Various resonance schemes of unstable particles and their verification at lepton colliders

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Outline

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- Summary

Propagator of particles

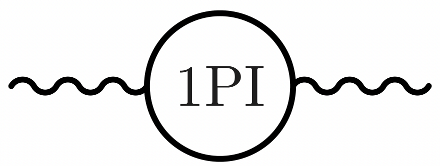
Energy behavior of the two-point function at tree level:

$$iG^0(p) = \frac{i}{p^2 - m^2} =$$

Resummed propagator of particles

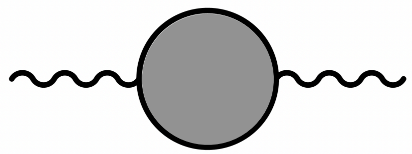
Self energy function: sum of all one-particle irreducible (1PI) contributions

$$i\Pi(p^2) =$$



Near its pole mass, the propagator is resummed to :

$$iG^R(p) = \frac{i}{p^2 - m^2 + \Pi(p^2)} =$$



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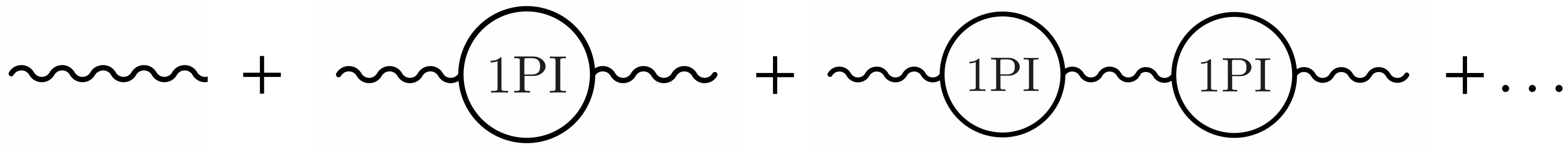


Unstable particles have $\mathrm{Im}\,\Pi \propto \Gamma \neq 0$.

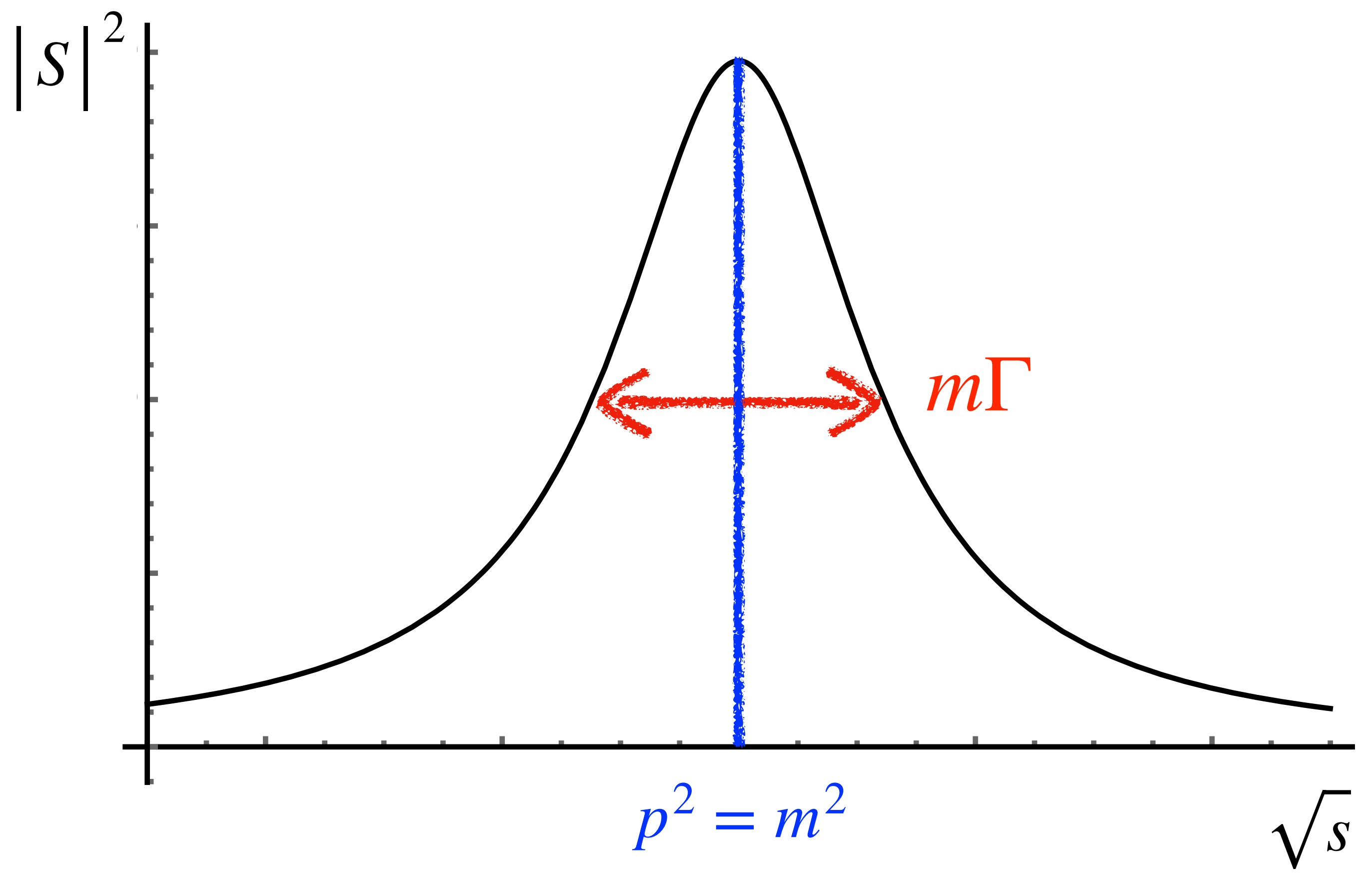
Can we expect any different results in different resonance schemes?
e.g. $\Gamma = \Gamma(p^2)$ vs constant Γ ?

Breit-Wigner scheme

Breit-Wigner resonance (BW)

constant decay rate from the self-energy function

$$\mathrm{Im}\,\Pi = m\,\Gamma$$



$$S \propto iG(p) = \frac{i}{p^2 - m^2 + im\Gamma} \quad \rightarrow \quad |S|^2 \propto \frac{1}{(p^2 - m^2)^2 + m^2\Gamma^2}$$

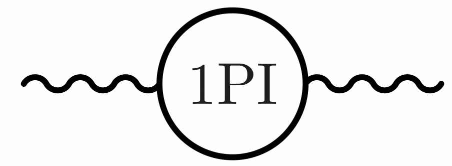
The pole masses of unstable particles are equivalent to m.

Theoretical scheme

Mass defined by the complex mass renormalization in QFT

For given self-energy function,

$$i\Pi(p^2) =$$



Expansion of the self-energy around μ^2 defined by

$$\mu^2 - m^2 + \Pi(\mu^2) \equiv 0 \quad \text{(complex mass renormalization)}$$

$$\Pi(p^2) = \Pi(\mu^2) + \Pi'(\mu^2) \cdot (p^2 - \mu^2) + \ldots$$

Resummed propagator:

$$iG^R(p) = \frac{i}{p^2 - m^2 + \Pi(p^2)} \quad \to \quad \frac{1}{1 + \Pi'(\mu^2)} \cdot \frac{i}{p^2 - \mu^2}$$

absorbed by couplings

Amplitude has a pole at $p^2 = \mu^2$,

and μ is generally a complex value, the solution of $\mu^2 - m^2 + \Pi(\mu^2) = 0$.

Expression of the complex mass

Fourier transformation of the amplitude in the rest frame of unstable particles

$$\tilde{S} \equiv \int \frac{dE}{2\pi} S \cdot e^{-iEt} \Big|_{\vec{p}=0} \propto \int \frac{dE}{2\pi} \frac{i}{p^2-\mu^2} \cdot e^{-iEt} \Big|_{\vec{p}=0} \propto e^{-i\mu t} \equiv e^{-imt} e^{-\Gamma t/2}$$

Decay mode

$$\left|\tilde{S}\right|^2 \propto e^{2\mathrm{Im}[\mu]\, t} = e^{-\Gamma t} \quad \rightarrow \quad \mu = m - \frac{i}{2}\Gamma$$

Energy behavior of the squared amplitude

$$\left|S\right|^2 \propto \left|\frac{i}{p^2-\mu^2}\right|^2 = \left[\left(p^2 - m^2 + \frac{\Gamma^2}{4}\right)^2 + m^2\Gamma^2\right]^{-1}$$

complex pole at $p^2 = \mu^2$ $\rightarrow$ real pole at $p^2 = m^2 - \frac{\Gamma^2}{4}$ in the theoretical scheme

The pole masses of unstable particles are NOT equivalent to m,

but shifted to $p^2 = m^2 - \Gamma^2/4$.

Energy-dependent-width scheme

Energy-dependent decay rate used in collider experiments

The decay rate from self-energy is not constant

$$\mathrm{Im}\,\Pi(p^2) = im\,\Gamma(p^2)$$

Energy behavior of amplitude in the ED scheme

$$S \propto iG^R(p) = \frac{i}{p^2 - m^2 + \Pi(p^2)} \quad \rightarrow \quad \frac{i}{p^2 - m^2 + im\Gamma(p^2)}$$

Assuming that decay products are massless, Γ is proportional to p^2:

$$\mathrm{Im}\,\Pi(p^2) \propto p^2 \quad \rightarrow \quad \Gamma(p^2) = \frac{p^2}{m^2}\Gamma(p^2 = m^2)$$

decay rate at $p^2 = m^2$

$$iG^R(p) = \frac{i}{p^2 - m^2 + im\Gamma(p^2)} = \frac{i}{p^2 - m^2 + ip^2\Gamma(m^2)/m}$$

Energy-dependent-width scheme

Corresponding squared amplitude

$$\left|S\right|^2 \propto \left[\left(p^2 - m^2\right)^2 + m^2\Gamma^2(p^2)\right]^{-1} = \left[\left(p^2 - m^2\right)^2 + \left(p^2\Gamma(m^2)/m\right)^2\right]^{-1}$$

$$= \underbrace{\left(1 + \frac{\Gamma^2}{m^2}\right)^{-1}} \left[\left(p^2 - \frac{m^2}{1 + \Gamma^2/m^2}\right)^2 + \frac{m^2\Gamma^2}{\left(1 + \Gamma^2/m^2\right)^2}\right]^{-1}$$

absorbed by couplings
and $\Gamma(m^2) = \Gamma$

Real pole at $p^2 = \dfrac{m^2}{1 + \Gamma^2/m^2}$ in the energy-dependent-width scheme

The pole masses of unstable particles are NOT equivalent to m.

but shifted to $p^2 = \dfrac{m^2}{1 + \Gamma^2/m^2}$.

Comparison between distinct schemes

	Breit-Wigner scheme	Theoretical scheme	energy-dependent scheme
$iG^R(p)$	$\frac{i}{p^2 - m_{\rm BW}^2 + i m_{\rm BW}\Gamma_{BW}}$	$\frac{i}{p^2 - m_{\rm Th}^2 + \Gamma_{\rm Th}^2/4 + i m_{\rm Th}\Gamma_{\rm Th}}$	$\frac{i}{p^2 - m_{\rm ED}^2 + i p^2\Gamma_{\rm ED}/m_{\rm ED}}$
real pole	$p^2 = m_{\rm BW}^2$	$p^2 = m_{\rm Th}^2 - \frac{\Gamma_{\rm Th}^2}{4}$	$p^2 = \frac{m_{\rm ED}^2}{1+\Gamma_{\rm ED}^2/m_{\rm ED}^2}$
mass conversion relation	$m_{\rm BW}$	$m_{\rm Th} \simeq m_{\rm BW}\left(1 + \frac{\Gamma_{\rm BW}^2}{8m_{\rm BW}^2}\right)$	$m_{\rm ED} \simeq m_{\rm BW}\left(1 + \frac{\Gamma_{\rm BW}^2}{2m_{\rm BW}^2}\right)$
decay rate conversion relation	$\Gamma_{\rm BW}$	$\Gamma_{\rm Th} \simeq \Gamma_{\rm BW}\left(1 - \frac{\Gamma_{\rm BW}^2}{8m_{\rm BW}^2}\right)$	$\Gamma_{\rm ED} \simeq \Gamma_{\rm BW}\left(1 + \frac{\Gamma_{\rm BW}^2}{2m_{\rm BW}^2}\right)$
		$\left(\Gamma_{\rm BW}^2/m_{\rm BW}^2 \ll 1\right)$	$\left(\Gamma_{\rm BW}^2/m_{\rm BW}^2 \ll 1\right)$

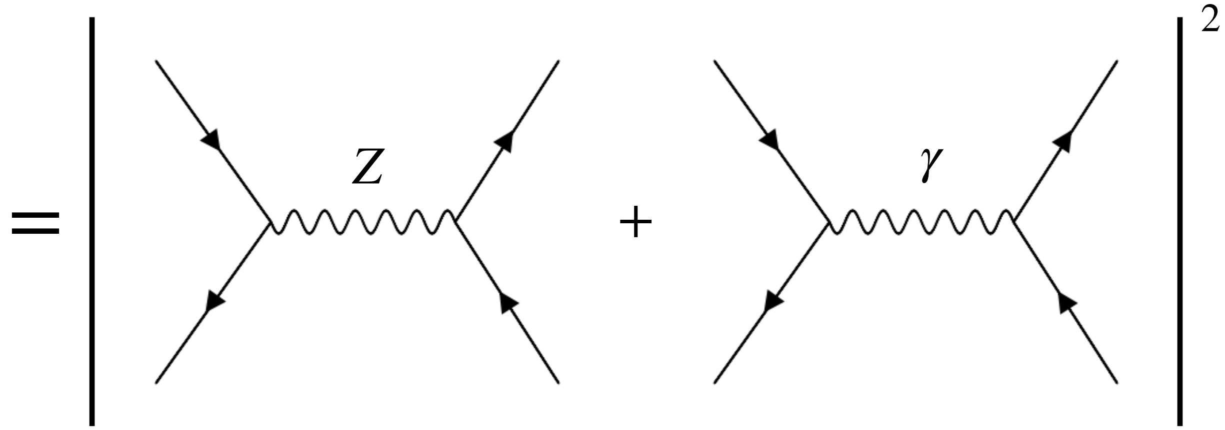
S. Willenbrock and G. Valencia, Phys. Lett. B 259, 373-376 (1991)

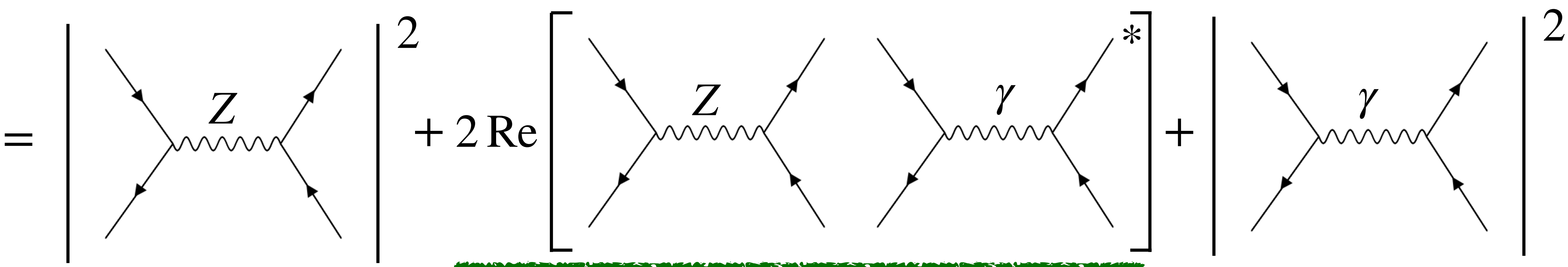
Question: Can we distinguish these three different schemes? YES

Z boson resonance

Z boson production cross-section at leading order

$$\sigma(e^+e^- \to f\bar{f}) \propto \left|\mathcal{M}_Z + \mathcal{M}_\gamma\right|^2 \propto \left|\frac{g_{Ze}\,g_{Zf}}{s - m_Z^2 + \ldots} + \frac{Q_e\,Q_f\,e^2}{s + i\epsilon}\right|^2$$





interference between Z and photon propagators

Cross-section formula

Z boson production cross-section at leading order

Scheme	$\left\|\mathcal{M}_Z\right\|^2$	$\mathcal{M}_Z\mathcal{M}_\gamma^* + \mathcal{M}_\gamma\mathcal{M}_Z^*$	$\left\|\mathcal{M}_\gamma\right\|^2$
BW	$\frac{g_{Ze,\mathrm{BW}}^2\, g_{Zf,\mathrm{BW}}^2}{\left(s-m_{\mathrm{BW}}^2\right)^2+m_{\mathrm{BW}}^2\Gamma_{BW}^2}$	$2\frac{g_{Ze,\mathrm{BW}}\, g_{Zf,\mathrm{BW}}}{\left(s-m_{\mathrm{BW}}^2\right)^2+m_{\mathrm{BW}}^2\Gamma_{BW}^2}\frac{Q_e\,Q_f\,e_{\mathrm{BW}}^2}{s}\left(s-m_{\mathrm{BW}}^2\right)$	$\frac{Q_e^2\,Q_f^2\,e_{\mathrm{BW}}^4}{s^2}$
Th	$\frac{g_{Ze,\mathrm{Th}}^2\, g_{Zf,\mathrm{Th}}^2}{\left(s^2-m_{\mathrm{Th}}^2+\Gamma_{\mathrm{Th}}^2/4\right)^2+m_{\mathrm{Th}}^2\Gamma_{\mathrm{Th}}^2}$	$2\frac{g_{Ze,\mathrm{Th}}\, g_{Zf,\mathrm{Th}}}{\left(s^2-m_{\mathrm{Th}}^2+\Gamma_{\mathrm{Th}}^2/4\right)^2+m_{\mathrm{Th}}^2\Gamma_{\mathrm{Th}}^2}\frac{Q_e\,Q_f\,e_{\mathrm{Th}}^2}{s}$ $\times\left(s^2-m_{\mathrm{Th}}^2+\Gamma_{\mathrm{Th}}^2/4\right)$	$\frac{Q_e^2\,Q_f^2\,e_{\mathrm{Th}}^4}{s^2}$
ED	$\frac{g_{Ze,\mathrm{ED}}^2\, g_{Zf,\mathrm{ED}}^2}{\left(s-m_{\mathrm{ED}}^2\right)^2+s^2\Gamma_{\mathrm{ED}}^2/m_{\mathrm{ED}}^2}$	$2\frac{g_{Ze,\mathrm{ED}}\, g_{Zf,\mathrm{ED}}}{\left(s-m_{\mathrm{ED}}^2\right)^2+s^2\Gamma_{\mathrm{ED}}^2/m_{\mathrm{ED}}^2}\frac{Q_e\,Q_f\,e_{\mathrm{ED}}^2}{s}\left(s-m_{\mathrm{ED}}^2\right)$	$\frac{Q_e^2\,Q_f^2\,e_{\mathrm{ED}}^4}{s^2}$

It looks complicated…

Cross-section formula

after using the conversion relations in the previous slide,

Scheme	$\left\|\mathscr{M}_Z\right\|^2$	$\mathscr{M}_Z\mathscr{M}_\gamma^* + \mathscr{M}_\gamma\mathscr{M}_Z^*$	$\left\|\mathscr{M}_\gamma\right\|^2$
BW	$\frac{g_{Ze}^2 g_{Zf}^2}{\left(s-m^2\right)^2+m^2\Gamma^2}$	$2\frac{g_{Ze}\,g_{Zf}}{\left(s-m^2\right)^2+m^2\Gamma^2}\frac{Q_e\,Q_f\,e^2}{s}\left(s-m^2\right)$	$\frac{Q_e^2\,Q_f^2\,e^4}{s^2}$
Th	$\frac{g_{Ze}^2 g_{Zf}^2}{\left(s-m^2\right)^2+m^2\Gamma^2}$	$2\frac{g_{Ze}\,g_{Zf}}{\left(s-m^2\right)^2+m^2\Gamma^2}\frac{Q_e\,Q_f\,e^2}{s}\left(s-m^2\right)$	$\frac{Q_e^2\,Q_f^2\,e^4}{s^2}$
ED	$\frac{g_{Ze}^2 g_{Zf}^2}{\left(s-m^2\right)^2+m^2\Gamma^2}$	$2\frac{g_{Ze}\,g_{Zf}}{\left(s-m^2\right)^2+m^2\Gamma^2}\frac{Q_e\,Q_f\,e^2}{s}\frac{\left(s-m^2-\Gamma^2\right)}{\sqrt{1+\Gamma^2/m^2}}$	$\frac{Q_e^2\,Q_f^2\,e^4}{s^2}$

$$m_{\mathrm{BW}} = m,\ \Gamma_{\mathrm{BW}} = \Gamma,\ g_{Ze}\,g_{Zf} = g_{Ze,\mathrm{BW}}\,g_{Zf,\mathrm{BW}} = g_{Ze,\mathrm{Th}}\,g_{Zf,\mathrm{Th}} = \frac{g_{Ze,\mathrm{ED}}\,g_{Zf,\mathrm{ED}}}{\sqrt{1+\Gamma_{\mathrm{ED}}^2/m_{\mathrm{ED}}^2}}$$

Z boson measurements

PDG values in BW scheme Phys. Rev. D 110, 030001 (2024)

$$m_Z = 91.1876 \pm 0.0021\,\text{GeV} \quad \rightarrow \quad \delta m/m \sim \mathcal{O}(10^{-5})$$

$$\Gamma_Z = 2.4955 \pm 0.0023\,\text{GeV} \quad \rightarrow \quad \delta\Gamma/\Gamma \sim \mathcal{O}(10^{-3})$$

from LEP data

Required precision level to distinguish BW and ED schemes

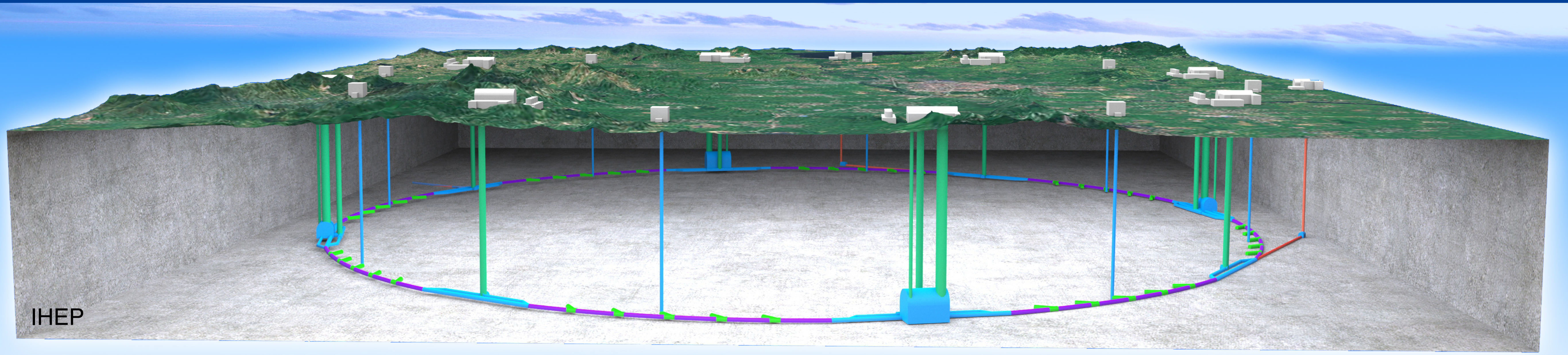
$$m_{\text{ED}} \simeq m_{\text{BW}}\left(1 + \frac{\Gamma_{\text{BW}}^2}{2m_{\text{BW}}^2}\right) \quad \text{and} \quad \Gamma_{\text{ED}} \simeq \Gamma_{\text{BW}}\left(1 + \frac{\Gamma_{\text{BW}}^2}{2m_{\text{BW}}^2}\right)$$

$$\rightarrow \quad \frac{\delta m}{m} = \frac{\delta\Gamma}{\Gamma} = \frac{\Gamma_Z^2}{2m_Z^2} \simeq 3.7 \times 10^{-4}$$ from the cross-section measurements

insufficient sensitivity at LEP: $\delta\Gamma/\Gamma \sim \mathcal{O}(10^{-3})$

Distinguishable at future lepton colliders?

Test at CEPC



Circular Electron Positron Collider (CEPC) CEPC Conceptual Design Report: Volume 2 - Physics & Detector

A future lepton collider proposed by the Chinese particle physics community
for precision measurements of Higgs, Z and W bosons and searches for BSM physics.

Test at CEPC (five data points for Z factory)

at Z pole: $\mathscr{L} = 100\,\mathrm{ab}^{-1}$ at $\sqrt{s} = m_Z \quad \rightarrow \quad \sim 3\times 10^{12}$ of Z events

off-Z pole: $\mathscr{L} = 1\,\mathrm{ab}^{-1}$ at $\sqrt{s} - m_Z = \pm 1,\ \pm 2\,\mathrm{GeV}$

Expected uncertainties at CEPC CEPC, Snowmass 2021

unc. lumi.: $\delta\mathscr{L}/\mathscr{L} = 5\times 10^{-5}$

unc. stat.: $\delta N/N \simeq 10^{-6}$

$\cdots$

BW vs ED Test at CEPC

Data points CEPC, Snowmass 2021

$$\mathscr{L} = 100\,\mathrm{ab}^{-1} \text{ at } \sqrt{s} \simeq 91.2$$

$$\mathscr{L} = 1\,\mathrm{ab}^{-1} \text{ at } \sqrt{s} \simeq 87.9,\ 90.2,\ 92.2,\ 94.3\,\mathrm{GeV}$$

Z line-shape scan and forward-backward asymmetry as observables

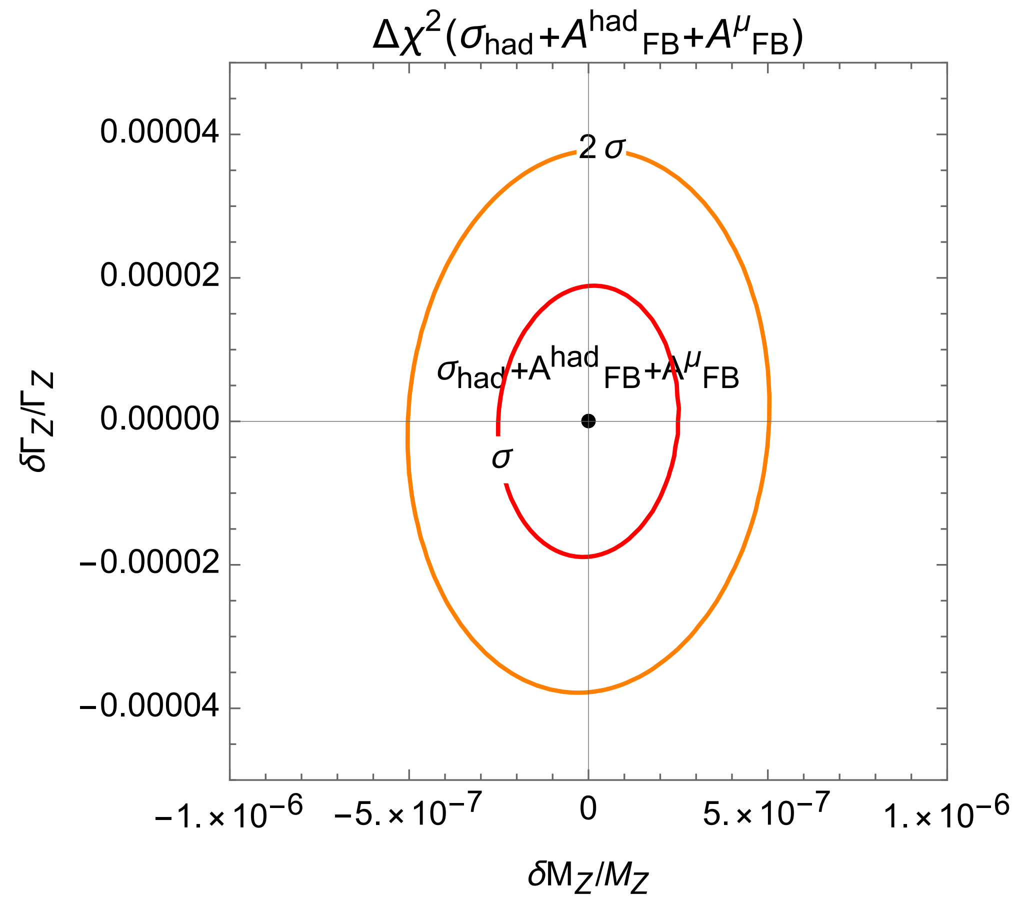
$$\mathscr{O}^i = \sigma_{\mathrm{had}},\ A^{\mu}_{\mathrm{FB}},\ A^{q=u,d,s,c,b}_{\mathrm{FB}} \qquad \left(A^f_{\mathrm{FB}} = \frac{\sigma^f_F - \sigma^f_B}{\sigma^f_F + \sigma^f_B} \right)$$

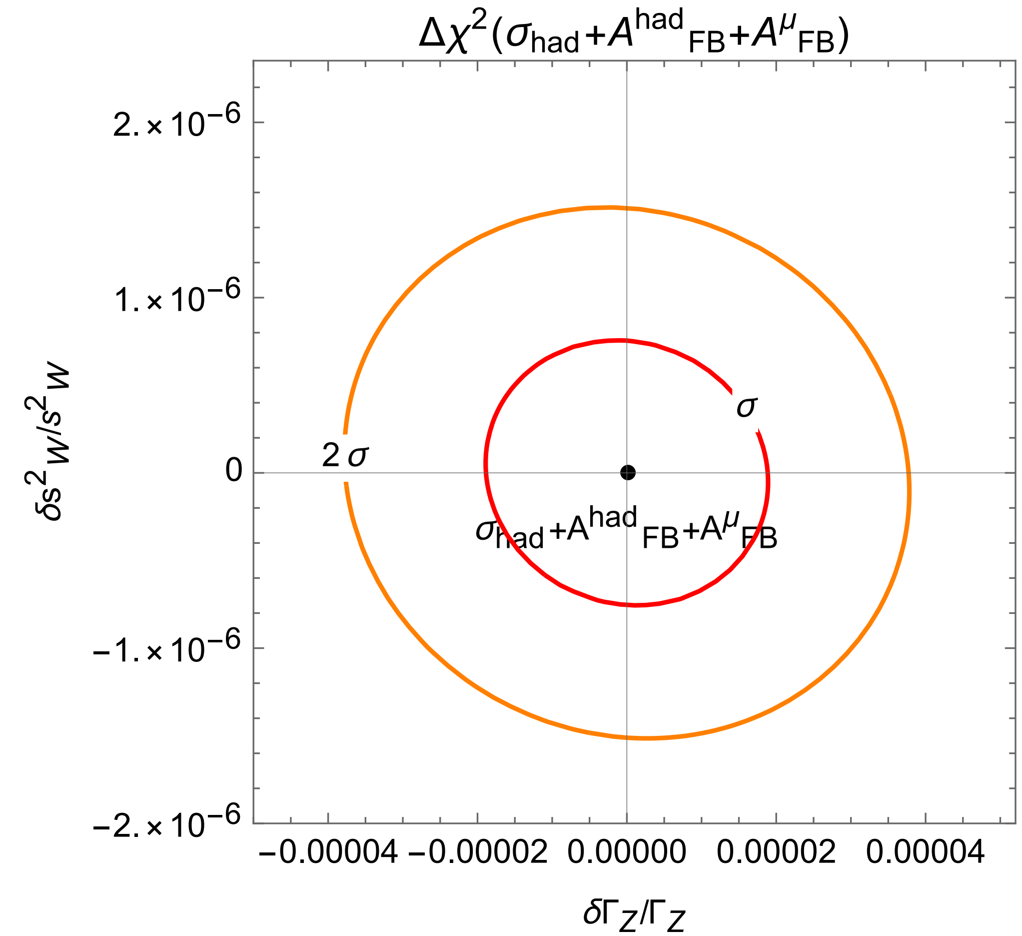
Chi-square fitting between BW and ED schemes

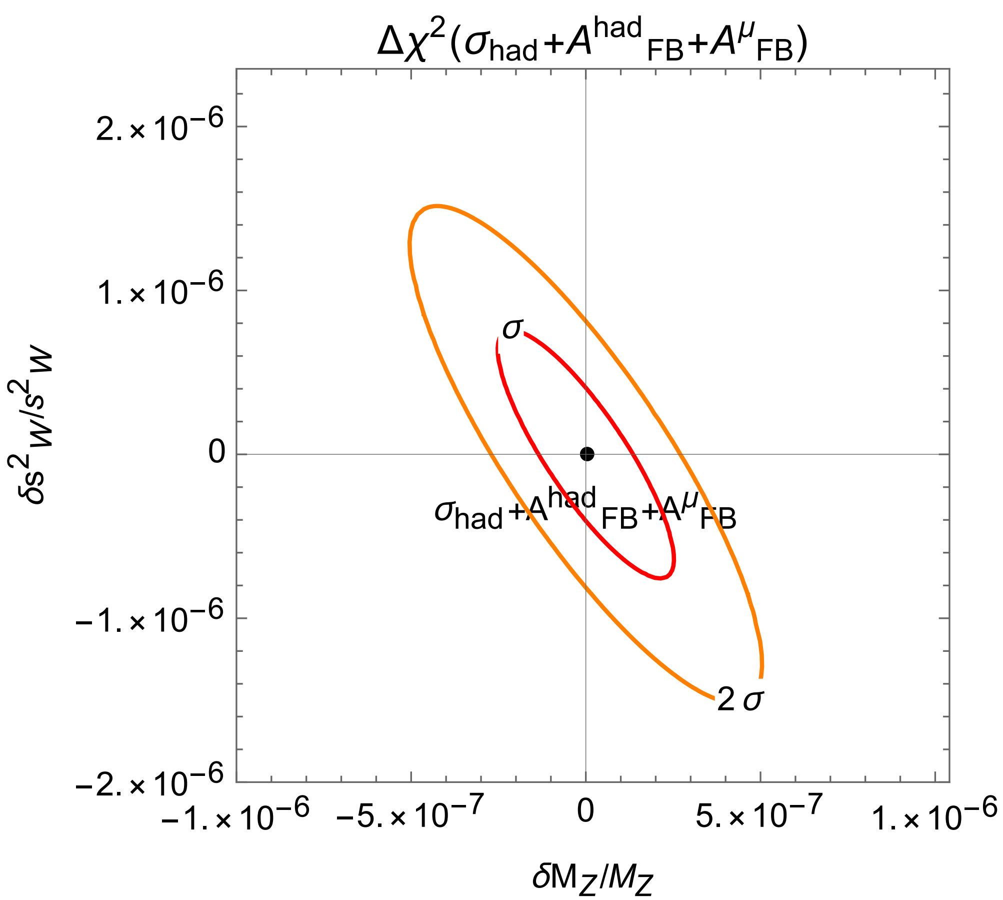
$$\chi^2 = \sum_i \left(\frac{\mathscr{O}^i_{\mathrm{BW}} - \mathscr{O}^i_{\mathrm{ED}}}{\delta \mathscr{O}^i_{\mathrm{ED}}} \right)^2$$ with fitting parameters $m_Z, \Gamma_Z,\ \sin^2\theta_W$

Chi-square minimum between BW and ED schemes

$$\chi^2_{\min} \simeq 5.1 \times 10^5$$







Fitting parameters: $m_Z, \Gamma_Z, \sin^2\theta_W$

Marginalization of the remaining parameter

$$\begin{matrix}\delta m/m \sim \mathcal{O}(10^{-6}) \\ \delta\Gamma/\Gamma \sim \mathcal{O}(10^{-5})\end{matrix} \quad < \quad \Gamma_Z^2/(2m_Z^2) \simeq 3.7\times10^{-4}$$

CPEC is expected to have sufficient precision to differentiate BW and ED schemes.

BW vs Th Test at CEPC

Opposite scaling behaviors of the decay rate

1. from BW-to-Th conversion relations

$$m_{\rm Th} \simeq m_{\rm BW}\left(1+\frac{\Gamma_{\rm BW}^2}{8m_{\rm BW}^2}\right) = m_{\rm BW}\,R_Z, \quad \Gamma_{\rm Th} \simeq \Gamma_{\rm BW}\left(1-\frac{\Gamma_{\rm BW}^2}{8m_{\rm BW}^2}\right) = \Gamma_{\rm BW}\,R_Z^{-1}$$

$$\mathrm{Im}\,\Pi_Z = m_{\rm BW}\Gamma_{\rm BW} = m_{\rm Th}\Gamma_{\rm Th}$$

What we actually measure from data

$\Gamma \propto m^{-1}$ regardless of BW or ED

2. from the leading order expression of the Z decay width in QFT

$$\Gamma(m) = \sum_f \frac{m}{24\pi}\left[g_L^2 + g_R^2 - \left(g_L^2+g_R^2-6g_Lg_R\right)\frac{m_f^2}{m_Z^2}\right]\sqrt{1-\frac{m_f^2}{m^2}} \propto m$$

BW vs Th Test at CEPC

Comparison between BW and Th schemes

$$m_{\mathrm{Th}} R_Z^{-1} \text{ vs } m_{\mathrm{BW}}, \quad \Gamma(m_{\mathrm{Th}}) R_Z \text{ vs } \Gamma_{\mathrm{BW}}$$

Expected sensitivities at CEPC CEPC, Snowmass 2021

$$\Delta m_Z = 0.1\,\mathrm{MeV}, \quad \Delta\Gamma_Z = 0.025\,\mathrm{MeV}$$

one-parameter fitting (m_{Th}) between BW and Th schemes

$$\chi^2 = \left(\frac{m_{\mathrm{Th}} R_Z^{-1} - m_{\mathrm{BW}}}{\Delta m_Z} \right)^2 + \left(\frac{\Gamma(m_{\mathrm{Th}}) R_Z - \Gamma_{\mathrm{BW}}}{\Delta\Gamma_Z} \right)^2$$

Chi-square minimum between BW and Th schemes

$\chi^2_{\min} \simeq 326$ with fixed $m_{\mathrm{BW}} = 91.1876\,\mathrm{GeV}$

Summary

- There are various resonance schemes describing unstable particles: Breit-Wigner, Theoretical, energy-dependent-width schemes as an S-matrix Ansatz.
- Using the chi-square fitting, we found that Z boson measurements at CEPC is expected to distinguish different resonance schemes with sufficient precision.
- Future work: numerical implementation via MadGraph5_aMC@NLO

Thank you for your attention.