JAGIELLONIAN UNIVERSITY IN KRAKÓW

Status of High Precision Calculations at e^+e^- Colliders

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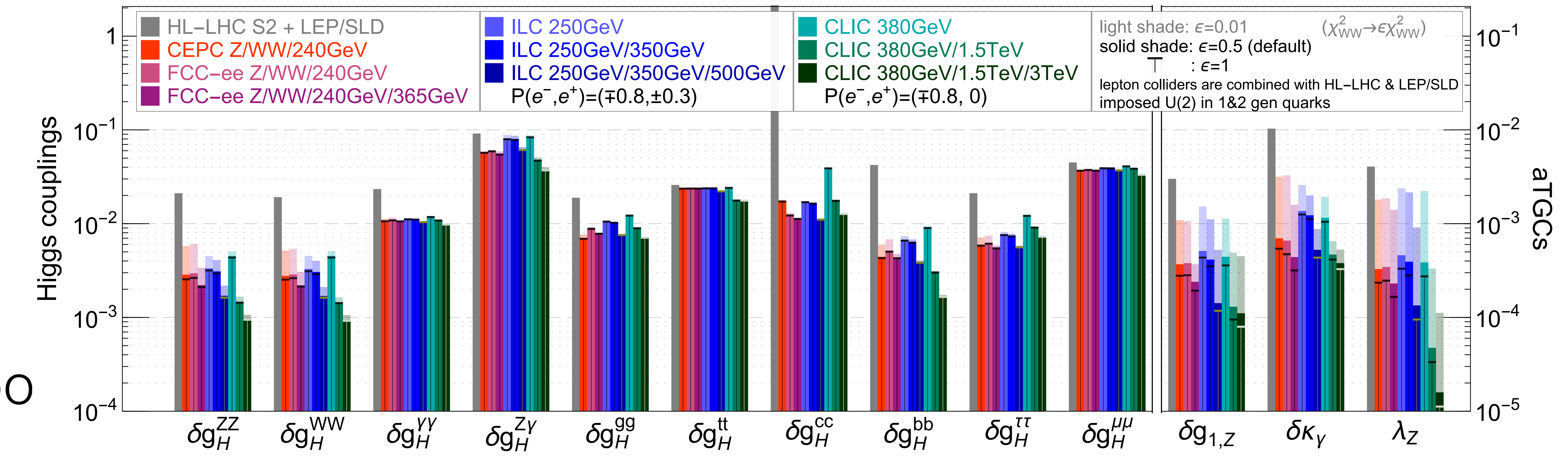
Disclaimer

The topics I cover are certainly not complete but chosen to avoid overlap with other talks

Physics Landscape at Higgs Factories

- ❖Higgs couplings measured to a few %
- ❖Self coupling with 50% precision
- ❖Top-quark pole mass uncertainty of 500 MeV
- ❖Flavour physics observables improved by abo one order of magnitude compared to today
- ❖ Improvement on direct Dark matter limits
- ❖ Possible surprises?

precision reach with different assumptions on $e^+e^- \to WW$ measurements



[J. De Blas *et al* **JHEP 12 (2019) 117**]

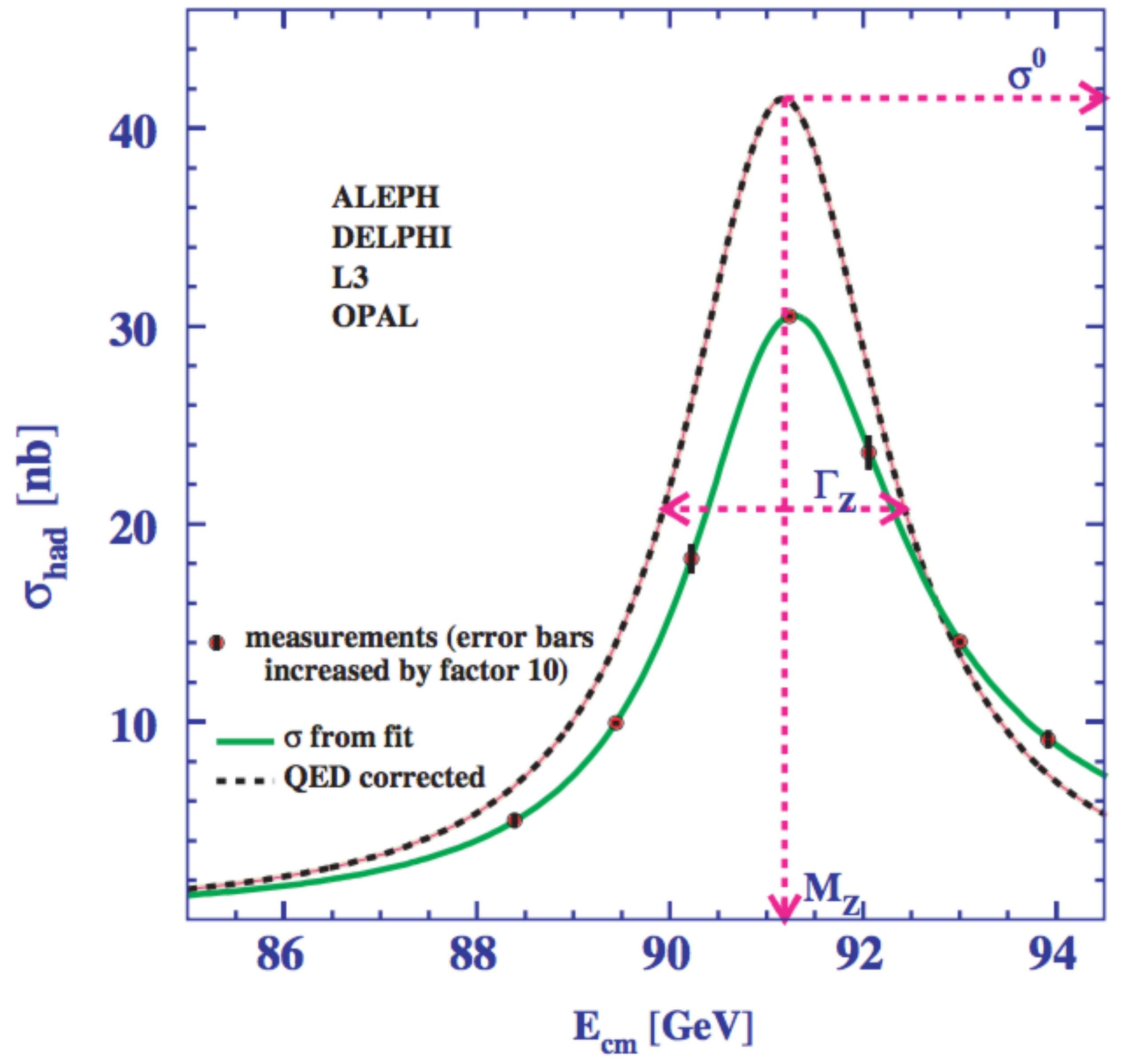
An electron-positron Higgs factory is the highest-priority next collider. -EUROPEAN STRATEGY FOR PARTICLE PHYSICS

Theory Requirements

- ❖Factor 5-200 reduction of experimental error
- ❖QED effects of 0.1% could be included in LEP error budget
- ❖Future colliders will deliver full LEP Statistics in minutes

Observable	Where from	Present (LEP)	FCC stat.	FCC syst	$\frac{\text{Now}}{\text{FCC}}$
M_Z [MeV]	Z linesh. [29]	$91187.5 \pm 2.1\{0.3\}$	0.005	0.1	3
Γ_Z [MeV]	Z linesh. [29]	$2495.2 \pm 2.1\{0.2\}$	0.008	0.1	2
$R_l^Z = \Gamma_h/\Gamma_l$	$\sigma(M_Z)$ [34]	$20.767 \pm 0.025\{0.012\}$	$6 \cdot 10^{-5}$	$1 \cdot 10^{-3}$	12
$\sigma^0_{\rm had}$[nb]	$\sigma^0_{\rm had}$ [29]	$41.541 \pm 0.037\{0.025\}$	$0.1 \cdot 10^{-3}$	$4 \cdot 10^{-3}$	6
N_ν	$\sigma(M_Z)$ [29]	$2.984 \pm 0.008\{0.006\}$	$5 \cdot 10^{-6}$	$1 \cdot 10^{-3}$	6
N_ν	$Z\gamma$ [35]	$2.69 \pm 0.15\{0.06\}$	$0.8 \cdot 10^{-3}$	$< 10^{-3}$	60
$\sin^2\theta_W^{eff} \times 10^5$	$A_{FB}^{lept.}$ [34]	$23099 \pm 53\{28\}$	0.3	0.5	55
$\sin^2\theta_W^{eff} \times 10^5$	$\langle\mathcal{P}_\tau\rangle, A_{\rm FB}^{pol,\tau}$ [29]	$23159 \pm 41\{12\}$	0.6	< 0.6	20
M_W [MeV]	ADLO [36]	$80376 \pm 33\{6\}$	0.5	0.3	12
$A_{FB,\mu}^{M_Z \pm 3.5\text{GeV}}$	$\frac{d\sigma}{d\cos\theta}$ [29]	$\pm 0.020\{0.001\}$	$1.0 \cdot 10^{-5}$	$0.3 \cdot 10^{-5}$	100

S.Jadach and M.Skrzypek, Eur. Phys. J.C 79, no.9, 756 (2019)



How to treat QED Corrections?

Collinear Resummation

- Collinear logs are resummed with universal PDF ($P_T = 0$)
- Recently matched to NLO
- Combined with Parton Shower to generate photon emissions
- Beyond NLO becomes tricky

Soft Resummation

- Soft logs resummed to infinite order using the YFS theorem
- Provides a robust scheme for the inclusion of real and virtual corrections at any order.

S.Frixone et.al JHEP 03 (2020)

Jadach et.al, Z.Phys.C 49 (1991) 577-584,Europhys. Lett.17(1992) 123–128

$$d\sigma(L,\hat{L}) = \alpha^k \sum_n^{\infty} \alpha^n \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \hat{\sigma}_{n,i,j} L^i \hat{L}^j$$

$$\hat{L} = \log\left(\frac{Q^2}{E_\gamma^2}\right) \qquad L = \log\left(\frac{Q^2}{m_e^2}\right)$$

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See talk by Giovanni Stagnitto

S.Frixone et.al JHEP 03 (2020)

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YFS Theorem

Yennie, Frautschi, and Suura showed how to reorder the entire perturbative series such that all IR divergences are resummed

It also provides an analytical treatment of the multi-photon phasespace

See talk by Z.Was

ANNALS OF PHYSICS: **13:** 379–452 (1961)

The Infrared Divergence Phenomena and High-Energy Processes*

D. R. YENNIE†

School of Physics, University of Minnesota, Minneapolis, Minnesota

S. C. FRAUTSCHI‡

Department of Physics, University of California, Berkeley, California

AND

H. SUURA

Department of Physics, Nihon University, Tokyo, Japan

A general treatment of the infrared divergence problem in quantum electrodynamics is given. The main feature of this treatment is the separation of the infrared divergences as multiplicative factors, which are treated to all orders of perturbation theory, and the conversion of the residual perturbation expansion into one which has no infrared divergence, and hence no need for an infrared cutoff. In the infrared factors, which are exponential in form, the infrared divergences arising from the real and virtual photons cancel out in the usual way. These factors can then be expressed solely in terms of the momenta of the initial and final charged particles and an integral over the region of phase space available to the undetected photons; they do not depend upon the specific details of the interaction. Electron scattering from a static potential is treated in considerable detail, and several other examples are discussed briefly. As an important byproduct of the general treatment, it is found that when the infrared contributions are separated in a particular way, they dominate the radiative corrections at high energies and together with certain "magnetic terms" and vacuum polarization corrections seem to give all the contributions proportional to ln (E/m). All of these corrections can be easily estimated (in most cases) simply from a knowledge of the external momenta of the charged particles; this then provides a very powerful and accurate way of estimating radiative corrections to high-energy processes.

* Supported in part by the U. S. Atomic Energy Commission, Contract AT(11-1)-50.

† Various refinements were added to the manuscript (particularly in Appendix C) during the academic year 1960–1961 while this author was a National Science Foundation Senior Fellow visiting the University of Paris. He is grateful to Professor M. M. Lévy for the hospitality afforded by the Laboratoire de Physique Théorique et Hautes Énergies at Orsay.

‡ Supported by National Science Foundation Grant.

YFS Master Equation

$$d\sigma = \sum_{n_\gamma=0}^{\infty} \frac{e^{Y(\Omega)}}{n_\gamma!} d\Phi_Q \left[\prod_{i=1}^{n_\gamma} d\Phi_i^\gamma \, \tilde{S}(k_i) \, \Theta(k_i, \Omega) \right] \left(\tilde{\beta}_0 + \sum_{j=1}^{n_\gamma} \frac{\tilde{\beta}_1(k_j)}{\tilde{s}(k_j)} + \sum_{\substack{j,k=1 \\ j<k}}^{n_\gamma} \frac{\tilde{\beta}_2(k_j, k_k)}{\tilde{s}(k_j)\tilde{s}(k_k)} + \cdots \right)$$

This expression contains **no approximations**.It does require any further matching. The accuracy is limited by how far you can calculate the betas

$$Y(\Omega) = \sum_{i<j} \mathscr{Re}\, B_{ij}(\Phi_n) + \tilde{B}_{ij}(\Phi_{n+1})$$

Virtual Emissions

Taking the soft limit allows us to factorise out amplitude

$$\tilde{\beta}_0^1(\Phi_n) = \mathscr{V}(\Phi_n) - \sum_{ij} \mathscr{D}_{ij}(\Phi_{ij})$$

IR Finite one-loop contribution

Full One-loop amplitude

YFS Subtraction term

Virtual Emissions

$$\tilde{\beta}_0^1(\Phi_n) = \mathscr{V}(\Phi_n) - \sum_{ij} \mathscr{D}_{ij}(\Phi_{ij})$$

Full One-loop amplitude

- Calculation of one-loop EW(QCD) corrections is essentially automated
- OpenLoops *Eur.Phys.J.C* 79 (2019) 10, 866
- Recola Comput.Phys.Commun. 214 (2017) 140-173
- MadLoop *JHEP* 05 (2011) 044
- GoSam *Eur.Phys.J.C* 74 (2014) 8, 3001
- BlackHat *Phys.Rev.D* 78 (2008) 036003

I can't calculate one-loop amplitudes but I can interface them

- Anonymous MC Author

Virtual Emissions

Automatically constructed in Sherpa using YFS algorithm

Convolution of the YFS form-factor with the born amplitude

$$\tilde{\beta}_0^1(\Phi_n) = \mathscr{V}(\Phi_n) - \sum_{ij} \mathscr{D}_{ij}(\Phi_{ij})$$

YFS Subtraction term

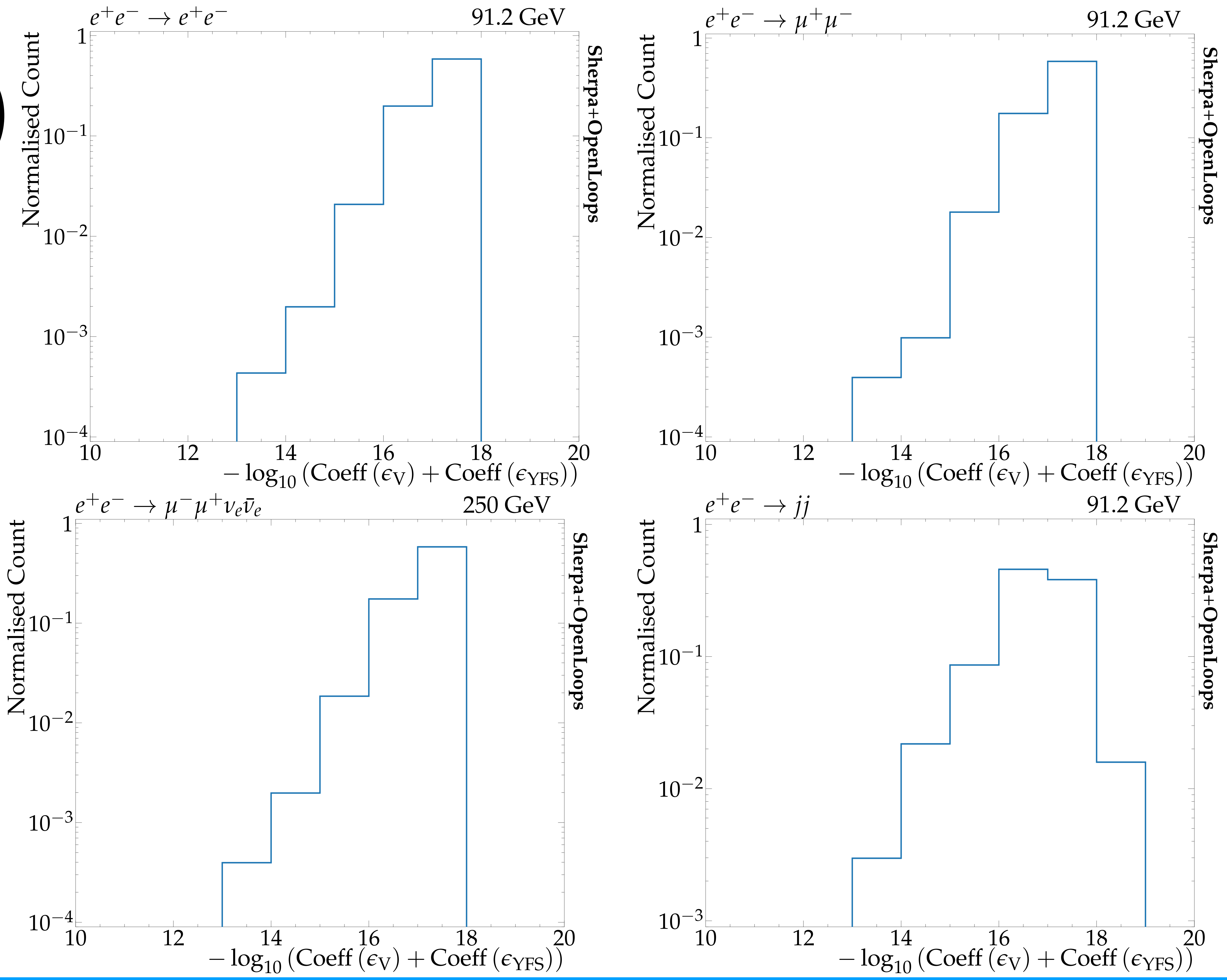
SciPost Phys. 13 (2022) 026

$$B_{ij} = -\frac{i}{8\pi^3} Z_i Z_j \theta_i \theta_j \int \frac{d^4k}{k^2} \left(\frac{2p_i\theta_i - k}{k^2 - 2(k \cdot p_i)\theta_i} + \frac{2p_j\theta_j + k}{k^2 + 2(k \cdot p_j)\theta_j} \right)^2$$

Virtual Corrections

$$\tilde{\beta}_0^1\left(\Phi_n\right) = \mathscr{V}(\Phi_n) - \sum_{ij} \tilde{\mathscr{D}}_{ij}\left(\Phi_{ij} \otimes \Phi_n\right)$$

Sherpa automatically constructs the subtraction terms while external tools provide the **IR divergent** one-loop amplitude



Pole cancellation for the virtual occur locally without input from the real emissions i.e Not via KLN theorem

Real Emissions

$$\tilde{\beta}_1^1\left(\Phi_{n+1}\right) = \frac{1}{2(2\pi)^3}\left|\mathcal{M}_0^{\frac{1}{2}}\left(\Phi_{n+1}\right)\right|^2 - \sum_{ij}\tilde{\mathcal{D}}_{ij}\left(\Phi_{ij+1}\otimes\Phi_n\right)$$

IR Finite contribution for real corrections

Real emission squared amplitude

YFS Subtraction term

Real Emissions

$$\tilde{\beta}_1^1\left(\Phi_{n+1}\right) = \frac{1}{2(2\pi)^3}\left|\mathscr{M}_0^{\frac{1}{2}}\left(\Phi_{n+1}\right)\right|^2 - \sum_{ij}\tilde{\mathscr{D}}_{ij}\left(\Phi_{ij+1}\otimes\Phi_n\right)$$

Real emission squared amplitude

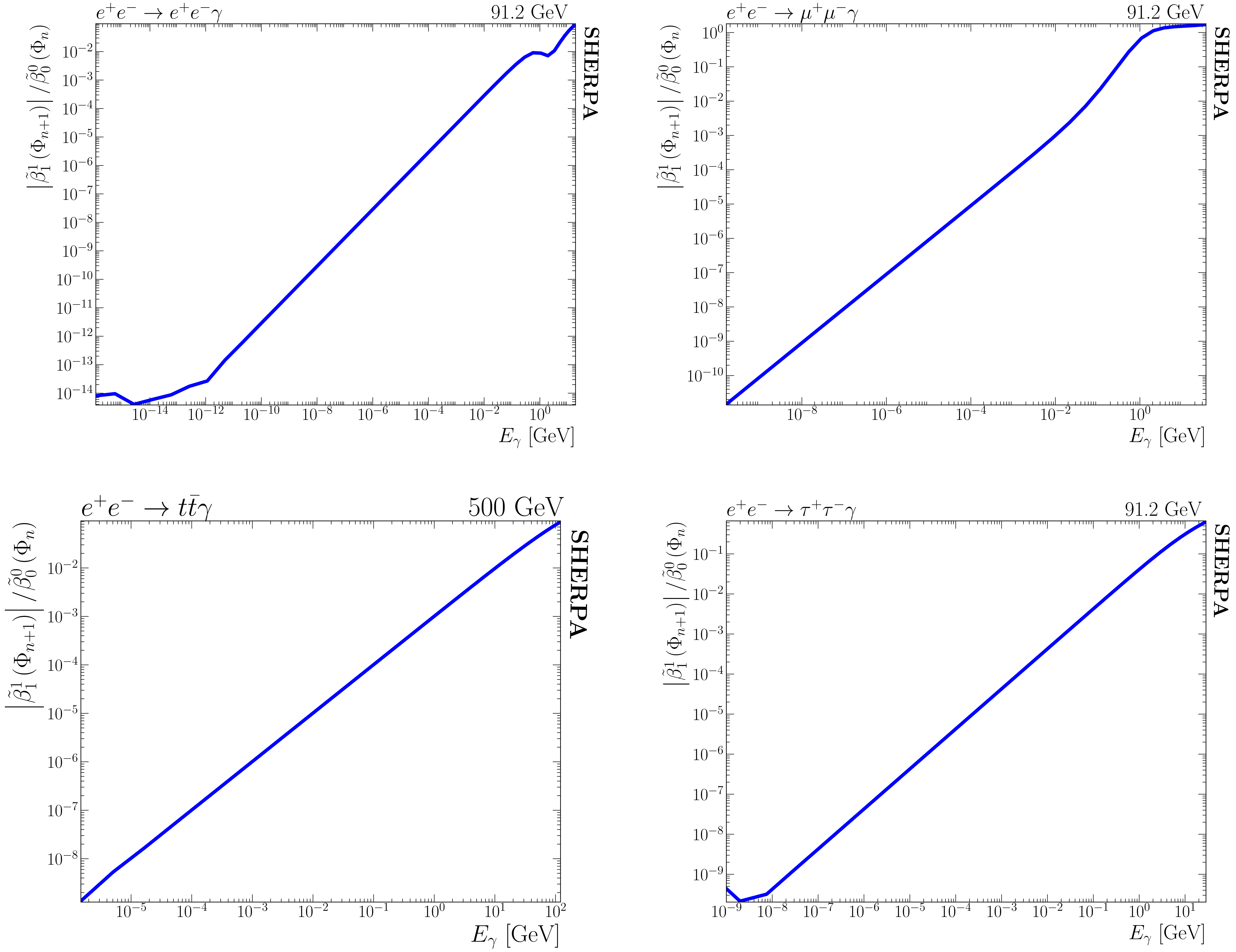
- Tree level amplitudes calculations fully automated
- Sherpa: Comix/Amegic
- Madgraph
- Whizard: O`Mega
- .....

Real Corrections

$$\tilde{\beta}_1^1\left(\Phi_{n+1}\right) = \mathscr{R}(\Phi_{n+1}) - \sum_{ij} \mathscr{D}_{ij}\left(\Phi_{ij+1} \otimes \Phi_n\right)$$

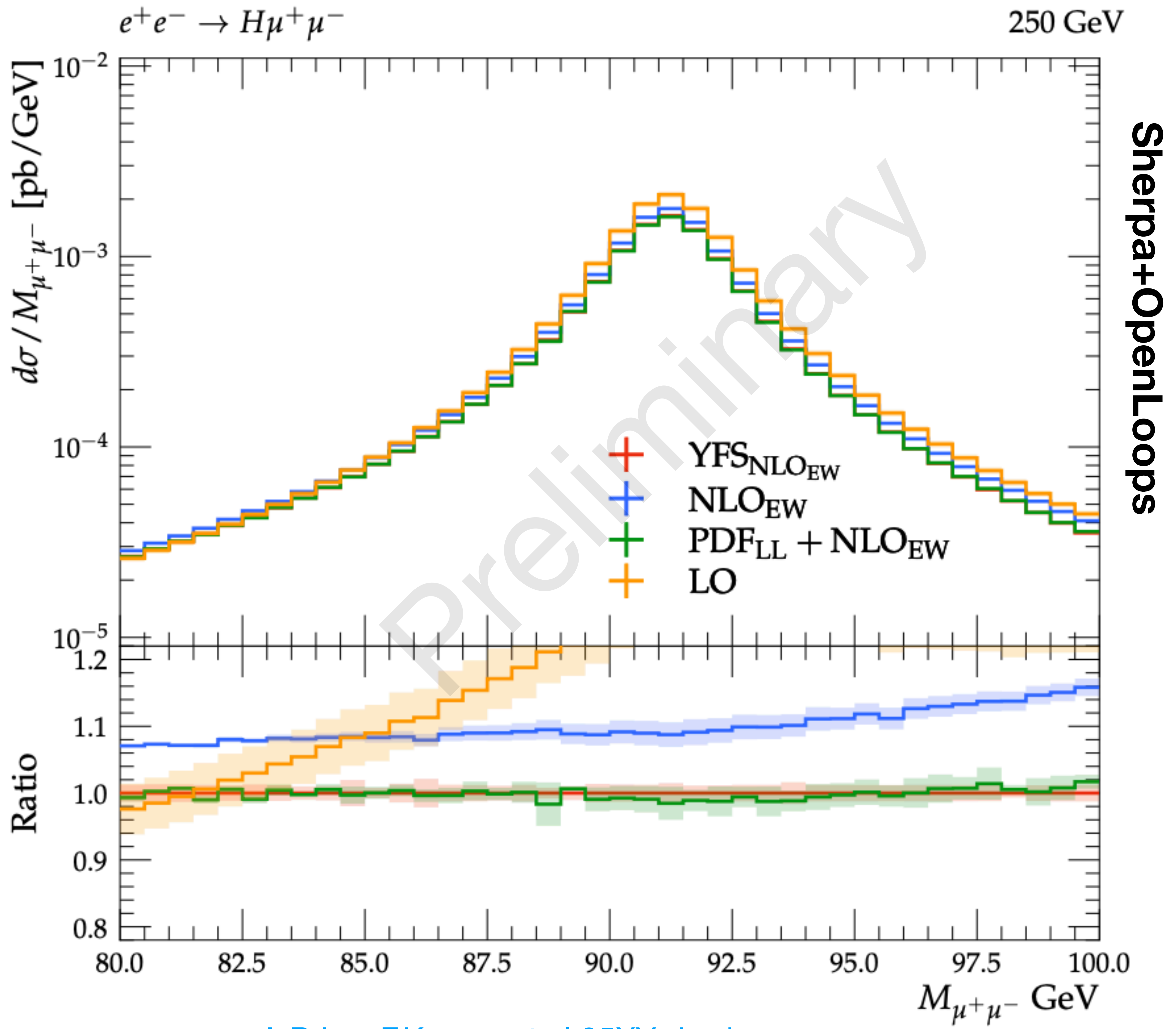
The real emissions are simple tree level amplitudes which can be calculated using standard methods in Sherpa

In the soft limit we see this contribution vanishes



NLO-EW Results

- YFS NLO results are compatible with Sherpa's collinear based NLO calculations
- Great consistency check as both approaches are based on very different algorithms
- Error bands are from varying the input scheme



A.Price, F.Krauss et al 25XY.abcd

See Juergen Reuter talk on Higgs Physics

Real-Virtual Emissions

YFS defined to all orders so we can look at NNLO corrections

$$\tilde{\beta}_1^2\left(\Phi_{n+1}\right) = \mathscr{RV}\left(\Phi_{n+1}\right) - \sum_{ij} \mathscr{D}_{ij}^{(1)}\left(\Phi_{ij+1} \otimes \Phi_n\right)$$

IR Finite one-loop contribution with additional real emission

Full One-loop amplitude

YFS Subtraction term

Real-Virtual Corrections

$$\tilde{\beta}^2_1\left(\Phi_{n+1}\right) = \mathscr{R}\mathscr{V}(\Phi_{n+1}) - \sum_{ij} \mathscr{D}^{(1)}_{ij}\left(\Phi_{ij+1} \otimes \Phi_n\right)$$

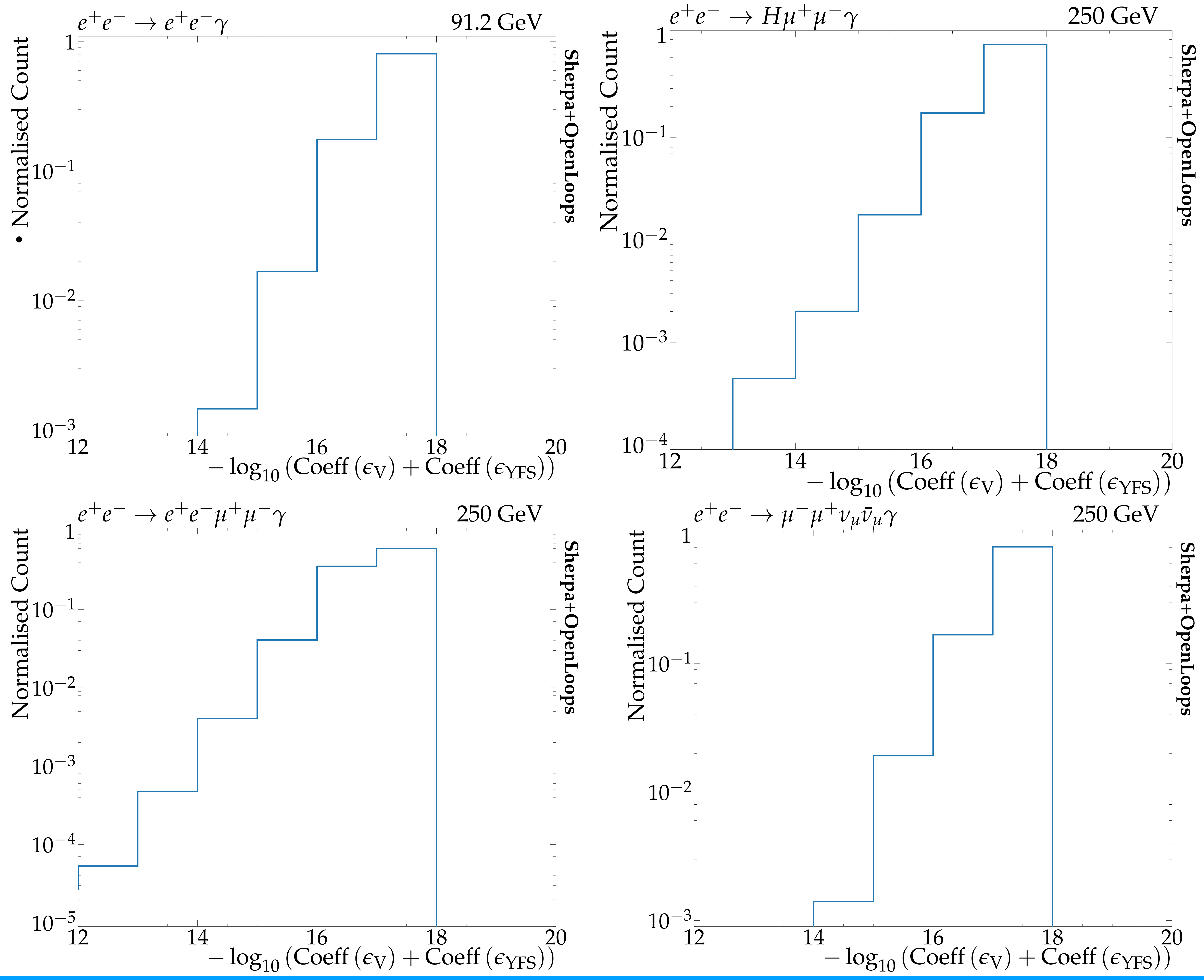
One-loop amplitudes again provided by external tool.

Sherpa again automatically constructs the subtraction term

These corrections contain no approximations e.g all masses are kept

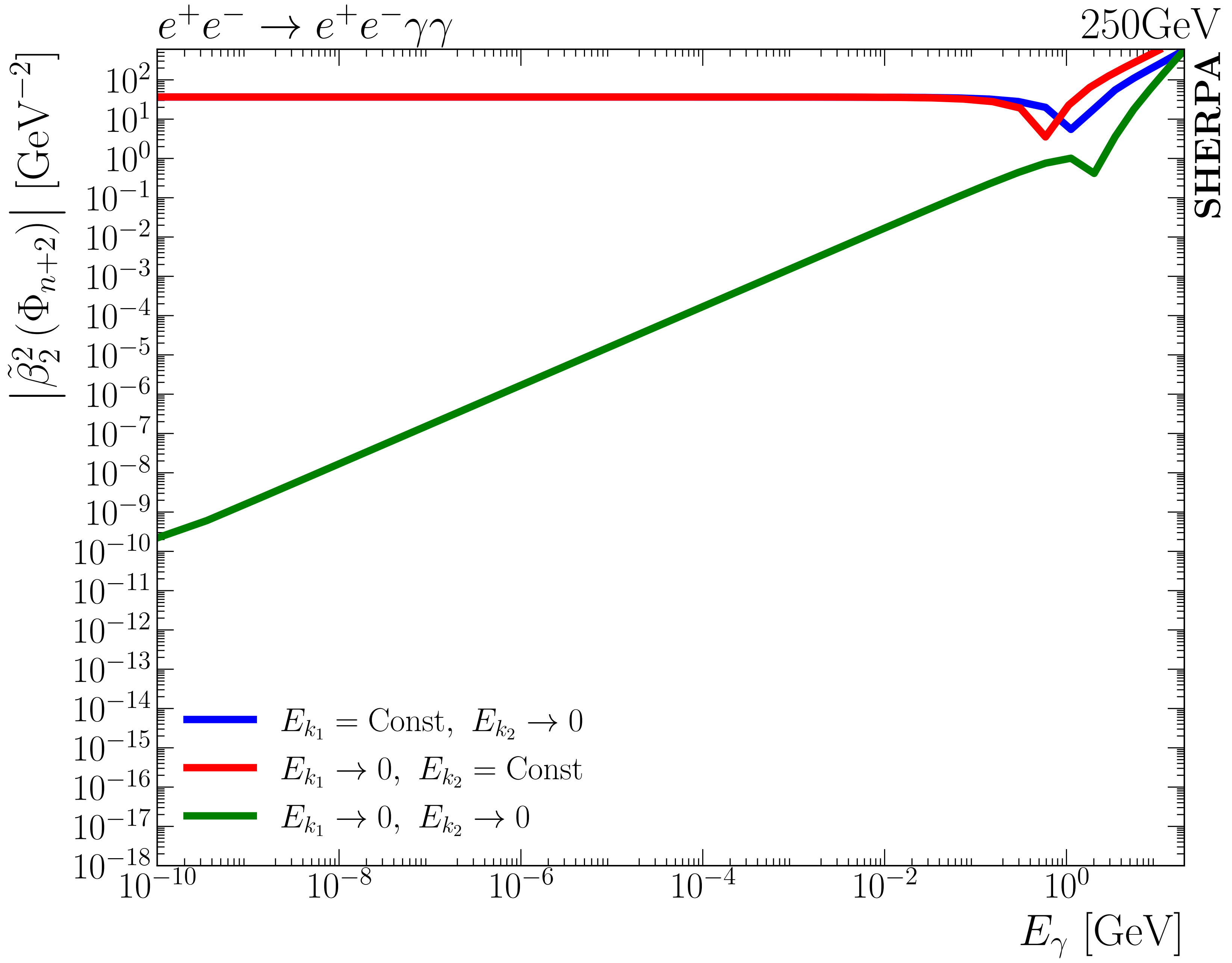
I can't calculate one-loop amplitudes but I can interface them

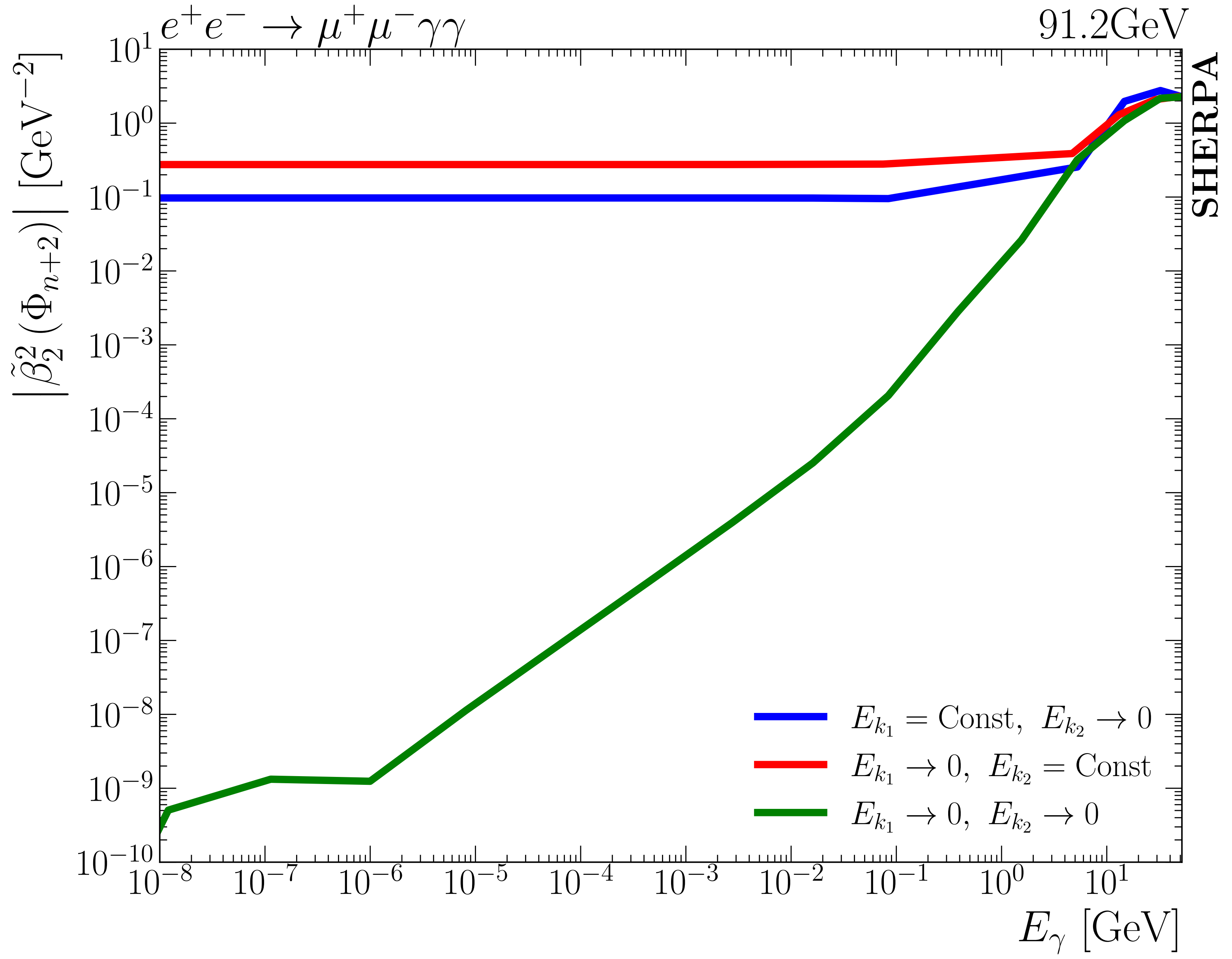
- Anonymous MC Author



Double Real Corrections

$$\tilde{\beta}_2^2\left(\Phi_{n+2}\right) = \mathscr{R}\mathscr{R}(\Phi_{n+2})$$
$$-\tilde{S}(k_1)\tilde{\beta}_1^1\left(\Phi_{n+1};k_2\right)$$
$$-\tilde{S}(k_2)\tilde{\beta}_1^1\left(\Phi_{n+1};k_1\right)$$
$$-\tilde{S}(k_1)\tilde{S}(k_2)\tilde{\beta}_0^0\left(\Phi_n\right)$$





By far the most complicated subtraction term

Double Virtual EW Corrections

Unfortunately, there is not automated tool for the calculation of double virtual correction.

GRIFFIN v1.0: $f\bar{f} \to f'\bar{f}'$ with NLO EW corrections and h.o. @ Z-pole

Future upgrades:

- Bbhabha scattering ($f = f'$)
- Higher-order off-resonance corrections, e.g.
  $\mathcal{O}(\alpha\alpha_s)$, Heller, v.Manteuffel, Schabinger, Spiesberger '20
  $\mathcal{O}(N_f\alpha^2)$ Bonciani et al. '21
- SMEFT $d{=}6$ operator effects
- W production and decay (a.k.a. charged-current DY)

Try out the code: **github.com/lisongc/GRIFFIN/releases**

Feedback welcome!

[Ayres Freitas LoopFest 2024](#)

[Tommaso Armadillo](#)

Double Virtual EW Corrections

Unfortunately, there is not automated tool for the calculation of double virtual correction.

But there is significant work underway

- Two-Loop QED Corrections to the Scattering of Four Massive Leptons — Phys.Rev.Lett. 132 (2024) 23
- Two-loop radiative corrections to e+e– → γγ* cross section — JHEP 11 (2023) 148
- Lepton-pair scattering with an off-shell and an on-shell photon at two loops in massless QED — JHEP 11 (2023) 041
- Two-Loop Electroweak Corrections with Fermion Loops to $e^+e^- \to ZH$ — Phys.Rev.Lett. 130 (2023) 3

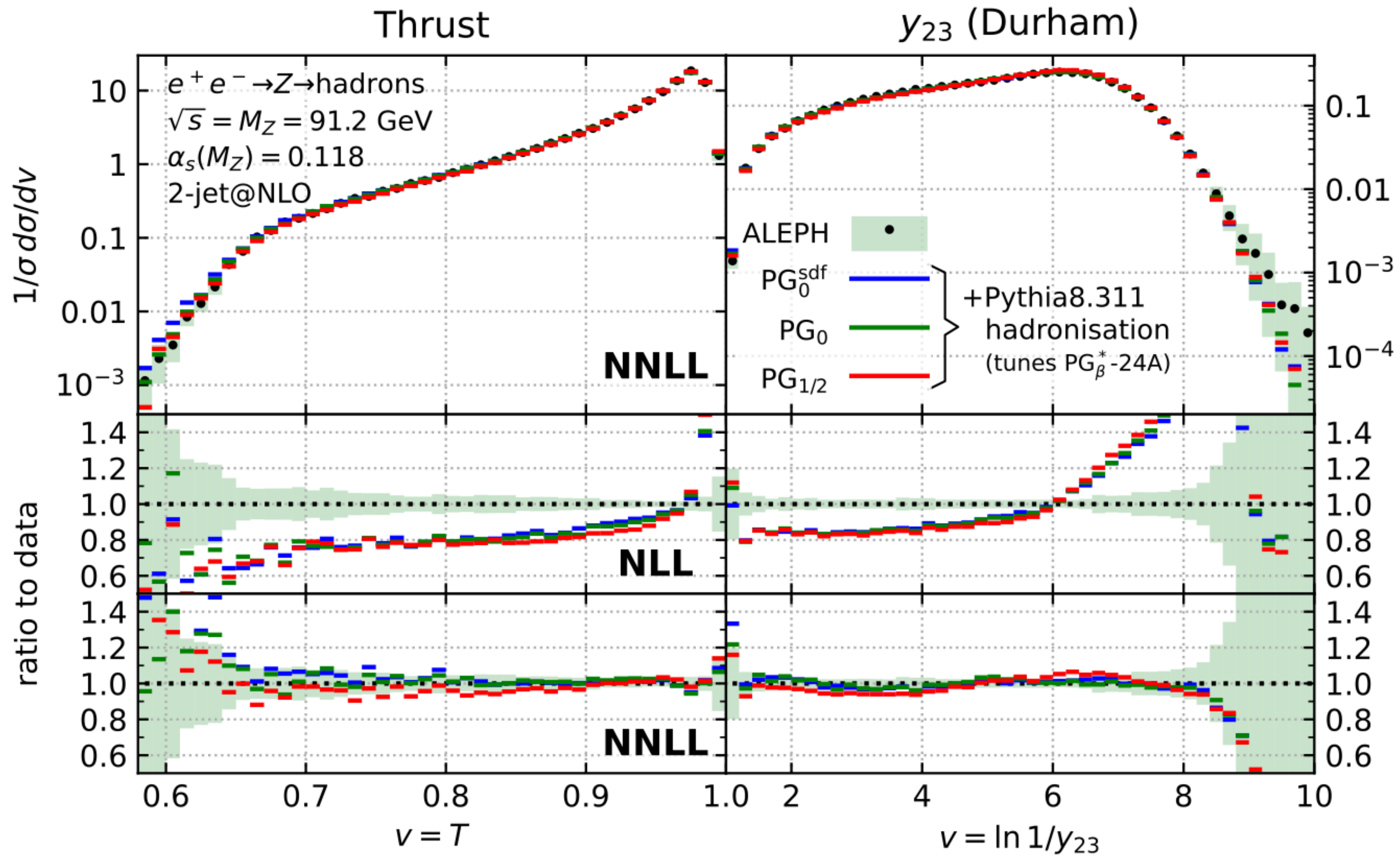
See talk by Tommaso Armadillo

QCD for e^+e^-: NLL Parton Showers

PanScales

van Beekveld et al

- Reproduce analytical resummation results
  - Global/Non-Global Event Shapes
  - Fragmentation/DGLAP evolution
- Ensure that the (N)NLL region is under control with improved kinematical mappings
- NNLL brings large corrections and improvement wrt data

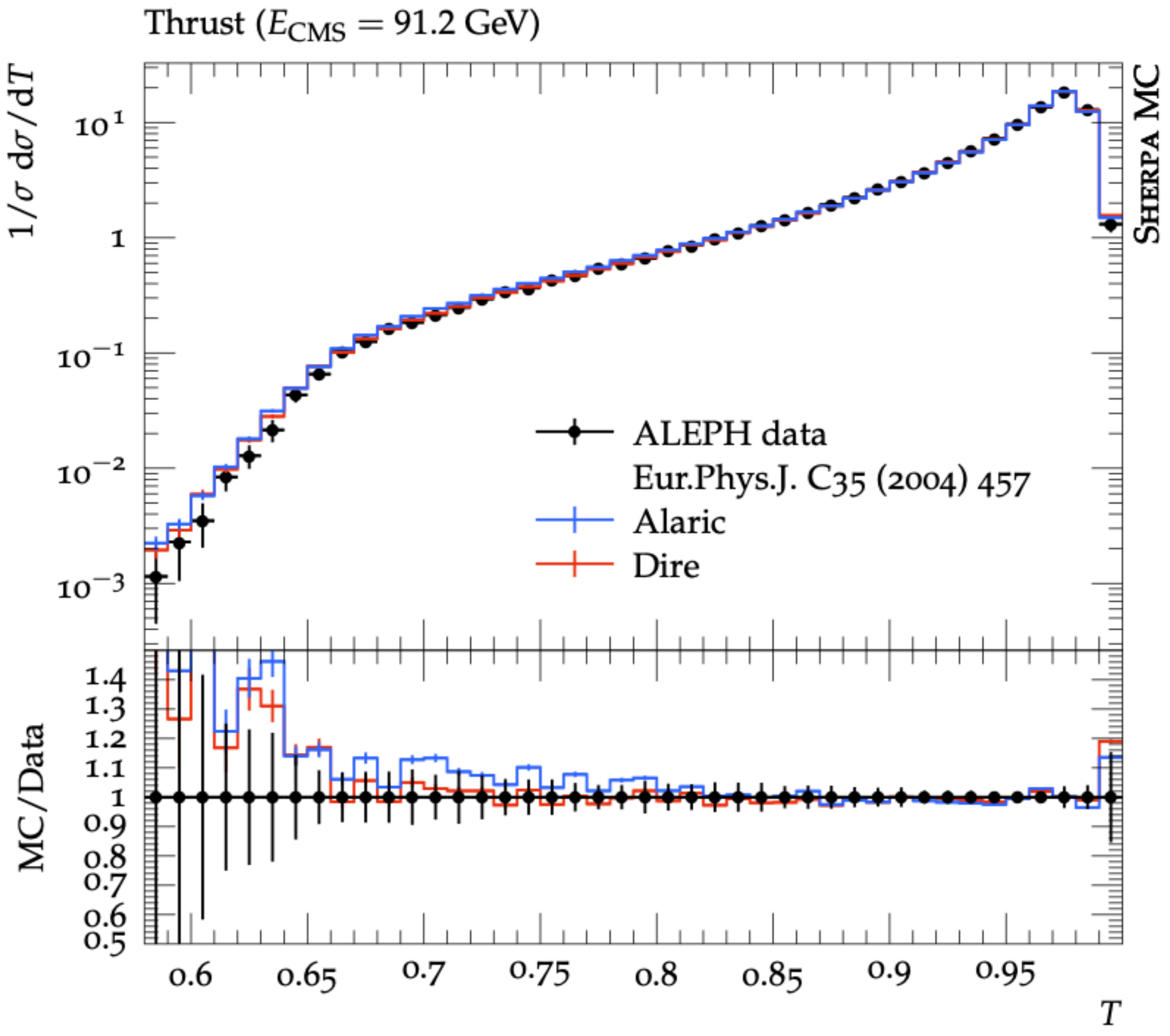


QCD for e^+e^-: NLL Parton Showers

ALARIC

[Herren, Höche, Krauss, Reichelt, Schönherr,’22]

- Explores connection between angular ordering and dipole showers
- Addresses NLL deficiencies found in recoil schemes of current dipole showers
- Multi-Jet Merging now available
- Matching NLO underway



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Nearly all QCD improvements at the LHC can be used for e^+e^-

See talks on Wednesday morning for more QCD

Conclusions

We have a lot of work to do to ensure our theory calculations do not hold back

e^+e^-

I can see a path for the theory community to achieve this but it will take huge effort by the community

But we have plenty of time !