
- The field evolves in time in the early universe.
- The mass-squared relaxes to a small negative value.
- The electroweak symmetry breaking stops the time-dependence.
- The small electroweak scale is fixed until today.

Caveats

The solution:

- is only technically natural.

- requires large field excursions (larger than the scale that cuts off loops).

- requires a very long period of inflation.

- can only push the cutoff up to 10^8 GeV.

Simplest Model

Standard Model plus QCD axion

$$\mathcal{L} \supset (-M^2 + g\phi)|h|^2 \qquad \cdots + \frac{\phi}{32\pi^2 f} G^{\mu\nu}\tilde{G}_{\mu\nu}$$

M cuts off SM loops.

Continuous shift symmetry broken completely by g.

The axion here is non-compact.

(The Abbott model with a coupling to the Higgs & QCD)

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Simplest Model

Standard Model plus QCD axion

$$\mathcal{L} \supset (-M^2 + g\phi)|h|^2 + gM^2\phi + g^2\phi^2 + \cdots + \Lambda^4 \cos\frac{\phi}{f}$$

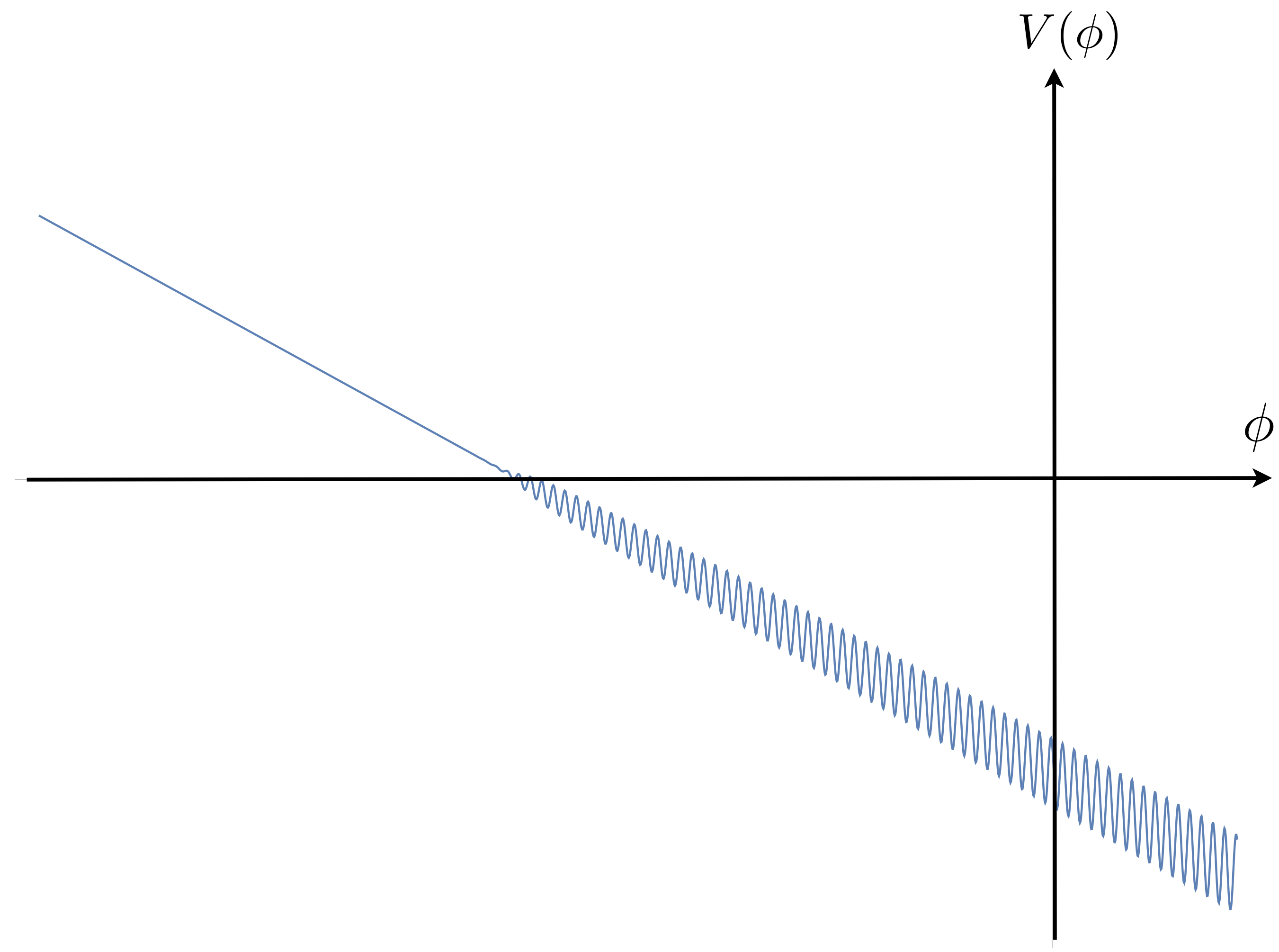
Continuous shift symmetry broken to discrete by non-perturbative effects.

Conservative effective field theory regime: $\phi \lesssim \dfrac{M^2}{g}$

(Assuming expansion of $V(g\phi)$ in powers of $\left(\frac{g\phi}{M^2}\right)$)

Chronology

- Take initial ϕ value such that $m_h^2 > 0$.
- During inflation, ϕ slow-rolls, scanning physical Higgs mass.
- ϕ hits value where ~ m_h^2 crosses zero.
- Barriers grow until rolling has stopped.



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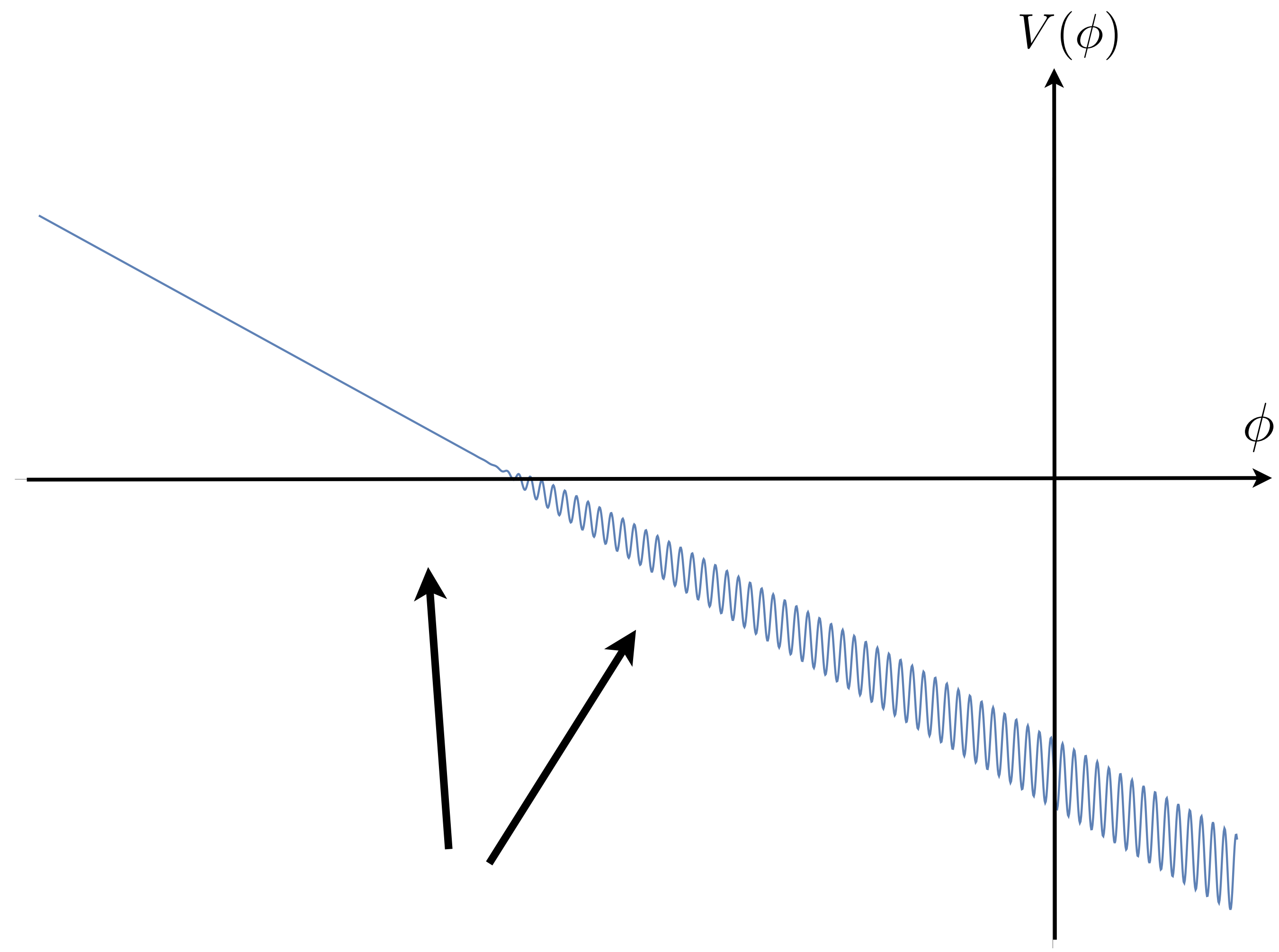

Key: Barriers grow because they depend on the Higgs vev.

Higgs vev and the Periodic Potential

Barrier height (axion potential) can be approximated in the chiral Lagrangian (2 flavors):

$$V_{\text{axion}}\left(\frac{\phi}{f}\right) \sim \Lambda^4 \cos\frac{\phi}{f}$$

Around the normal EW scale: $\Lambda^4 \sim f_\pi^2 m_\pi^2 \left(\frac{\min(m_u, m_d)}{m_u + m_d}\right)$

$$m_\pi^2 \propto (y_u + y_d)\langle h \rangle$$

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Barrier height grows with the Higgs vev.

Parameter Requirements

φ stops rolling and Higgs vev stops growing when slope turns around:

$$\partial_\phi(gM^2\phi + \Lambda^4 \cos(\phi/f)) \sim 0$$

or

$$\boxed{gM^2 f \sim \Lambda^4}$$

$$\Lambda^4 \sim 100 \text{ MeV}$$

fixed parameters → $gM^2 f$

changes with Higgs vev → Λ^4

$$gM^2 f \sim f_\pi^2 \mu (y_u + y_d)\langle h \rangle$$

Parameter Requirements

1) Vacuum energy density during inflation $> M^4$

$$\boxed{H_{\text{infl}} > \frac{M^2}{M_{\text{pl}}}}$$

2) Classical rolling dominates: $\frac{\dot{\phi}}{H_{\text{infl}}} > H_{\text{infl}}$

$$\boxed{H_{\text{infl}}^3 < gM^2}$$

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$$H_{\rm infl}^3 < g M^2$$

Plugging in for g, and using 1) and 2):

$$M^6 < \frac{\Lambda^4 M_{\rm pl}^3}{f}$$

Bound on cutoff...

$$M < 10^7 \text{ GeV} \left(\frac{10^9 \text{ GeV}}{f} \right)^{1/6}$$

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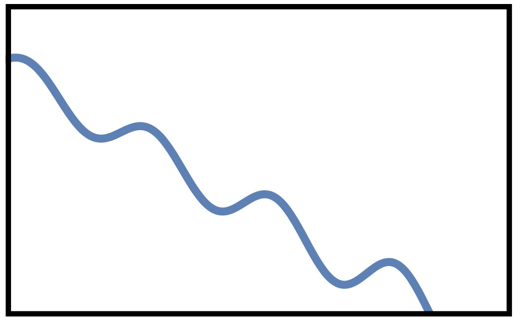
$$\theta_{\text{QCD}} \simeq \pi/2$$



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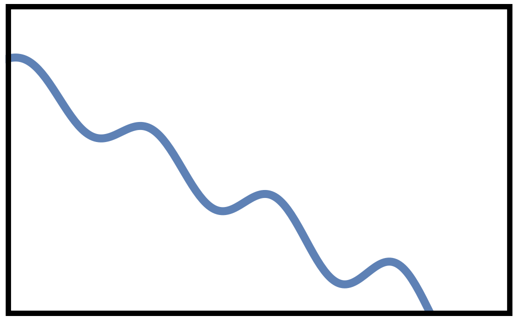


Prediction: $d_n \simeq few \times 10^{-16} e\,\text{cm}$

Solve Strong CP (1)

Usual solutions don't quite work.

Dynamical one -- Drop the slope:

$$\mathcal{L} \supset (-M^2 + g\phi)|h|^2 + \kappa\sigma^2\phi + gM^2\phi + \cdots + \Lambda^4 \cos\frac{\phi}{f}$$

inflaton - drops at end of inflation

$$gM^2 \simeq \theta \times \kappa\sigma^2 \longrightarrow$$

$$gM^2 f \sim \theta\,\Lambda^4$$

$$H_{\text{infl}} > \theta^{-\frac{1}{2}}\,\frac{M^2}{M_{\text{pl}}}$$

$$H_{\text{infl}}^3 < \theta^{-1} g M^2$$

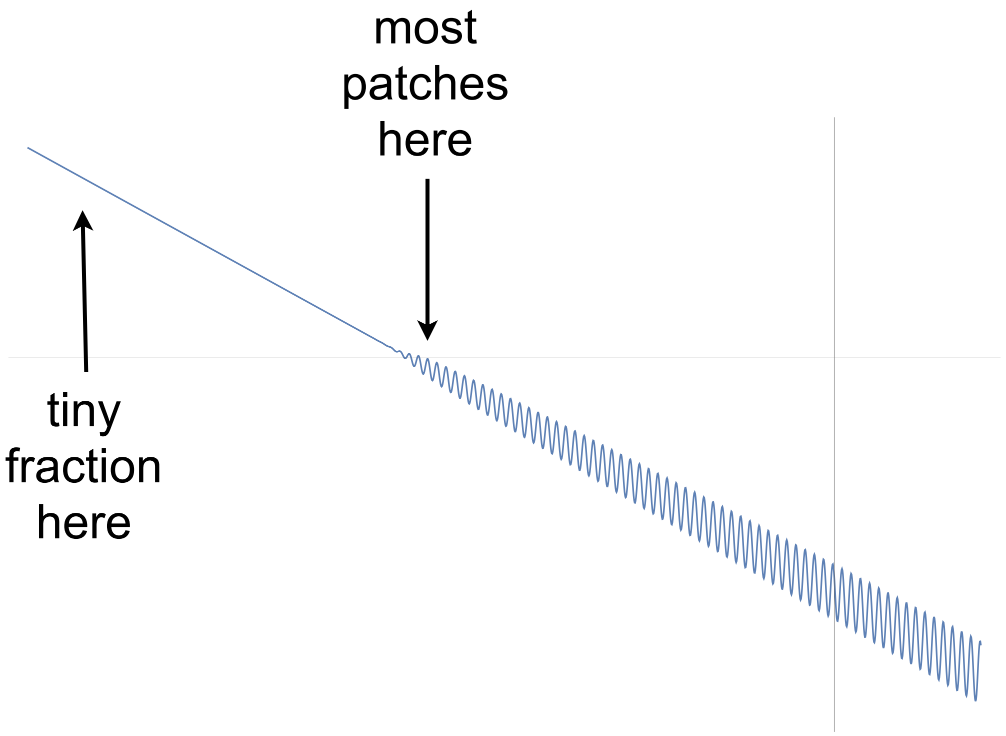
Bound on cutoff!

$$M^6 < \theta^{\frac{3}{2}} \; \frac{\Lambda^4 M_{\rm pl}^3}{f}$$

or

$$M < 30 \text{ TeV} \left(\frac{\theta}{10^{-10}}\right)^{\frac{1}{4}} \; \left(\frac{10^9 \text{ GeV}}{f}\right)^{\frac{1}{6}}$$

Quantum vs. Classical evolution



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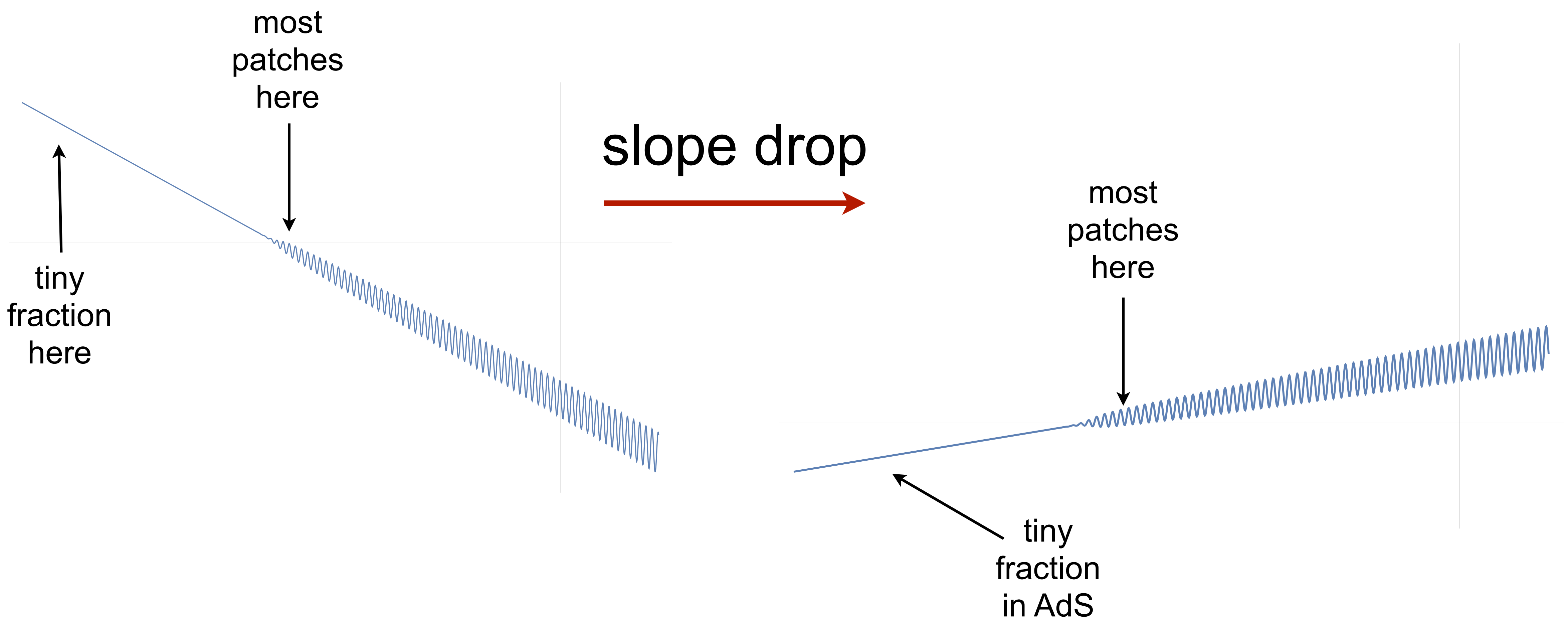

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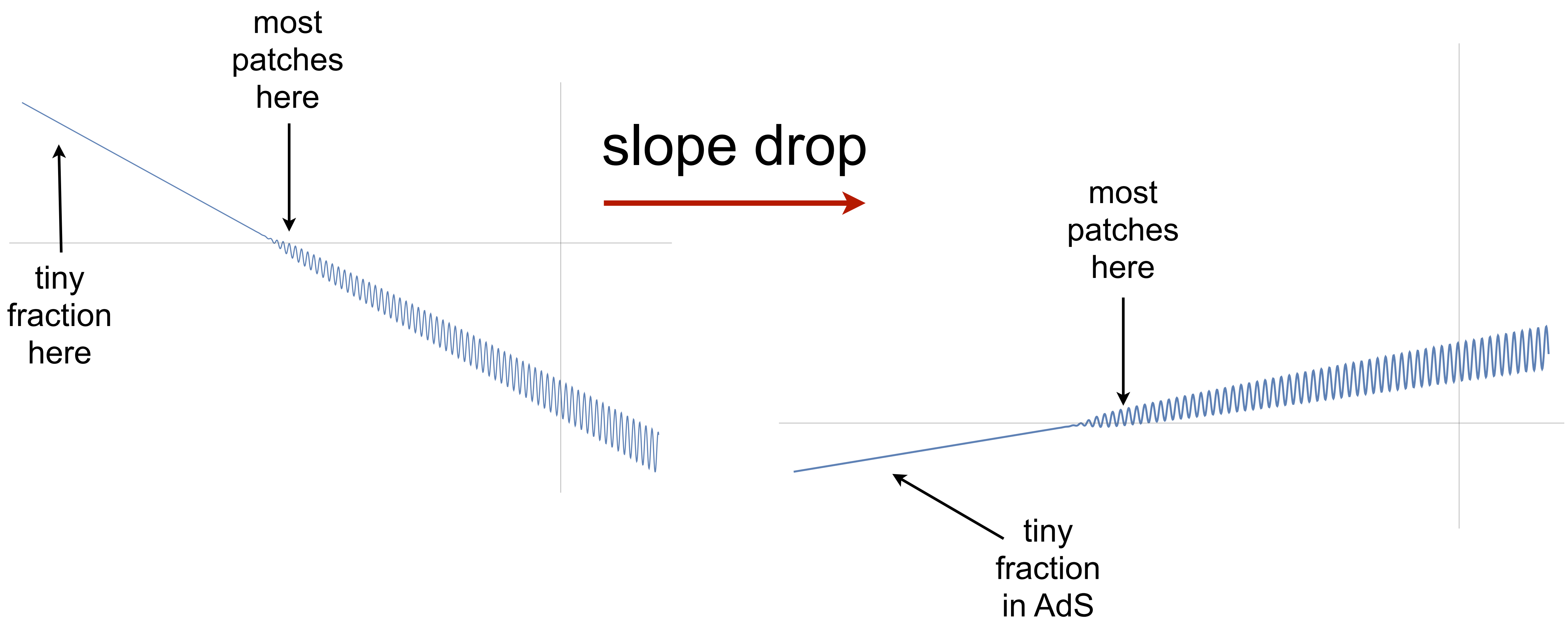

If we remove this constraint, upper bound on Hubble comes from requiring barriers to form:

$$H_{\rm infl} < \Lambda$$

Weaker bound on cutoff!

$$M^2 < \theta^{\frac{1}{2}} \Lambda M_{\rm pl}$$

or

$$M < 1000 \text{ TeV} \left(\frac{\theta}{10^{-10}} \right)^{\frac{1}{4}}$$

Solve Strong CP (2)
(Model 2)

Use a different strong group and couple ϕ to $G'^{\mu\nu}\tilde{G}'_{\mu\nu}$.

The Higgs must change the barrier heights: Add fermions

	$SU(3)$
L, N	$\Box$
L^c, N^c	$\overline{\Box}$

$$\mathcal{L} \supset m_L L L^c + m_N N N^c + y h L N^c + \tilde{y} h^\dagger L^c N$$

Model 2

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The Higgs must change the barrier heights: Add fermions

$$\mathcal{L} \supset m_L L L^c + m_N N N^c + y h L N^c + \tilde{y} h^\dagger L^c N$$

assume: $m_L \gg f_{\pi'} \gg m_N$

NDA: $\Lambda^4 \simeq 4\pi f_{\pi'}^3 m_{N_1}$

(lightest neutral fermion)

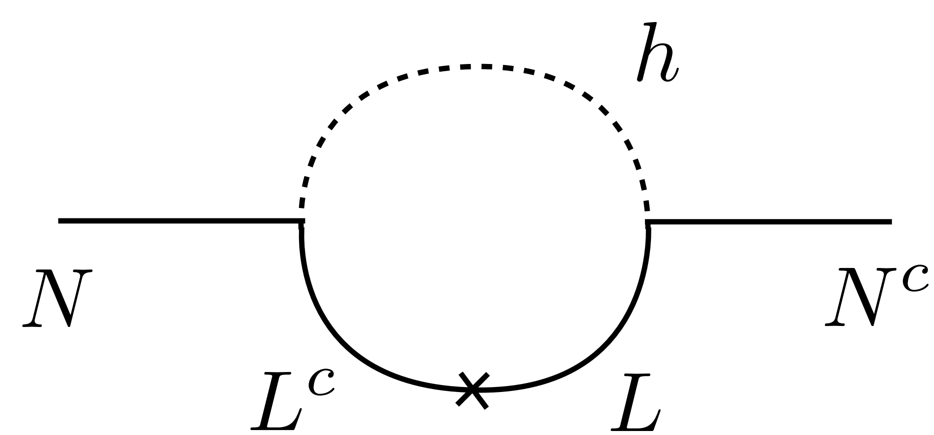
Model 2

Use a different strong group and couple ϕ to $G'^{\mu\nu}\tilde{G}'_{\mu\nu}$.

$$\mathcal{L} \supset m_L L L^c + m_N N N^c + y h L N^c + \tilde{y} h^\dagger L^c N$$

Higgs induced: $\delta m_{N_1} \simeq \frac{y\tilde{y}\langle h \rangle^2}{m_L}$

"Bare": $m_N \gtrsim \frac{y\tilde{y}}{16\pi^2} m_L \log \frac{M}{m_L}$



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Require: $$m_L < \frac{4\pi\langle h\rangle}{\sqrt{\log M/m_L}}$$

Bounds: $$m_L \gtrsim 250\ \mathrm{GeV}$$

Bound on cutoff (Model 2)

$$M < (\Lambda^4 {M_{\rm pl}}^3)^{\frac{1}{7}} \left(\frac{M}{f}\right)^{\frac{1}{7}}$$

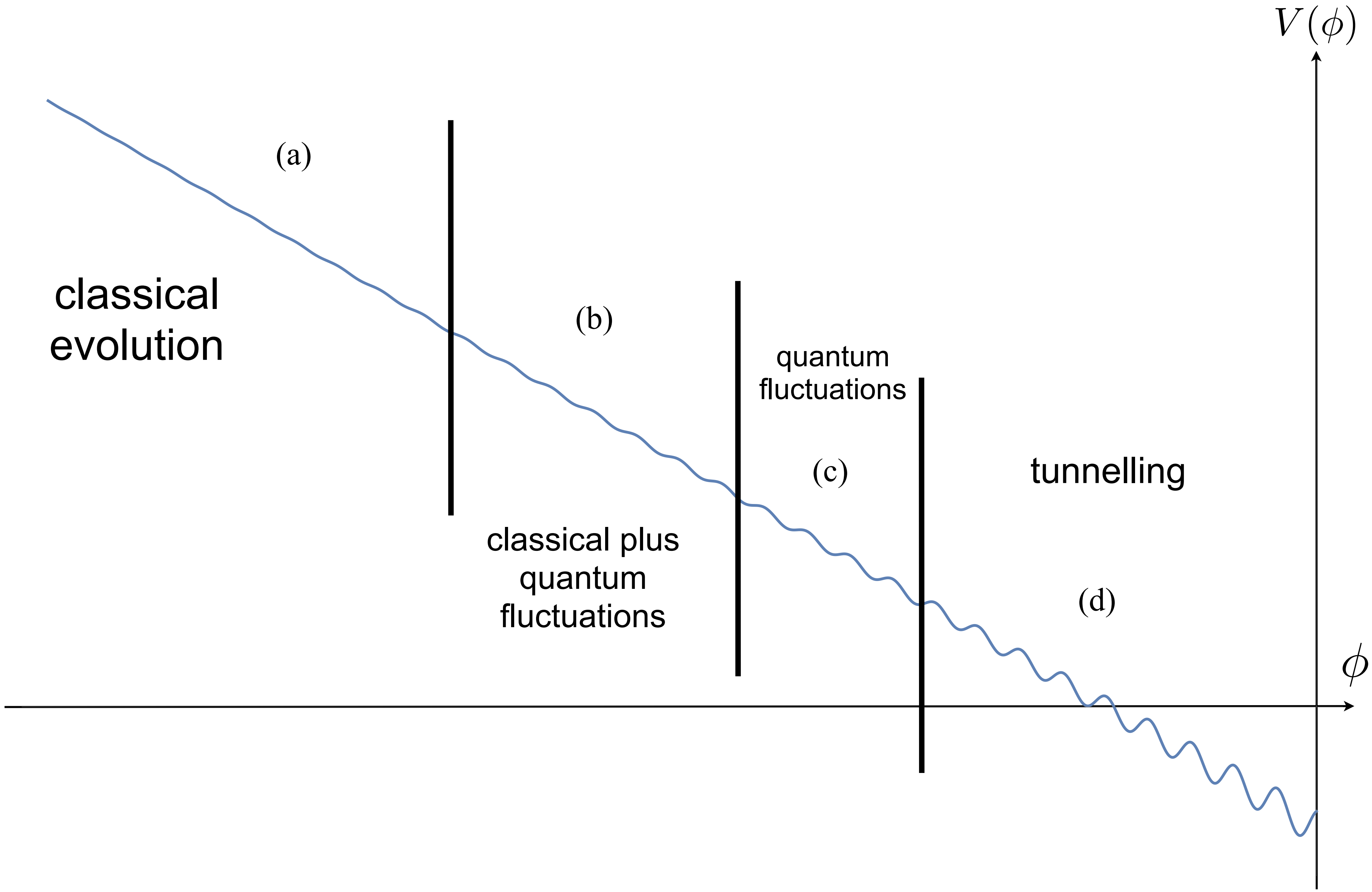
or

$$M < 3 \times 10^8 \text{ GeV} \left(\frac{f_{\pi'}}{30 \text{ GeV}}\right)^{\frac{3}{7}} \left(\frac{y\tilde{y}}{10^{-2}}\right)^{\frac{1}{7}} \left(\frac{250 \text{ GeV}}{m_L}\right)^{\frac{1}{7}} \left(\frac{M}{f}\right)^{\frac{1}{7}}$$

Bounds from Higgs decays, EWP

Constraints weaker due to loops

End of Roll



Need to end up in vacua that lives longer than the age of the universe since reheating

Inflation

To achieved the relaxed value,
inflation has to last long enough:

$$\Delta\phi \sim \frac{\dot{\phi}}{H_{\rm infl}} N \sim \frac{\partial_\phi V}{H_{\rm infl}^2} N \sim \frac{g M^2}{H_{\rm infl}^2} N$$

We require:

$$\Delta\phi \gtrsim \left(\frac{M^2}{g}\right)$$

$$N \gtrsim \frac{H_{\rm infl}^2}{g^2} \sim 10^{48}, 10^{37}$$ (Model 1,2 saturated)

Inflation

Single field: $V(\Phi) = m^2\Phi^2$

$$N = \int H dt \sim \int \frac{H^2}{\partial_\Phi V} d\Phi \sim \frac{\Phi_i^2}{M_{\rm pl}^2}$$

Classical rolling:

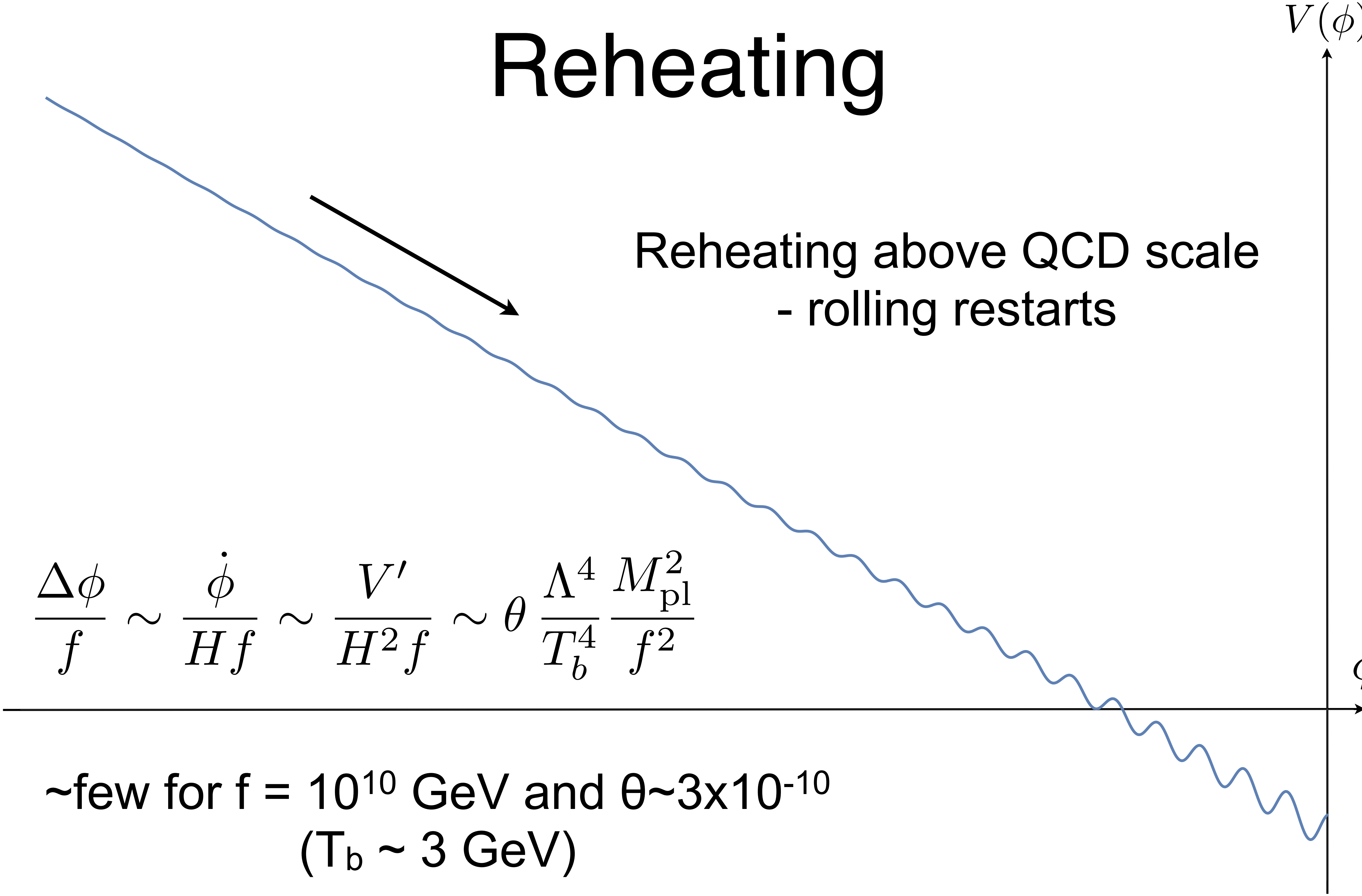
$$\frac{\dot{\Phi}}{H_{\rm infl}} < H_{\rm infl} \longrightarrow \frac{m\Phi_i^2}{M_{\rm pl}^3} < 1 \longrightarrow V(\Phi_i) < \frac{M_{\rm pl}^4}{N}$$

$$\longrightarrow N < \left(\frac{M_{\rm pl}}{M}\right)^4 (\times\theta)$$

$$N \gtrsim \frac{H_{\rm infl}^2}{g^2} \longrightarrow M < 10^5, 10^{8.75}\ {\rm GeV}$$

Reheating requires additional dynamics (e.g., hybrid)

Reheating



Reheating above QCD scale
- rolling restarts

$$\frac{\Delta\phi}{f} \sim \frac{\dot{\phi}}{Hf} \sim \frac{V'}{H^2 f} \sim \theta\,\frac{\Lambda^4}{T_b^4}\,\frac{M_{\rm pl}^2}{f^2}$$

~few for f = 10^{10} GeV and θ~3x10^{-10}
(T_b ~ 3 GeV)

(Rel)axion DM?

~few for f = 10^{10} GeV and θ~3x10^{-10}

$$\theta_0 \sim \left(\frac{10^{10}\ \mathrm{GeV}}{f}\right)^2 \left(\frac{\theta_{QCD}}{3\times 10^{-10}}\right)$$

for f < 10^{10} GeV, axion rolls over barriers initially, extra kinetic energy can add to DM abundance.

Observables

QCD model: Small parameter space

- (Rel)axion: May be dark matter, **with different abundance prediction from vacuum misalignment**.
- Observable neutron EDM favored.
- **Coupling to the Higgs**: (tiny)
    - New force experiments
    - Background oscillations of SM mass scales (if DM)
- Low-scale inflation (no primordial tensor modes in the CMB)

Low energy precision measurements to test this solution to the hierarchy problem!

Observables

non-QCD model: weak-scale physics

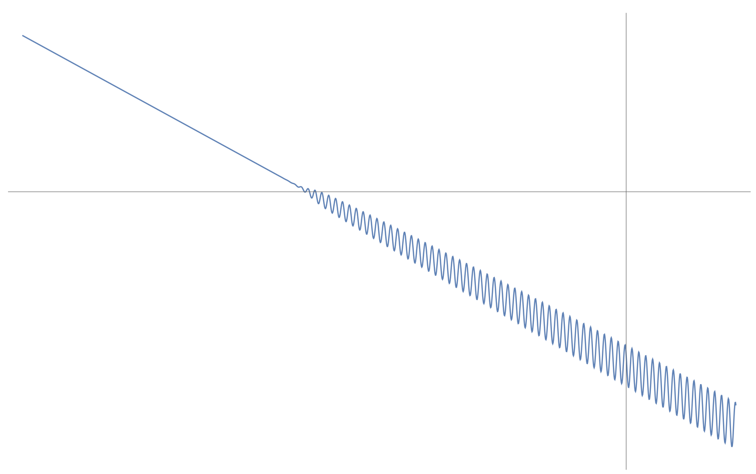
- (Rel)Axion: Still be dark matter, with different abundance prediction from vacuum misalignment, **as well as mass prediction/couplings**
- **Fermions with electroweak quantum numbers**
- Coupling to the Higgs:
  - **New force experiments**
  - **Oscillations of SM mass scales, e.g. m_e (if DM)**
- Low-scale inflation (no primordial tensor modes in the CMB)

Low energy precision measurements to test this solution to the hierarchy problem!

Relaxion Conditions

Self-organized criticality?

- Dissipation - Dynamical evolution of Higgs mass (field) must stop. **Hubble friction**.
- Self-similarity - Cutoff-dependent quantum corrections will choose an arbitrary point where the Higgs mass is cancelled. **Periodic axion**.



- Higgs back-reaction - EWSB must stop the evolution at the appropriate value. **Yukawa couplings**.
- Long time period - There must be a sufficiently long time period during the early universe for scanning. **Inflation**.

To Do

- Phenomenology:
  - Dark matter / cosmological predictions
  - Collider predictions
  - New forces
- New low-energy experimental ideas (CASPEr)
- UV completion (axion monodromy?)
- Better Inflation models
- Better models/higher cutoff