Pion interaction cross section sensitivity

Ryotaro Tsuchii

Science Tokyo

Pion interaction meeting
April 4, 2025

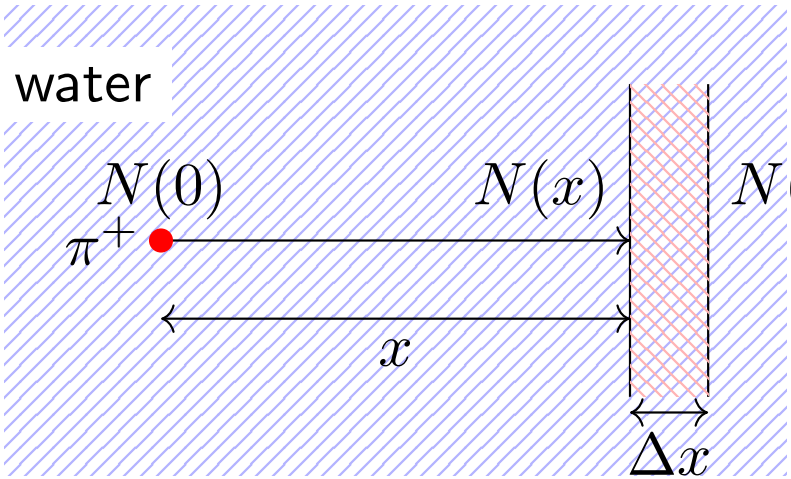
Introduction

- I evaluated uncertainties of the mesurement of the pion interactions.
- First, I checked the method of the cross section calculation in WCTE:
  - Assuming thinner target:

    $\#$ of signal $=$ total $\#$ of entry$\times$number density$\times$thickness of target$\times\sigma$.

  - The WCTE detector is the thick water target.
    - we have to calculate the cross sections using reconstructed vertex.

Calculation of cross section for each interaction



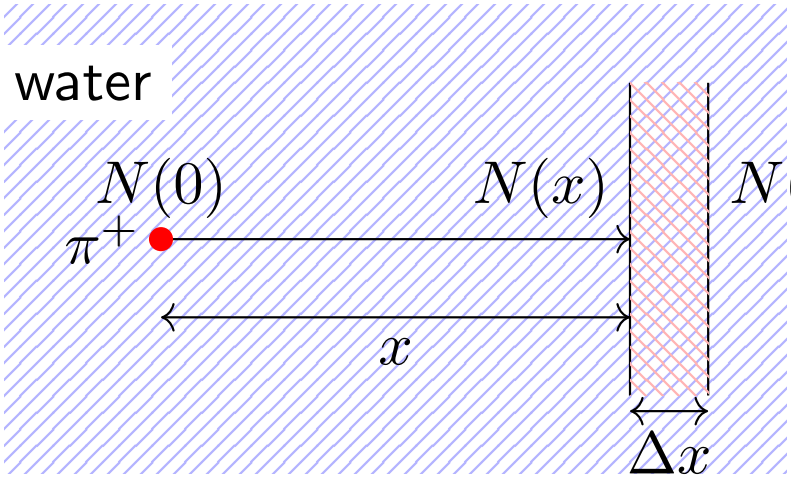
- x: length from injection point
- $N(x)$: # of surviving pions.

- # of reacted events in Δx:

$$N(x+\Delta x) - N(x) = -N(x) n \sigma(x) \Delta x.$$

  - n: number density of water ($\rho N_{\mathrm{A}}/18$).
  - $\sigma(x)$: variable of total cross section is momentum and these momentum correlate with energy loss in travel

Calculation of cross section for each interaction



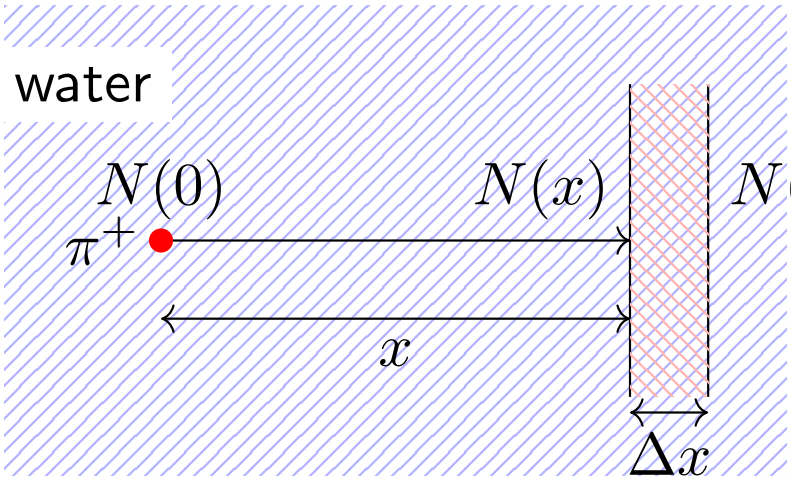
- x: length from injection point
- $N(x)$: # of surviving pions.

- # of CX events in Δx:

$$\Delta N_{\mathrm{CX}}(x) = N(x) n \sigma_{\mathrm{CX}}(x) \Delta x$$

$$\Longrightarrow \sigma_{\mathrm{CX}}(x) = \frac{1}{n\Delta x} \times \frac{\Delta N_{\mathrm{CX}}(x)}{N(x)}.$$

Calculation of cross section for each interaction

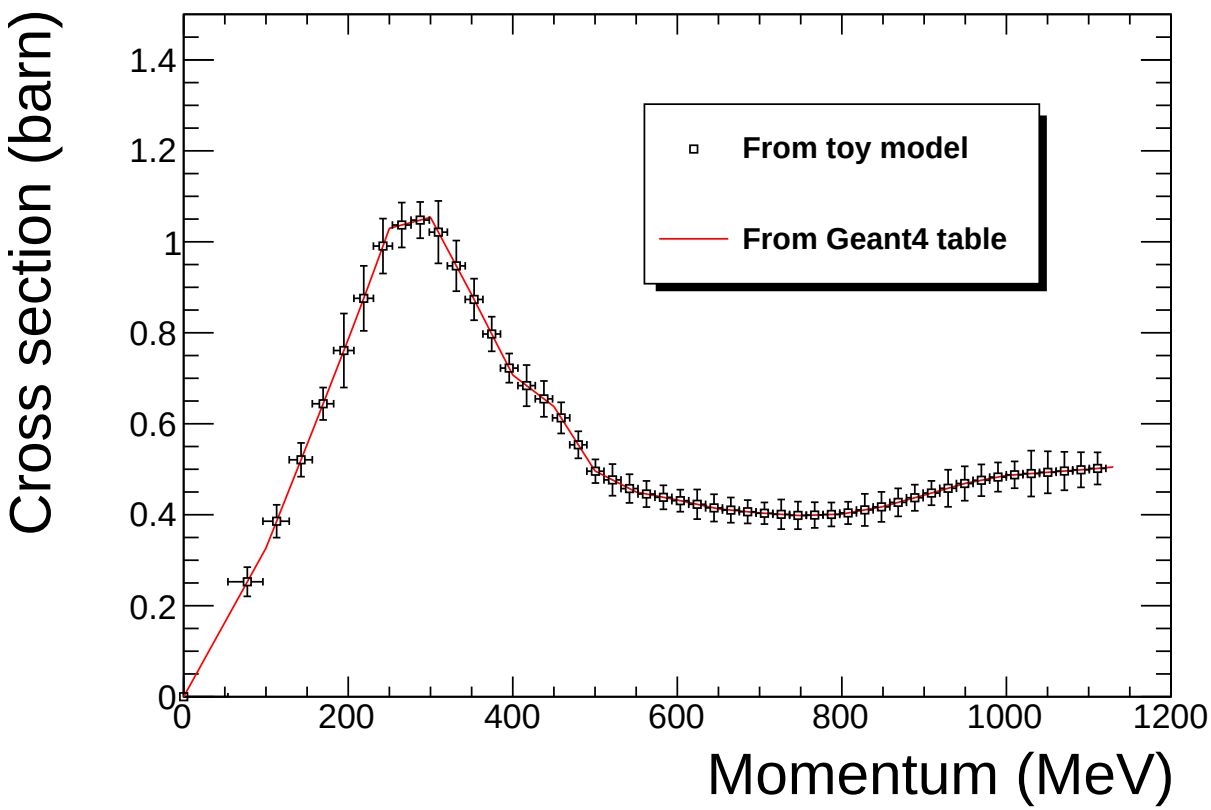


$$\sigma(x) = \frac{1}{n\Delta x}\left(1 - \frac{N(x+\Delta x)}{N(x)}\right)$$

$$\sigma_{\mathrm{CX}}(x) = \frac{1}{n\Delta x} \times \frac{\Delta N_{\mathrm{CX}}(x)}{N(x)}$$

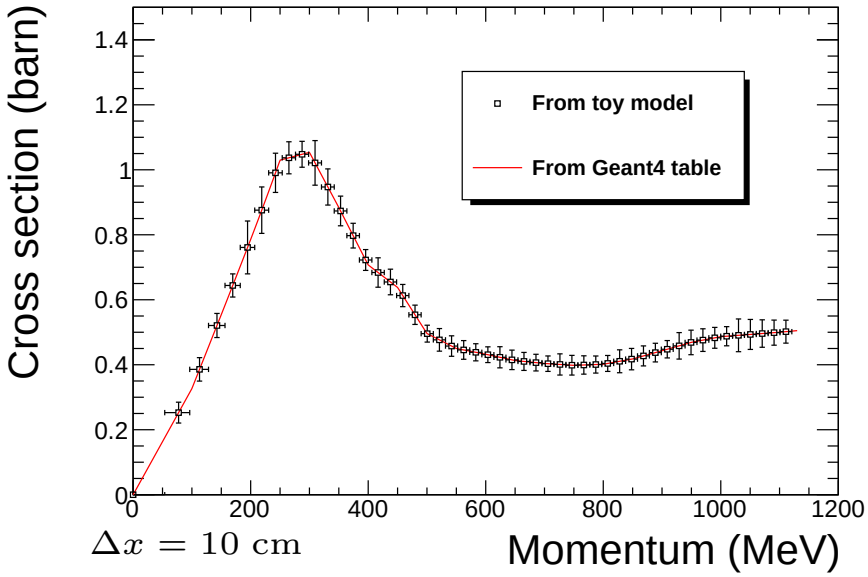
- I evaluated the effect of Δx and $N(x)$ on the cross-section measurement by using toy model.

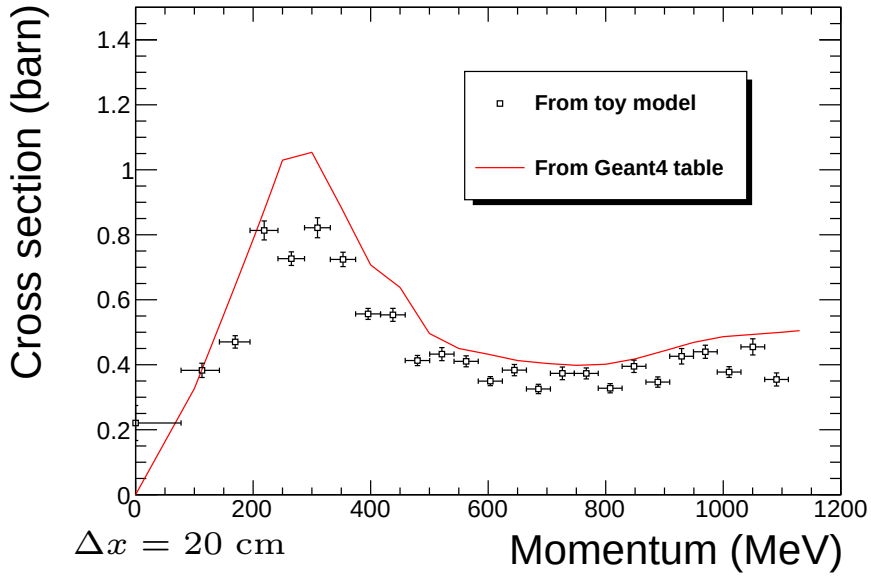
Uncertainty evaluation by toy model

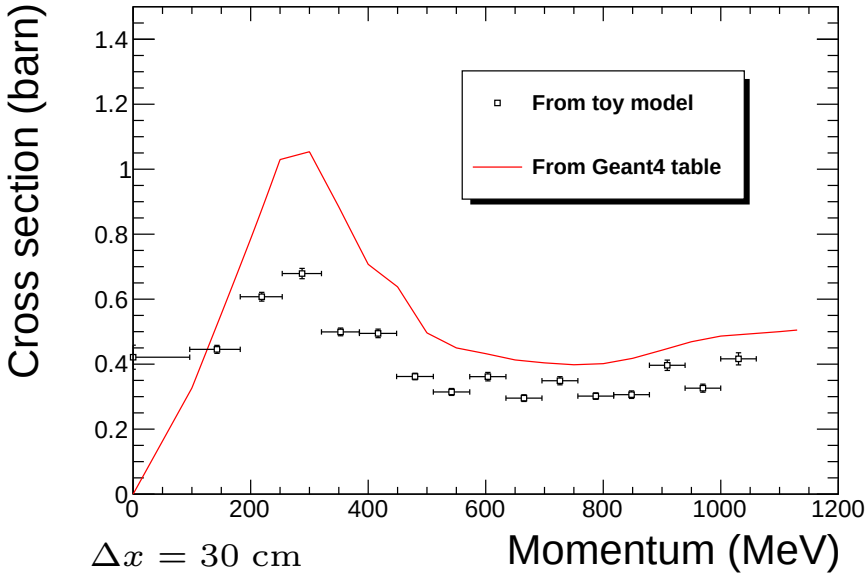


- The cross sections that could be measured and their errors were calculated by the toy model:
  - assumed that the true vertex could be obtained and that the pion did not decay. (Details are in the backup.)
- Inject 10^3 pions of kinetic energy 100, 200,$\cdots$, 1000 MeV each.
- $\Delta x = 10$ cm

Change Δx







- Change Δx $(10,\ 20,\ 30\ \mathrm{cm})$.
- As Δx is increased, the center values deviate from the true in regions of large cross sections.
  - The resolution of the vertex reconstruction should be about 10 cm.

Change Δx

- Causes of calculation results depending on Δx.
  - # of surviving pions in Δx:
    - small Δx:

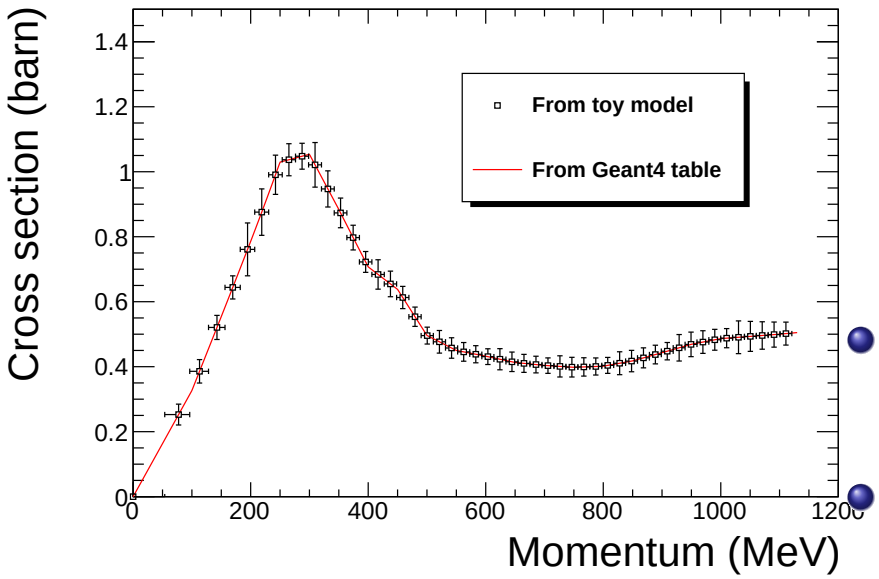
$$N(x+\Delta x) - N(x) = -N(x) n\sigma(x)\Delta x.$$

    - large Δx:

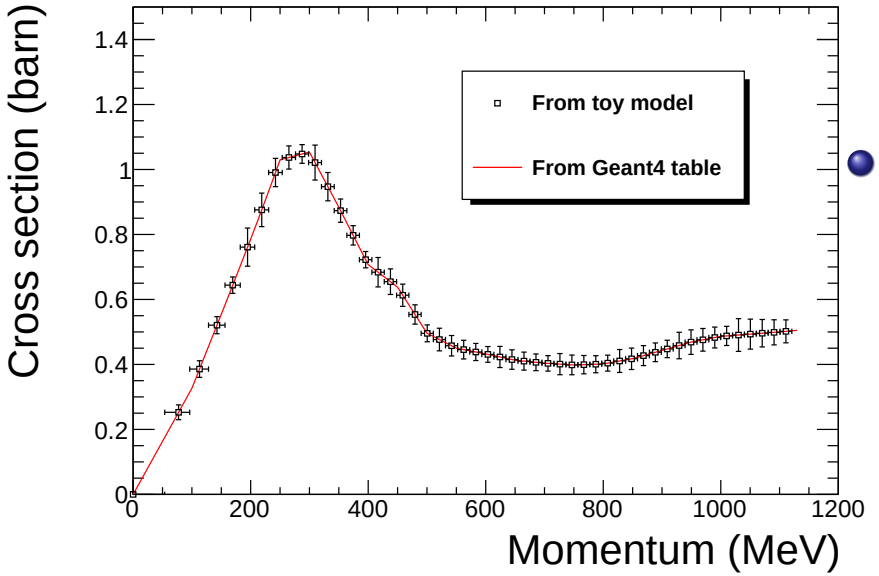
$$N(x+\Delta x) - N(x) = -\int_{x}^{\mathrm{x+\Delta x}} N(x')n\sigma(x')\mathrm{d}x'.$$

  - This integral can be approximated to the above equation when Δx is small, but does not work when Δx is large.

Change ratio of # of pions



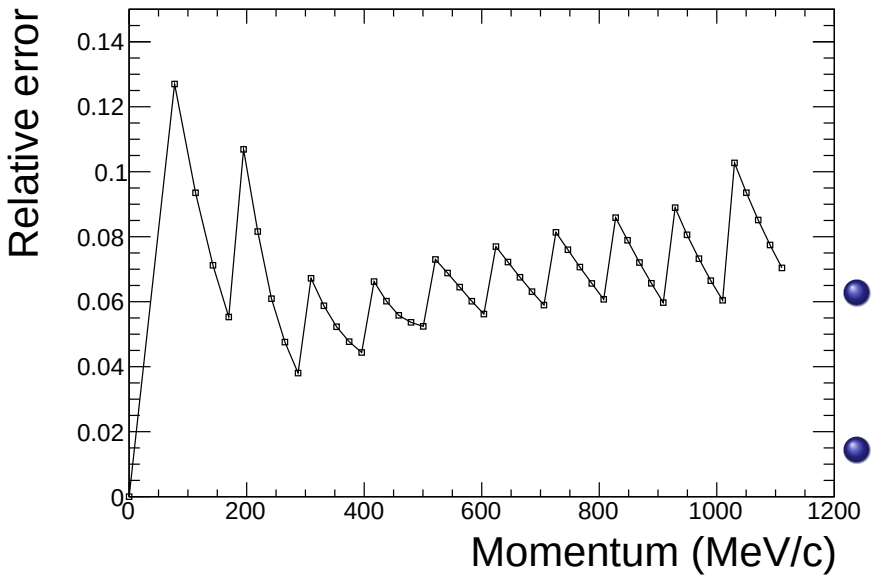
$N_0 = 10^3$ for all the energies.



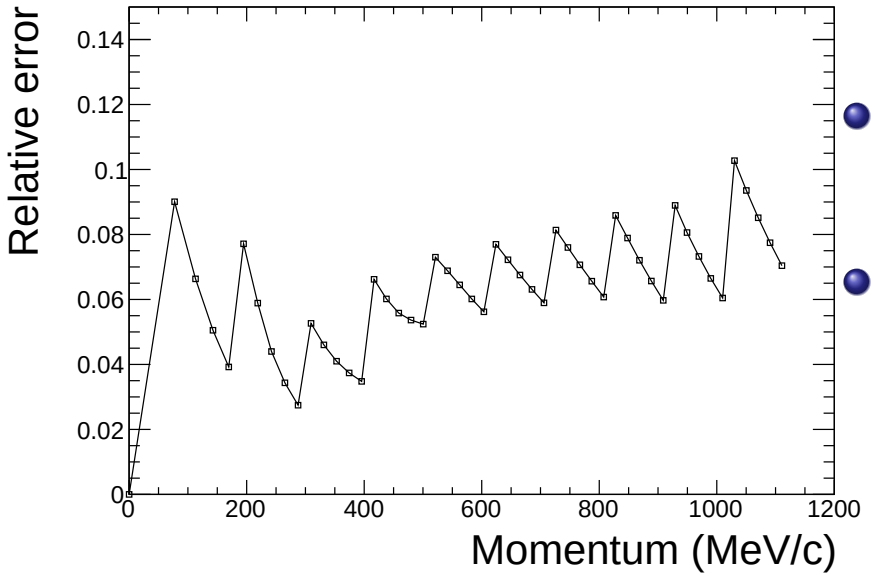
$N_0 = 2 \times 10^3$ for 100 300 MeV

- The error bars around $300\ \mathrm{MeV/c}$ are larger.
- Changed N_0 of the kinetic energies of $100,\ 200,\ 300\ \mathrm{MeV}$ from 10^3 to 2×10^3.
  - Momentum of $195,\ 310,\ 416\ \mathrm{MeV/c}$.
- Errors are decreased on the low momentum region.

Change ratio of # of pions



$N_0 = 10^3$ for all the energies.



$N_0 = 2 \times 10^3$ for 100-300 MeV

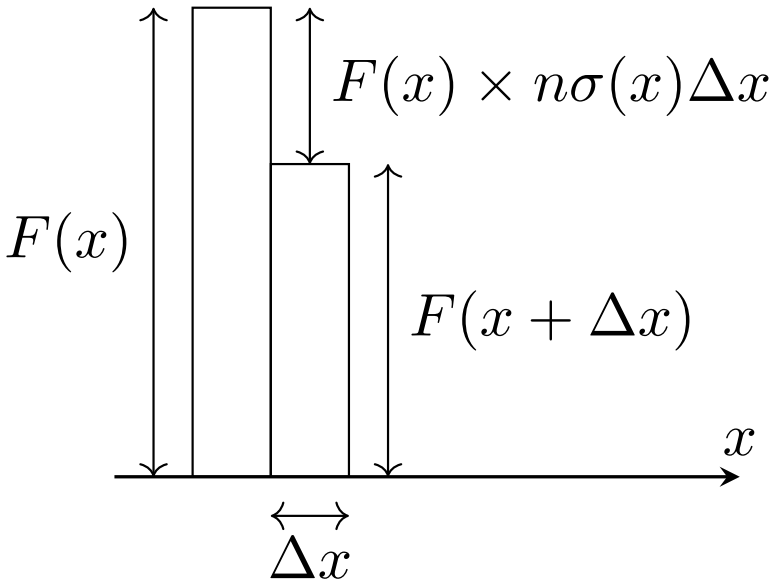
- The error bars around $300\ \mathrm{MeV/c}$ are larger.
- Changed N_0 of the kinetic energies of $100,\ 200,\ 300\ \mathrm{MeV}$ from 10^3 to 2×10^3.
  - Momentum of $195,\ 310,\ 416\ \mathrm{MeV/c}$.
- Errors are decreased on the low momentum region.
- Relative errors = errorbar/σ.

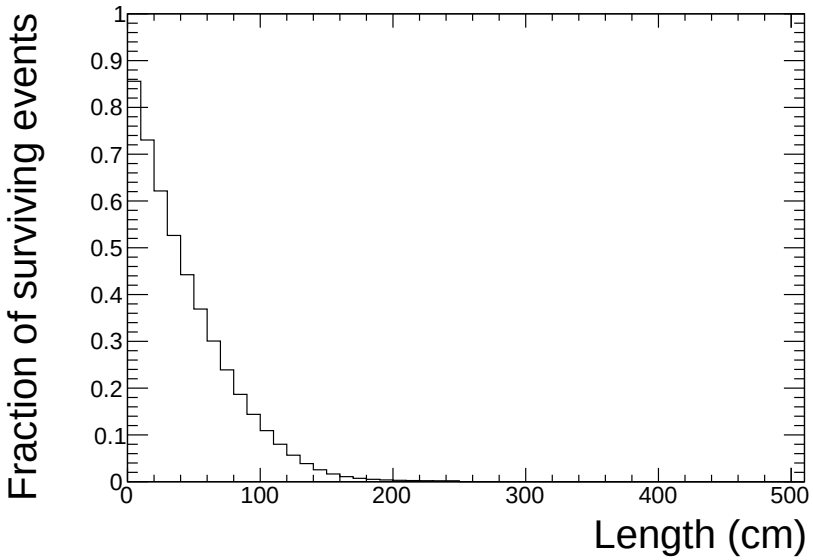
Summary

- Evaluated uncertainties from a method of calculation and # of pions.
  - Checked the method of the cross-section calculation for WCTE the thick target.
  - Made the toy model of pion interaction.
  - Checked the Δx (width for approximation) dependence of uncertainty.
  - Checked the #-of-pions dependence of uncertainty for each momenta.
- ToDo:
  - Evaluate a effect of pion decay.
    - take into account pion decay if this effect is not small.
  - Install effects of vertex reconstruction performance.
  - Make suggestion for pion beam configuration of the WCTE.

appendix

appendix

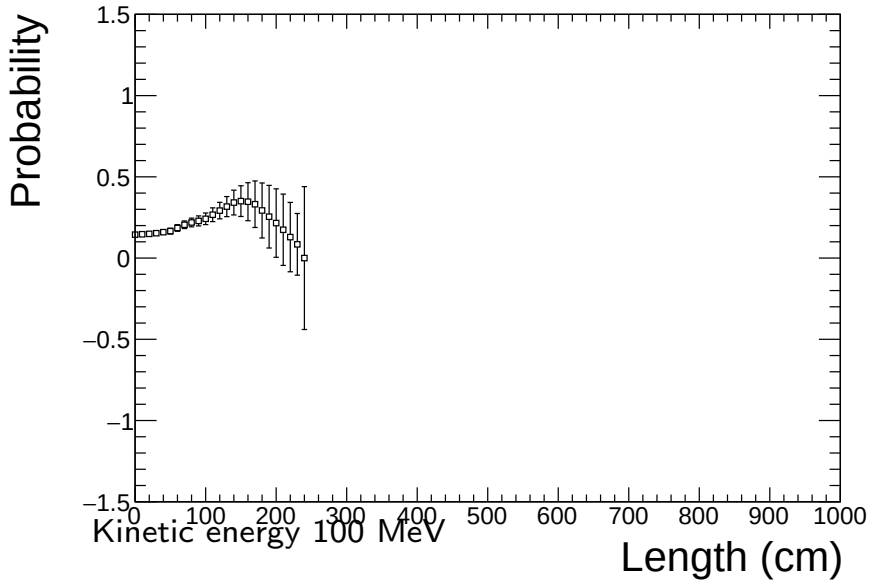


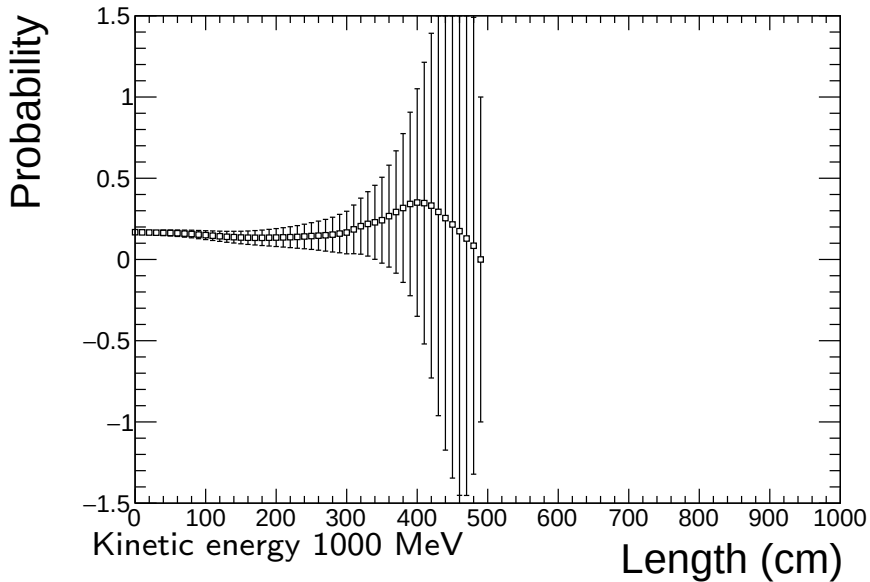


Kinetic energy 500 MeV

- Prepare a toy model for evaluation of the uncertainties.
  - $F(x)$: fraction of # of particles that have survived after running the length x to # of injection pions.
  - For kinetic energies $(100,\ 200,\ \cdots, 1000\ \mathrm{MeV})$ of pions at the time of injection.
  - For each $\Delta x = 10\ \mathrm{cm}$ forward:
    1. Calculate the energy at x calculated from the energy loss $(2\ \mathrm{MeV/cm})$.
    2. Get a reaction cross section of the energy from the Geant4 table.
    3. Calculation $F(x+\Delta x)$.

appendix





- Calculate the probabilities from the distribution for some Δx as:

$$p(x) = \frac{N(x) - N(x + \Delta x)}{N(x)}$$

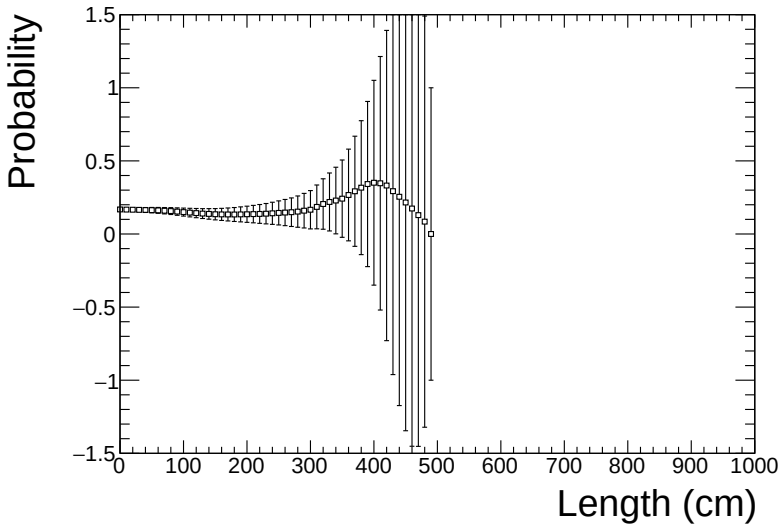
- Stat. uncertainty:

$$\delta p(x) = \sqrt{\frac{p(1-p)}{N(x)}}$$

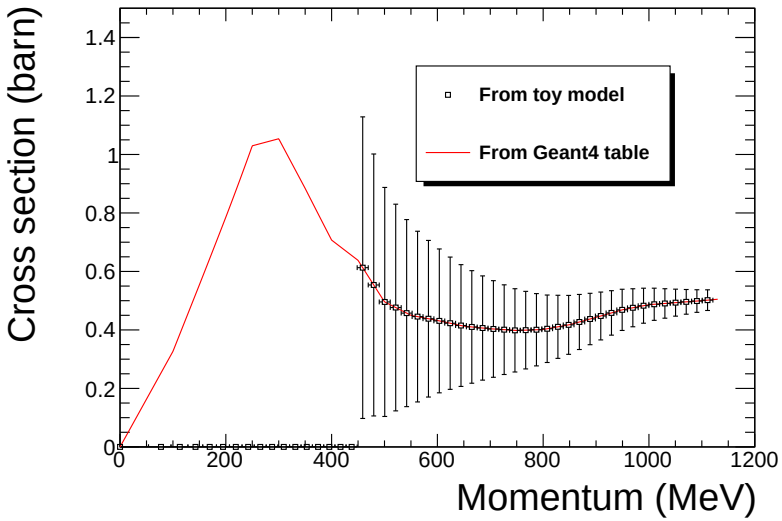
based on binomial distribution.

  - $N(x) = N_0 F(x)$.
  - the initial value N_0 is 1000 events in these figure.

appendix



convert

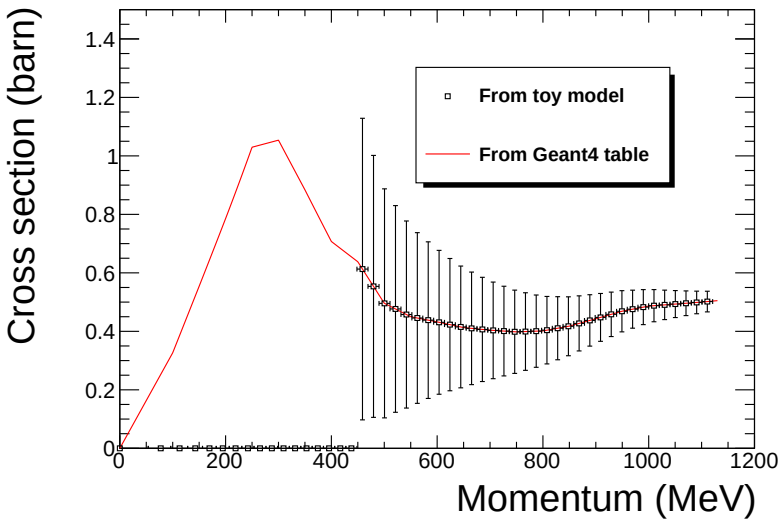


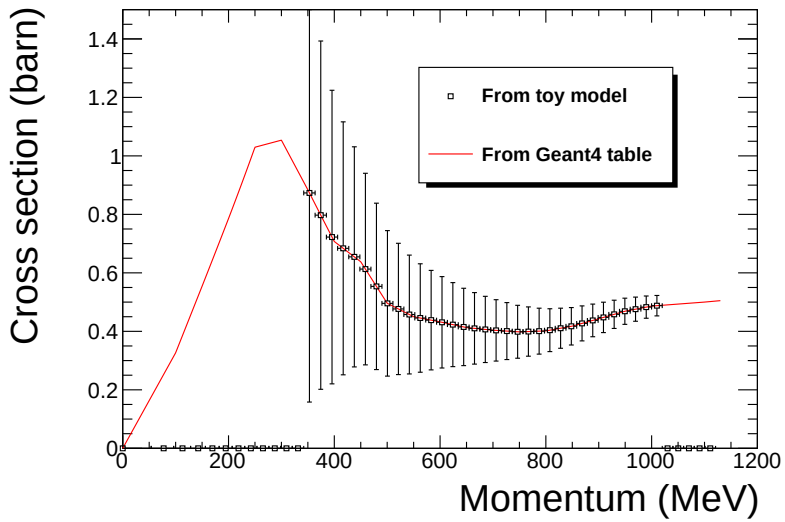
- Convert length to momentum.
  - Assume energy loss as $2\ \mathrm{MeV/cm}$.
- Convert probabilities to cross sections.

$$\sigma = \frac{p}{n\Delta x}$$

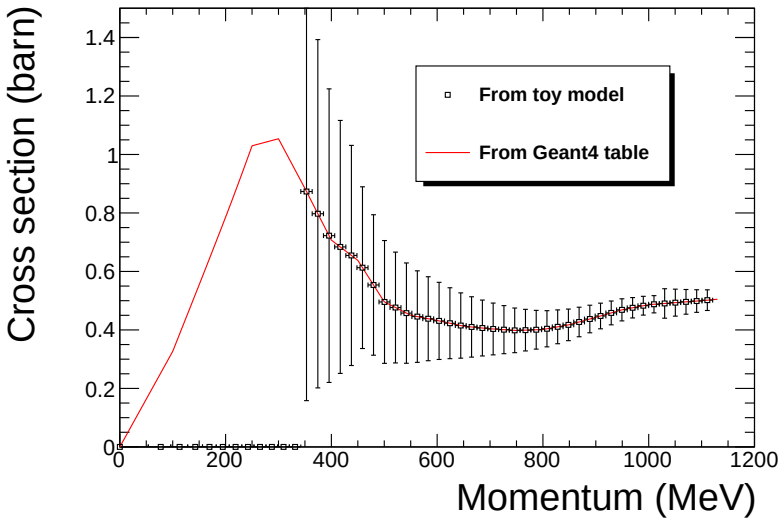
  - $\Delta x = 10\ \mathrm{cm}$ in the bottom figure.

appendix





combine



- Combine the center values and the error bars for each point.
  - all the initial kinetic energies: $100,\ 200,\ 300,\ 400,\ 500,\ 600,\ 700,\ 800,\ 900,\ 1000$ MeV. (momentum: $195,\ 310,\ 417,\ 521,\ 624,\ 726,\ 828,\ 929,\ 1030$ MeV/c)