Bridging the nHz gravitational wave gap via Moon's physical librations

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Moon's movement

- Tidally locked. It shows some oscillations: libration motion.
- Different causes: Optical, parallax and **physical librations**.
- Different modes: **longitude**, latitude, wobbling.
- Physical libration's amplitude (1 arcsec) is not fully understood[1] and precisely measured due to LLR[2].

[1] Rambaux and Williams 2011.
[2] See for example Slava G. Turyshev and Hahn 2023.

Libration motion basics

- Due to the interaction of the Earth's gravitational potential with **Lunar Quadrupole Moment** $\longrightarrow$ Correction to the potential $\propto 1/r^3$.

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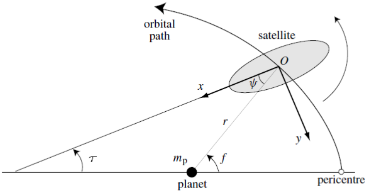
- Then Euler eq's of motion take the following form (Murray 2000):

$$\ddot{\tau} - \frac{3}{2}\frac{B-A}{C}\frac{Gm_s}{r^3}\sin(2\psi) = 0 \qquad (1)$$



- By **averaging** (1) over the orbital period:

$$\ddot{\tau} + \omega_0^2 \sin\tau = 0 \qquad (2)$$

Libration motion basics: Damping mechanisms

- Anelastic damping (Eckhardt 1993). Complex Love number $k(1 - i\kappa)$
- Viscous fluid model (Yoder and Hutchinson 1981; Poisson and Will 2014).
- This modifies our EoM

$$\ddot{\tau} = -\omega_0^2(1 + i\kappa)\tau \simeq -\omega_0^2 \left(1 + i\frac{\kappa}{2}\right)^2 \tau$$

- **Estimates give a τ_{damp} of 10^4 yr (anelastic) or 10^5 yr (viscous).**
- For simplicity we can write the EoM as:

$$\ddot{\tau} + 2b\dot{\tau} + \omega_0^2\tau = 0 \qquad (3)$$

.

Key question and hypothesis

- Previous concerns account only for *maintaining* the Moon's libration motion for a (relatively) short time. **What has initiated this movement?**
- Several explanations have been proposed: asteroid impact, orbital resonances...
- Our study explores the possibility of new physics:

Key question and hypothesis

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Gravitational Wave Resonance

Cartoon Figure (I)

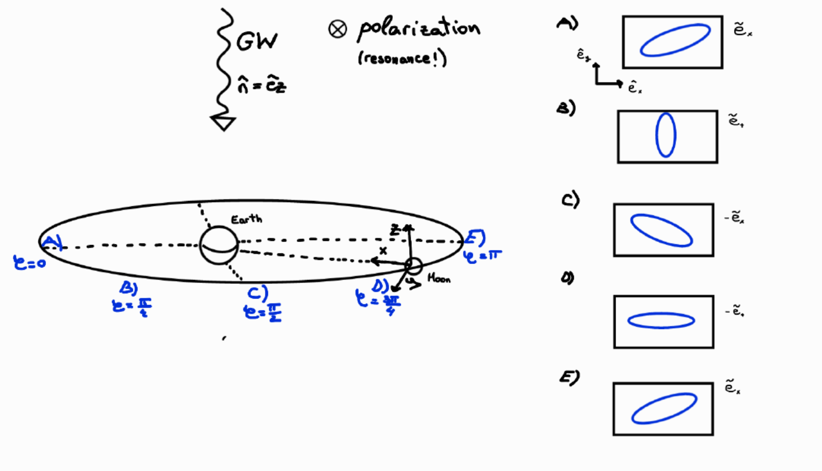


Figure: Cartoon picture representing the situation for ⊥ direction and × polarization.

GWs as a direct perturbation force

In Fermi normal coordinates, **the perturbative term** (Force/unit mass) can be calculated from linearised GR, obtaining

$$\delta \ddot{r}^i = \frac{1}{2} \ddot{h}_{ij}^{\mathrm{TT}} r^j \quad (4)$$

The **perturbation is sinusoidal** with amplitude:

$$A_F = \frac{h_0 m_s a^2 \omega_g^2}{2C}$$

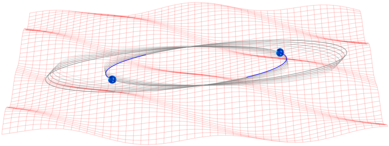
or $A_F e^2$ (depending on the source) as we will see later.

Other effects of GW

- **Modifying the principal moments of inertia** and specifically $(B - A)/C$. The calculated amplitude A_I (Rosat and Majstorović 2021) is:

$$\frac{A_I}{A_F} = \left(\frac{B - A}{C}\right)^{-1} \frac{4k_2 R_s^5 \omega_0^2}{3Gm_s a^2} \sim 10^{-11} \tag{5}$$

- **Modifying the orbital parameters:** a, e, i... (Blas and Jenkins 2022) which affect eq. (1). Second-order perturbation.



Obtaining the Equations of Motion

From Euler's equation:

$$N_z = \dot{\omega}_z - \cancel{(A-B)\omega_x\omega_y} = yF_x - xF_y \tag{6}$$

As previously stated, we add the GWs effect as a perturbation force.

$$\delta\ddot{r}^i = \frac{1}{2}\,\ddot{h}_{ij}^{\mathrm{TT}}\,r^j \quad \text{with} \quad (h_{ij} = h_0\epsilon_{ij}e^{i\omega_g t}) \tag{7}$$

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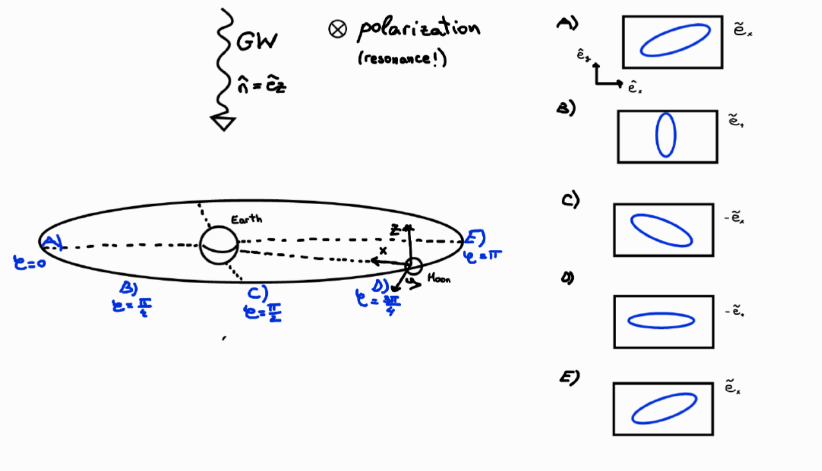
Here we can make the distinction between the different incoming directions of the GWs $\parallel$, $\perp$ and also between the polarizations ϵ_{ij}^{+}, $\epsilon_{ij}^{\times}$.

Cartoon Figure (II)



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EoM for perpendicular direction and cross polarization

Taking into account the Moon's rotation on its own axis (ω_{rev}), the polarisation tensor reads as follows: (similar approach as Amicis et al. n.d.)

$$\epsilon_{ij}^{\times} = \begin{pmatrix} -\sin(2\omega_{rev}t) & \cos(2\omega_{rev}t) & 0 \\ \cos(2\omega_{rev}t) & \sin(2\omega_{rev}t) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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Averaging Euler's EoM over the orbital period, one obtains:

$$\ddot{\tau} - \frac{1}{2}\omega_0^2 \sin(2\tau) + A_F e^2 \cos(\omega_g t)\cos(2\tau) = 0 \tag{8}$$

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And, since libration's amplitude $\simeq 1$ arcsec $\ll 1$, eq. (8) becomes (+ adding the damping term)

$$\boxed{\ddot{\tau} + 2b\dot{\tau} + \omega_0^2 \tau + A_F e^2 \cos(\omega_g t) = 0} \tag{9}$$

Searching for an astronomical source

- Central Mechanism: Incoming GW in the right direction and polarization, with $f_g \simeq f_0 = 11\,\mathrm{nHz}$ resonates with Moon's Libration and induce a high libration amplitude. A first estimate for h_0 is given by

$$A_{\mathrm{max}} \simeq \frac{A_F}{\kappa \omega_0^2} \qquad \longrightarrow \qquad h_{\mathrm{required}} \simeq \frac{4C}{m_s a^2 \omega_0 e^2} \frac{A_{\mathrm{max}}}{T_{\mathrm{damp}}}$$

  For the Moon and a punctual source, the required h_0 for $A_{\mathrm{max}} \simeq 1\,\mathrm{arcsec}$ is between 2×10^{-12} and 4×10^{-14}.

- What could be the origin of these GW? How strong does the source have to be? Do they exist? **Possible new window in the GW's Astronomy era.**

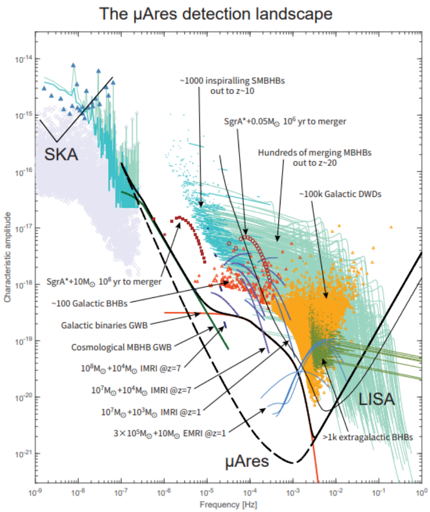
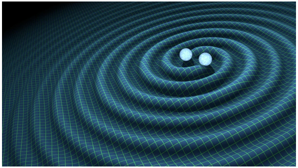


Figure: GW detection landcape. Figure from Sesana et al. 2021.

A candidate: Supermassive black hole binaries (I)

During the inspiral, SMBHB emit GWs with increasing frequency until they merge (chirping).



From conservation of energy (without back reactions corrections $\rightarrow$ approximation), we have that (Maggiore 2007)

$$\omega_g = \left(\omega_{g,0}^{-8/3} - \frac{32}{5} 2^{1/3} \left(\frac{GM_c}{c^3} \right)^{5/3} t \right)^{-3/8} \qquad (10)$$

The strain of the incoming GW also varies with time:

$$h_c(t_{\text{obs}}^{\text{ret}}) = \frac{4}{d_L} \left(\frac{G\mathcal{M}_c}{c^2} \right)^{5/3} \left(\frac{\pi f_{\text{gw}}^{(\text{obs})}(t_{\text{obs}}^{\text{ret}})}{c} \right)^{2/3} \qquad (11)$$

A candidate: Supermassive black hole binaries (II)

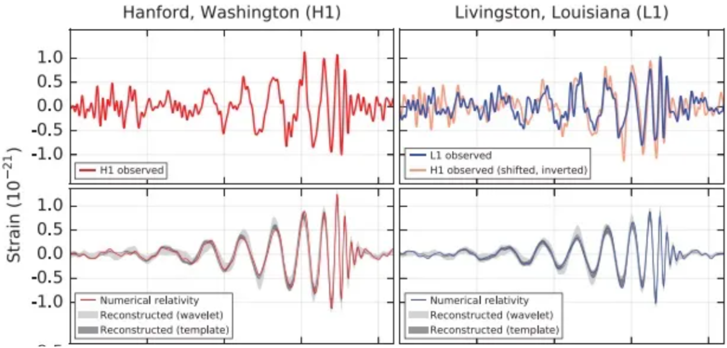


Figure: Graphic obtained from LIGO measures

EoM for GWs from SMBHB

With this source, the EoM is

$$\boxed{\ddot{\tau} + 2b\dot{\tau} + \omega_0^2\tau + A(t)\cos(\omega_g(t)\,t) = 0} \tag{12}$$

where $\omega_g(t)$ is given by eq. (10) and $A(t) = \epsilon\,\mathcal{M}_c^{5/3}\omega_g^{2/3}$, with ϵ being a constant (fixing distance).

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where $\omega_g(t)$ is given by eq. (10) and $A(t) = \epsilon\,\mathcal{M}_c^{5/3}\omega_g^{2/3}$, with ϵ being a constant (fixing distance). Two approaches:

- Analytical approximation (for low mass regime). Predicts some characteristics of the simulations: peak amplitude and the respective resonant frequency.
- Numerically for different ranges of masses.

A) Analytical approach (low mass regime $M \lesssim 10^8\, M_\odot$)

- For low masses $\longrightarrow$ Resonance window really large (ω_g varies really slowly) $\longrightarrow$ stationary solution.

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- For low masses $\longrightarrow$ Resonance window really large (ω_g varies really slowly) $\longrightarrow$ stationary solution.
- We look for a solution of the style

$$\tau(t) = \mathrm{Re}\left[\mathcal{A}(t)\, e^{i\omega_g(t)t}\right] \tag{13}$$

where $\mathcal{A}(t)$ represents the envelope of the motion.

- Solving for $\mathcal{A}(t)$ provides us with useful information for better understanding the numerical simulation:
    1. The time at which the system reaches maximum amplitude is $t_{max} \simeq \frac{1}{2}\, t_0$.
    2. The maximum amplitude is $A_{max} \propto \mathcal{M}_c^{5/3}, h_c$ and $\simeq 2A_0$.

B) Simulations

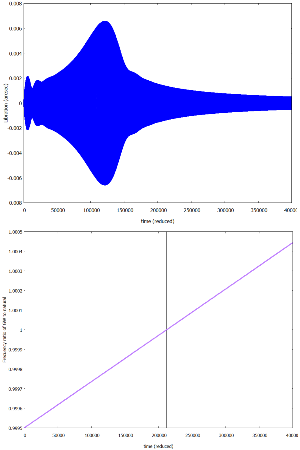

Figure: Libraions for a BHMB of $10^7\ M_\odot$ at a distance of $1\,\mathrm{Mpc}$. The plots have been generated with the code in C.

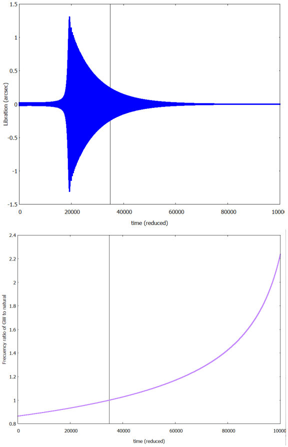

Figure: Librations for a BHMB of $10^9\ M_\odot$ at a distance of $1\,\mathrm{Mpc}$. The plots have been generated with the code in C.

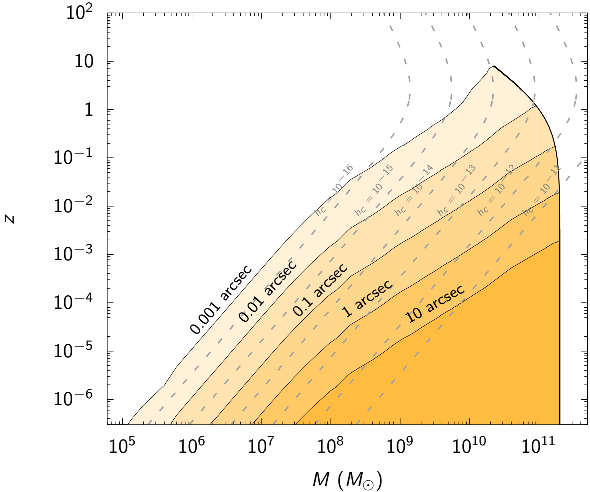

Figure: Curves of constant induced libration amplitude in a redshift z vs total mass M diagram (assuming equal masses). The actual noise on the Moon's libration is of some milliarsceconds. The curve in the range 10^{10} $M_\odot$ to 10^{11} $M_\odot$ is subject to numerical uncertainty. The cutoff for the shaded area is due to the restriction that the system does not reach the resonance frequency (actually, the line should be farther away, since maximum amplitude occurs at earlier frequencies). We have also drawn the lines that correspond to a constant characteristic strain h_c, at observed frequency equal to the resonant one. Note that this SNR is generated for two identical SMBHs.

Do these SMBHB exist?

Distribution of the number of masses capable of generating a certain libration amplitude

Theoretical models and NANOGrav's PTA data. See Sato-Polito, Zaldarriaga, and Quataert 2024 and Sato-Polito and Zaldarriaga 2024.
In this last paper, they estimate $dN/d\log h_s^2$, and we can obtain $dN/d\log M$ as a function of M such that they generate a certain amplitude of libration:

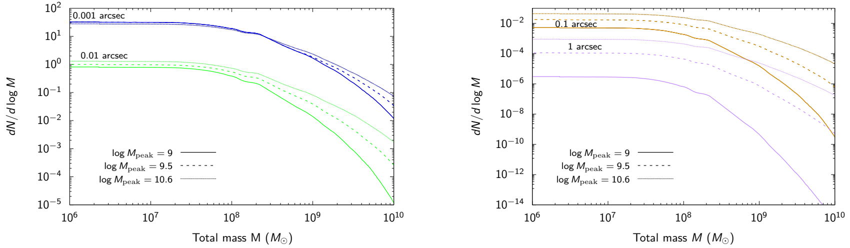

Figure: Distribution of the number of sources as a function of mass.

Conclusions

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3. From now on, we can use the Moon as a *gravitational wave detector* in the nHz range with a two-fold purpose:
    - Setting up constraints for some events, such as the distribution of SMBHBs.
    - Detecting new powerful events in the upcoming years.

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4. New experimental devices will replace the current LLR, thus uncertainties in the measure of the librations will improve in the incoming years.
5. New ideas have appeared and are still to be explored...

Future prospects

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3. Can other Moon's libration modes (e.g. latitude) be due to GWs resonance?
4. Other causes of Moon's libration: Dark Matter Transients (PBHs of asteroid size) that interacted with the Moon (Tran et al. 2024; Cuadrat-Grzybowski et al. 2024).

Thank you!

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