Quantum gravity footprints and where to find them

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The state of the art: the standard models

Special relativity is a theory of *spacetime*.

Postulates of special relativity

- **Principle of relativity**: The laws of physics take the same form in all inertial reference frames.
- **Speed of light constancy**: The speed of light in vacuum is the same in all inertial reference frames.

- There is *no preferred reference frame*.
- Inertial frames are connected by *Lorentz transformations*.
- Time dilation, length contraction and relative simultaneity.

The **standard model of particle physics** is a theory about the *electromagnetic, weak and strong interactions*.

Standard model of particle physics

- It combines special relativity and quantum mechanics using the relativistic quantum field theory framework.

- *Lagrangian formalism* and perturbative approach.
- Particles are represented by excitations of *quantum fields*.
- Fields interact through *local products* (related to energy-momentum conservation).

General relativity is a theory about the *gravitational interaction*.

General relativity

- It is a relativistic classical (i.e. non-quantum) field theory.
- **Principle of equivalence**: locally, the laws of physics for freely-falling reference frames reduce to those of inertial frames in special relativity.

- Gravitational interaction couples *universally*.
- Thus, it can be interpreted as a consequence of *spacetime geometry*.
- *Spacetime evolves* according to the matter and energy content.

We expect that **at the Planck scale**,

$$E_{\mathrm{Pl}} = \sqrt{\frac{\hbar\, c^5}{G}} \approx 1.22 \times 10^{28}\,\mathrm{eV}\,, \tag{1}$$

$$l_{\mathrm{Pl}} = \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \times 10^{-35}\,\mathrm{m}\,, \tag{2}$$

gravitational and quantum effects are *equally relevant*,
and *spacetime* is expected to reveal its *quantum nature*.

The most wanted:
a quantum gravity theory

WANTED

QUANTUM GRAVITY THEORY

- Possible hideout at the Planck scale, vicinity of black holes and the early universe
- Suspected of being part of a unified theory of everything
- **Be aware: may leave subtle footprints at lower energies**

FUNDAMENTAL OR EFFECTIVE APPROACHES

Fundamental approaches to a quantum gravity theory have struggled to *recover the classical limit* and to make *experimental predictions*:

- Asymptotic safety,
- Loop quantum gravity,
- Causal set theory,
- Group field theory,
- Causal dynam. triangulations,
- Non-commutative geometry,
- Emergent gravity,
- many more...

A similar situation arises for "theory of everything" candidates:

- String theory.

One can try, instead, an **effective approach**.

Any quantum gravity theory needs to go beyond the current concept of spacetime, and thus **beyond** special relativity and **Lorentz invariance**.

Beyond Lorentz symmetry

The **symmetry is broken**

The **symmetry is deformed**

Lorentz invariance violation

Doubly special relativity

Give me a break:
Lorentz invariance violation

Lorentz invariance violation is an effective framework.

Lorentz invariance violation

- Lorentz symmetry is no longer an exact symmetry.
- There is **no principle of relativity**.
- It is characterized by parameters (Λ_{LIV}) defined in an specific **(privileged) reference frame**.

- It allows an EFT description, the *standard model extension*, which preserves gauge invariance, energy-momentum conservation, etc.
- It can include CPT violation.

The free terms of the Lagrangian give rise to *non-universal* **modified energy-momentum relations** (MDR),

$$E^2 = m^2 + p^2 \left[1 \pm \left(\frac{p}{\Lambda_{\mathrm{LIV}}}\right)^n\right] . \quad (3)$$

This is the starting point of many *bottom-up approaches*, allowing one to go beyond the standard model extension.

Non-universal modified energy-momentum relations can lead to **propagation anomalies**:

- Anomalous velocity for massless particles.
- Different velocities for opposite photon helicities (if CPT is violated, $n = 1$).
- Anomalous neutrino oscillations (if flavour dependent modifications are allowed).

These corrections *accumulate with the travelled distance*, producing observable effects.

Non-universal modified energy-momentum relations, and the absence of a relativity principle, can lead to **interaction anomalies** with respect to special relativity:

- Allowing SR-forbidden interactions in certain energy ranges.
- Forbidding SR-allowed interactions in certain energy ranges.
- Anomalous energy dependence of decay widths and cross sections.

Thresholds are very sensitive to small corrections, producing observable effects.

If it ain't broke, deform it: doubly special relativity

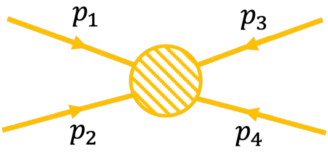
Doubly special relativity is a deformation of special relativity.

Postulates of doubly special relativity

- It goes beyond SR preserving the **principle of relativity**.
- There are **two observer-independent scales**: speed scale (c) and energy scale (Λ_{DSR}).

- There is *no preferred reference frame*.
- It necessarily includes a *deformation of the conservation laws* and multi-particle Lorentz transformations.
- It may also include a deformation of the energy-momentum relation and single-particle Lorentz transformations.

The sum of momenta is not conserved anymore,
but a non-commutative **composition of momenta** is,
which can be interpreted as going beyond locality of interactions.



$$p_1 \oplus p_2 = p_3 \oplus p_4\,. \tag{4}$$

A universal modified energy-momentum relation can lead to **propagation anomalies**:

- Anomalous velocity for massless particles.
- ~~Helicity-dependent velocities~~ and ~~anomalous neutrinos oscillation~~ are forbidden because the MDR is universal.

These corrections *accumulate with the travelled distance*, producing observable effects.

A universal modified energy-momentum relation and/or a modified conservation law, can lead to **interaction anomalies**:

- ~~Allowing SR-forbidden processes~~ or ~~forbidding SR-allowed processes~~ is prohibited by the principle of relativity, but thresholds can still be modified with respect to SR.
- Anomalous energy dependence of decay widths and cross sections.

There is no amplification effect, so producing observable effects *requires at least* $E \sim \Lambda_{DSR}$.

Potential footprints: gamma-ray phenomenology

① **Vacuum birefringence**

② **Time delays**

③ **Vacuum pair emission**

④ **Splitting**

⑤ **Breit-Wheeler**

⑥ **Bethe-Heitler**

① Vacuum birefringence (LIV, n=1)

Different velocities for opposite photon helicities produce **rotation of the polarization plane** of linearly polarized light [Gubitosi:2009eu],

$$\Delta\theta_{\mathrm{LIV}} = \frac{E^2}{\Lambda_{\mathrm{LIV}}}\frac{I_1(z)}{H_0}\,, \quad \text{with} \quad I_1(z) = \int_0^z dz'\,\frac{(1+z')}{\sqrt{\Omega_m(1+z')^3+\Omega_\Lambda}}\,. \quad (5)$$

The absence of detection discourages the linear scenario.

Bounds from [Gotz:2014vza]

- $\Lambda^{(1)}_{\mathrm{LIV}}/E_{\mathrm{Pl}} \gtrsim 10^{16}$.
- Forbidden process for quadratic (CPT-even) LIV and DSR.

② Time delays (LIV)

An *energy-dependent velocity* leads to a **delay in the arrival time** of photons of different energies emitted simultaneously [Jacob:2008bw],

$$\Delta t_{\rm LIV} = \frac{n+1}{2}\frac{\Delta E^n}{\Lambda_{\rm LIV}^n}\frac{I_n(z)}{H_0}\,, \quad \text{with} \quad I_n(z) = \int_0^z dz'\,\frac{(1+z')^n}{\sqrt{\Omega_m(1+z')^3+\Omega_\Lambda}}\,. \quad (6)$$

Bounds from [LHAASO:2024lub], [MAGIC:2020egb], [Vasileiou:2013vra]

- $\Lambda^{(1)}_{\rm LIV}/E_{\rm Pl} \gtrsim 10$ for the linear case.
- $\Lambda^{(2)}_{\rm LIV}/E_{\rm Pl} \gtrsim 10^{-8}$ for the quadratic case.

② Time delays (DSR)

An *energy-dependent velocity* leads to a **delay in the arrival time** of photons of different energies emitted simultaneously.

How to combine the *deformed transformations* with the *expansion of the universe* is still an open question. E.g., [Amelino-Camelia:2023srg],

$$\Delta t_{\mathrm{DSR}} = \frac{\Delta E}{E_{\mathrm{Pl}}} \frac{1}{H_0} \int_0^z dz' \frac{(1+z')}{h(z')} \left[\eta_1 + \eta_2 \, J_2(z') + \eta_3 \, J_3(z') \right] , \qquad (7)$$

with

$$\begin{aligned} J_2(z) &= 1 - \left(1 - \frac{H(z)}{(1+z)} \int_0^z \frac{dz'}{H(z')} \right)^2 \\ J_3(z) &= 1 - \left(1 - \frac{H(z)}{(1+z)} \int_0^z \frac{dz'}{H(z')} \right)^4 \end{aligned} \qquad (8)$$

③ Photon vacuum pair emission (superluminal LIV)

A *superluminal MDR* makes the photon unstable, allowing it to decay through **vacuum pair emission** ($\gamma \to e^- + e^+$), for photons with

$$E > E_{\mathrm{th}} = (4 m_e^2 \Lambda_{\mathrm{LIV}}^n)^{1/(n+2)} \,. \tag{9}$$

The decay width, well above the threshold, is given by [Rubtsov:2012kb]

$$\Gamma_{\mathrm{LIV}} \propto \alpha_{\mathrm{EM}} \, E \left(\frac{E}{\Lambda_{\mathrm{LIV}}} \right)^n \,. \tag{10}$$

Bounds from [Chen:2021hen]

- $\Lambda_{\mathrm{LIV}}^{(1)}/E_{\mathrm{Pl}} \gtrsim 10^5$ for the superluminal linear case.
- $\Lambda_{\mathrm{LIV}}^{(2)}/E_{\mathrm{Pl}} \gtrsim 10^{-4}$ for superluminal quadratic case.
- Forbidden process in DSR and subluminal LIV.

④ Photon Splitting (superluminal LIV)

A *superluminal MDR* makes the photon unstable, allowing it to **split into three photons** ($\gamma \to 3\,\gamma$).

This process has no threshold, and the decay width, well below the VPE threshold, is given by [Gelmini:2005gy]

$$\Gamma_{\mathrm{LIV}} \propto \alpha_{\mathrm{EM}}^4\, E \left(\frac{E}{m_e}\right)^8 \left(\frac{E}{\Lambda_{\mathrm{LIV}}}\right)^{5n}, \tag{11}$$

Bounds from [LHAASO:2021opi]

- $\Lambda^{(1)}_{\mathrm{LIV}}/E_{\mathrm{Pl}} \gtrsim 10^5$ for the superluminal linear case.
- $\Lambda^{(2)}_{\mathrm{LIV}}/E_{\mathrm{Pl}} \gtrsim 10^{-3}$ for the superluminal quadratic case.
- Forbidden process for DSR and subluminal LIV.

⑤ Anomalous Breit-Wheeler (subluminal LIV)

A *subluminal MDR* makes interactions with the CMB and EBL more difficult ($\gamma + \gamma_{\text{soft}} \to e^+ + e^-$), leading to a **more transparent universe**.

In special relativity,

$$\sigma_{\text{BW}}(\bar{s}) = \frac{4\pi\alpha_{\text{EM}}^2}{8m_e^2\bar{s}}\left[\left(2+\frac{2}{\bar{s}}-\frac{1}{\bar{s}^{\,2}}\right)\ln\left(\frac{1+\sqrt{1-1/\bar{s}}}{1-\sqrt{1-1/\bar{s}}}\right)-\left(2+\frac{2}{\bar{s}}\right)\sqrt{1-1/\bar{s}}\right], \tag{12}$$

with $\bar{s} = E\omega(1-\cos\theta)/(4m_e^2)$. For LIV, the usual approach is to take

$$\sigma_{\text{LIV}}^{(\text{R1})} = \sigma_{\text{BW}}(\bar{s}\to\bar{\tau}), \quad \text{with} \quad \bar{\tau} = \bar{s} - E^4/(4m_e^2\Lambda_{\text{LIV}}^2) \tag{13}$$

Bounds from [Lang:2018yog]

- $\Lambda_{\text{LIV}}^{(1)}/E_{\text{Pl}} \gtrsim 10$ for the subluminal linear case.
- $\Lambda_{\text{LIV}}^{(2)}/E_{\text{Pl}} \gtrsim 10^{-7}$ for the subluminal quadratic case.

⑤ Anomalous Breit-Wheeler (subluminal LIV)

A more appropriate *replacement rule* is

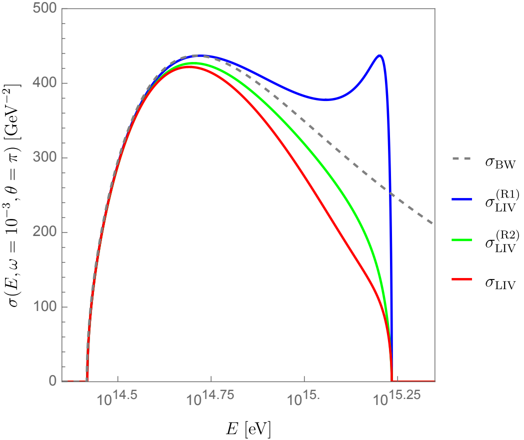
$$\sigma_{\mathrm{LIV}}^{(\mathrm{R2})} = \frac{1}{8 m_e^2 \bar{s}}\, \mathcal{F}_{\mathrm{BW}}(\bar{s} \to \bar{\tau})\,, \quad \text{with} \quad \bar{\tau} = \bar{s} - \bar{\mu}\,. \tag{14}$$

Nevertheless, one should use the *explicit result* [Carmona:2024thn],

$$\begin{aligned} \sigma_{\mathrm{LIV}} = & \frac{4\pi\alpha_{\mathrm{EM}}^2}{8 m_e^2 \bar{s}} \left[\left(2 + \frac{2\bar{\tau}(1-2\bar{\mu})}{(\bar{\tau}+\bar{\mu})^2} - \frac{(1-\bar{\mu})}{(\bar{\tau}+\bar{\mu})^2} \right) \right. \\ & \left. \times \ln\left(\frac{1+\sqrt{1-1/\bar{\tau}}}{1-\sqrt{1-1/\bar{\tau}}} \right) - \left(2 + \frac{2\bar{\tau}(1-4\bar{\mu})}{(\bar{\tau}+\bar{\mu})^2} \right) \sqrt{1-1/\bar{\tau}} \right], \end{aligned} \tag{15}$$

with $\bar{\mu} = E^4/(4 m_e^2 \Lambda_{\mathrm{LIV}}^2)$.

⑤ Anomalous Breit-Wheeler (subluminal LIV)



The use of the explicit result provides bounds that are 20%-30% stronger.

⑤ Anomalous Breit-Wheeler (DSR)

DSR does not provide a QFT framework for the explicit computation of decay widths and cross sections.

For a model without single-particle deformation, an appropriate *replacement rule* can provide a good approximation [Carmona:2025fdu],

$$\sigma_{\mathrm{DSR}} = \frac{1}{8m_e^2\bar{s}}\,\mathcal{F}_{\mathrm{BW}}(\bar{s}\to\bar{\tau})\,, \quad \text{with} \quad \bar{\tau} = (p_1 \oplus p_2)^2/(4m_e^2) \qquad (16)$$

⑥ Anomalous Bethe-Heitler (subluminal LIV and DSR)

A *subluminal MDR* makes the **shower development** ($\gamma + Z \to Z + e^+ + e^-$) more difficult.

The cross section, for $n = 2$ and in the limit $E^2(E/\Lambda)^2 \gg 4m_e^2$, is given by [Rubtsov:2012kb]

$$\sigma_{\text{LIV}} \propto \alpha_{\text{EM}}^3 \frac{Z^2}{E^2(E/\Lambda)^2} \times \left(2 \log \frac{1}{\alpha_{\text{EM}} Z^{1/3}} + \frac{1}{2} \log \frac{E^2(E/\Lambda)^2}{m_e^2}\right) \log \frac{E^2(E/\Lambda)^2}{m_e^2} \tag{17}$$

Bounds from [Rubtsov:2016bea]

- $\Lambda_{\text{LIV}}^{(2)}/E_{\text{Pl}} \gtrsim 10^{-6}$ for the subluminal quadratic case.
- Linear LIV and DSR scenarios not tested yet.

Potential footprints: neutrino phenomenology

1. **Anomalous oscillations**
2. **Time delays**
3. **Vacuum pair emission**
4. **Splitting**

① Anomalous oscillations (LIV)

If each mass eigenstate has a *different dispersion relation*,

$$E_i^2 = m_i^2 + p_i^2 \left[1 + \delta_i^{(n)} \left(\frac{E}{\Lambda_{\mathrm{LIV}}} \right)^n \right] , \tag{18}$$

the **oscillation probability** acquires a *new energy dependence*.
For instance, for $n = 0$, [Motie:2012qj],

$$P_{\nu_\alpha \to \nu_\beta} = \sin^2(2\theta) \sin^2 \left(\frac{\Delta m^2 L}{4E} + \frac{\Delta \delta_i^{(0)} LE}{4} \right) \tag{19}$$

Bounds from [Maccione:2011fr], [Ahrens:2007zzc]

- $\Delta \delta_i^{(0)} \lesssim 10^{-27}$ for the order-zero case.

② Time delays (LIV and DSR)

Neutrinos can be treated as *massless* in time delay analysis. We can still use the Jacob & Piran formula [Jacob:2008bw] for the LIV case,

$$\Delta t_{\rm LIV} = \frac{n+1}{2}\frac{\Delta E^n}{\Lambda^n_{\rm LIV}}\frac{I_n(z)}{H_0}\,, \quad \text{with} \quad I_n(z) = \int_0^z dz' \frac{(1+z')^n}{\sqrt{\Omega_m(1+z')^3+\Omega_\Lambda}}\,. \quad (20)$$

The DSR case presents the same difficulties we encountered with photons (combination of deformed transformations with expansion).

Bounds from [Wei:2016ygk]

- $\Lambda^{(1)}_{\rm LIV}/E_{\rm Pl} \gtrsim 1$ for the linear case.
- $\Lambda^{(2)}_{\rm LIV}/E_{\rm Pl} \gtrsim 10^{-8}$ for the quadratic case.

③ Neutrino vacuum pair emission (superluminal LIV)

A *superluminal MDR* makes the neutrino unstable, allowing it to undergo **vacuum pair emission** ($\nu \to \nu + e^- + e^+$) for neutrinos with

$$E > E_{\text{th}} = (4 m_e^2 \Lambda_{\text{LIV}}^n)^{1/(n+2)} . \tag{21}$$

The decay width, well above the threshold, is given by [Carmona:2022dtp]

$$\Gamma_{\text{LIV}} \propto G_F^2 \, E^5 \left(\frac{E}{\Lambda_{\text{LIV}}} \right)^{3n} . \tag{22}$$

However, the process is subdominant with respect to neutrino splitting (for ν_μ and ν_τ), except for the case $n = 0$ ($\Gamma_{\text{LIV}} \propto G_F^2 \, E^5 \, \delta_{\text{LIV}}^{(0)\,3}$).

Bounds from [KM3NeT:2025mfl]

- $\delta_{\text{LIV}}^{(0)} \lesssim 10^{-22}$ for the superluminal order-zero case.
- Forbidden process for DSR and subluminal LIV.

④ Neutrino splitting (superluminal LIV)

A *superluminal MDR* makes the neutrino unstable, allowing it to **split into three neutrinos** ($\nu \to \nu + \nu + \bar{\nu}$).

This process has a small threshold, above which the decay width is given by [Carmona:2022dtp],

$$\Gamma_{\mathrm{LIV}} \propto G_F^2 \, E^5 \left(\frac{E}{\Lambda_{\mathrm{LIV}}} \right)^{3n} . \tag{23}$$

Bounds from [Satunin:2025uui]

- $\Lambda_{\mathrm{LIV}}^{(1)}/E_{\mathrm{Pl}} \gtrsim 10^{11}$ for the superluminal linear case.
- $\Lambda_{\mathrm{LIV}}^{(2)}/E_{\mathrm{Pl}} \gtrsim 1$ for the superluminal quadratic case.
- Forbidden process for DSR, subluminal LIV and order-zero LIV.

Conclusions and take-home messages

- Thanks to the amplification provided by the very high-energy astroparticles, *small deviations from special relativity can produce observable effects*.
- The detection (or no detection) of these effects of new physics can *guide the theoretical development* in the right direction.

Summary of bounds

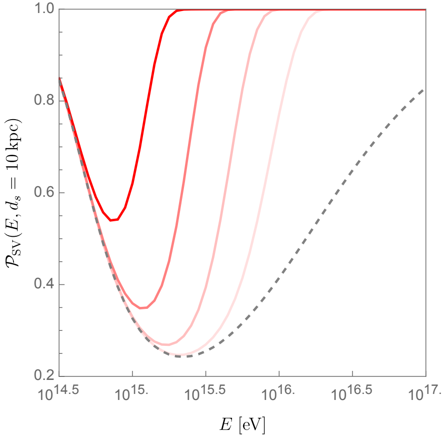
	$\Lambda_{\rm LIV}^{(1)}/E_{\rm Pl}$	$\Lambda_{\rm LIV}^{(2)}/E_{\rm Pl}$	$\Lambda_{\rm DSR}$
γ VB	10^{16}	✗	✗
γ TD	10	10^{-8}	?
γ VPE	10^{5}	10^{-4}	✗
γ Spl	10^{5}	10^{-3}	✗
γ BW	10	10^{-7}	?
γ BH	?	10^{-6}	?

	$\Lambda_{\rm LIV}^{(1)}/E_{\rm Pl}$	$\Lambda_{\rm LIV}^{(2)}/E_{\rm Pl}$	$\Lambda_{\rm DSR}$
ν TD	1	10^{-8}	?
ν VPE	10^{11}	1	✗
ν Spl	10^{11}	1	✗

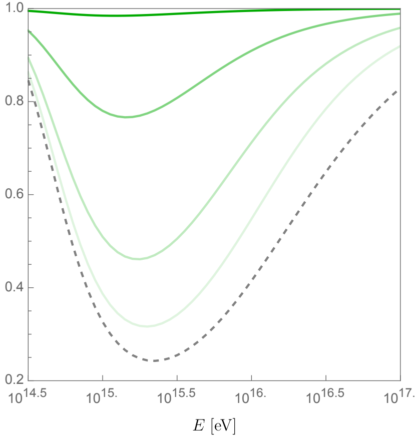
VB=Vacuum Birefringence, TD=Time Delays, VPE=Vacuum Pair Emission, Spl=Splitting, BW=Breit-Wheeler, BH=Bethe-Heitler

- There are several bounds for LIV, but *the DSR scenario is mainly unexplored*!
- Even when *LIV and DSR* produce the same type of phenomenon, the effects are *clearly distinguishable*.

Anomalous BW comparison

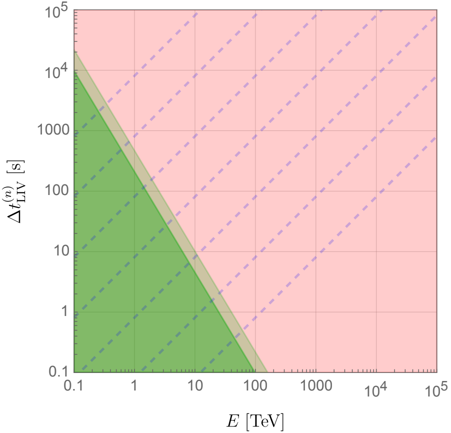


Subl. LIV

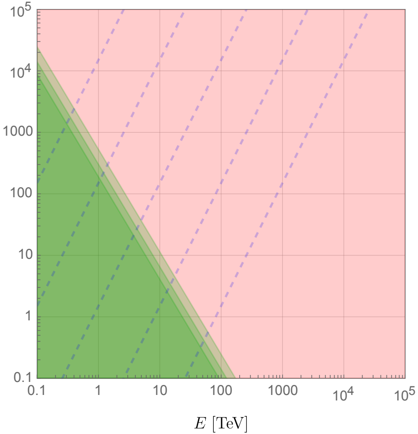


DSR

- The studies should take into account that *several of these effects can be present at the same time* (for instance, neutrino time delays and neutrino decay/splitting).



Sup. LIV $n = 1$



Sup. LIV $n = 2$

- For more information and references, check the review written by the *COST Action CA18108 QG-MM* [Addazi:2021xuf].
- And stay tuned for new results from new COST Action CA23130 BridgeQG.

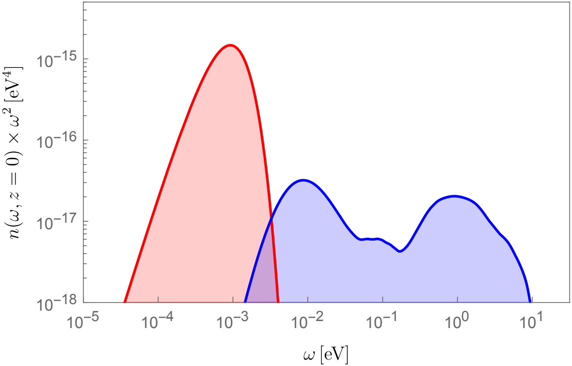
Thanks for your attention

Extra slides

The **cross section** is proportional to the integral of the squared amplitude, with a constant $\mathcal{K}$ which only depends on the initial *free states*

$$\sigma \propto \underbrace{\int [d\mathcal{PS}]\, |\mathcal{A}|^2}_{\doteq\, \mathcal{F}} \quad \to \quad \sigma = \frac{1}{\mathcal{K}} \times \mathcal{F}\,. \tag{24}$$

The deformed transformations lead to **relative locality**.



Gamma rays around TeV can interact with the **EBL** [Saldana-Lopez:2020qzx]. For those around PeV, the **CMB** dominates.

$\Lambda/\text{eV} = 10^{12}$