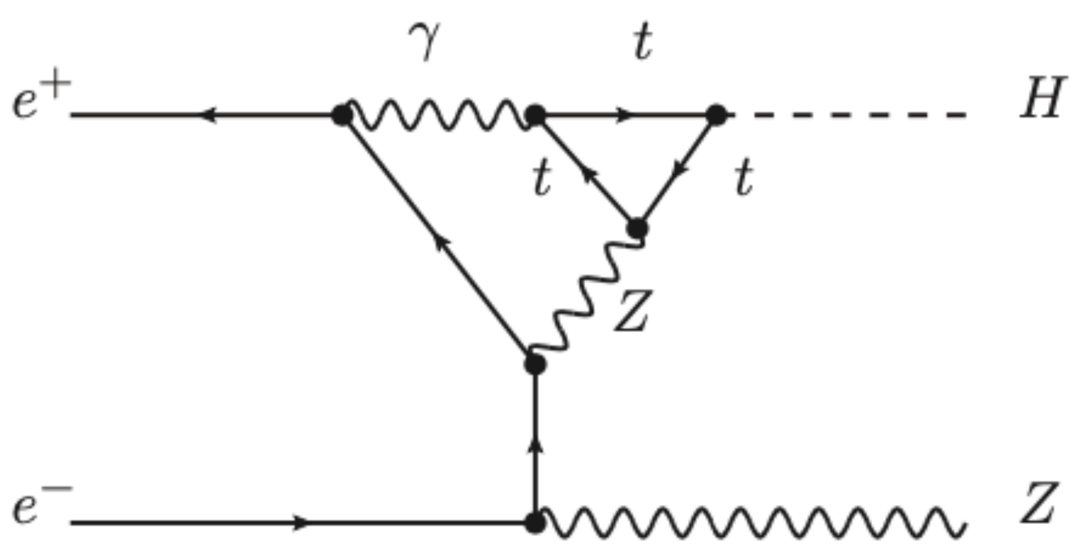
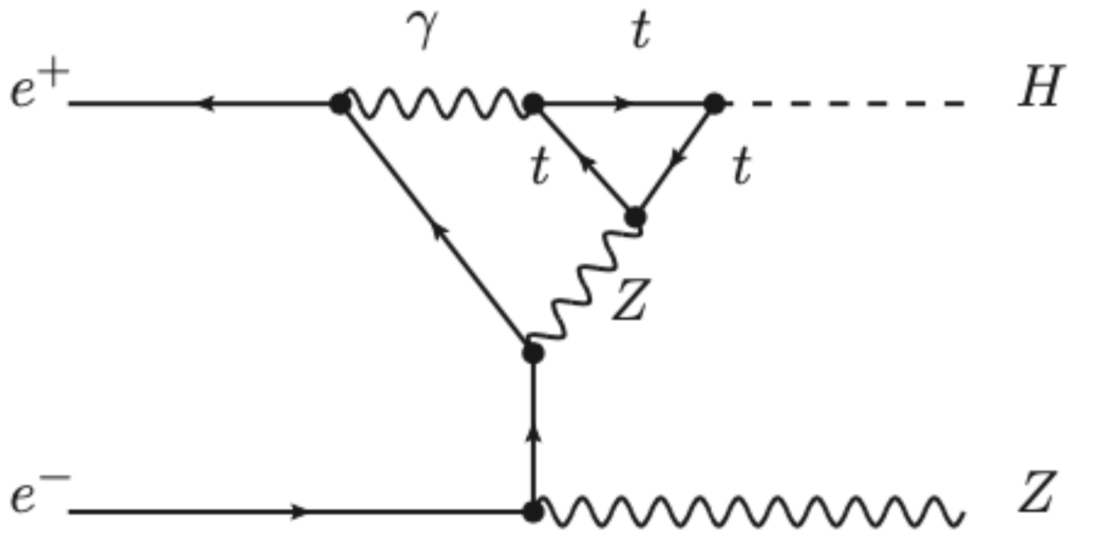
Status of Higgs Theory Predictions at e^+e^- colliders





Narodowe Centrum Nauki
Opus

DESY. HELMHOLTZ

Jürgen R. Reuter

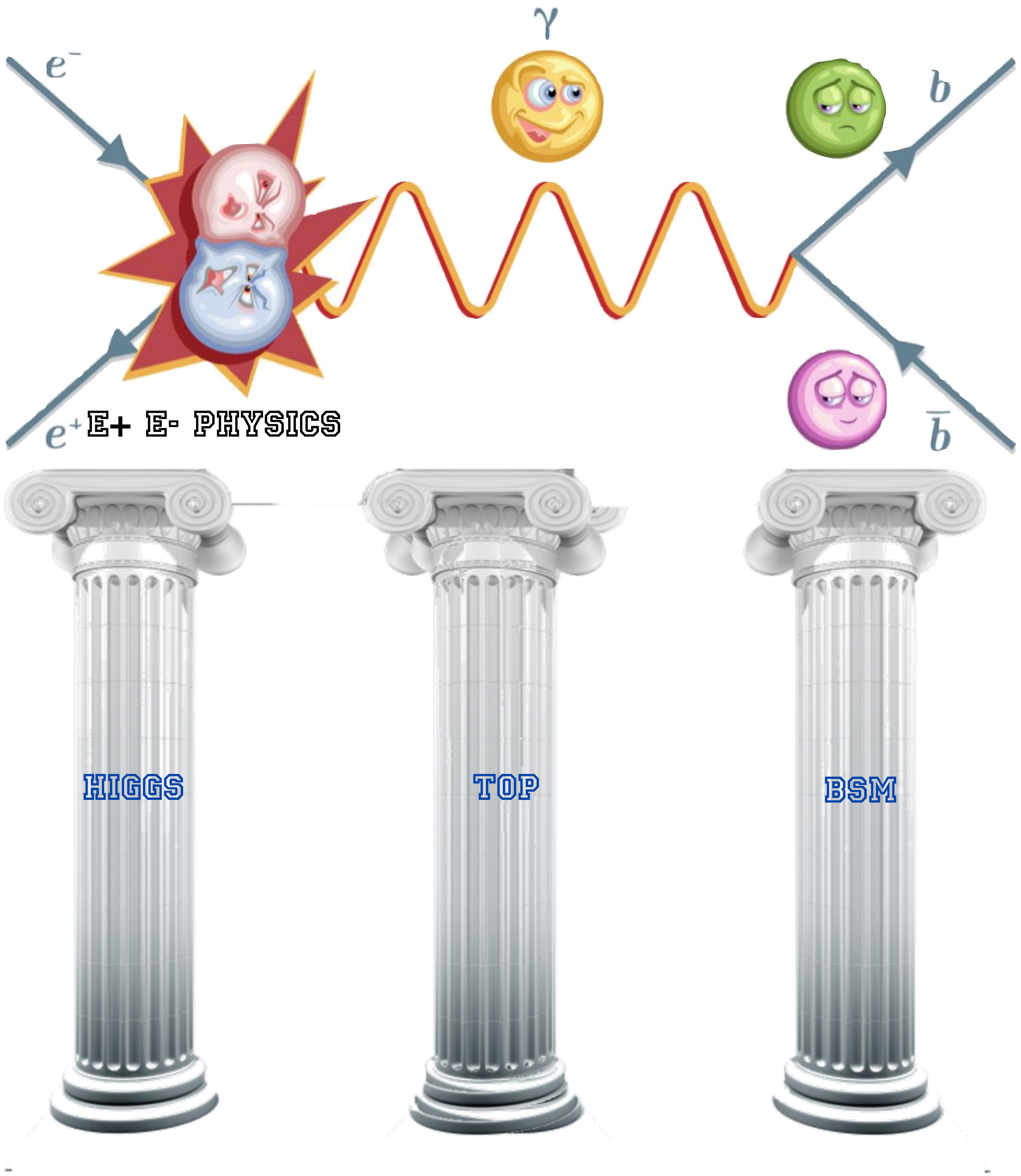
Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

EAJADE
Europe-America-Japan Accelerator Development Exchange Programme

DESY.

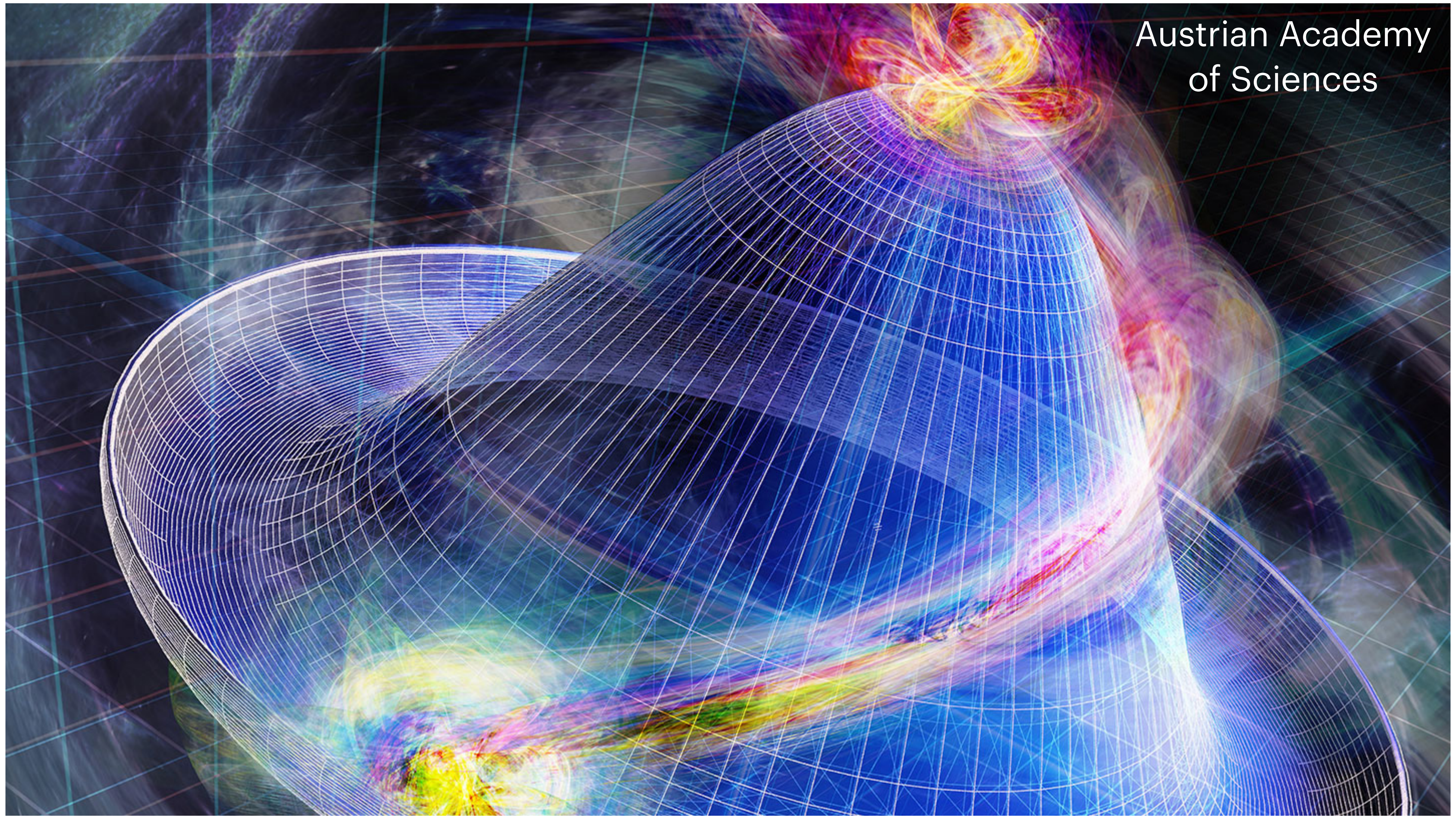
Motivation



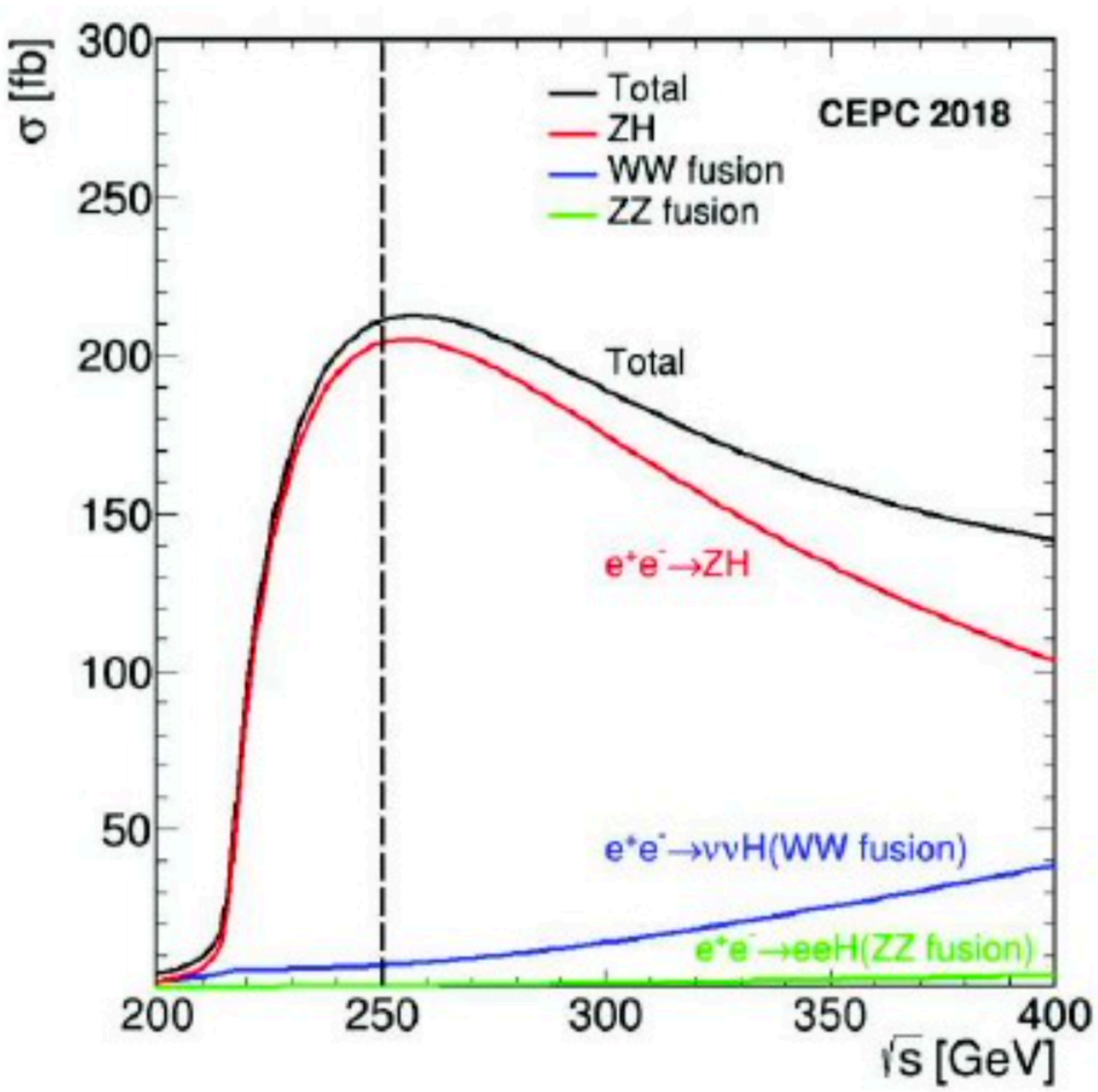
Higgs sector most peculiar phase transition in the universe!

Cornerstone of all "Higgs factory" programs!

Potentially best path towards the "undiscovered country"



Motivation



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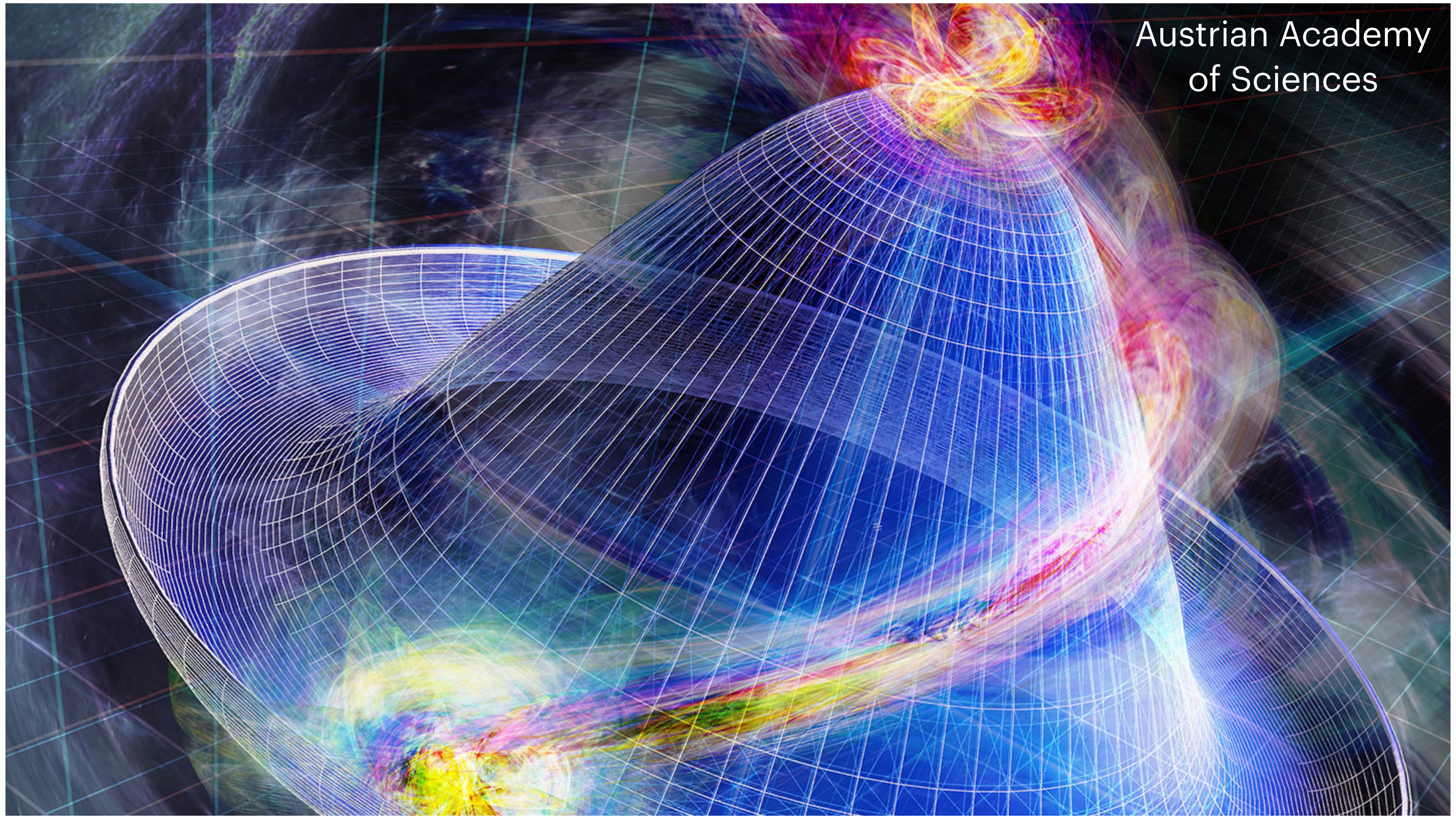

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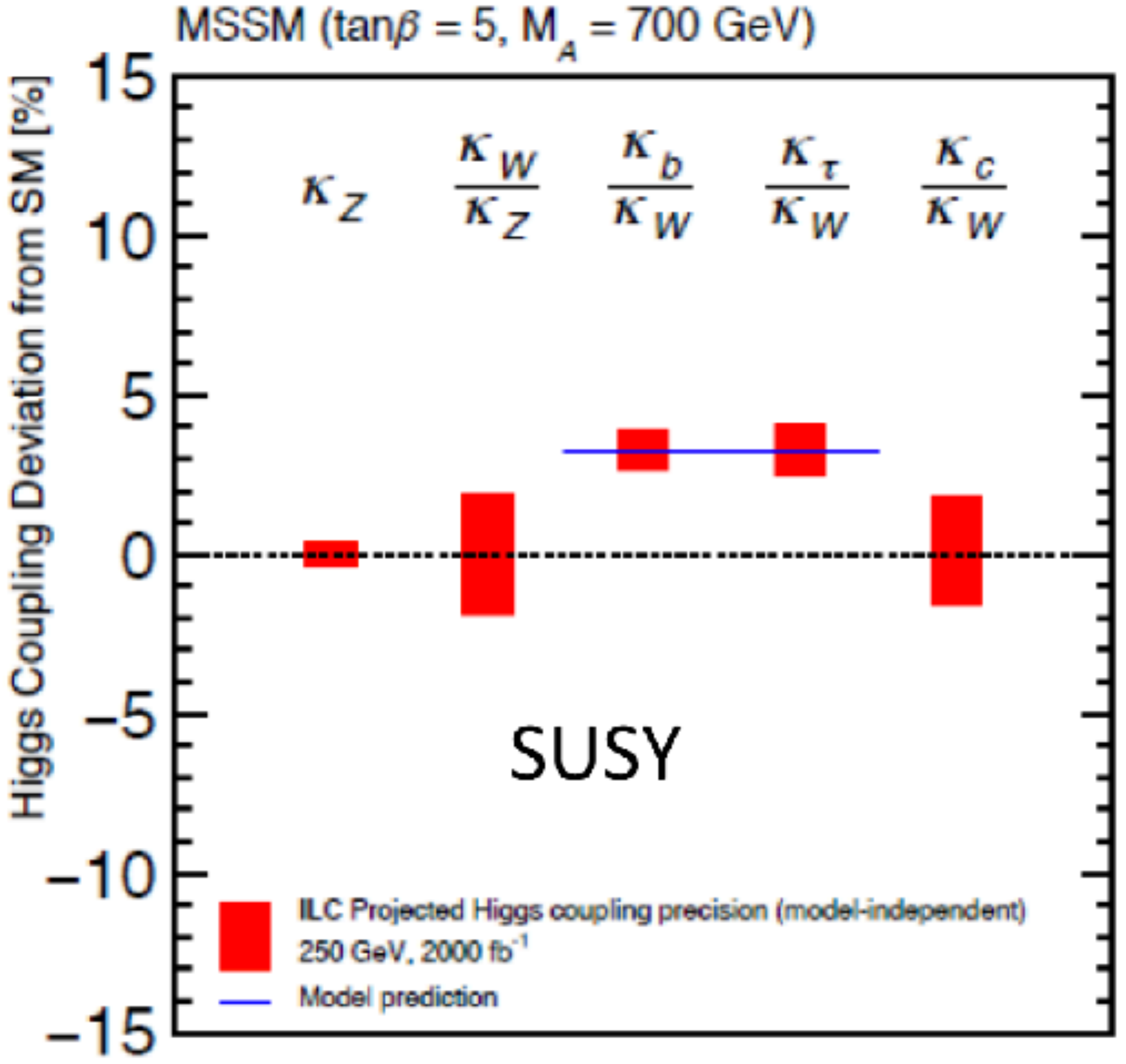

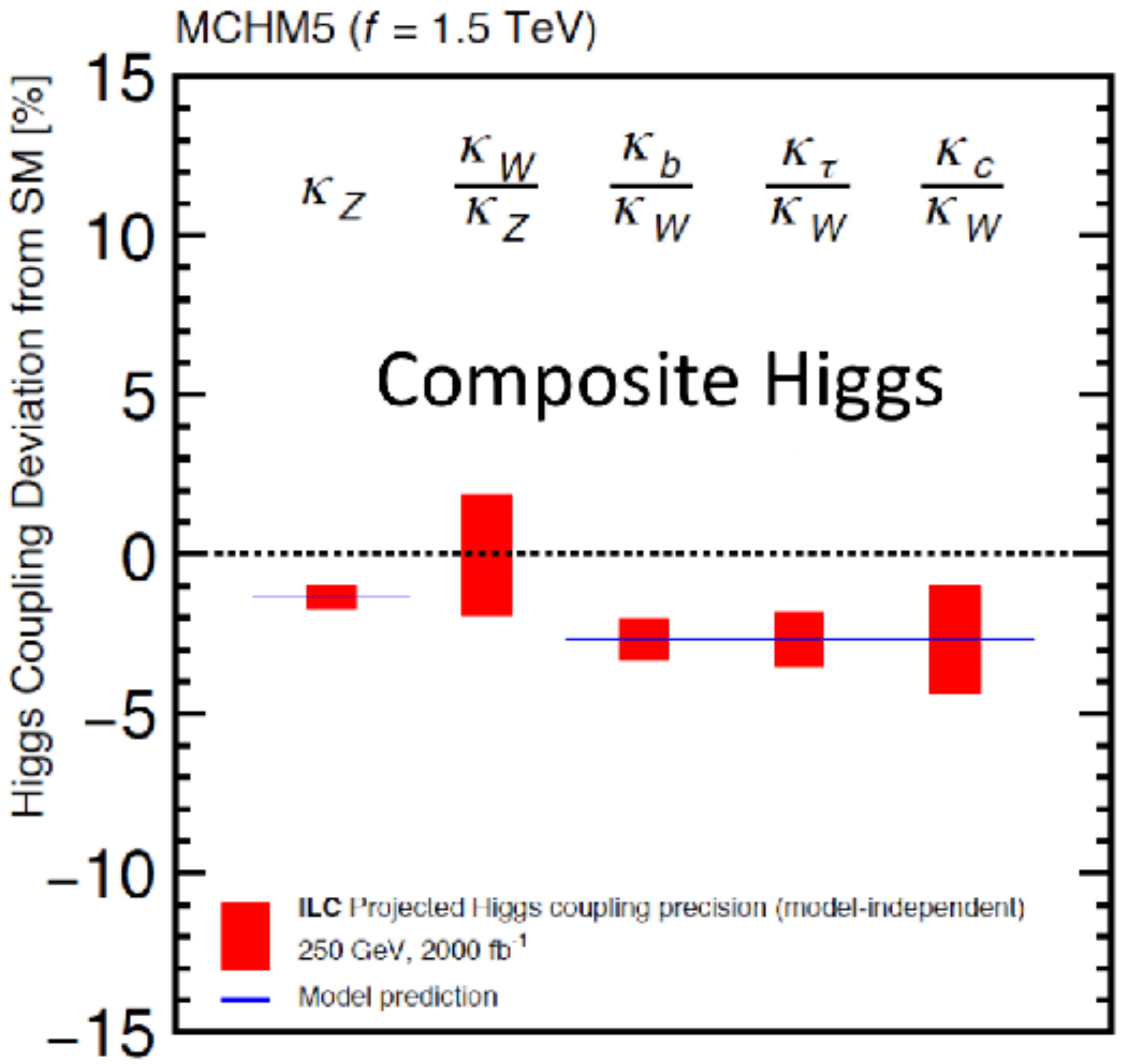

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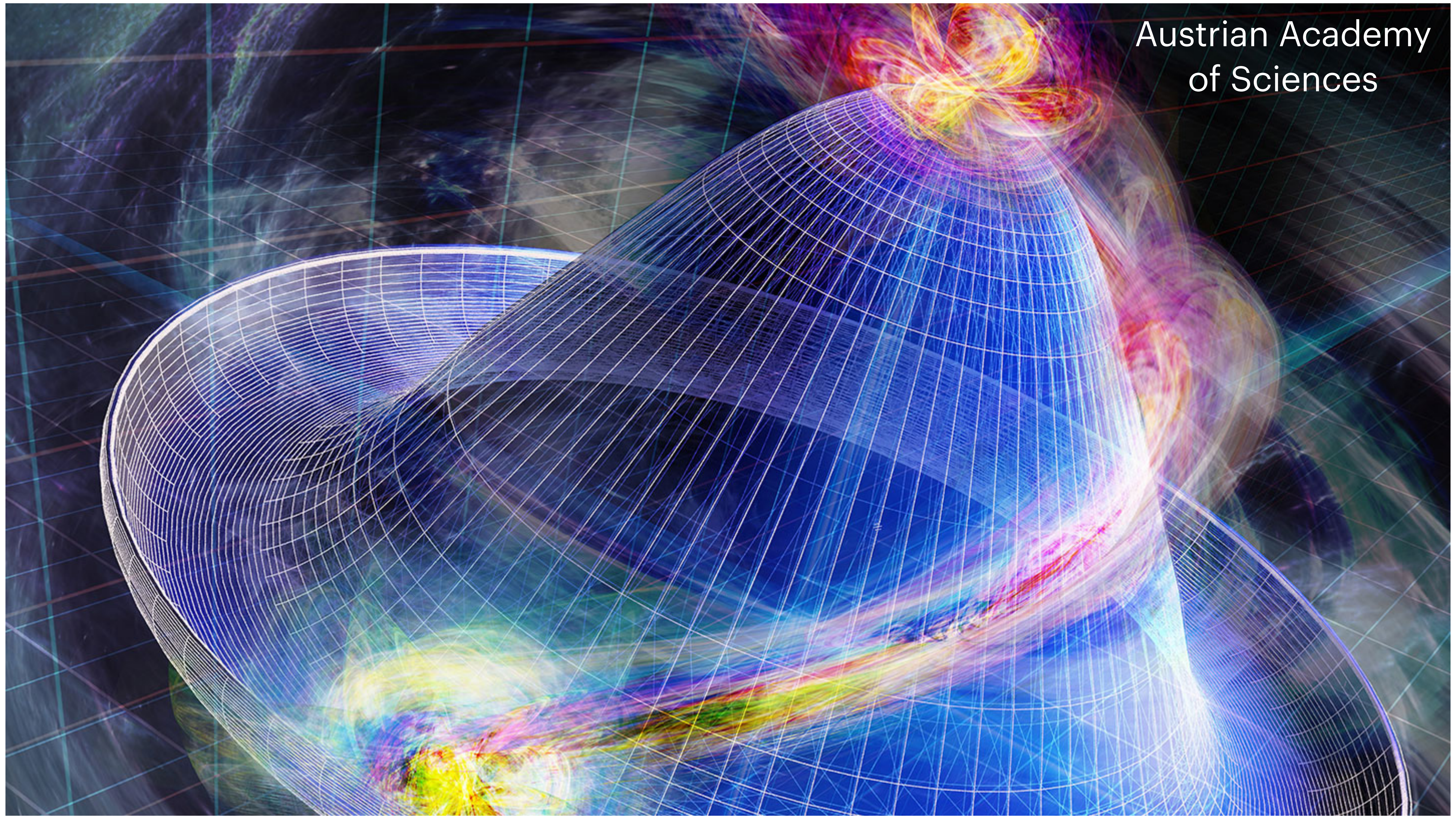
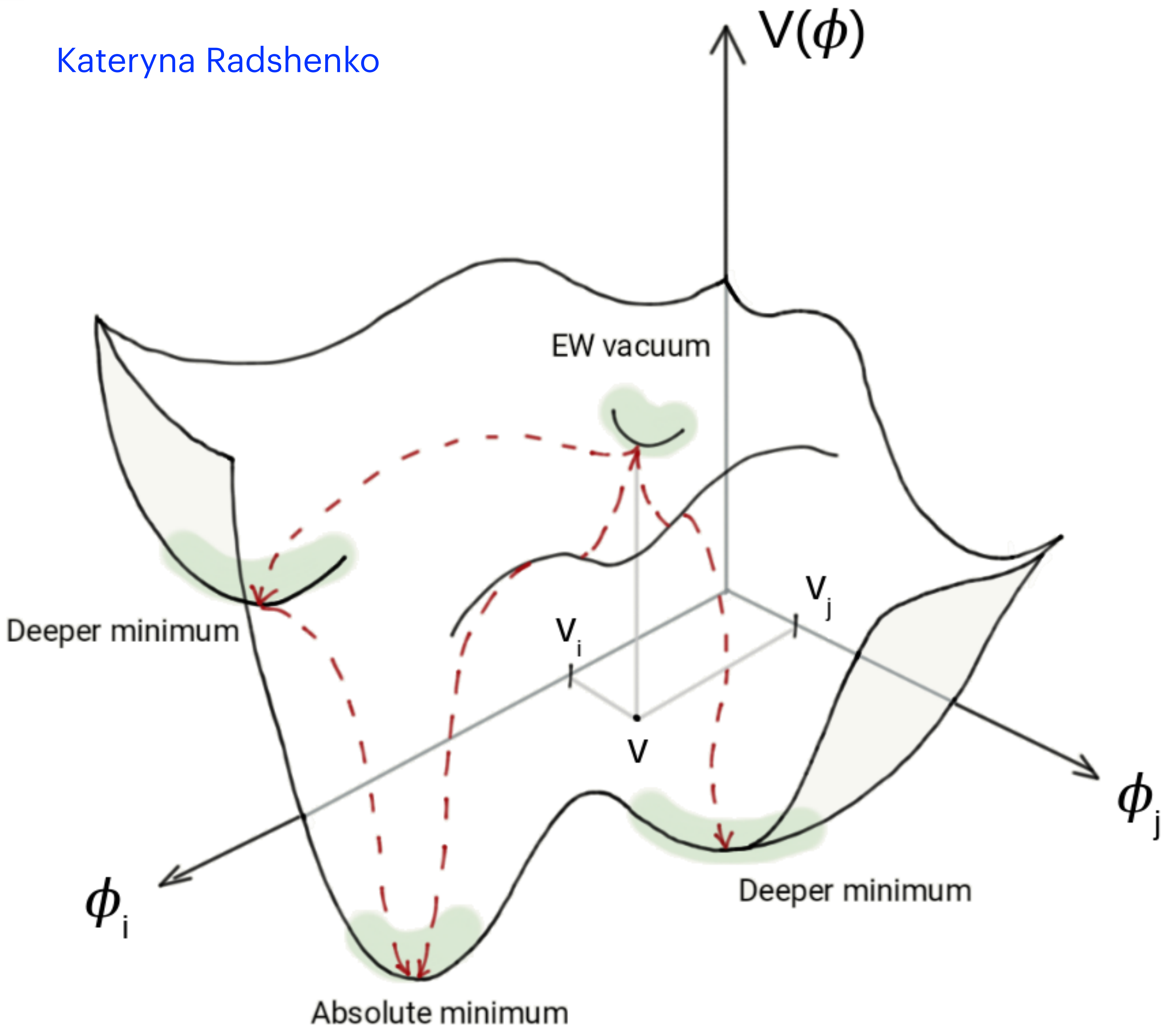


Motivation

Kateryna Radshenko



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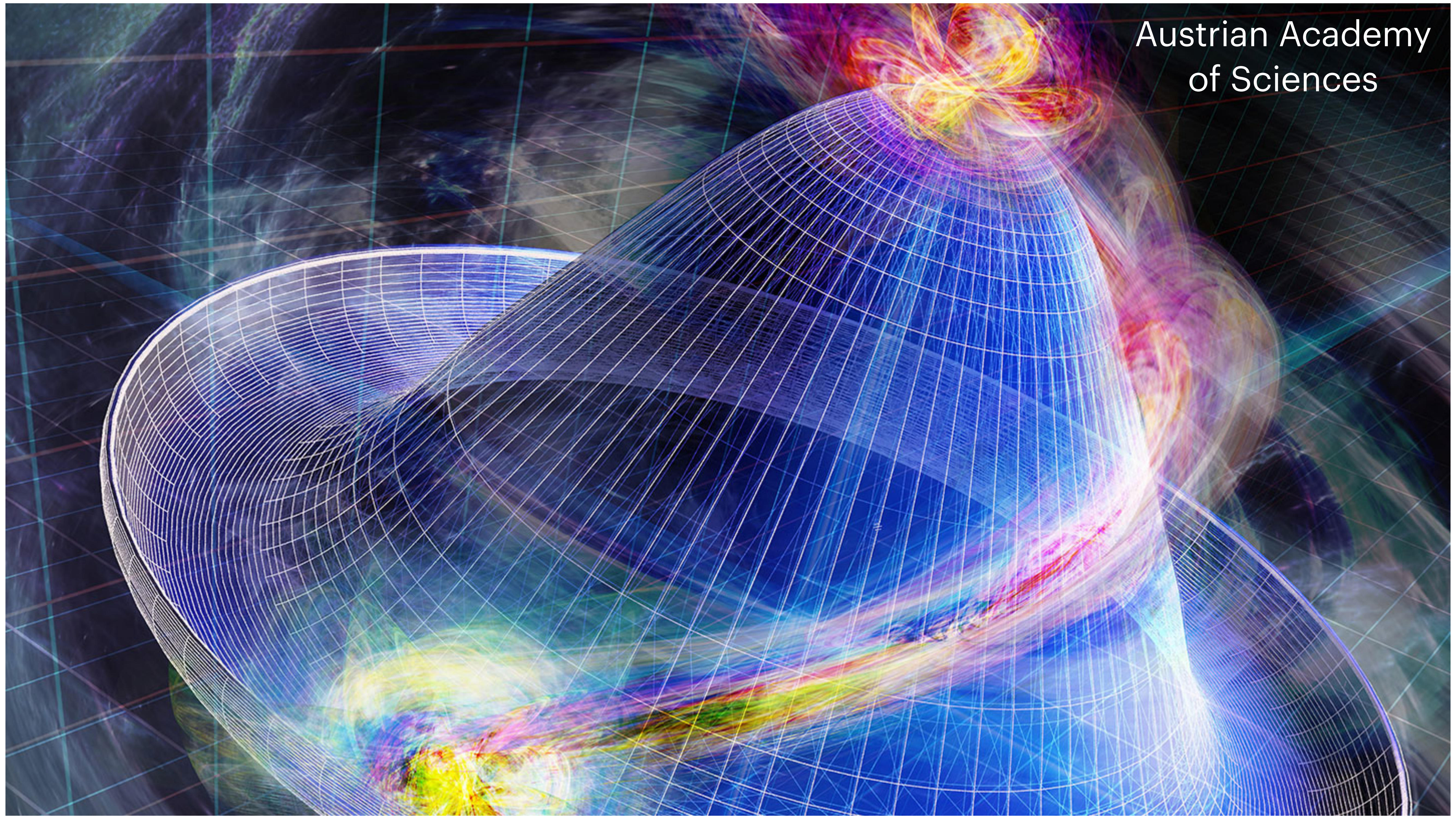
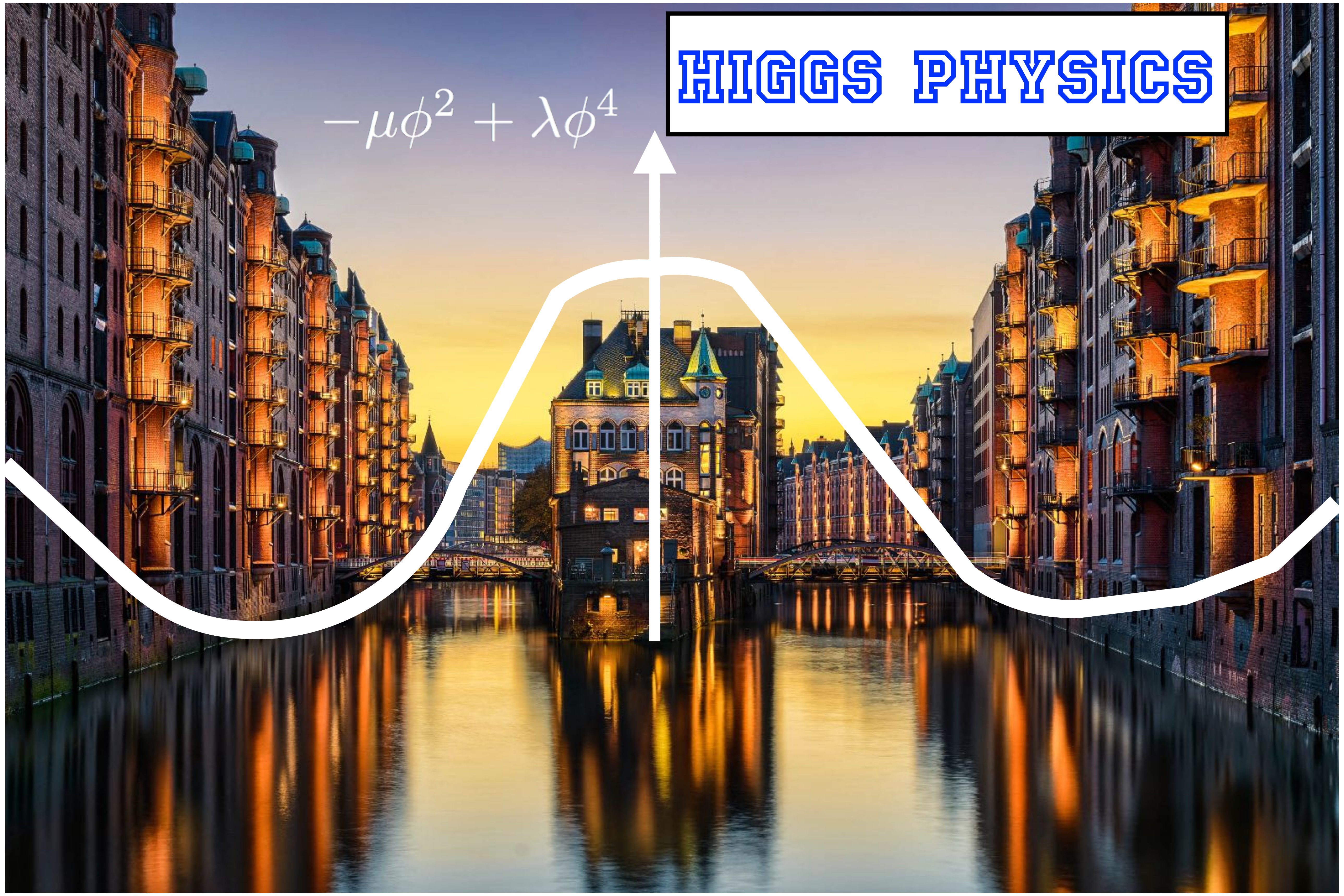


Higgs Theory precision landscape



- LHC HXSWG / LHC EWWG / LHC EFTWG:
  1101.0593, 1201.3084, 1307.1347, 1610.07922
- LCGenG: focus on complete SM samples
- FCC-ee theory effort: CERN workshops '18-'22: 1906.05379
- US Snowmass CSS 2021 Reports:
  2203.11110, 2209.08078, 2209.14872 etc.

- ECFA HTEF WS:

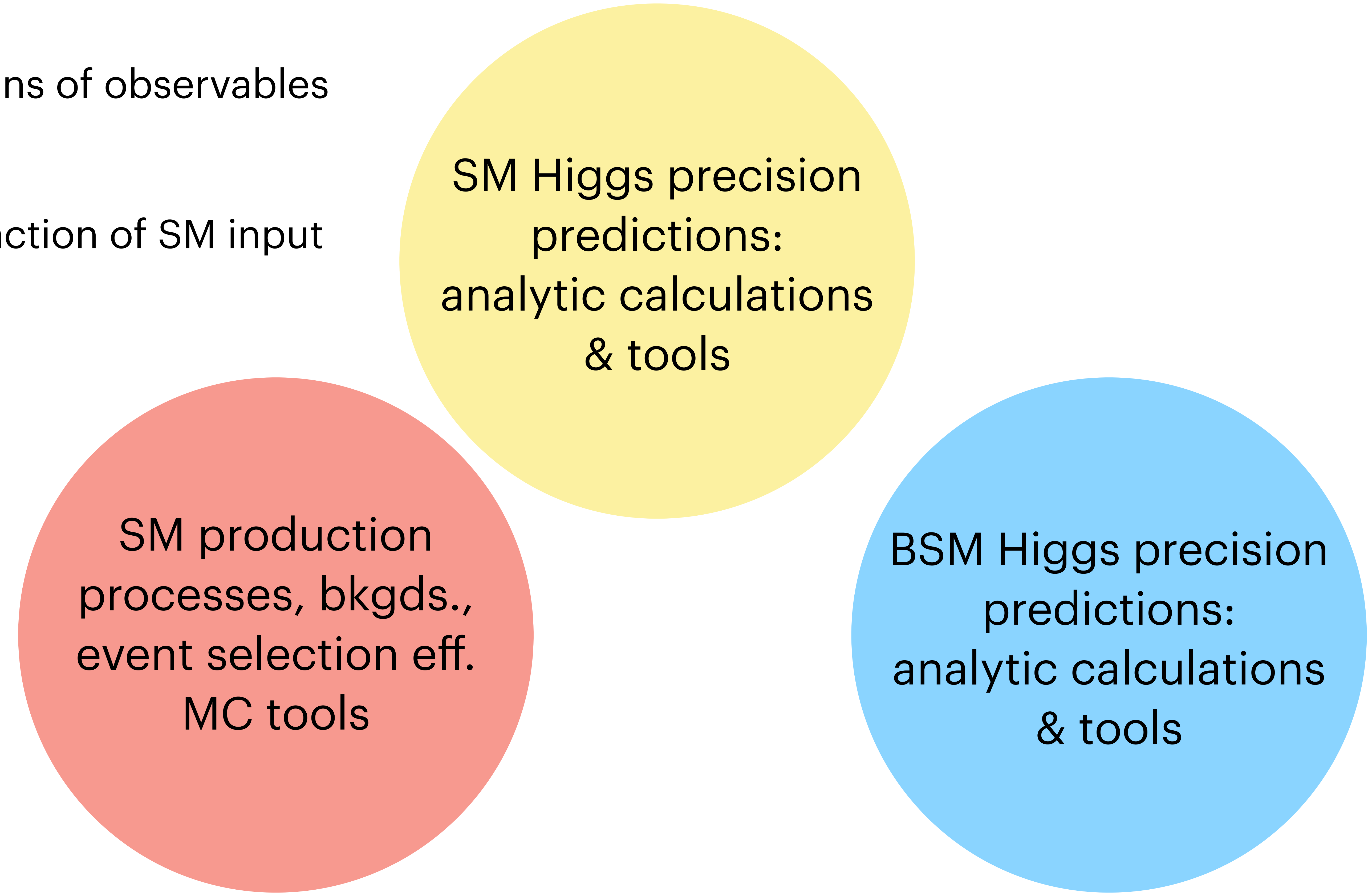
  ECFA HTEF Report: https://cds.cern.ch/record/2920434/

  Simulation/MCs 11/21 https://indico.cern.ch/event/1078675/

  Precision Calc. 05/22 https://indico.cern.ch/event/1140580/

Higgs Theory Precision Landscape

- **Intrinsic uncertainties:**
  missing higher-order calculations of observables
- **Parametric uncertainties:**
  imperfect knowledge/data extraction of SM input



- **Fully differential MC simulations:**
  NLO QCD/EW matched to QCD/QED showers
- **Estimation of efficiencies and systematics:**
  Simulation of full signal + backgrounds
- **Calculations in BSM Higgs models:**
  Higgs sector calculations towards full predictions
- **BSM Higgs as SM Higgs deviations:**
  Framework of SMEFT/HEFT, needed at NLO EW

Theoretical Uncertainties

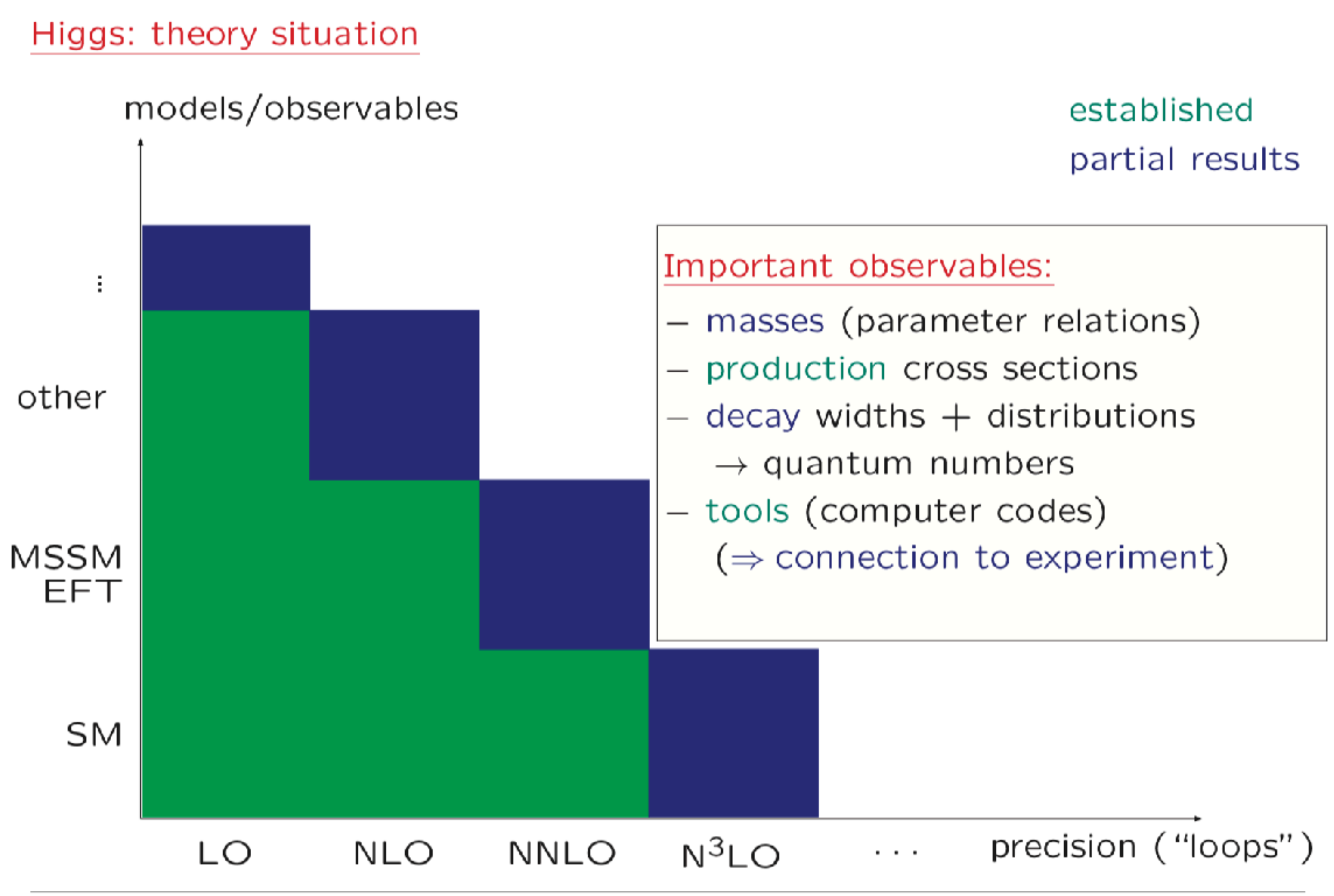
- Comparison of EWPOs / HPOs with SM to **probe new physics**
  $\longrightarrow$ multi-loop corrections in full SM

- Extraction of EWPOs / HPOs (**pseudo-observables**) from **real observables**
  $\longrightarrow$ backgrounds (in full SM), QED/QCD, MC tools

- "Other" eletroweak parameters (**"input" parameters**)
  $\longrightarrow$ m_t, α_s, etc. extracted from other processes

- Strip loop amps. of group theory / mass ratios / multiplicities / couplings. → $\mathcal{O}(1)$
- Extrapolate to higher orders from geometric series (beware of renormalons)
- Scale dependence for missing higher order corrections (QCD, $\overline{\text{MS}}$, less useful for EW)
- Compare differences in renormalisation schemes (e.g. On-Shell vs. $\overline{\text{MS}}$)

Higgs Precision Calculations



from S. Heinemeyer

Parametric Higgs decay uncertainties, Lepage/McKenzie/Peskin, 1404.0319

Partial width	QCD	electroweak	total
$H \to b\bar{b}/c\bar{c}$	$\sim 0.2\%$	$< 0.3\%$	$< 0.4\%$
$H \to \tau^+\tau^-/\mu^+\mu^-$	–	$< 0.3\%$	$< 0.3\%$
$H \to gg$	$\sim 3\%$	$\sim 1\%$	$\sim 3.2\%$
$H \to \gamma\gamma$	$< 0.1\%$	$< 1\%$	$<1\%$
$H \to Z\gamma$	$\lesssim 0.1\%$	$\sim 5\%$	$\sim 5\%$
$H \to WW/ZZ \to 4f$	$< 0.5\%$	$< 0.3\%$	$\sim 0.5\%$

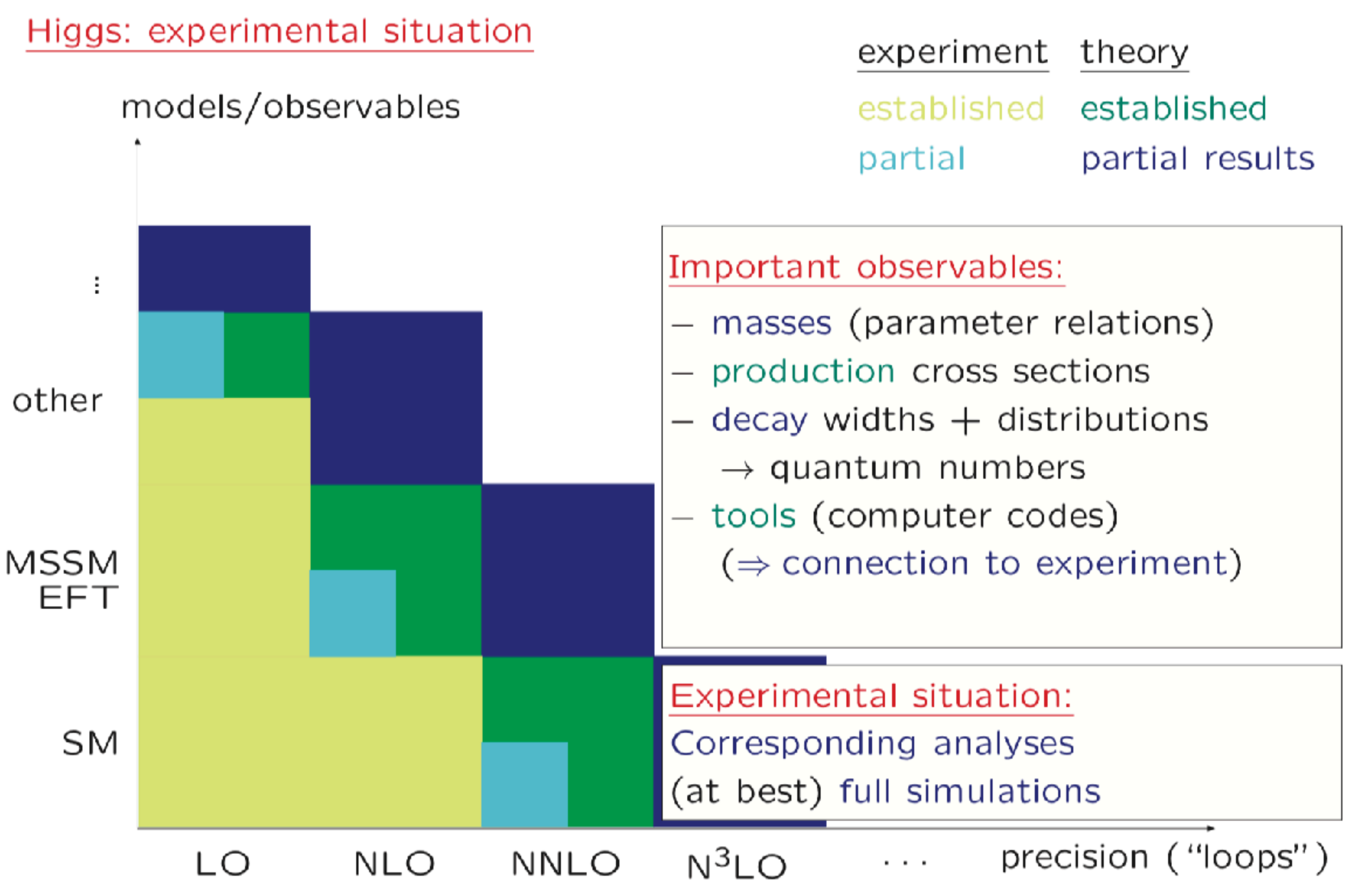
Intrinsic Higgs decay uncertainties, LHCHXSWG

decay	para. m_q	para. α_s	para. M_H
$H \to b\bar{b}$	1.4%	0.4%	–
$H \to c\bar{c}$	4.0%	0.4%	–
$H \to \tau^+\tau^-$	–	–	–
$H \to \mu^+\mu^-$	–	–	–
$H \to gg$	$< 0.2\%$	3.7%	–
$H \to \gamma\gamma$	$< 0.2\%$	–	–
$H \to Z\gamma$	–	–	2.1%
$H \to WW$	–	–	2.6%
$H \to ZZ$	–	–	3.0%

- Full NLO EW exists for $ee \to ZH$, Denner/Dittmaier/Roth/Weber, hep-ph/0311089
  $ee \to \nu\nu H$ Belanger/Boudjema/Fujimoto/Ishikawa/Kaneki/Kato/Shimizu, hep-ph/0212261

5-10% NLO corrections

Higgs Precision Calculations



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  ee → *ννH* Belanger/Boudjema/Fujimoto/Ishikawa/Kaneki/Kato/Shimizu, hep-ph/0212261
  - Full 2-loop for *ee* → *ZH* available Chen/Guan/He/Li/Liu/Ma, 2209.14953
  - Missing NNLO EW corrections [*2→2, 2→3*] : intrinsic uncertainty 1%
  - Compared to experimental uncertainty of 0.5-1.0%

5-10% NLO corrections

NNLO EW hard task for VBF !

Higgs Branching Ratios

ILC/CEPC/FCC-ee projections

Higgs decays	red = updated compared to Snowmass 21/22				
	future projection (conservative)		**future projection (agressive)**		
	th.err. estimate	additional orders available	th.err. estimate	additional orders available	comments
bb/cc	0.2	a^2+a*as	0.1	as^5	unc. dominated by N3LO mixed EW-QCD
ττ/μμ	<0.1	a^2+a*as	0.05 ??	af^2+af*as	what is dominant remaining error? how to estimate?
WW/ZZ	0.3	as^2			
gg	1.0	as^4	0.5	as^5+a*as	unc. dominated by a^2 (3-loop)
γγ					
Zγ					

Notation:		
a = \alpha_ew		
as = \alpha_s		
at = y_t^2/(4 pi)	(ew. corr. enhanced by top mass)	
af = \alpha_ew with closed fermion loop		

A. Freitas,
Physics Preparatory Group,
June 2025

Parametric uncertainties

Parametric uncertainties

SM predictions for Higgs decays need measured input parameters

Reviews: 1906.05379, 2012.11642

Numerical impact of input parameter uncertainties:

	$\delta m_t = 0.5$ GeV	$\delta\alpha_s = 0.001$	$\delta(\Delta\alpha) = 10^{-4}$	FCC-ee exp.
M_W [MeV]	3	0.7	2	0.4
$\sin^2\theta^\ell_{eff}$ [10^{-5}]	1.5	0.3	3.5	0.4
Γ_Z [MeV]	0.1	0.5	0.1	0.025
$\Gamma[h \to gg]$ [%]	<0.2	3	–	1.8

To keep impact subdominant for FCC-ee/CEPC precision studies, would need:

$\delta m_t < 50$ MeV

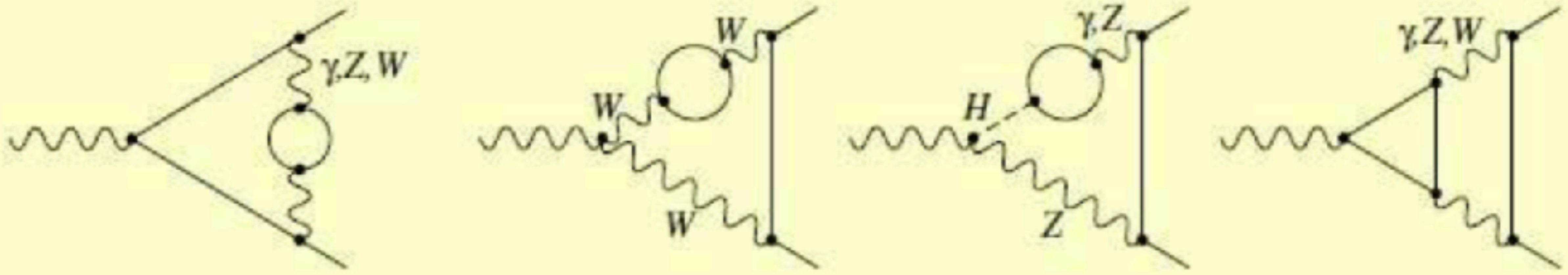
$\delta\alpha_s < 5 \times 10^{-5}$

$\delta(\Delta\alpha_s) < 10^{-5}$

Parametric uncertainties

Parametric uncertainties

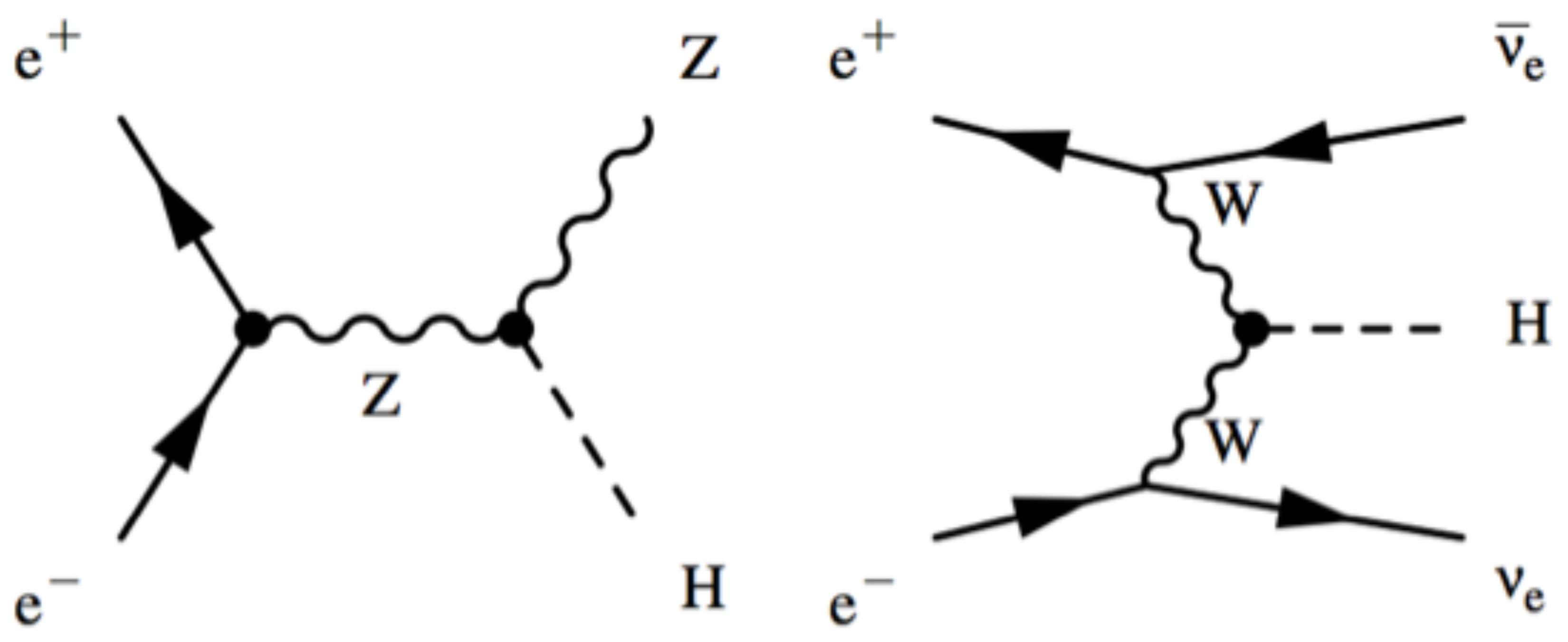
Known corrections to Δr, $\sin^2\theta^f_{\text{eff}}$, $g_{V\,f}$, $g_{A\,f}$:



- Complete NNLO corrections (Δr, $\sin^2\theta^\ell_{\text{eff}}$) — Freitas, Hollik, Walter, Weiglein '00; Awramik, Czakon '02; Onishchenko, Veretin '02; Awramik, Czakon, Freitas, Weiglein '04; Awramik, Czakon, Freitas '06; Hollik, Meier, Uccirati '05,07; Degrassi, Gambino, Giardino '14
- "Fermionic" NNLO corrections ($g_{V\,f}$, $g_{A\,f}$) — Czarnecki, Kühn '96; Harlander, Seidensticker, Steinhauser '98; Freitas '13,14
- Partial 3/4-loop corrections to ρ/T-parameter
  $\mathcal{O}(\alpha_t\alpha_s^2)$, $\mathcal{O}(\alpha_t^2\alpha_s)$, $\mathcal{O}(\alpha_t\alpha_s^3)$ — Chetyrkin, Kühn, Steinhauser '95; Faisst, Kühn, Seidensticker, Veretin '03; Boughezal, Tausk, v. d. Bij '05; Schröder, Steinhauser '05; Chetyrkin et al. '06; Boughezal, Czakon '06

  $(\alpha_t \equiv \frac{y_t^2}{4\pi})$

Higgs production channels



Prefactor estimates: [del(x) = relative cross-section correction of O(x)]
del(a^3) = del(a^2) * a/pi*nf, where a = g^2/(4*pi), nf = 12 = # of charged fermion species
del(a^2*as) = del(a^2) * as/pi*2*CF (heuristic factor 2 to account for diagram combinatorics)
del(a*as^2) = del(a*as) * as/pi*2*CA (heuristic factor 2 to account for diagram combinatorics)

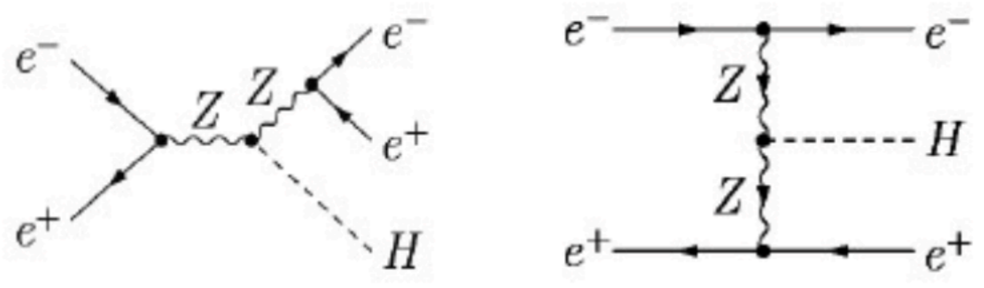
Higgs production		(slide 8+9)	red = updated compared to Snowmass 21/22		
	current		**future projection (conservative)**		
	th.err. estimate	avail. orders	th.err. estimate	additional orders available	comments
HZ [%]	0.3	a+a*as+af*a	<0.1	a^2	estimate based on prefactors (see below)
WW fusion [%]	~1	a			

A. Freitas,
Physics Preparatory Group,
June 2025

From
arXiv:2404.11441

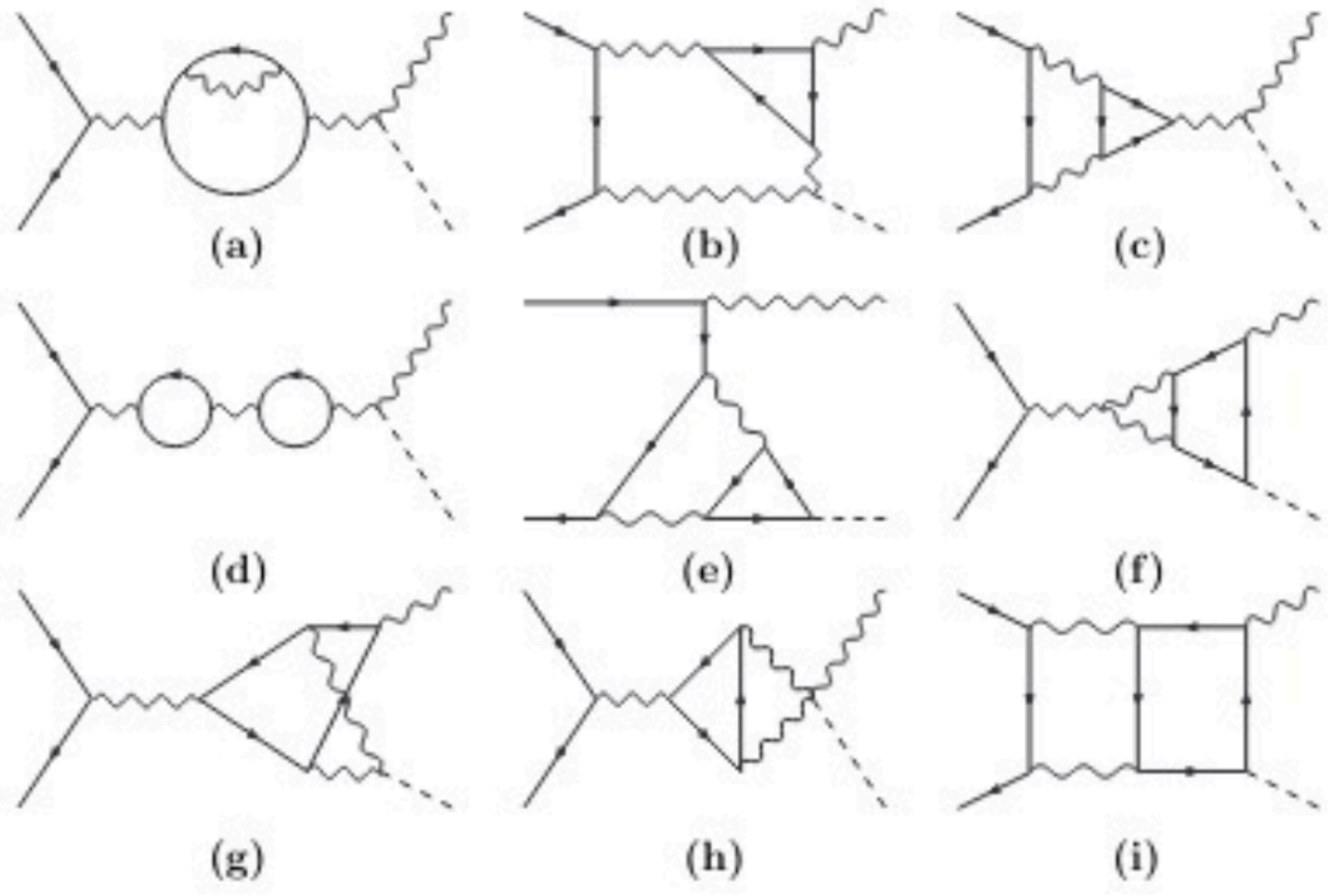
Technology for $\mathcal{O}(\alpha)$ with off-shell Z-boson available

Boudjema et al. '04
Denner, Dittmaier, Roth, Weber '03



The Higgsstrahlung process

- Higgsstrahlung main Higgs production mechanismus below ca. 500 GeV
- Need for NNLO corrections: O(1%) precision and better
- Expected experimental precision on cross section:
  CEPC: 0.5% [1811.10545], FCC-ee: 0.4% [EPJ ST 228(2019) 261], LCF: 0.62% [2503.19983]
- On-shell Higgsstrahlung (SM) available at NNLO EW Freitas/Song(/Xie), 2101.00308, 2209.07612, 2305.16547



	$\alpha(0)$ scheme	G_μ scheme
$\sigma^{\rm LO}$ [fb]	223.14	239.64
$\sigma^{\rm NLO}$ [fb]	229.78	232.46
$\sigma^{\rm NNLO,EW\times QCD}$ [fb]	232.21	233.29

Gong ea., 2016; Chen/Feng/Jia/Sang, 2018

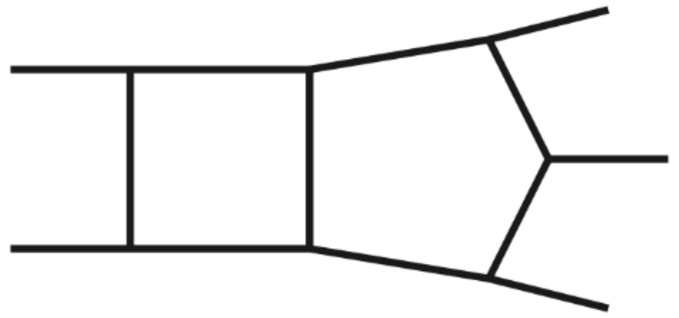
Semi-numerical approach for EW 2-loop problems

Virtual corrections — (N)NNLO master integrals

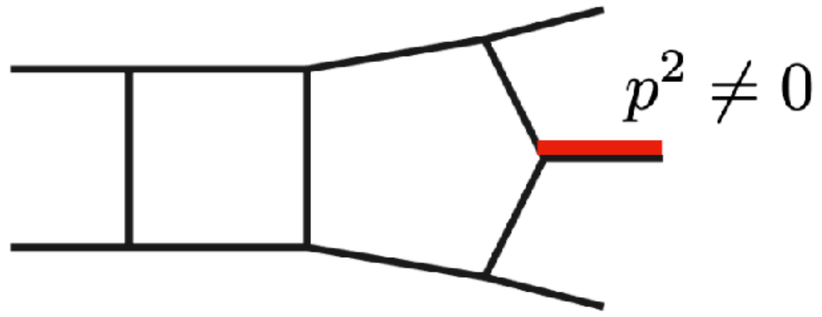
G. Heinrich, DESY Theory Workshop talk, 09/2022

Current multi-loop frontiers:

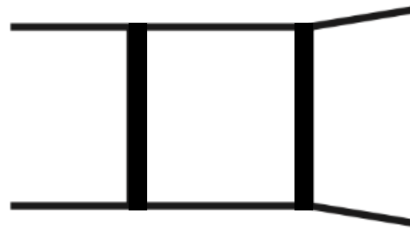
massless 5-point functions

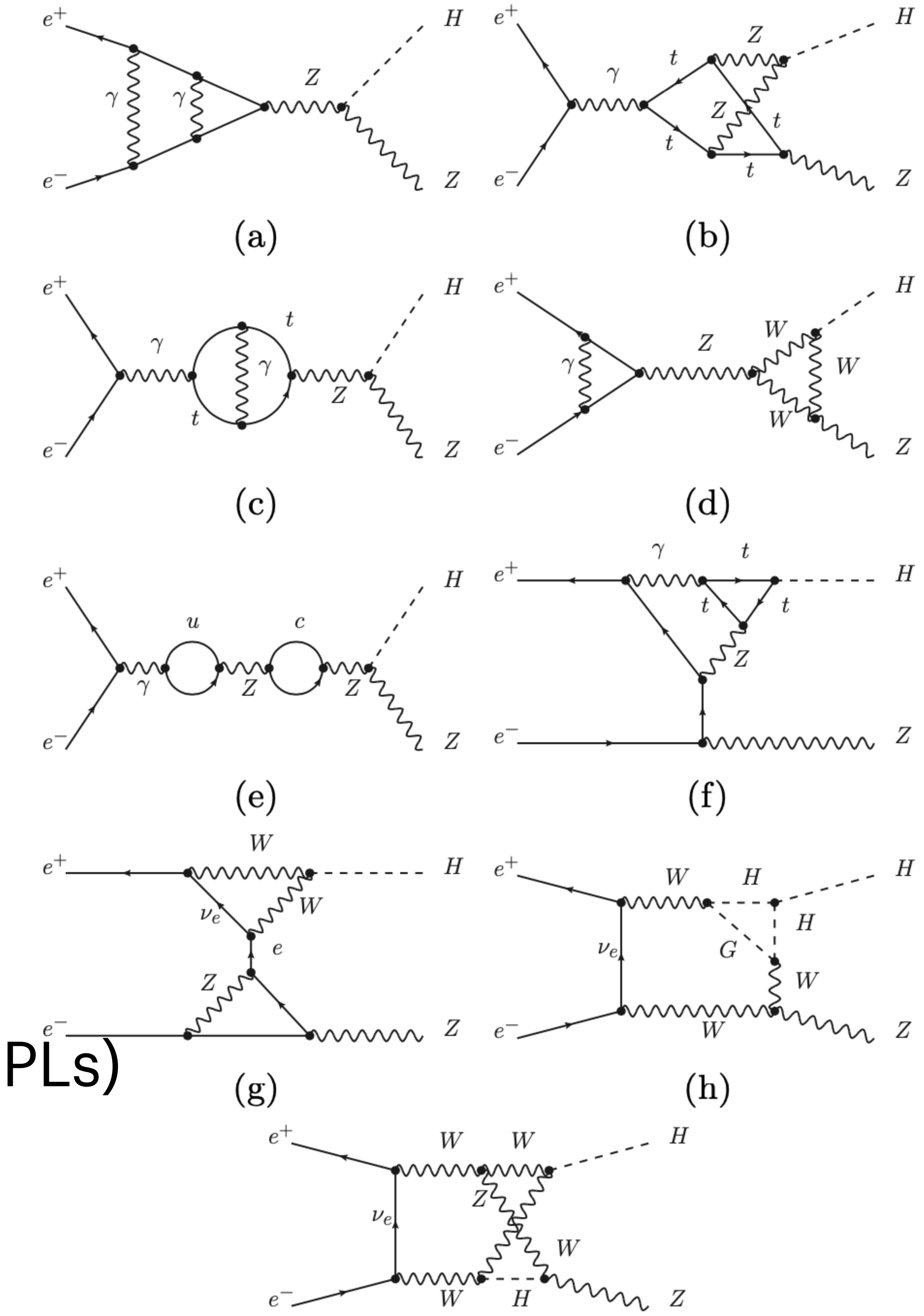


massless 5-point functions, 1 off-shell line



4-point functions w/ massive propagator(s)





- ☑ NNLO EW highly complicated: many mass scales (m_Z, m_W, m_H, m_t)
- ☑ Tensor & IntegrationByParts reduction to master integrals
- ☑ Analytic solution via diff.eq. (DE): Fire6, FireFly, LiteRed, FiniteFlow, Caravel
- ☑ (Semi-)Numerical solution of DE: DiffExp, AMFlow
- ☑ For NNLO EW: importance of viable γ_5 scheme

Abreu, Bonciani, Duhr, Gluza, Henn, Hirschi, Kossower, Liu, Ma, von Manteuffel, Panzer, Pezaro, Sotnikov, Stöckinger, Vicini, Weinzierl, Weißwange, Zoller amm.

- ☐ No analytic 2-loop w/ mass. propagators yet: unknown generalized functions (beyond HPLs)
- ☐ Combination of numerical and analytical methods needed
- ☐ W.i.p.: automation of 2-loop virtuals, Openloops2loop, Griffin, McMule

The Higgsstrahlung process @ NNLO EW

Scheme dependence:

	$\alpha(0)$ scheme	G_μ scheme
$\sigma^{\rm LO}$ [fb]	223.14	239.64
$\sigma^{\rm NLO}$ [fb]	229.78	232.46
$\sigma^{\rm NNLO,EW\times QCD}$ [fb]	232.21	233.29
$\sigma^{\rm NNLO,EW}$ [fb]	233.86	233.98

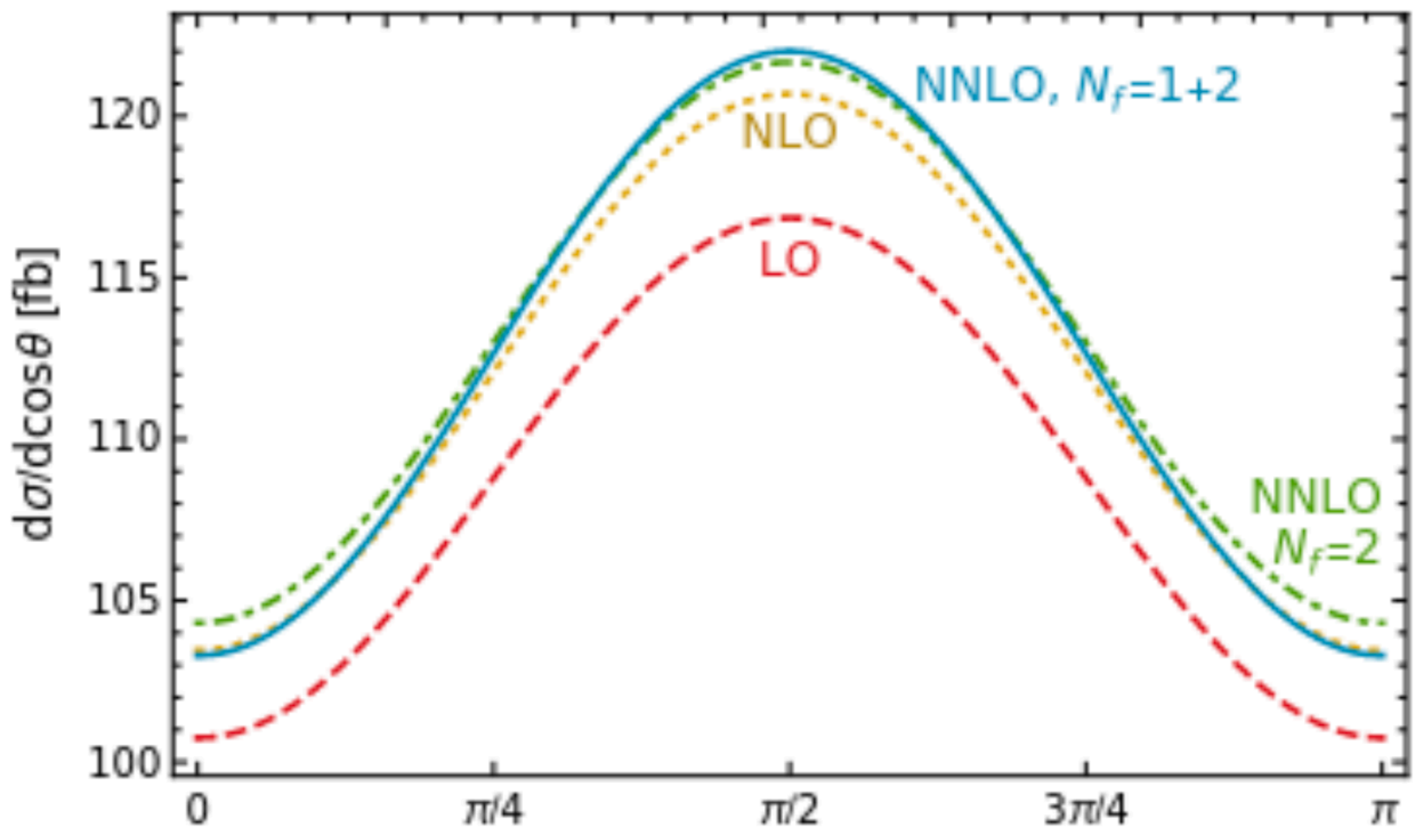
Very good agreement between the two schemes

$\alpha(0)$ scheme:

$$\alpha = e^2/(4\pi)$$
$$g = \frac{e}{\sin\theta_W} = \frac{e}{\sqrt{1 - m_W^2/m_Z^2}}$$

G_μ scheme:

$$\frac{G_\mu}{\sqrt{2}} = \frac{g^2}{8m_W^2}(1 + \Delta r).$$



- ☑ Fermionic 2-loop corrections dominant
- ☑ Scheme-dependence massively reduced
- ☑ Numerical precision limited, sufficient for applications
- ☑ Main theory uncertainty: missing bosonic corrections

Difference btw. $\alpha(0)$ and G_μ schemes	0.12 fb (0.05%)
$\lvert\mathcal{M}_{(1,\rm bos)}\rvert^2$	0.65 fb (0.3%)

↪ Talk Alan Price

Exclusive (BSM) Higgs processes

- NLO QCD ⊕ EW automated: Sherpa, MG5, Whizard
- Fixed-order N(N)LO, resummation and matching in MCs
- Need $e^+e^- \to 2f,\ 3f,\ 4f,\ 5f,\ 6f$ @ NLO QCD ⊕ EW

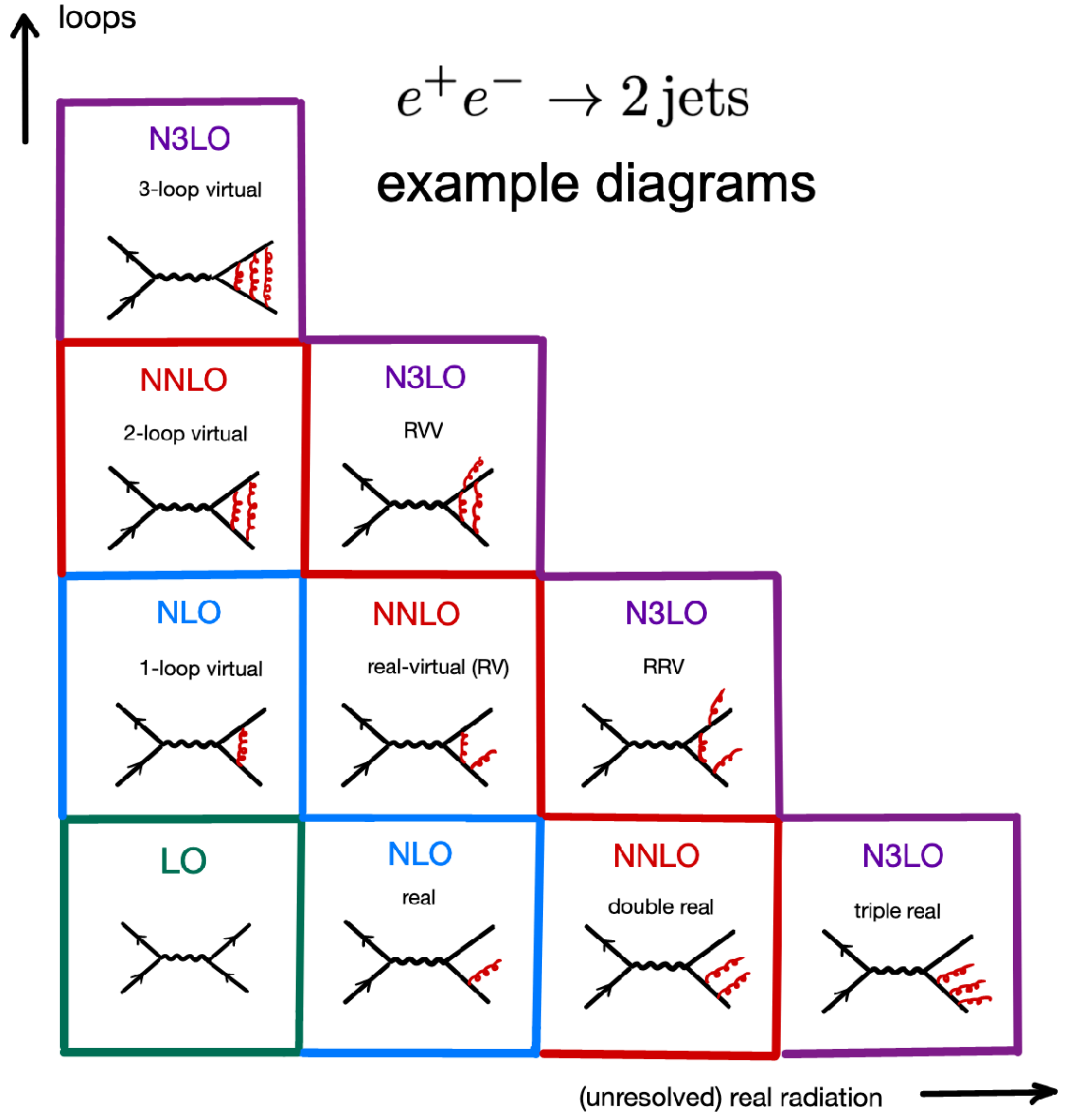
  (arbitrary cuts, fully differential)

SM Higgs (here with polarization)

$e^+e^- \to HZ$ $\sqrt{s}$ [GeV]	MCSANCee [199] $\sigma^{\rm tot}_{\rm LO}$ [fb]	$\sigma^{\rm tot}_{\rm NLO}$ [fb]	WHIZARD+RECOLA $\sigma^{\rm tot}_{\rm LO}$ [fb]	$\sigma^{\rm tot}_{\rm NLO}$ [fb]	$\delta_{\rm EW}$ [%]	$\sigma^{\rm sig}$ (LO/NLO)
250	225.59(1)	206.77(1)	225.60(1)	207.0(1)	−8.25	0.4/2.1
500	53.74(1)	62.42(1)	53.74(3)	62.41(2)	+16.14	0.2/0.3
1000	12.05(1)	14.56(1)	12.0549(6)	14.57(1)	+20.84	0.5/0.5

$e^+e^- \to 2\,\mathrm{jets}$

example diagrams



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SM Higgs (here with polarization)

$e^+e^- \to HZ$ $\sqrt{s}$ [GeV]	MCSANCee [199] σ^{tot}_{LO} [fb]	MCSANCee [199] σ^{tot}_{NLO} [fb]	WHIZARD+RECOLA σ^{tot}_{LO} [fb]	WHIZARD+RECOLA σ^{tot}_{NLO} [fb]	WHIZARD+RECOLA δ_{EW} [%]	σ^{sig} (LO/NLO)
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Bredt/Kilian/JRR/Stienemeier, 2208.09438

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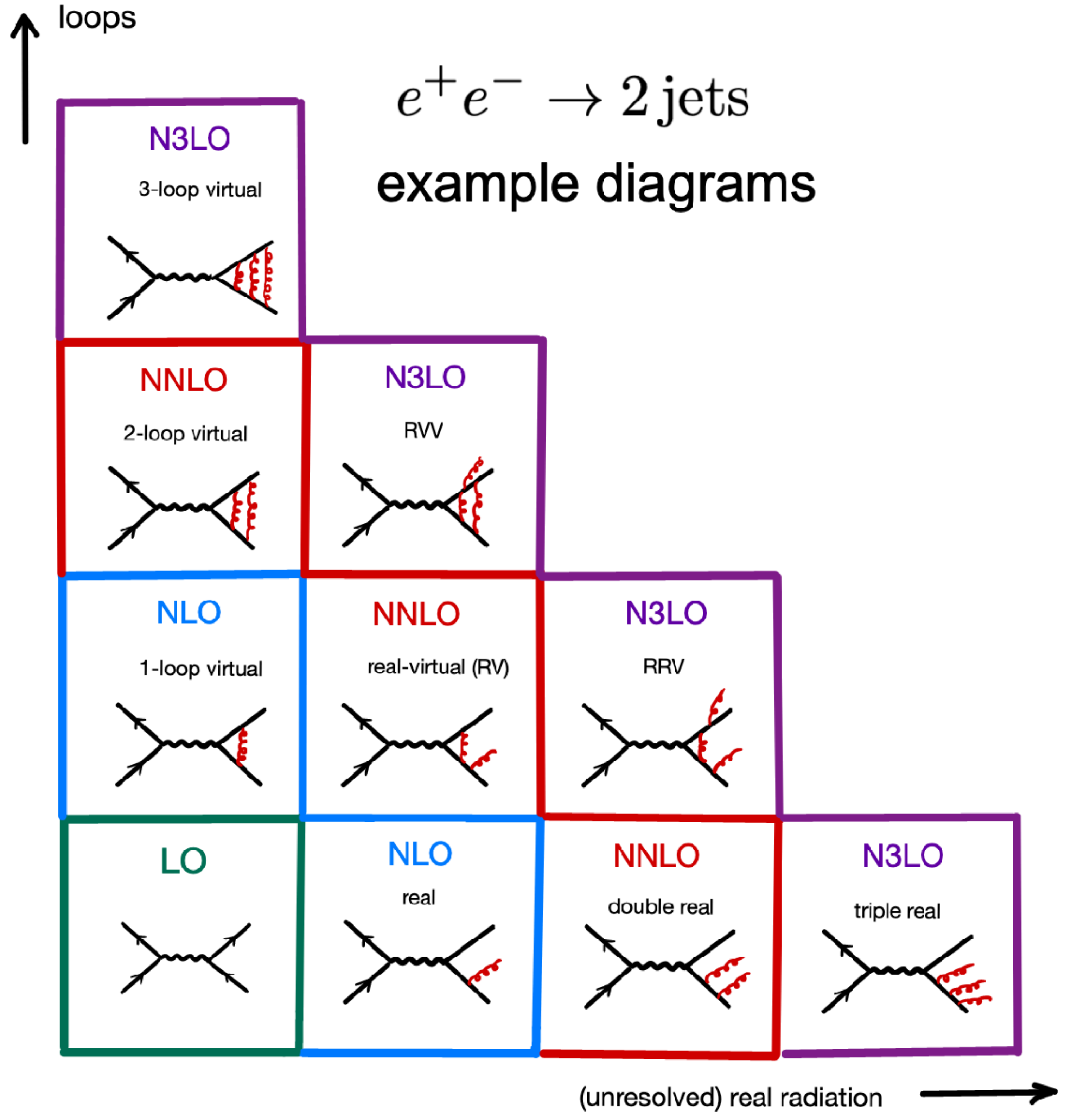
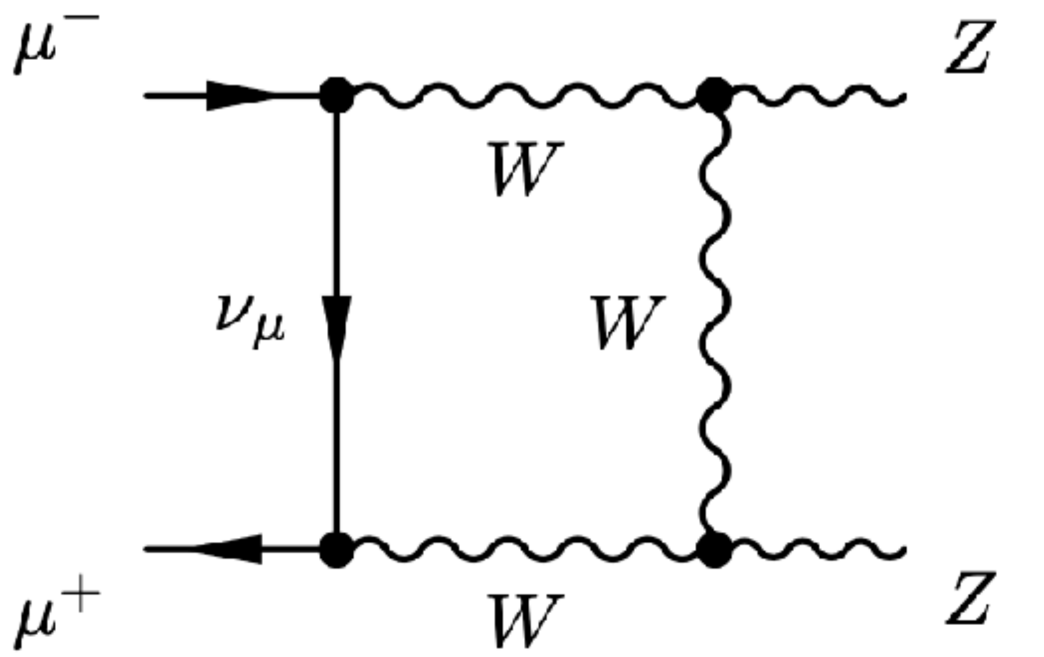
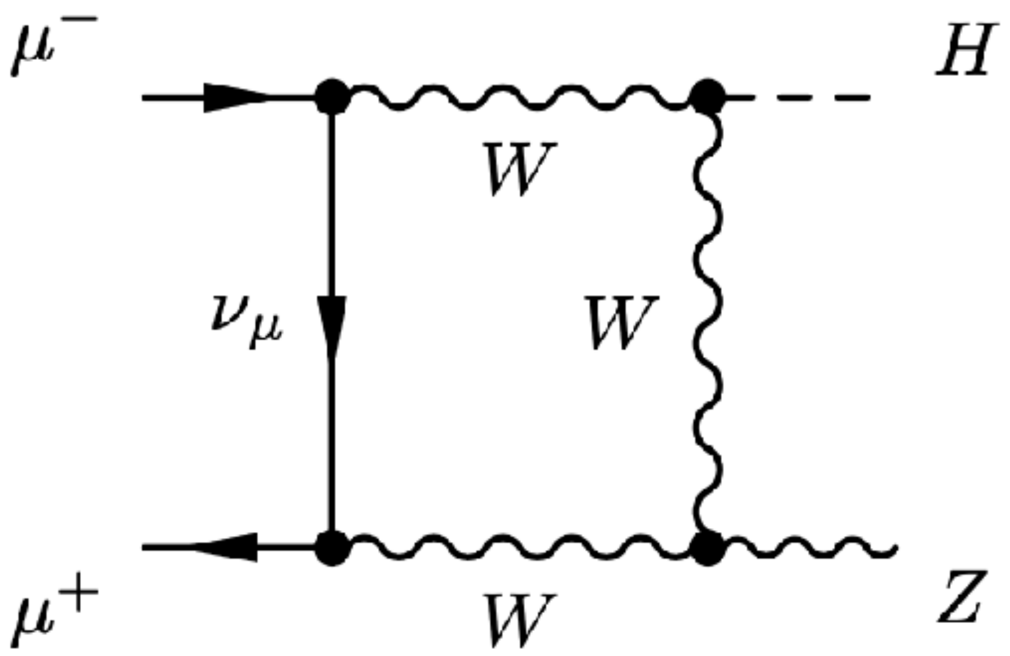


$\mu^+\mu^- \to X, \sqrt{s} = 3$ TeV	σ^{incl}_{LO} [fb]	σ^{incl}_{NLO} [fb]	δ_{EW} [%]
W^+W^-	$4.6591(2) \cdot 10^2$	$4.847(7) \cdot 10^2$	+4.0(2)
ZZ	$2.5988(1) \cdot 10^1$	$2.656(2) \cdot 10^1$	+2.19(6)
HZ	$1.3719(1) \cdot 10^0$	$1.3512(5) \cdot 10^0$	−1.51(4)
HH	$1.60216(7) \cdot 10^{-7}$	$5.66(1) \cdot 10^{-7}$ *	

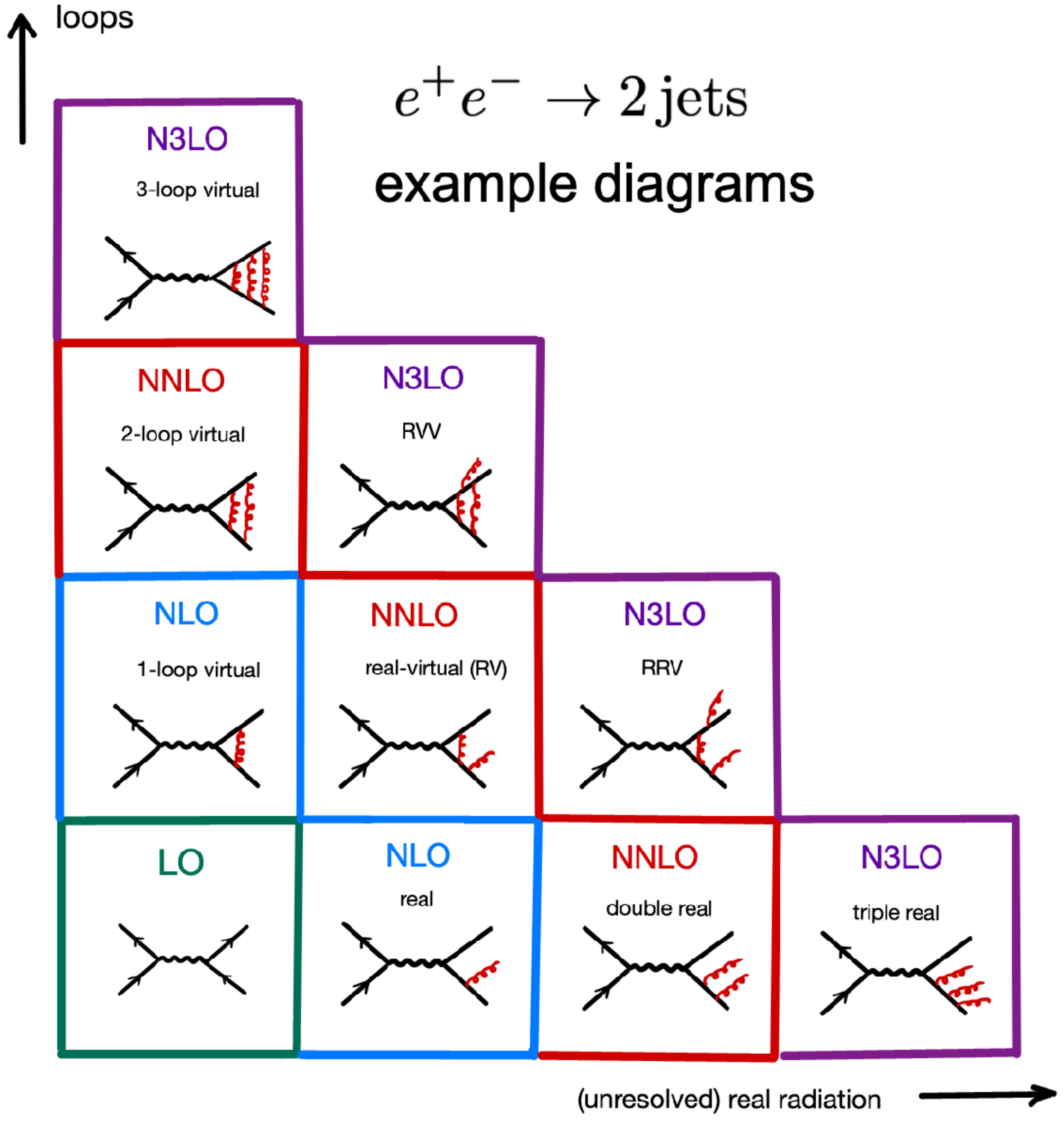




Exclusive (BSM) Higgs processes

- NLO QCD ⊕ EW automated: Sherpa, MG5, Whizard
- Fixed-order N(N)LO, resummation and matching in MCs
- Need $e^+e^- \to 2f,\ 3f,\ 4f,\ 5f,\ 6f$ @ NLO QCD ⊕ EW

  (arbitrary cuts, fully differential)



SM Higgs (here with polarization)

$e^+e^- \to HZ$	MCSANCee [199]		WHIZARD+RECOLA			σ^{sig}
$\sqrt{s}$ [GeV]	$\sigma^{\text{tot}}_{\text{LO}}$ [fb]	$\sigma^{\text{tot}}_{\text{NLO}}$ [fb]	$\sigma^{\text{tot}}_{\text{LO}}$ [fb]	$\sigma^{\text{tot}}_{\text{NLO}}$ [fb]	δ_{EW} [%]	(LO/NLO)
250	225.59(1)	206.77(1)	225.60(1)	207.0(1)	−8.25	0.4/2.1
500	53.74(1)	62.42(1)	53.74(3)	62.41(2)	+16.14	0.2/0.3
1000	12.05(1)	14.56(1)	12.0549(6)	14.57(1)	+20.84	0.5/0.5

Bredt/Kilian/JRR/Stienemeier, 2208.09438

$\mu^+\mu^- \to X, \sqrt{s} = 3$ TeV	$\sigma^{\text{incl}}_{\text{LO}}$ [fb]	$\sigma^{\text{incl}}_{\text{NLO}}$ [fb]	δ_{EW} [%]
W^+W^-	$4.6591(2) \cdot 10^2$	$4.847(7) \cdot 10^2$	+4.0(2)
ZZ	$2.5988(1) \cdot 10^1$	$2.656(2) \cdot 10^1$	+2.19(6)
HZ	$1.3719(1) \cdot 10^0$	$1.3512(5) \cdot 10^0$	−1.51(4)
HH	$1.60216(7) \cdot 10^{-7}$	$5.66(1) \cdot 10^{-7}$ *	

Validation against: Denner/Dittmaier/Roth/Weber, hep-ph/0301189

process	cross section @ LO [fb]	NLO EW [fb]
$e^+e^- \to H + \nu_e\bar{\nu}_e$	84.9752	75.61601 ± 0.26
$e^+e^- \to H + \nu_l\bar{\nu}_l$ with $l = \mu, \tau$	3.9246278	4.396657 ± 0.0058
$e^+e^- \to H + \nu\bar{\nu}$ (with OpenLoops)	92.824	84.409 ± 0.26
$e^+e^- \to H + \nu\bar{\nu}$ (with Recola)	92.72 ± 0.11	$84.82^* \pm 0.13$
$e^+e^- \to H + \nu\bar{\nu}$ in Ref. [3]	92.64 ± 0.002	85.01 ± 0.08

Note: G_μ scheme, $m_H = 115$ GeV, $\sqrt{s} = 500$ GeV

↪ Talk Alan Price

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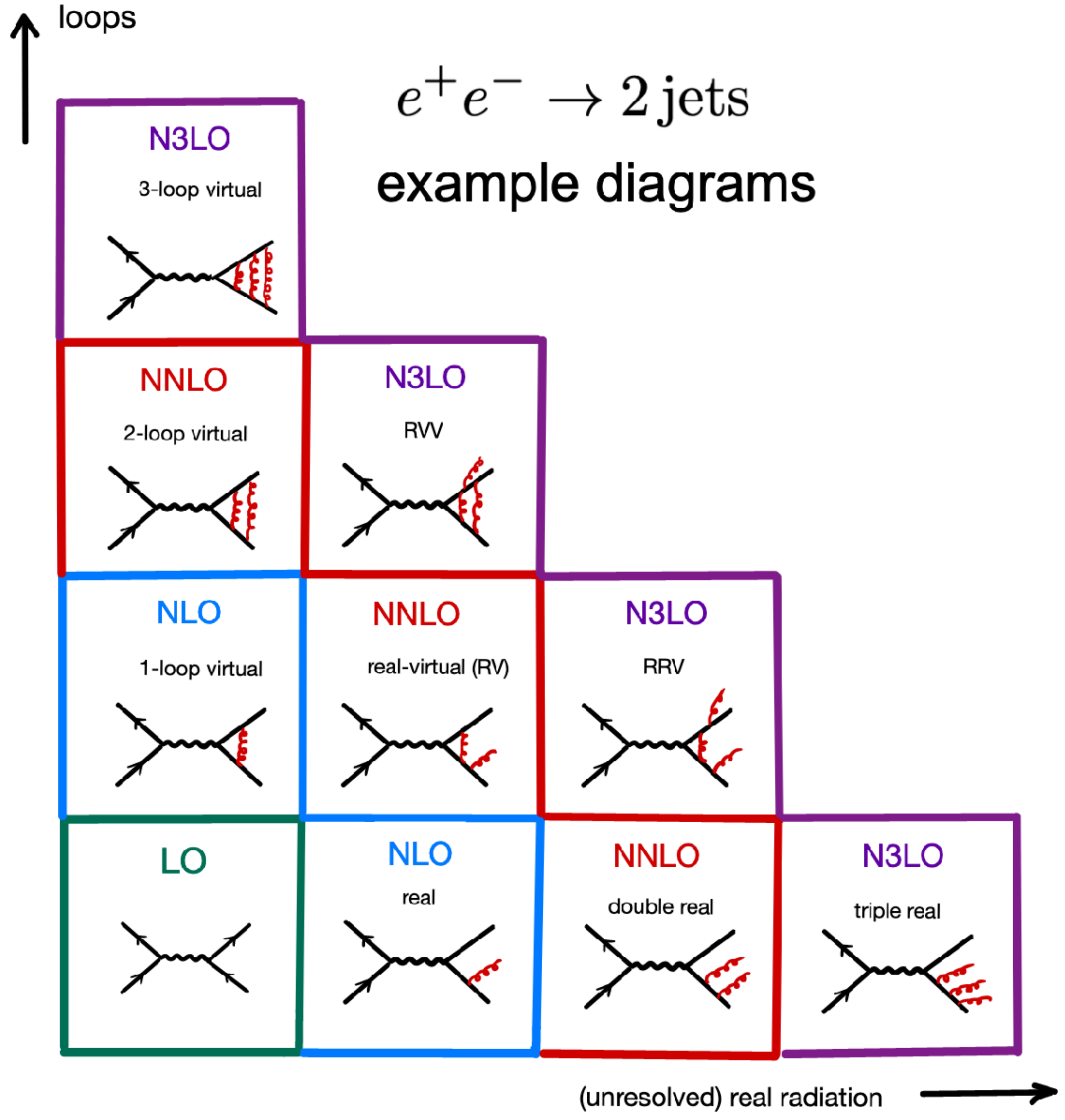

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2HDM, $e^+e^- \to H\tau^+\tau^-$, Bredt/Höfer/Iguro/Ma/JRR/Zhang, *w.i.p.*

parameters	LO [fb]	NLO EW [fb]	correction [%]
SM	4.028(5)	4.152(13)	3.10 ± 0.29
$\tan(\beta) = 1$ for type-II	4.022(5)	4.164(12)	3.52 ± 0.26
$\tan(\beta) = 30$ for type-II	4.022(5)	4.084(11)	1.54 ± 0.25
$\tan(\beta) = 0.5$ for type-II	4.022(5)	4.185(12)	4.05 ± 0.26
$\tan(\beta) = 1$ for type-X	4.022(5)	4.163(12)	3.52 ± 0.26
$\tan(\beta) = 40$ for type-X	4.022(5)	4.081(12)	1.47 ± 0.25

↪ also Talk Victor Maura

Trilinear Higgs coupling calculations

Bahl/Braathen/Gabelmann/Kanemura/Mühlleitner/Radshenko/Weiglein amm.

Braathen ea.; 2305.03015

- ☑ (Only?) trace towards EW symmetry breaking
- ☑ Still very large deviations from SM possible (300-400%)
- ☑ Calculation of triple Higgs form factor (“coupling”) in specific models
- ☑ NLO corrections very important to determine parameter points in model space
- ☑ Automated calculations possible based on UFO models
- ☑ Renormalization done automatically (non-trivial!)
- ☑ Connected also to additional Higgs bosons

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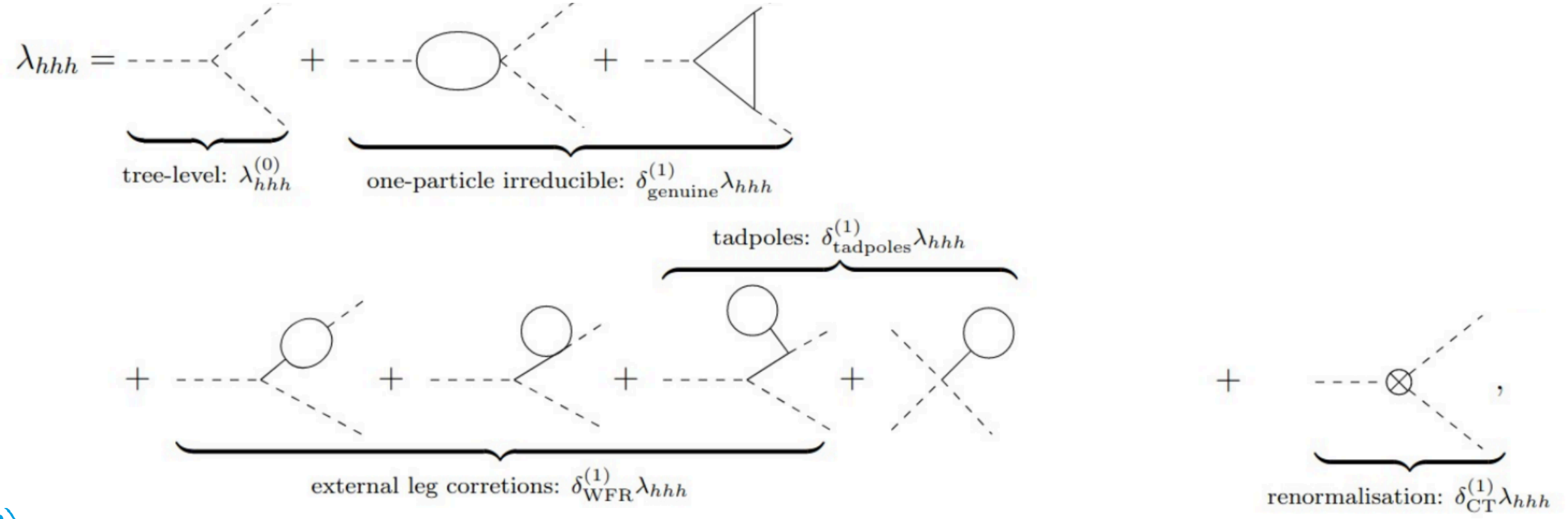

Solid lines:
- *scalars,*
- *fermions,*
- *gauge/vector bosons,*
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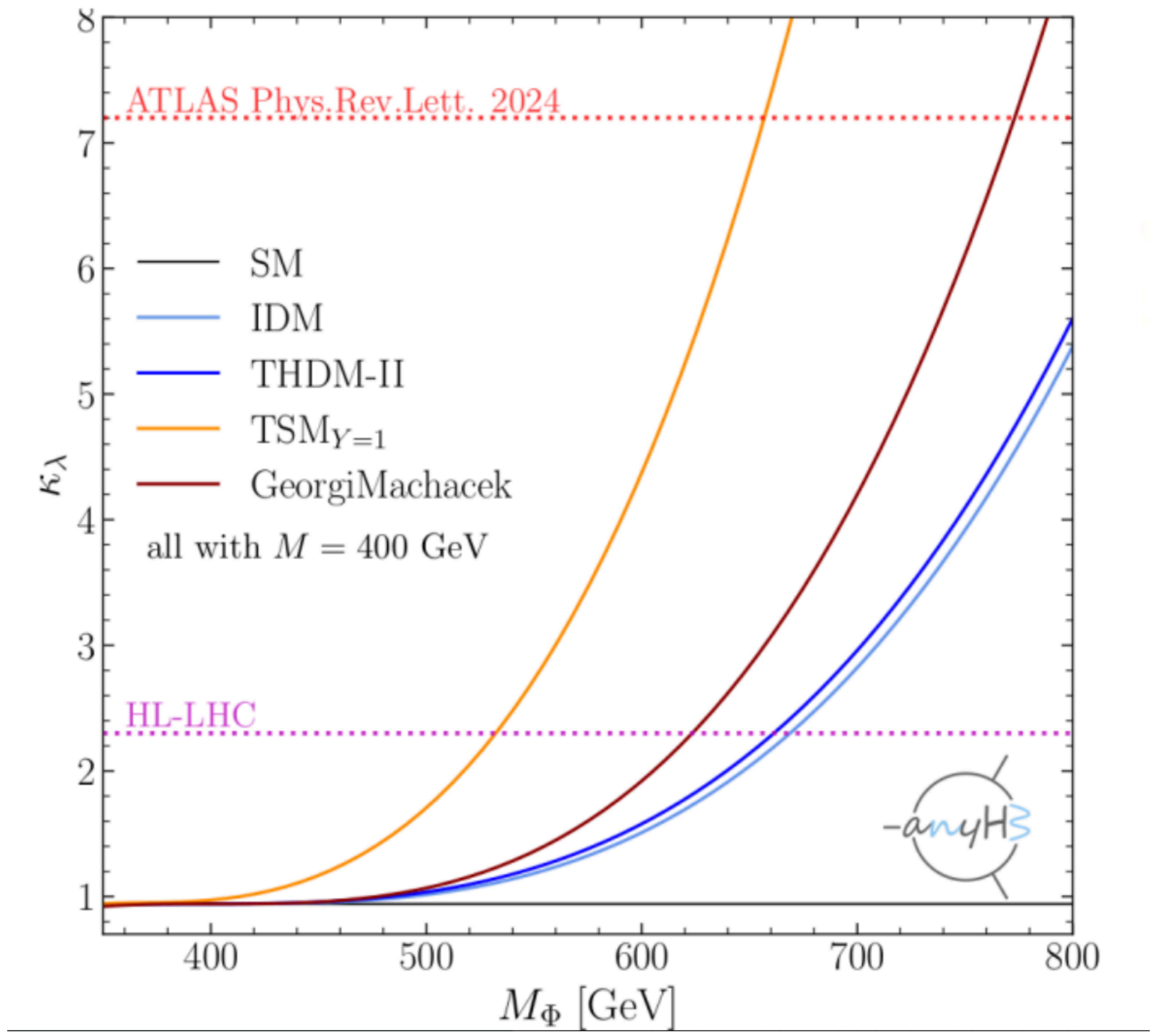

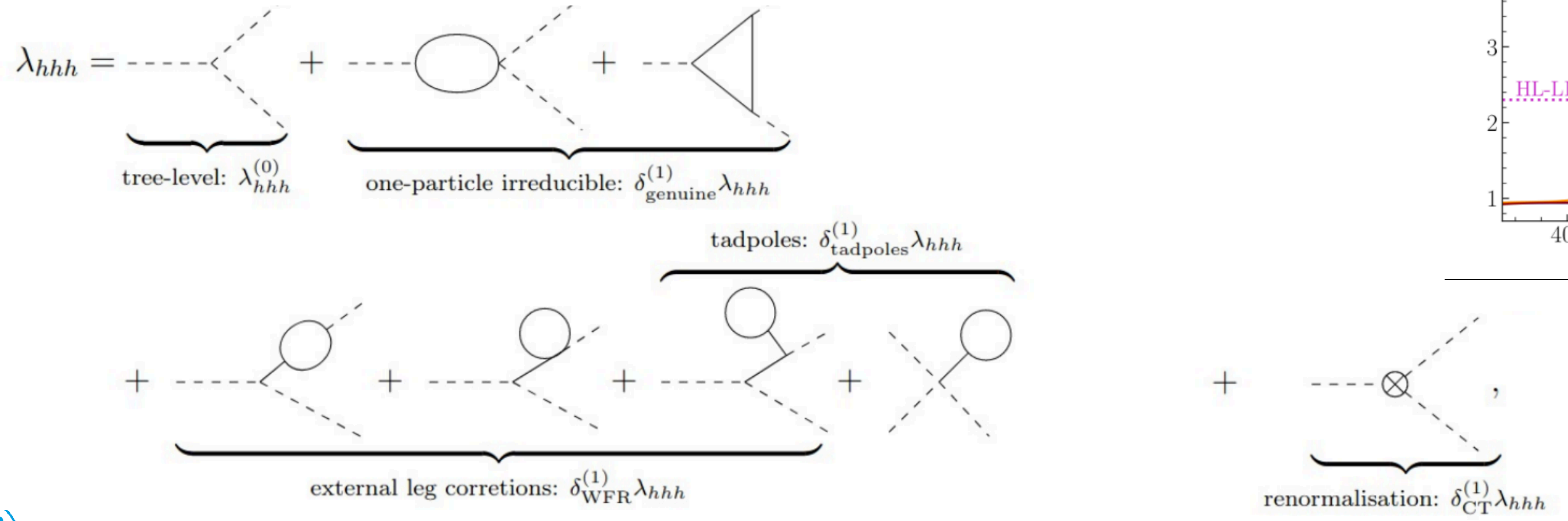

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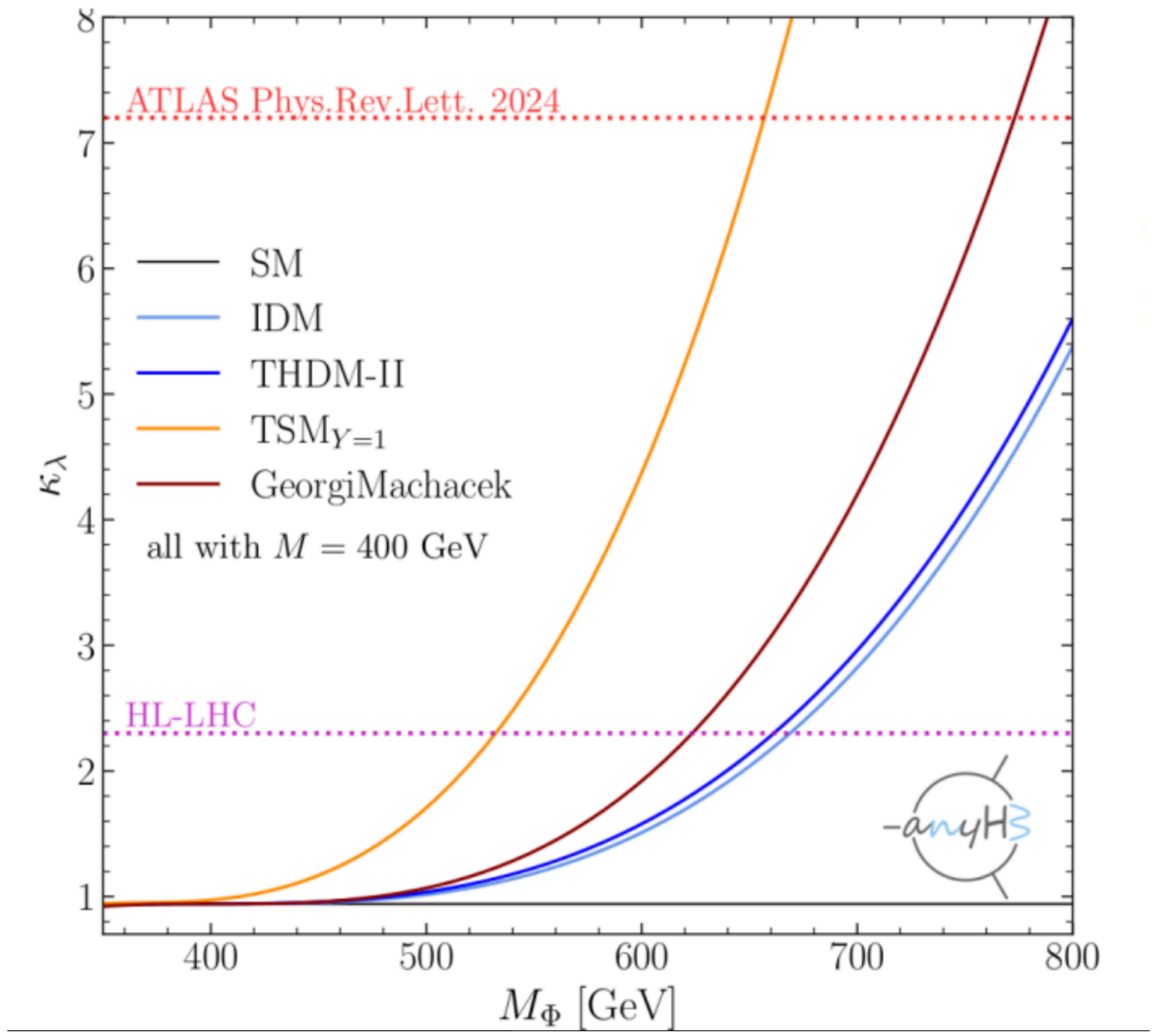
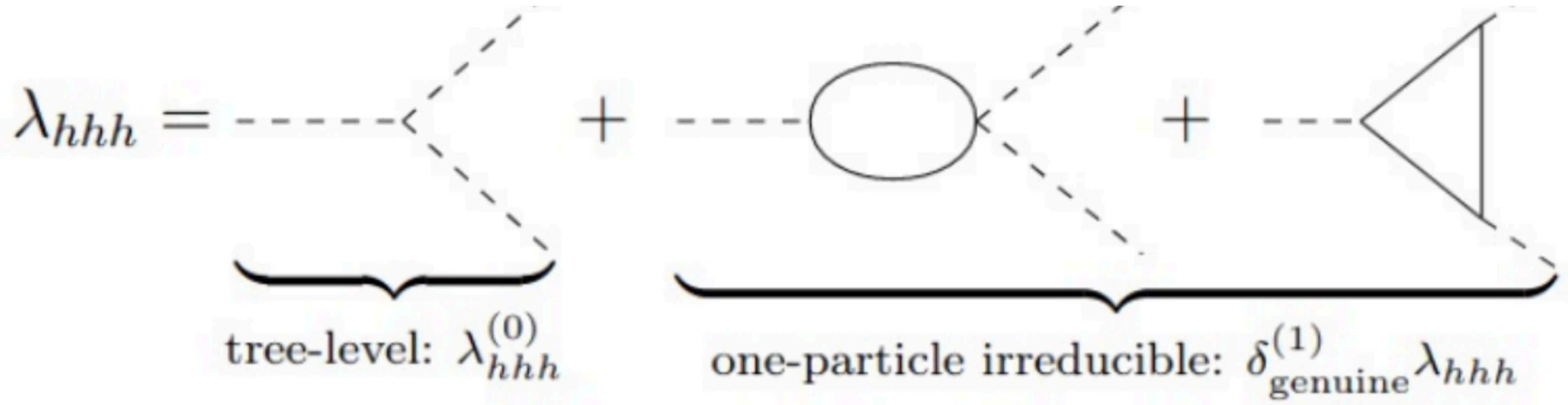
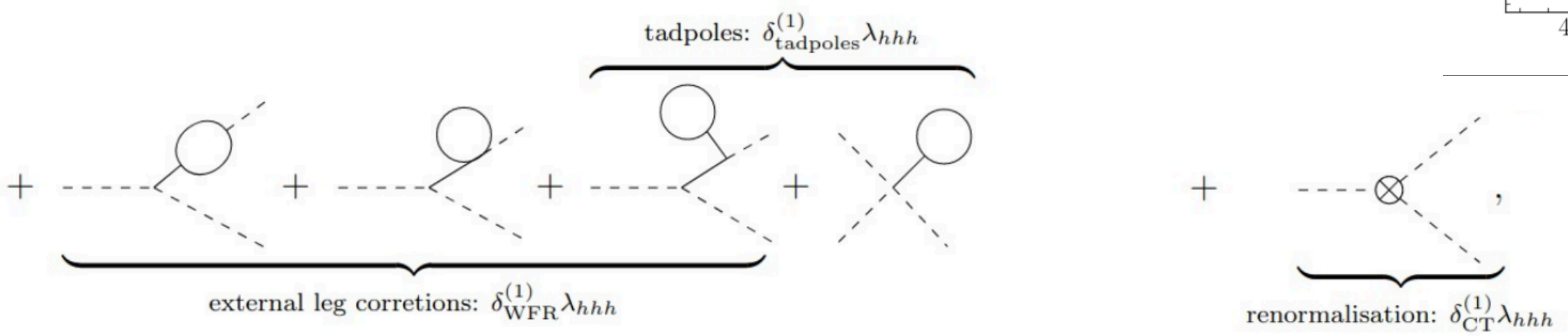


2-loop results start to become available!

Bahl/Braathen/Gabelmann/Paßehr, 2503.15645





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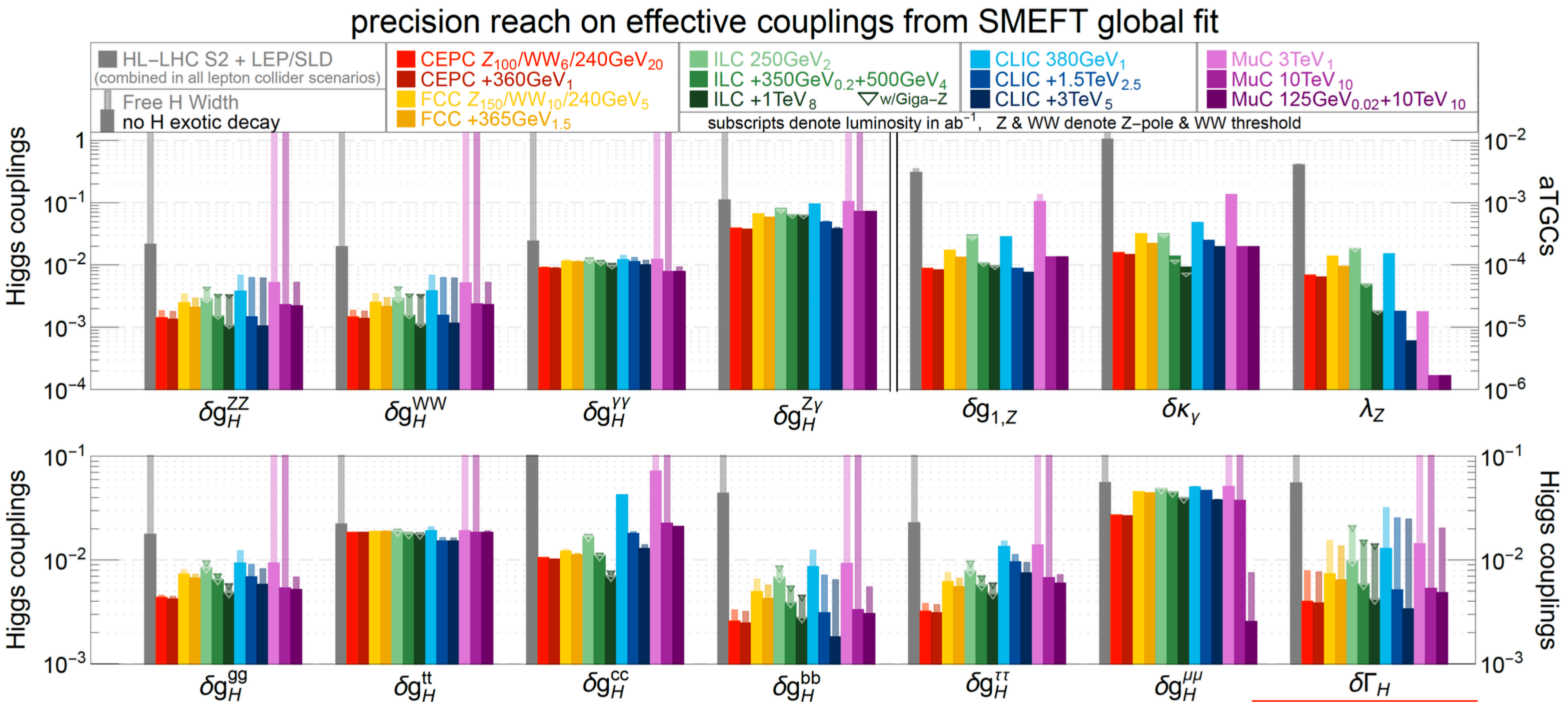
Higgsstrahlung @ NLO SMEFT

Model-agnostic BSM search



Higgsstrahlung @ NLO SMEFT

Model-agnostic BSM search Asteriadis/Dawson/Giardino/Szafron, 2409.11466, 2406.03557; Dawson/Giardino/Forslund. 2411.08952

At tree level, depends on $C_{\phi D},\ C_{\phi\Box},\ C_{\phi e},\ C^{1}_{\phi l},\ C^{3}_{\phi l},\ C_{ll}$

- $C_{\phi D}$, $C_{\phi\Box}$: Linear rescaling of ZZH vertex
- $C_{\phi e}$: eeZ vertex
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- C_{ll}: Relationship between v and G_F

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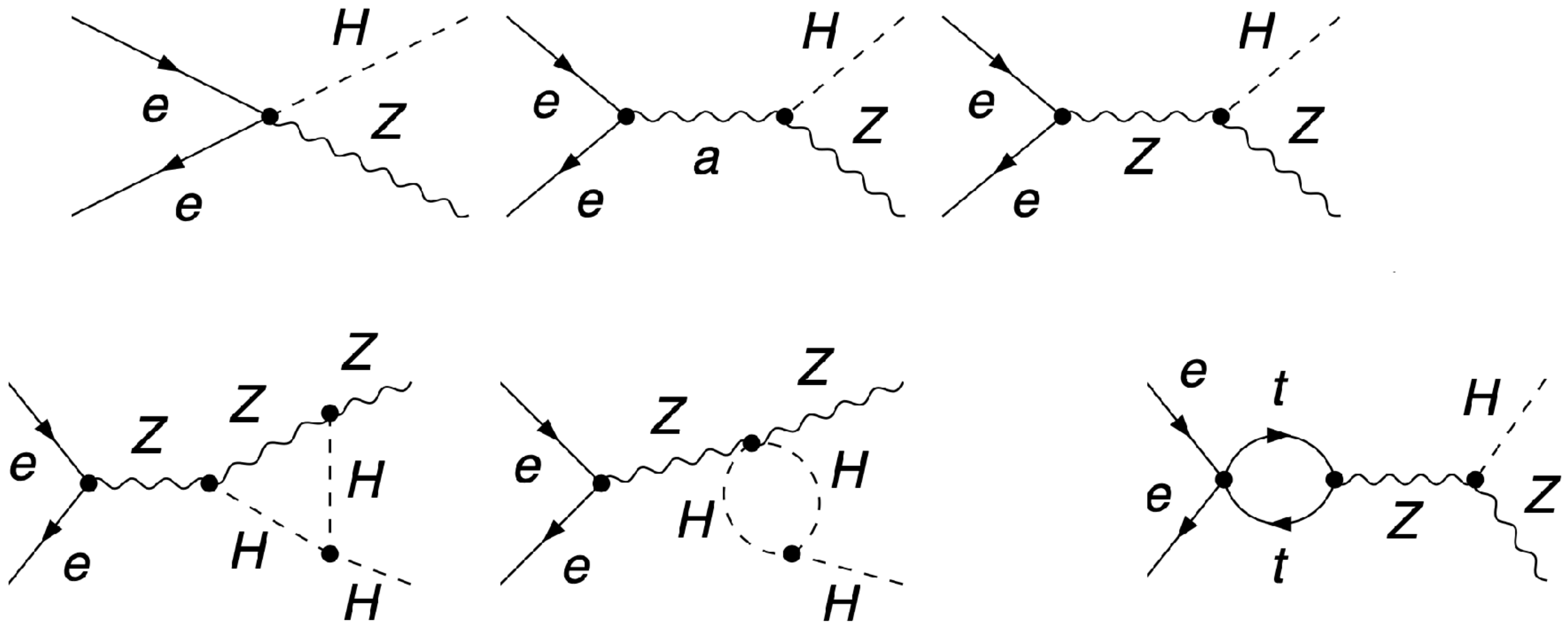

Higgs trilinear

4-fermion operators, $C_{eu}[1133]$

Higgsstrahlung @ NLO SMEFT

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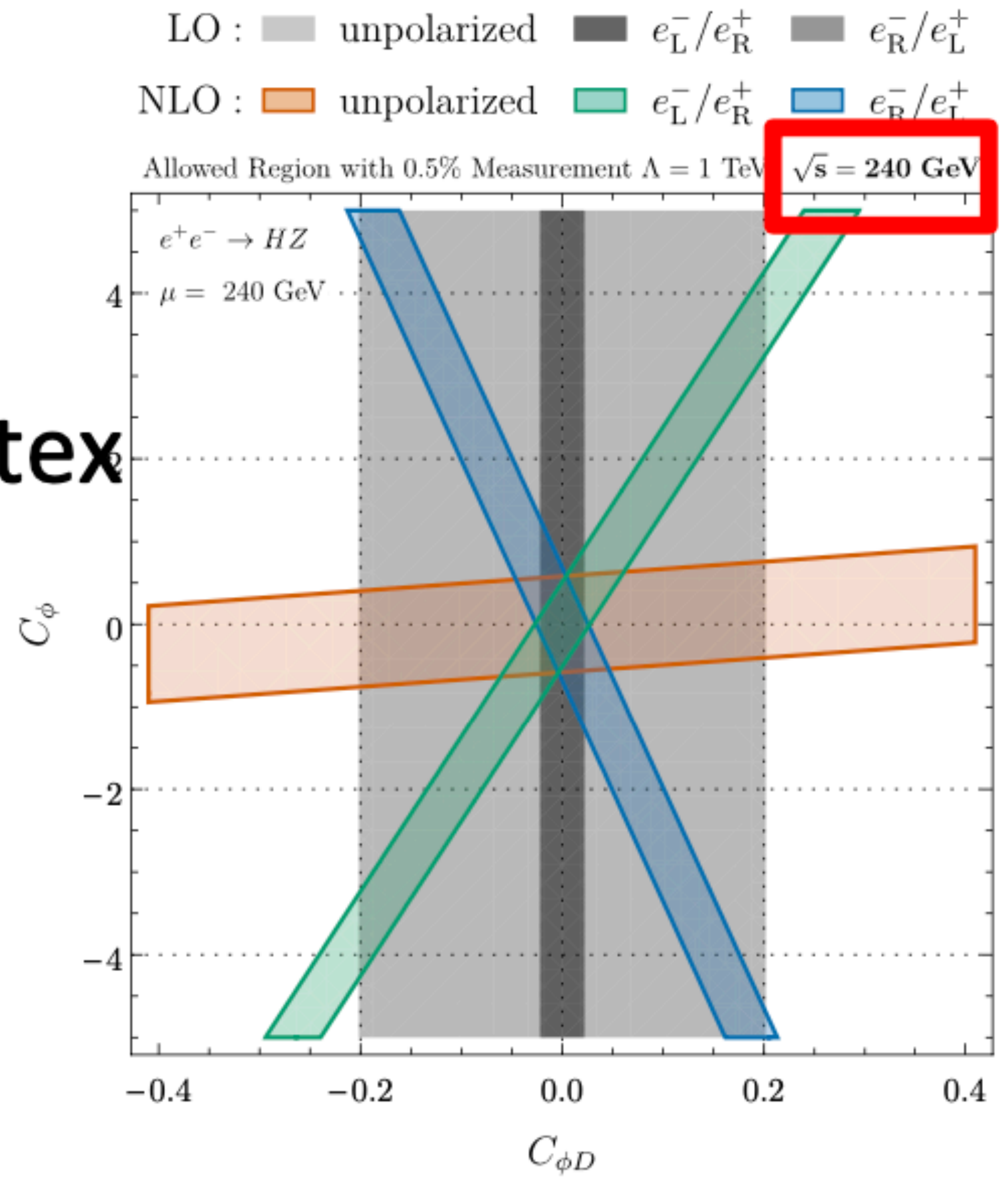
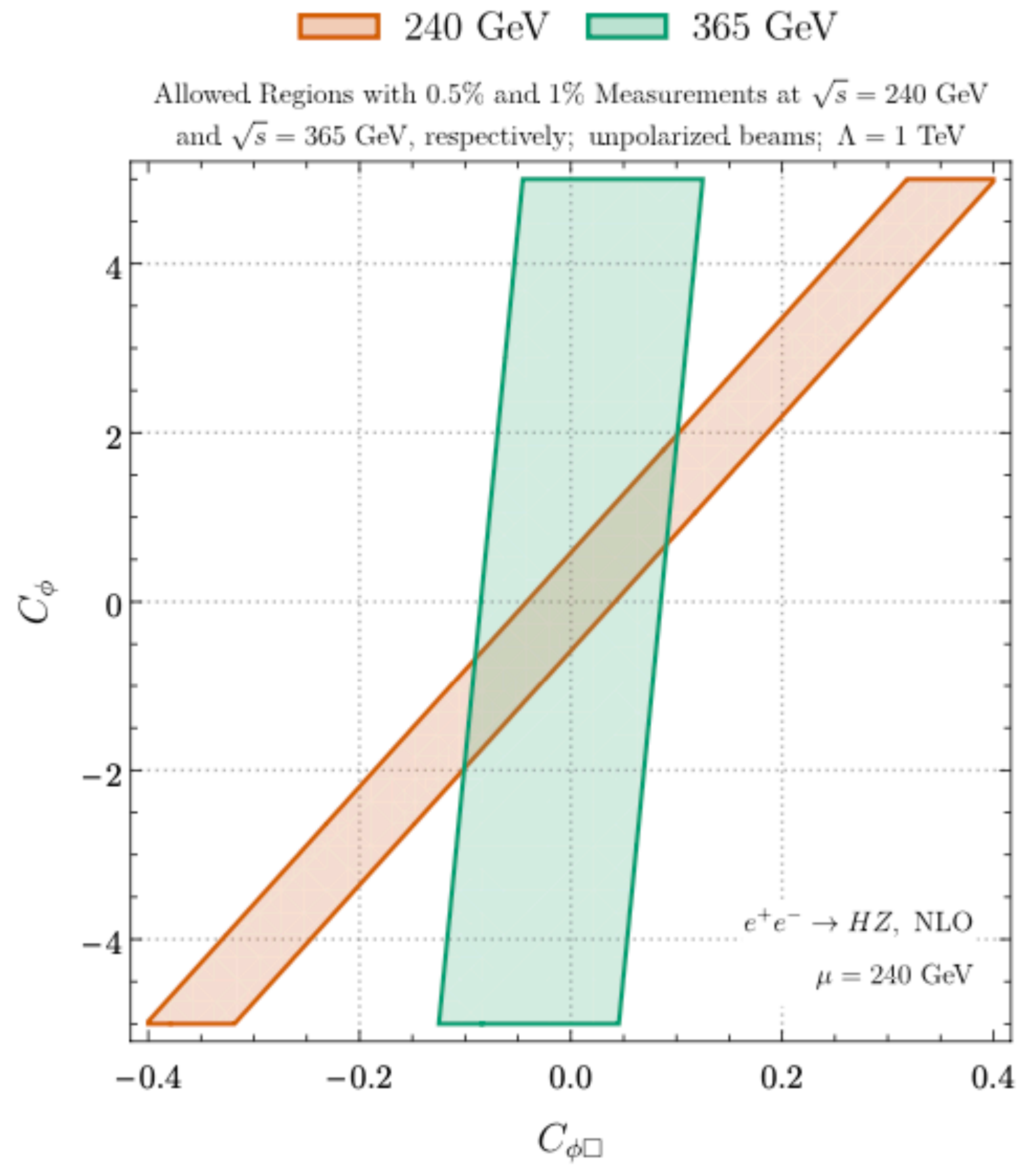
H^3 vertex





Contributes to ZZH vertex

Higgsstrahlung @ NLO SMEFT

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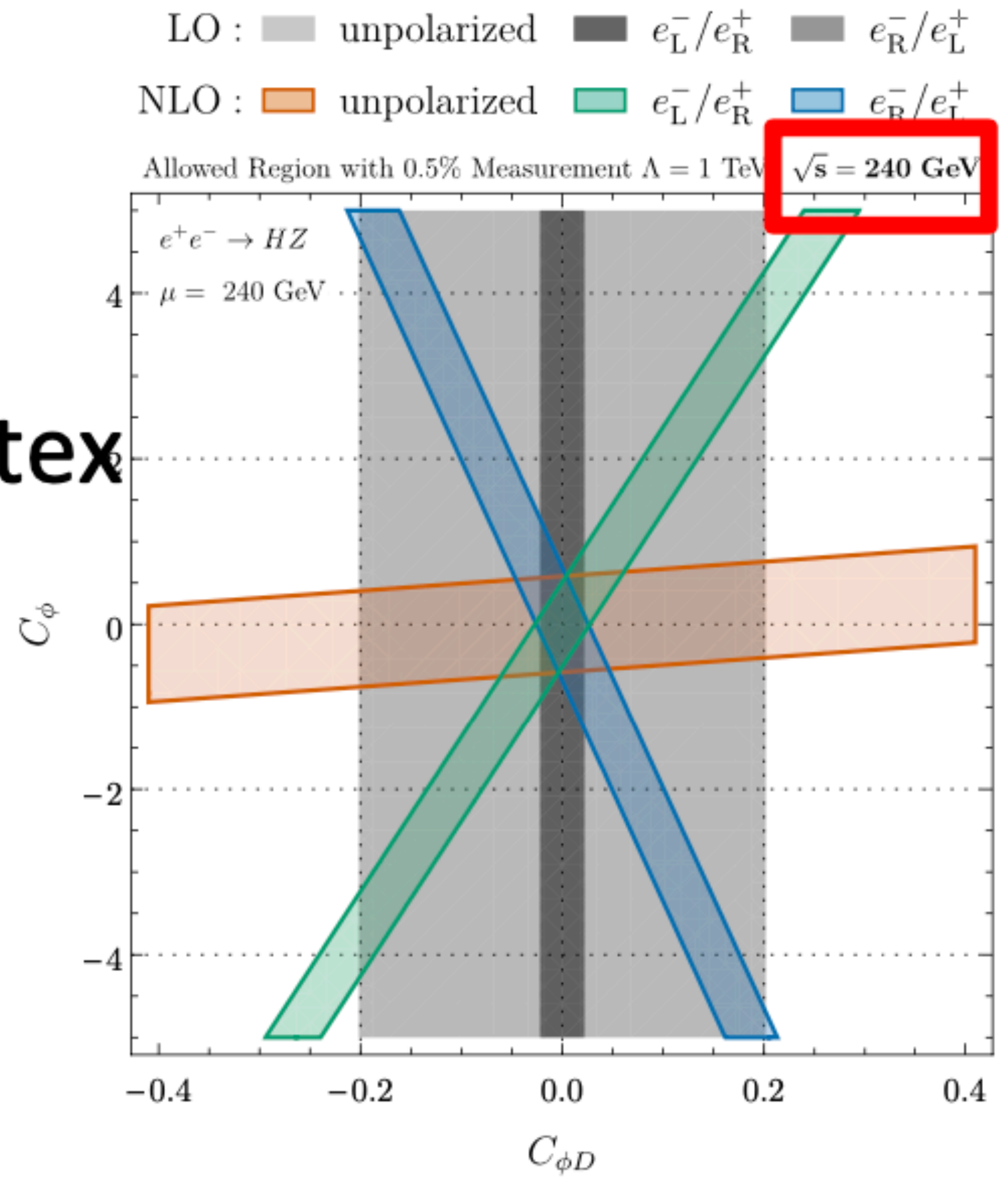
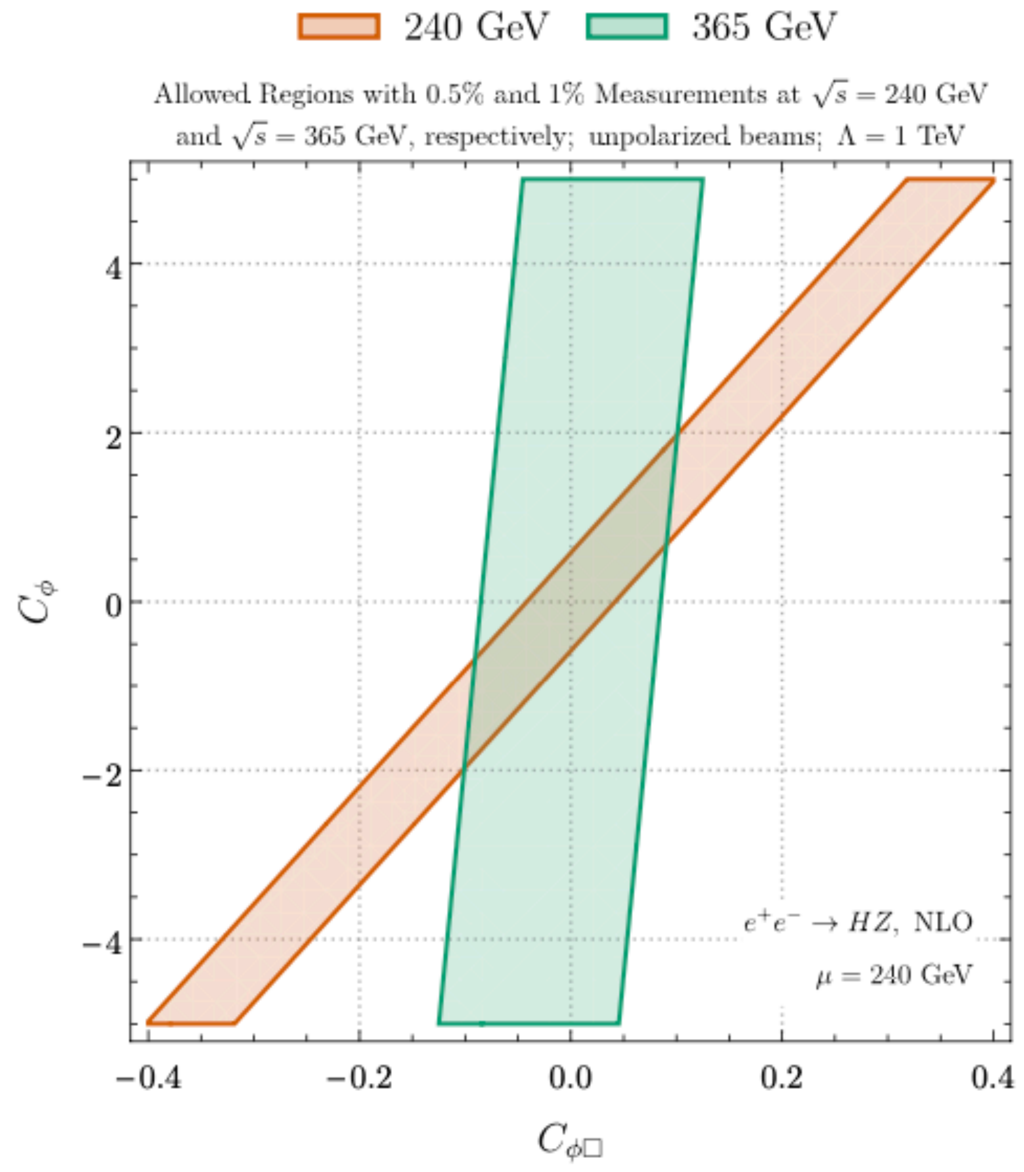
☑ Wilson coeffcients at order $C_i/(\Lambda^2 16\pi^2)$

- Complete NLO EW SMEFT for $e^+e^- \to HZ$
- NLO little impact on single parameter fits
- Large correlations of operators that appear only at NLO

H³ vertex



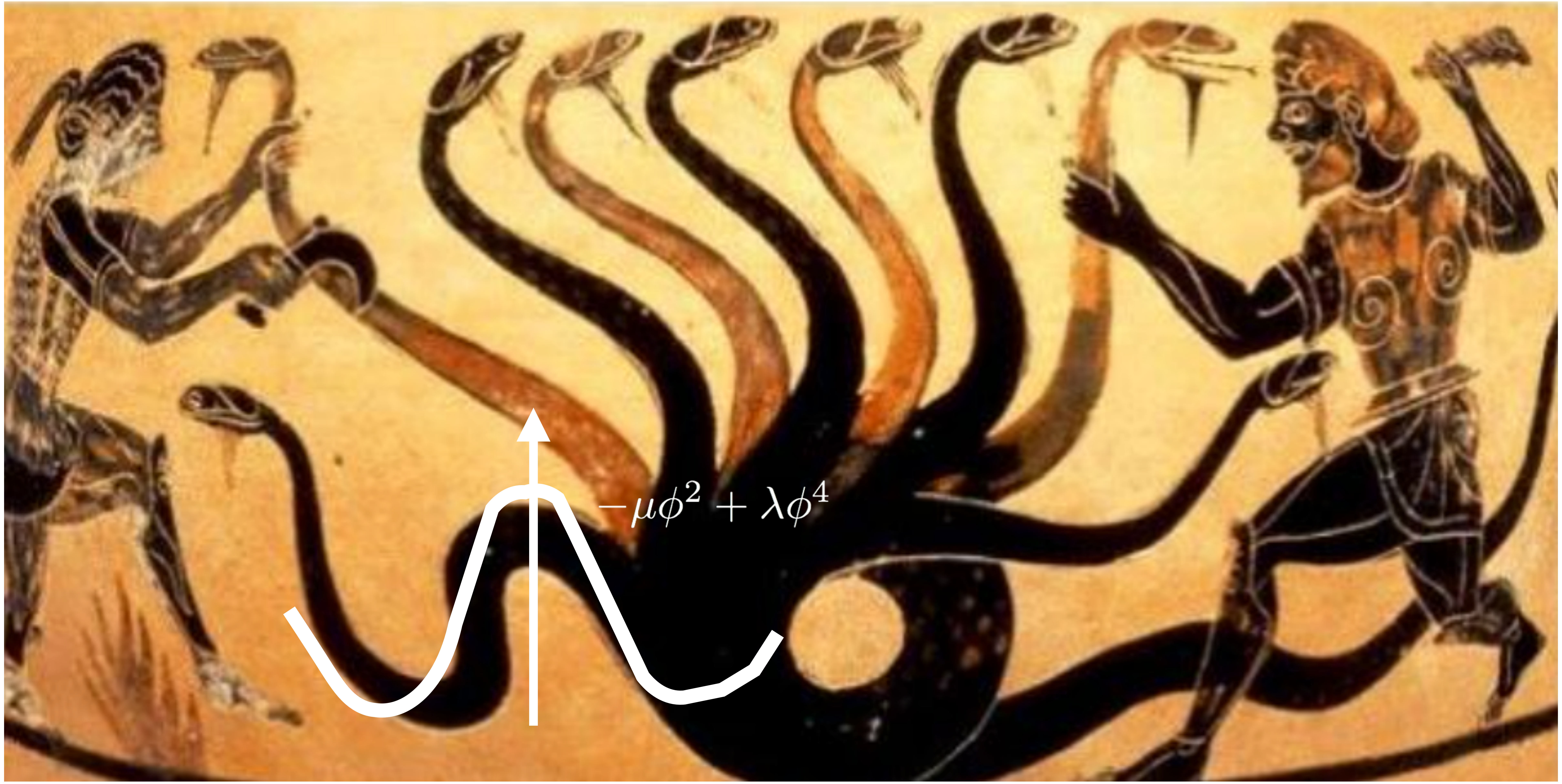


Contributes to ZZH vertex

Summary & Conclusions

- Spectacular experimental Higgs (+EW) precision program in e^+e^- collisions
- Most measurements allow per-cent down to (sub-) per-mil level precision
- **Hard theoretical work needed to match this precision!**
- NNLO (2-loop) EW slowly getting closer; still a long way to go for full processes
- Higgs precision program needs: production processes NNLO, decays @ min. 3-loop
- Massive 2- & 3-loop diagrams: PDE, sector decomposition, Mellin methods etc.
- Calculations in EFTs (SMEFT/HEFT) at least at the NLO EW level needed; cross correlations
- Calculations in specific models (extended Higgs sectors, MSSM, etc.): scheme dependence
- Focus on specific "effects", e.g. trilinear Higgs coupling; full NLO model calculations starting
- "Exclusive frontier": $2 \to 4, 6, (8)$ NLO SM, NLO $e^\pm$ PDFs, QED showers/matching
- Tools, tools, tools: community must value and support codes (loops, MC, fits)

Higgs Precision is reconciling Loops and Legs



Getty Villa, Pacific Palisades, Etruscan, 525 BC

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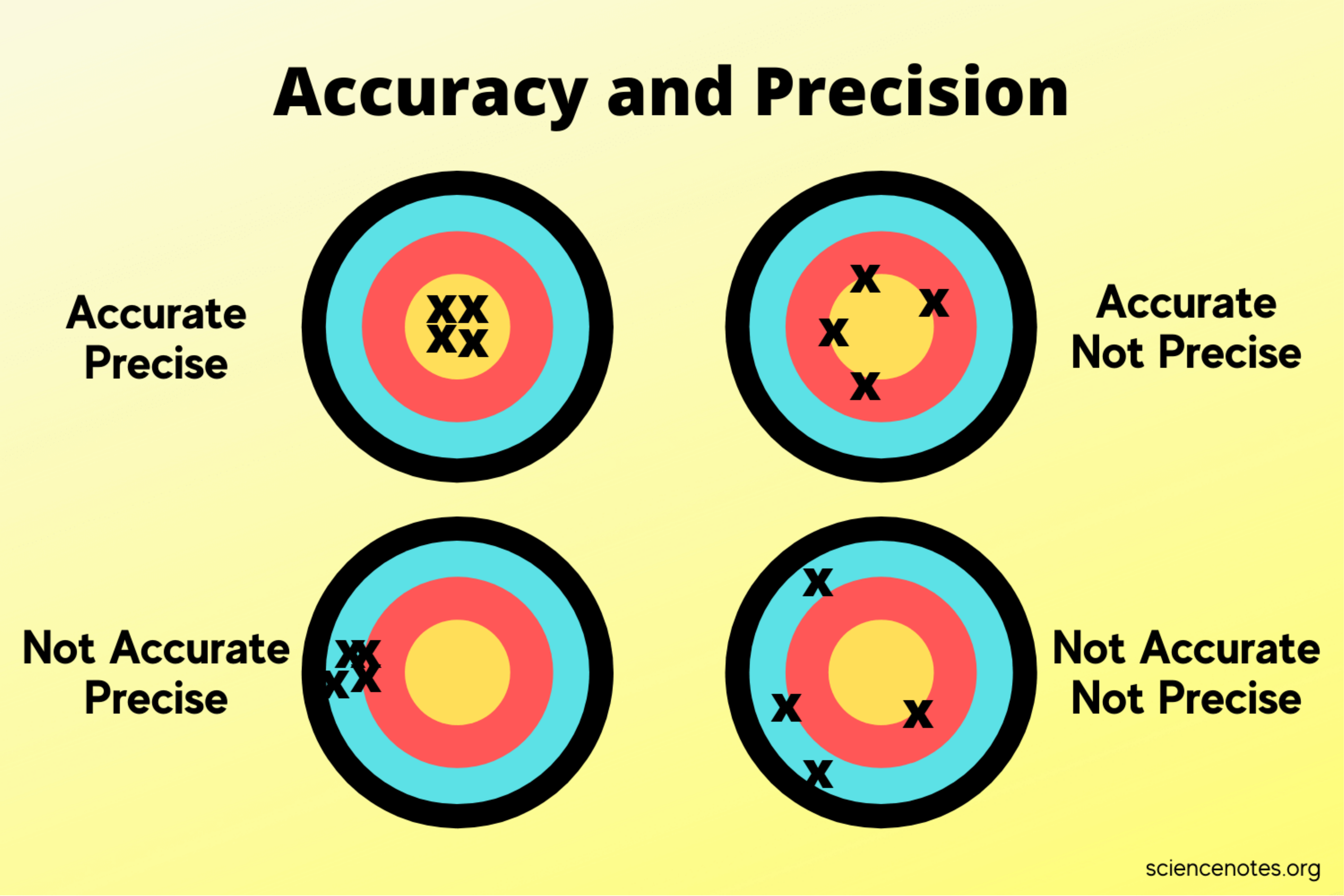


BACKUP

Electroweak Precision Physics

$$\sigma^0_{\text{had}} = \sum_q \sigma_q(M_Z^2),$$

$$\Gamma_Z = \sum_f \Gamma[Z \to f\bar{f}], \qquad \text{(from a fit to } \sigma_f(s) \text{ at various values of } s\text{)}$$

$$R_\ell = \left[\textstyle\sum_q \sigma_q(M_Z^2)\right]/\sigma_\ell(M_Z^2), \qquad (\ell = e, \mu, \tau)$$

$$R_q = \sigma_q(M_Z^2)/\left[\textstyle\sum_q \sigma_q(M_Z^2)\right], \qquad (q = b, c)$$

$$A^f_{\text{FB}} = \frac{\sigma_f(\theta < \frac{\pi}{2}) - \sigma_f(\theta > \frac{\pi}{2})}{\sigma_f(\theta < \frac{\pi}{2}) + \sigma_f(\theta > \frac{\pi}{2})} \equiv \frac{3}{4}\mathcal{A}_e\mathcal{A}_f,$$

$$A^f_{\text{LR}} = \frac{\sigma_f(P_e < 0) - \sigma_f(P_e > 0)}{\sigma_f(P_e < 0) + \sigma_f(P_e > 0)} \equiv \mathcal{A}_e|P_e|.$$

$$\mathcal{A}_f = \frac{1 - 4|Q_f|\sin^2\theta^f_{\text{eff}}}{1 - 4|Q_f|\sin^2\theta^f_{\text{eff}} + 8(Q_f\sin^2\theta^f_{\text{eff}})^2}.$$

$$\text{total} = \sqrt{\text{experimental}^2 + \text{parametric}^2} + \text{intrinsic}$$

$\mathcal{O}(\alpha\alpha_s^2)$ complete
$\mathcal{O}(\alpha^2\alpha_s)$ fermionic
$\mathcal{O}(\alpha^3)$ double-fermionic
$\mathcal{O}(\alpha_t\alpha_s^3)$ 4-loop

Quantity	FCC-ee	Current intrinsic error		Projected intrinsic error
M_W [MeV]	0.5–1 ‡	4	$(\alpha^3, \alpha^2\alpha_s)$	1
$\sin^2\theta^\ell_{\text{eff}}$ [10^{-5}]	0.6	4.5	$(\alpha^3, \alpha^2\alpha_s)$	1.5
Γ_Z [MeV]	0.1	0.4	$(\alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2)$	0.15
R_b [10^{-5}]	6	11	$(\alpha^3, \alpha^2\alpha_s)$	5
R_l [10^{-3}]	1	6	$(\alpha^3, \alpha^2\alpha_s)$	1.5

Theoretical uncertainties for *WW* threshold don't match exp. precision: 3 GeV uncertainty

Beneke/Falgari/Schwinn/Signer/Zanderighi, 0707.0773; Actis/Beneke/Falgari/Schwinn, 0807.0102; C. Schwinn, in 1905.05078

needed: full 2-loop corr. $e^+e^- \to W^+W^-$ and $W \to ff$, ISR & matching (later); 3-loop Coulomb-enhanced

Recent efforts in $e^+e^- \to f\bar{f}$ (2-loop, logarithmic corr.)

Blümlein/de Freitas/Raab/Schönwald, 1901.08018, 1910.05759, 2003.14283, 2004.04287

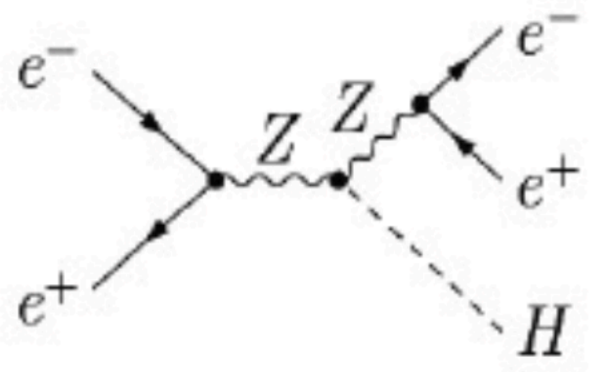
Parametric Uncertainties

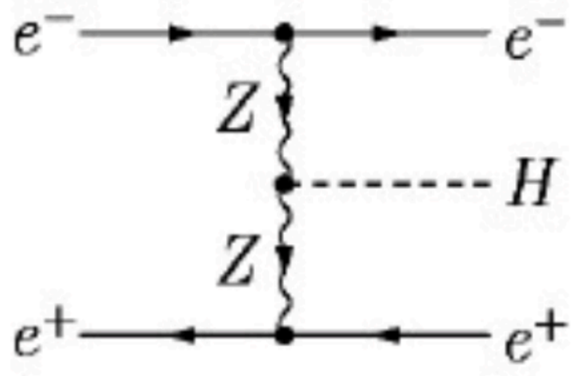
Parametric uncertainties

- M_H: Higgsstrahlung at threshold, 10 MeV uncertainty, leptonic recoil, minor theory uncertainties
- M_Z: Z lineshape, ~0.1 MeV exp., QED ISR+ISR/FSR, EW box diagrams, Jadach/Skrzypek/Pietrzik, 1999
- $\alpha_s\,(M_Z)$: global fit of overconstrained EW pseudo-observables at Z pole, non-perturbative uncertainties
- $m_t^{MS}\,(m_t)$: N^{3}LO QCD/NNLO EW, resummed NNLL, 4-loop mass translation., off-shell corr.
  Beneke ea., 1506.06864/1711.10429, Hoang ea. 1309.6323, Marquard ea. 1502.01030, Chokoufé et al. 1609.03390, Bach ea. 1712.02220
- $m_{c/s}^{MS}\,(m_{c/s})$: lattice QCD, sum rules, NNLO jet ratios. 1404.0319, 1401.7035, 0907.2110, 1411.3132, 1504.07638
- $\Delta\alpha$: extracted from $e^+e^- \to$ *hadrons* , τ *decays* (BESIII, VEPP-2000, Belle II), radiative return
  Proposal for direct measurement below/above Z pole: subtract EW from QED corrections
  available @ 1-loop; needed fermionic 2-loop corr., $\mathcal{O}(\alpha^2, \alpha^2\alpha_s)$ corr. $\Longrightarrow\ 10^{-4}$
  2-/3-loop box diagrams: full $\mathcal{O}(\alpha\,\alpha_s^2)$, double-fermionic $\mathcal{O}(\alpha^3)$ corr. $\Longrightarrow 10^{-5}$

hZ production:

- $\mathcal{O}(\alpha)$ corr. to hZ production and Z decay — Kniehl '92; Denner, Küblbeck, Mertig, Böhm '92; Consoli, Lo Presti, Maiani '83; Jegerlehner '86; Akhundov, Bardin, Riemann '86
- Technology for $\mathcal{O}(\alpha)$ with off-shell Z-boson available — Boudjema et al. '04; Denner, Dittmaier, Roth, Weber '03





- Can be combined with h.o. ISR QED radiation — Greco et al. '17
- $\mathcal{O}(\alpha\alpha_s)$ corrections — Gong et al. '16; Chen, Feng, Jia, Sang '18
- $\mathcal{O}(N_f\alpha^2)$ corrections — Freitas, Song '22 [also see Chen, Guan, He, Li, Liu, Ma '22]

Theory error: $\Delta_{\text{th}} \lesssim \mathcal{O}(0.3\%)$ (mostly from non-fermionic NNLO)