Beyond GR Tests with the Einstein Telescope

IFAE Seminar

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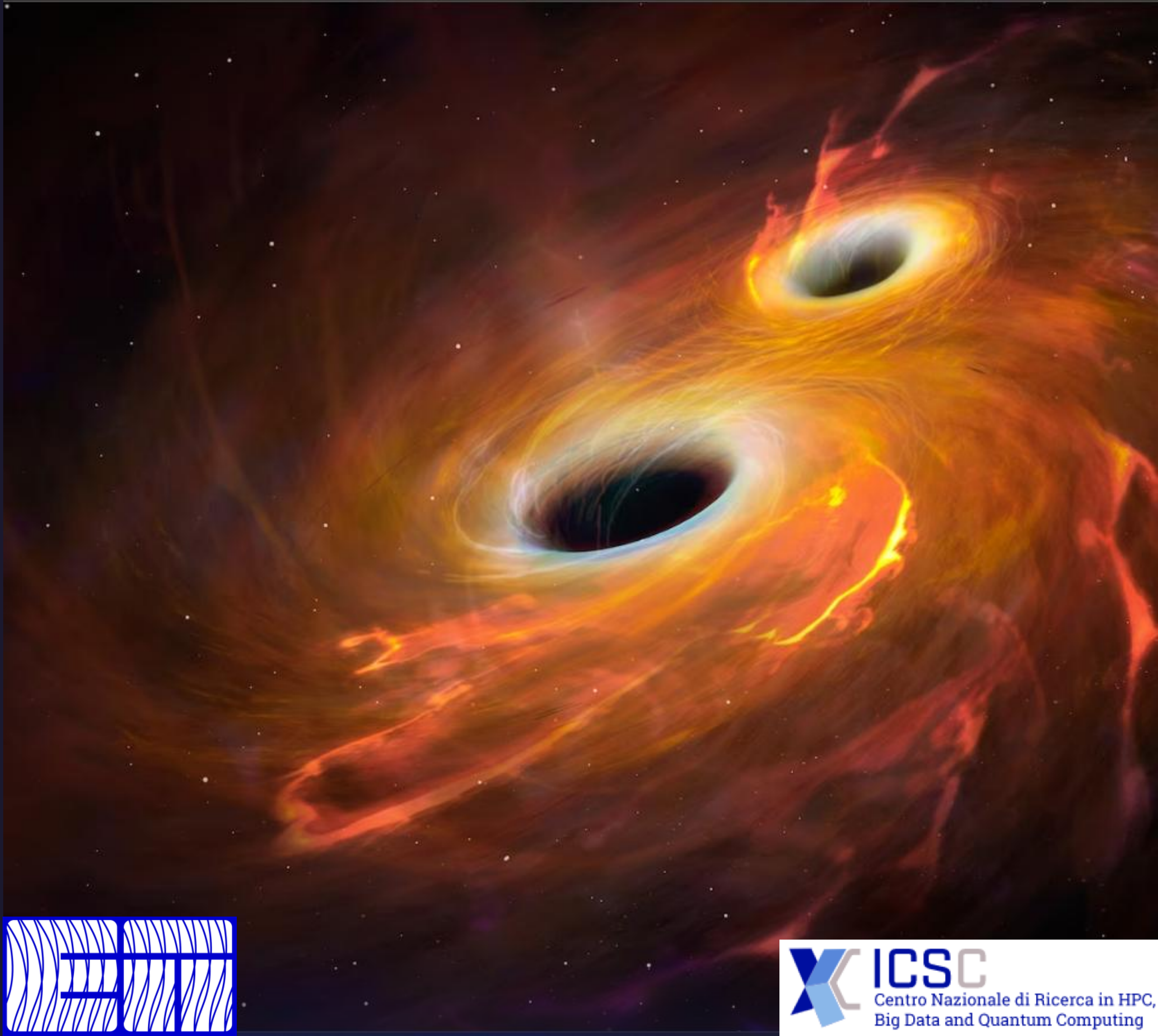
13/6/25

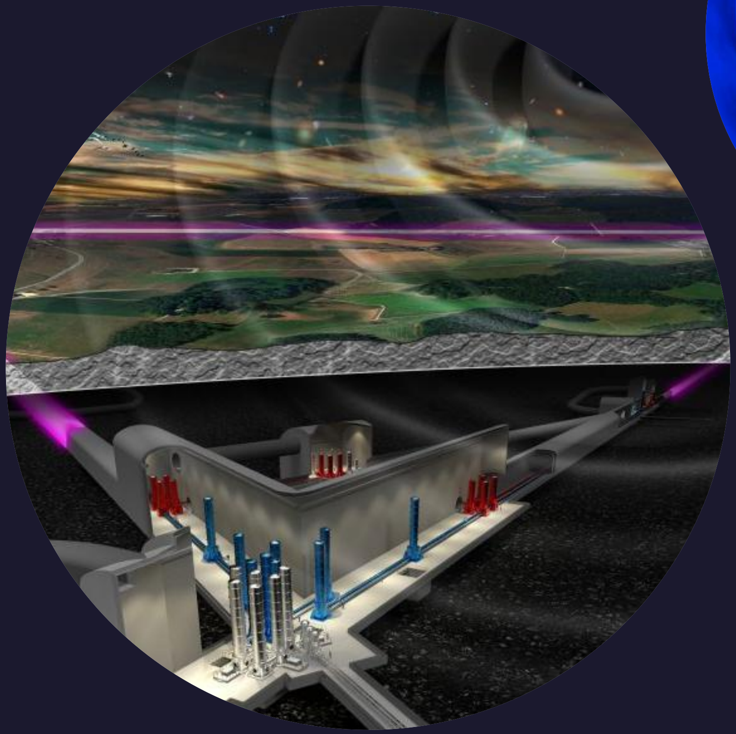


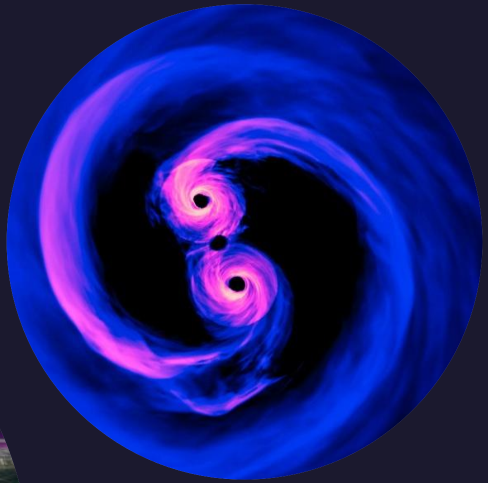
ICSC
Centro Nazionale di Ricerca in HPC, Big Data and Quantum Computing

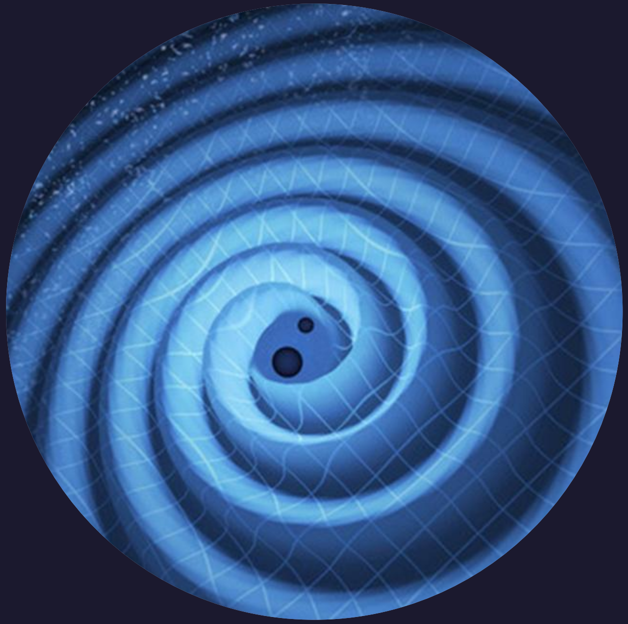
Collaborators

All the work presented was done in collaboration with:

- Stefano Anselmi, Unipd
- Walter Del Pozzo, Unipi
- Matteo Pegorin, Unipd
- Mauro Pieroni, IEM
- Joachim Pomper, Unipi
- Alessandro Renzi, Unipd
- Angelo Ricciardone, Unipi



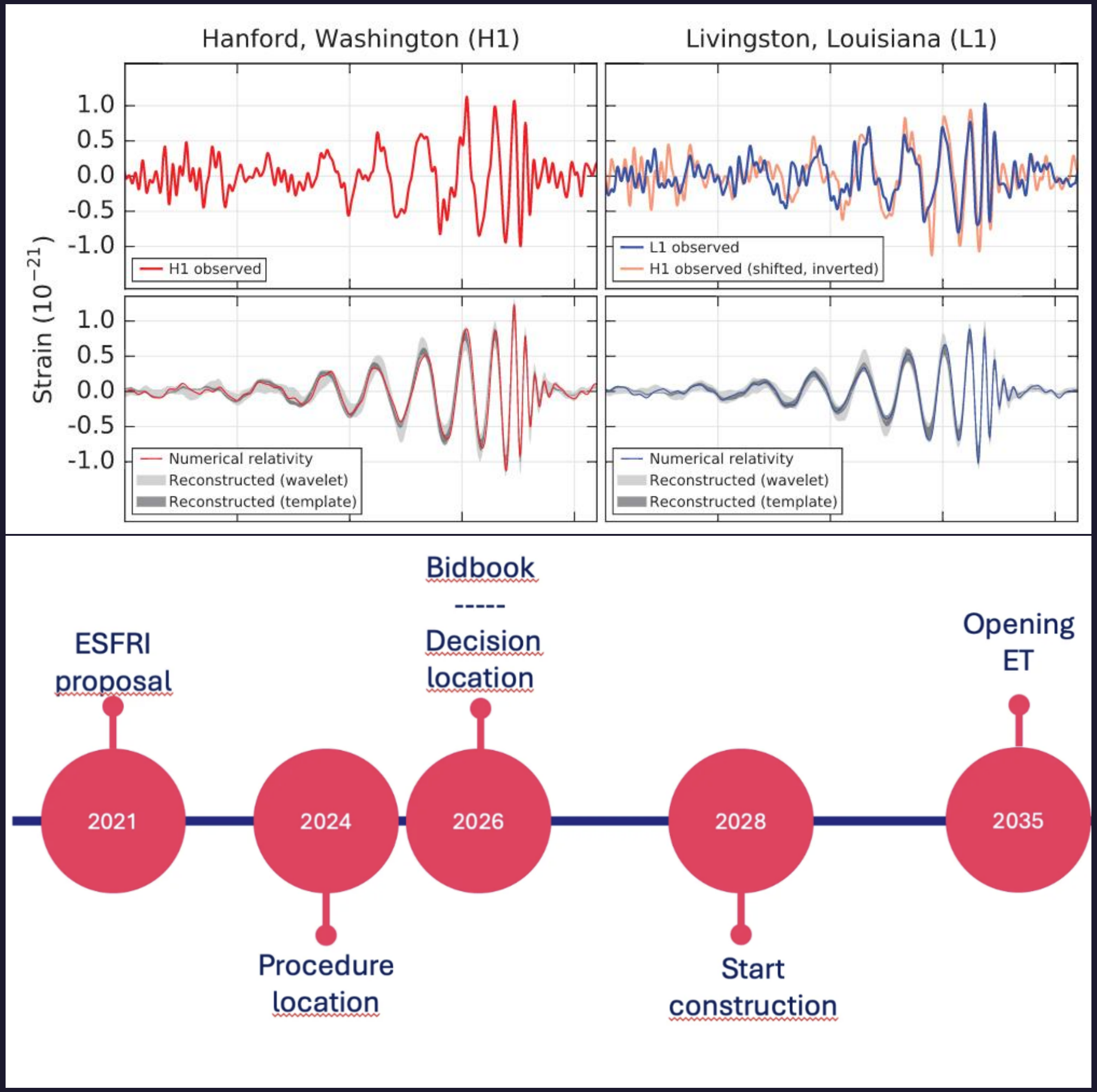




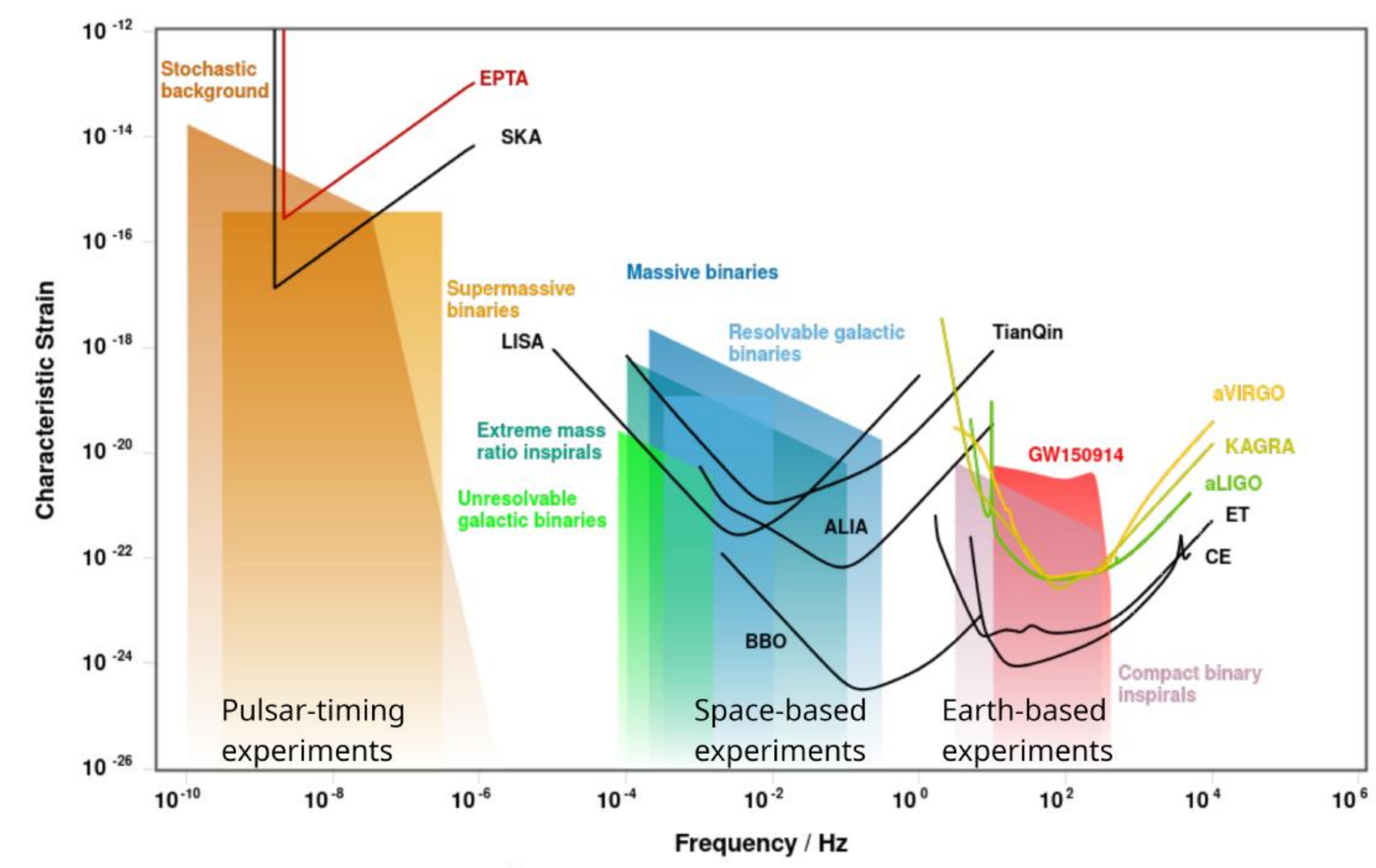
The dawn of GW astronomy

- Are we getting closer to noon? -

- The first GW detection was almost 10 years ago
- During the O4 run from Ligo Virgo Kagra (LVK) detected more than 200 sources in 2 years
- PTA discovery of a Stochastic GW background (SGWB)
- ET procedure location is ongoing
- LISA entered the hardware implementation phase (B2)



Present and future GW experiment



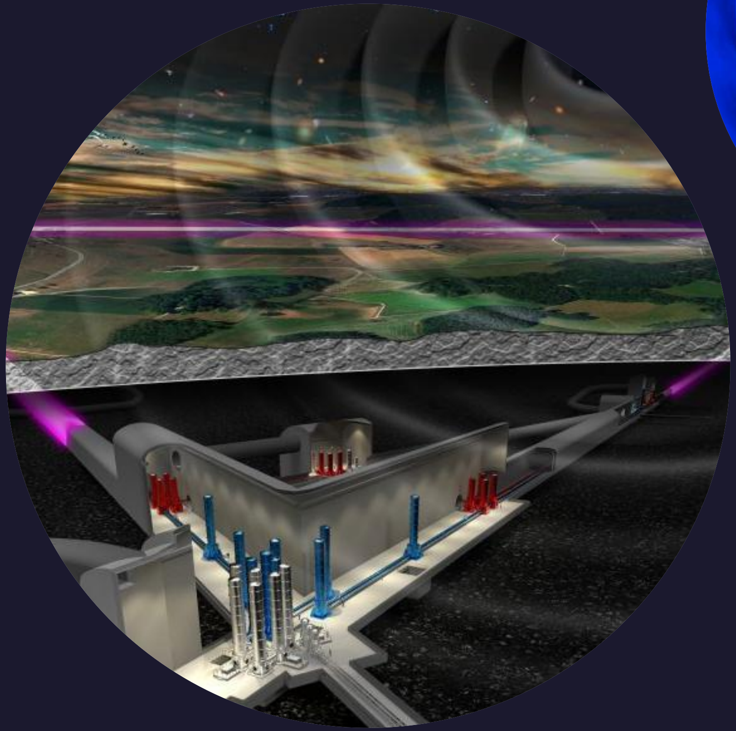
* Figure adapted from GWPlotter

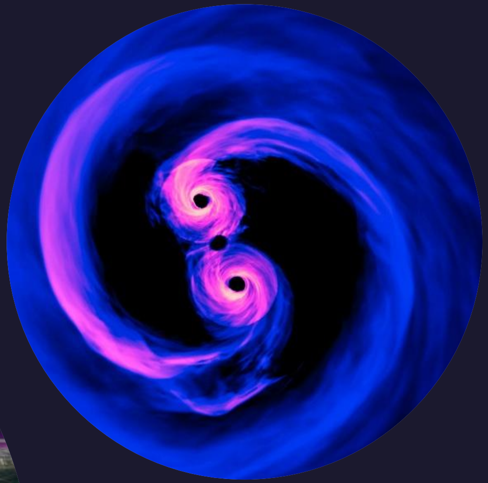
Different types of detectors will probe different frequency bands (and sources).

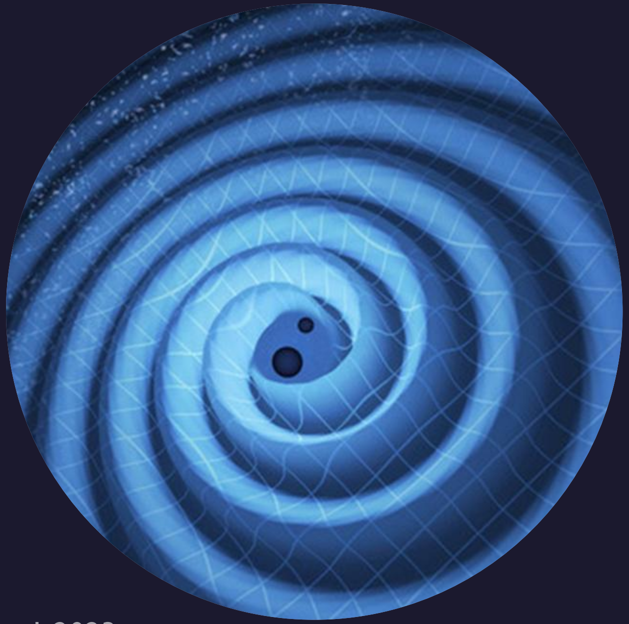
High-frequency GW experiments are missing from the plot

Einstein Telescope

- ET will be part of the third generation of ground-based interferometers
- Will be operative in the late 2030s
- The design is still under scrutiny (triangular vs 2L) [1-2]



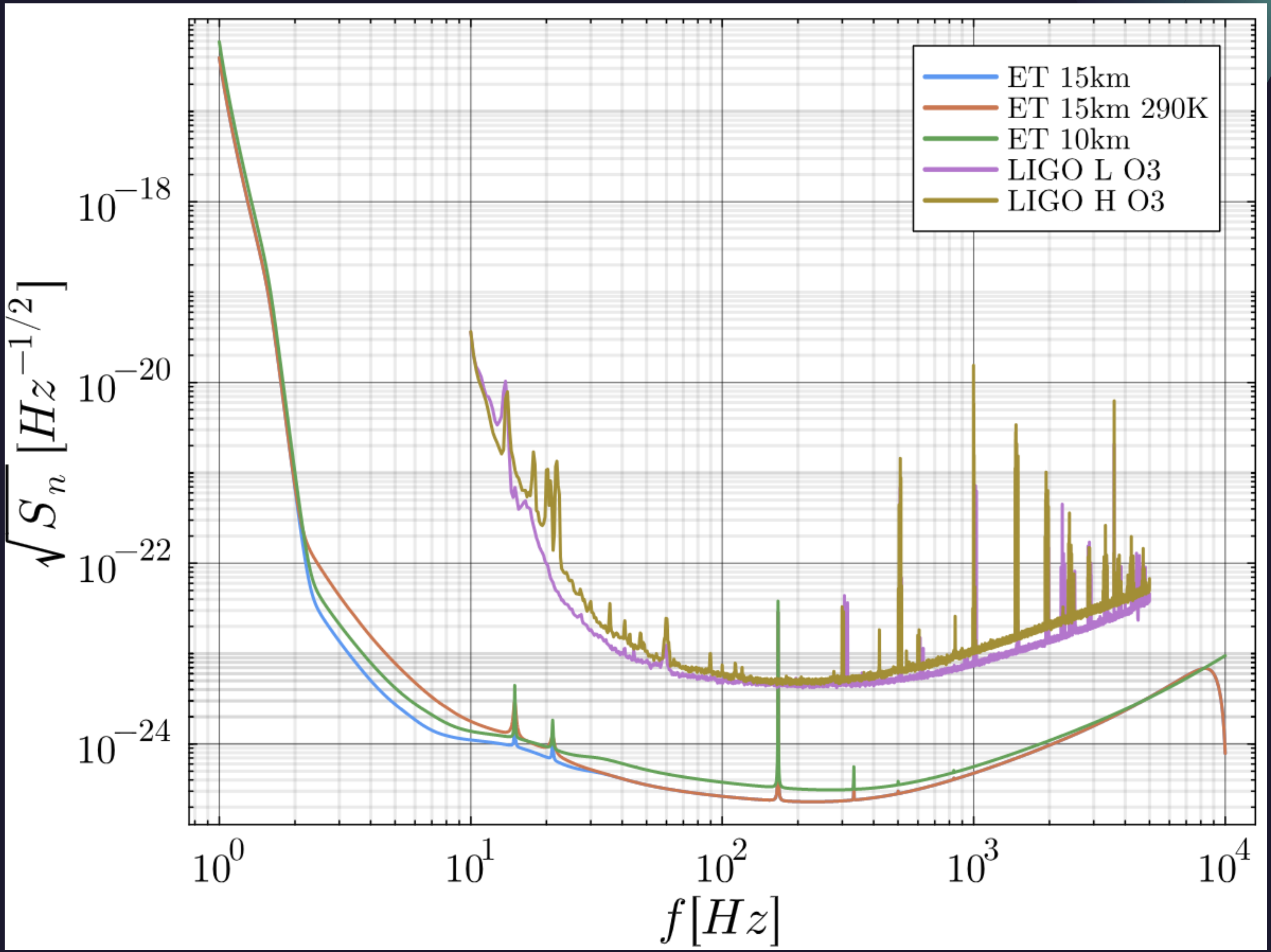




[1] Branchesi et al. 2023
[2] Abac et al, 2025

What ET brings to the table

- Larger frequency space
- Detections up to 50k BBH per year
- Detections up to $z \sim 10$
- SGWB: $\Omega_{\mathrm{GW}} \sim 10^{-12}$



Waveforms

- The measured data consists of the GW signal plus a noise

$$d(t) = s(t) + n(t)$$

- Where the signal is the projection of the strain onto the detector

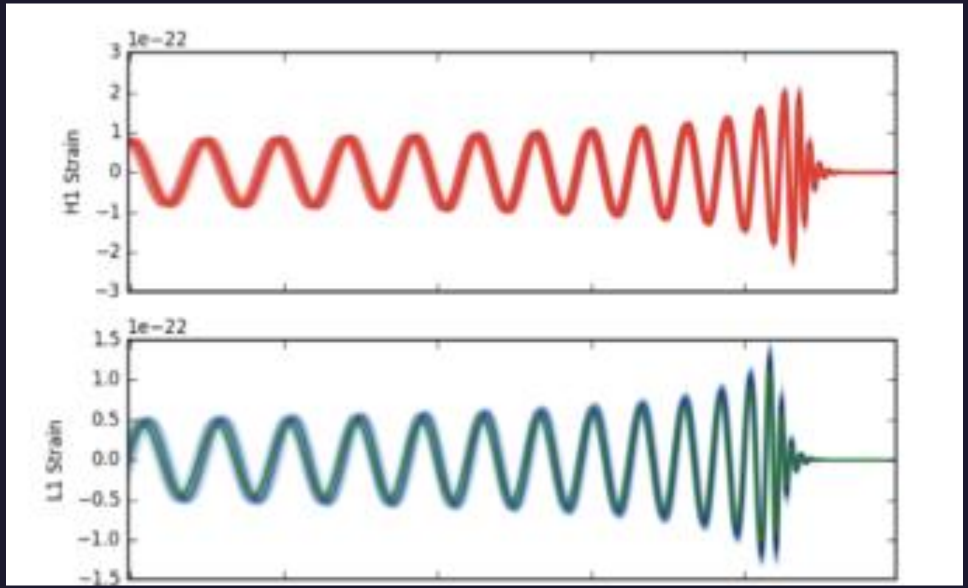
$$s = h_+ e_{+ij} D^{ij} + h_\times e_{\times ij} D^{ij} \equiv h_+ F_+ + h_\times F_\times$$

- A waveform is a function that associates to the source parameters the, in time or frequency domain

- Here we use the IMRPhenom waveforms [3-8] (up to XHM) that work in the frequency domain.

$$h = \sum_k A_k(f) e^{i\Phi(f)}$$

[3] Khan et al. 2015
[4] London et al. 2018
[5] García-Quirós et al. 2020
[6] Pratten et al. 2020a
[7] Pratten et al. 2020b
[8] Dietrich et al. 2019

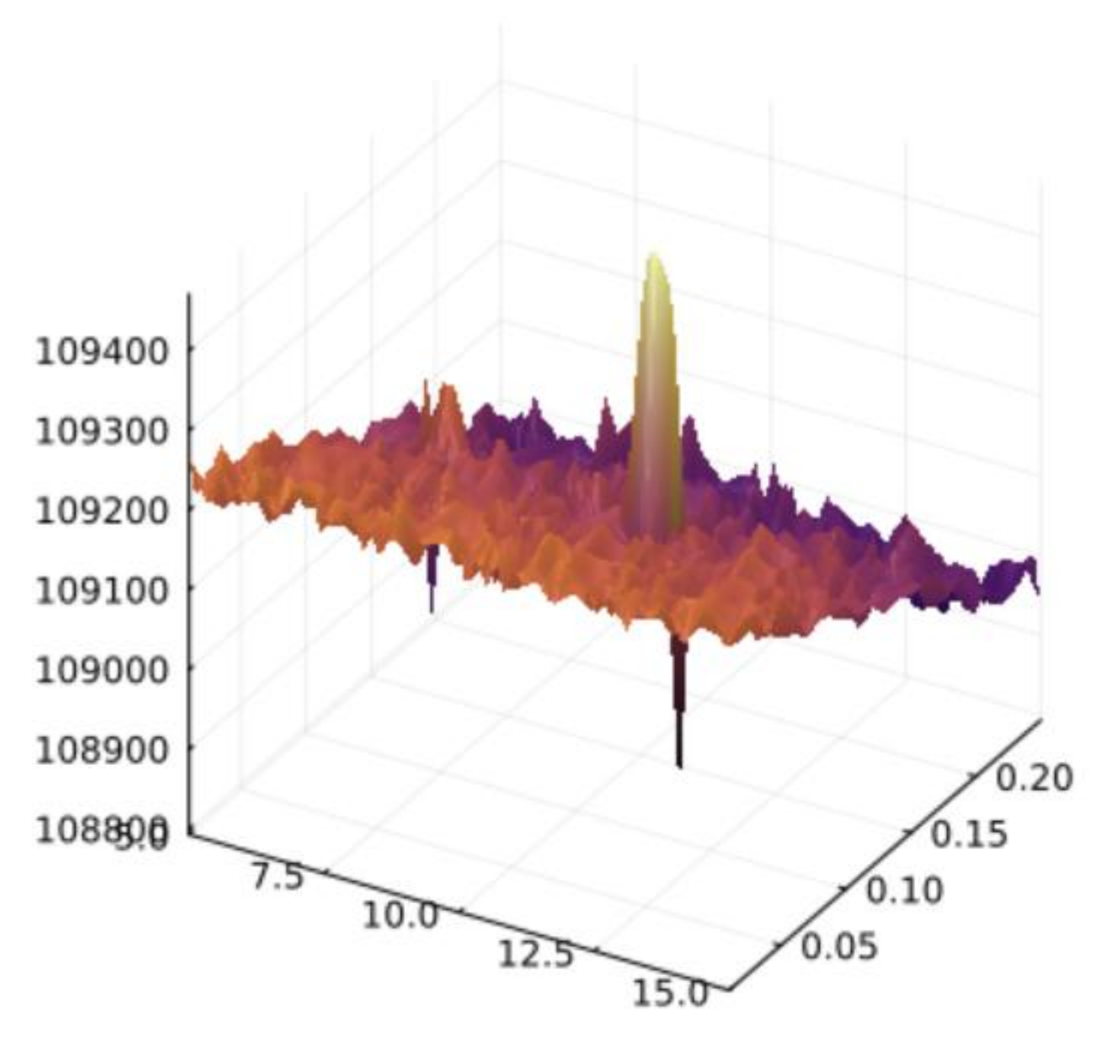


Bayesian analysis

- The gaussian likelihood is expressed as

$$\mathcal{L}(d \mid \boldsymbol{\theta}) \propto \exp\{-(d - s(\boldsymbol{\theta}) \mid d - s(\boldsymbol{\theta}))/2\}$$

- It is very hard to sample (extremely multimodal and long evaluation)
- In LVK nested sampling is used



Fisher Matrix

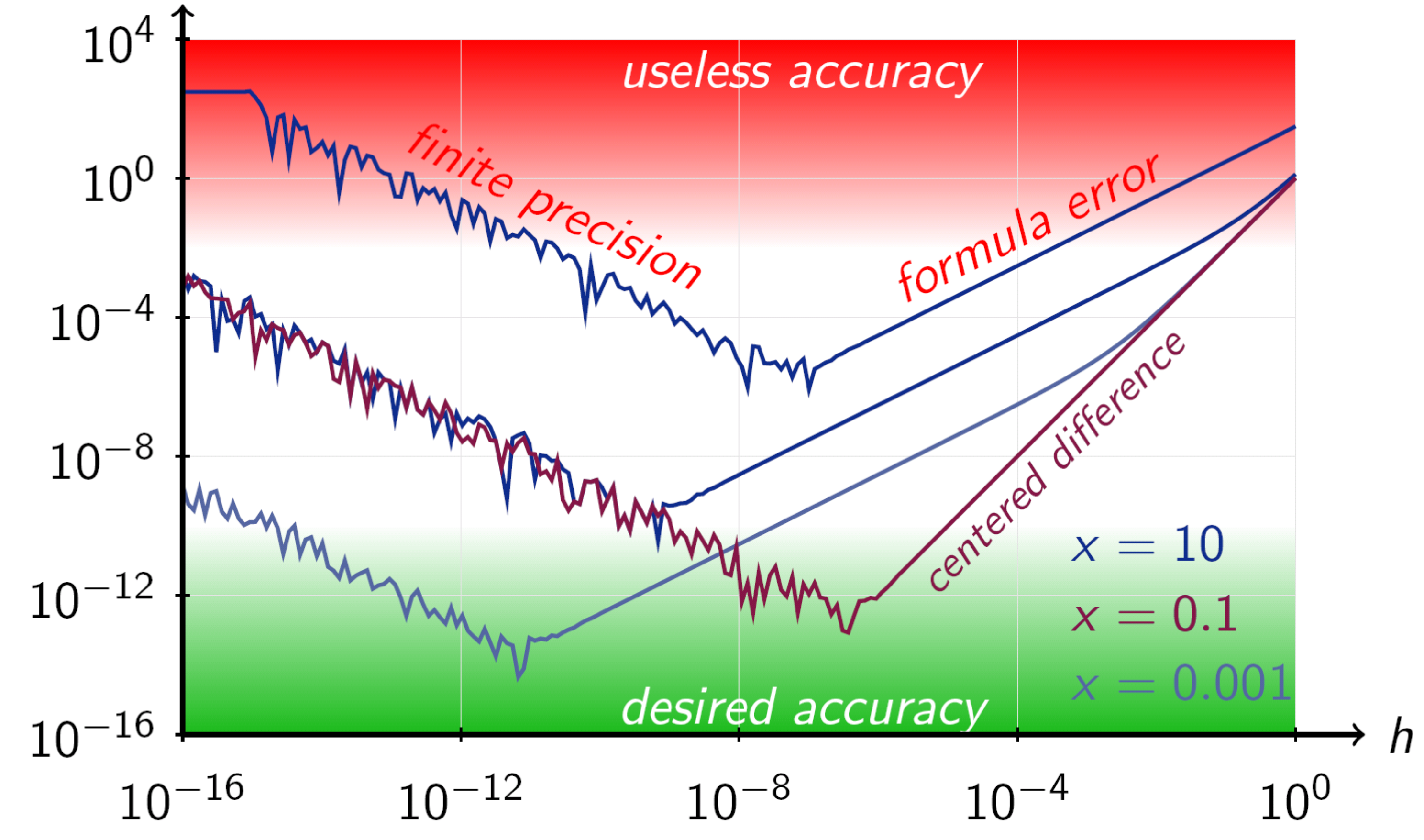
- To compute the Fisher Matrix which is defined as

$$\Gamma_{ab} = -\langle \partial_a \partial_b \log \mathcal{L}(d|\theta) \rangle \big|_{\theta=\theta_0} = (h_a \mid h_b)$$

  where we need to derive the likelihood of the data realization
- Each derivative requires calling the expensive waveform function at least a few times (for numerical diff methods)

Numerical derivatives

- There is an optimal step size to maximize the accuracy
- The step size is usually dependent on both the function and the point where the derivative is calculated
- There are better alternatives to finite difference methods



Credits: Wikipedia

Automatic Differentiations

```julia
x = Dual(1.0, 1.0, 0.)
y = Dual(2.0, 0.0, 1.)
z = x*y + 3y^2
```

✓ 1.6s

```
Dual{Nothing}(14.0,2.0,13.0)
```

Automatic Differentiations

Variable = [value, ∂_xvariable, ∂_yvariable]

$$
\begin{aligned}
\langle u, u' \rangle + \langle v, v' \rangle &= \langle u + v, u' + v' \rangle \\
\langle u, u' \rangle - \langle v, v' \rangle &= \langle u - v, u' - v' \rangle \\
\langle u, u' \rangle * \langle v, v' \rangle &= \langle uv, u'v + uv' \rangle \\
\langle u, u' \rangle / \langle v, v' \rangle &= \left\langle \frac{u}{v}, \frac{u'v - uv'}{v^2} \right\rangle \quad (v \neq 0)
\end{aligned}
$$

Automatic Differentiations

- Accurate (at machine level)
- If a derivative exists, it will find it
- Very fast (2x the evaluation of the target function in our case)
- Adopted from ML

Julia

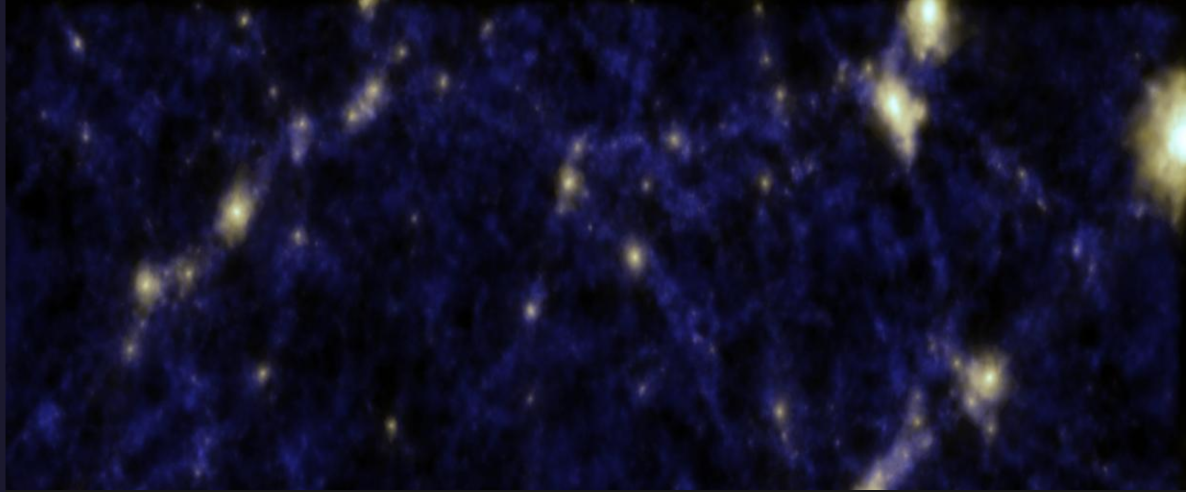
- Compiled (Python is interpreted)
- Designed for scientific computing
- As easy to write as Python

```python
def factorial(n):
    if n == 0 or n == 1:
        return 1
    else:
        return n * factorial(n - 1)

# Example usage
num = 5
print(f"The factorial of {num} is {factorial(num)}")
```

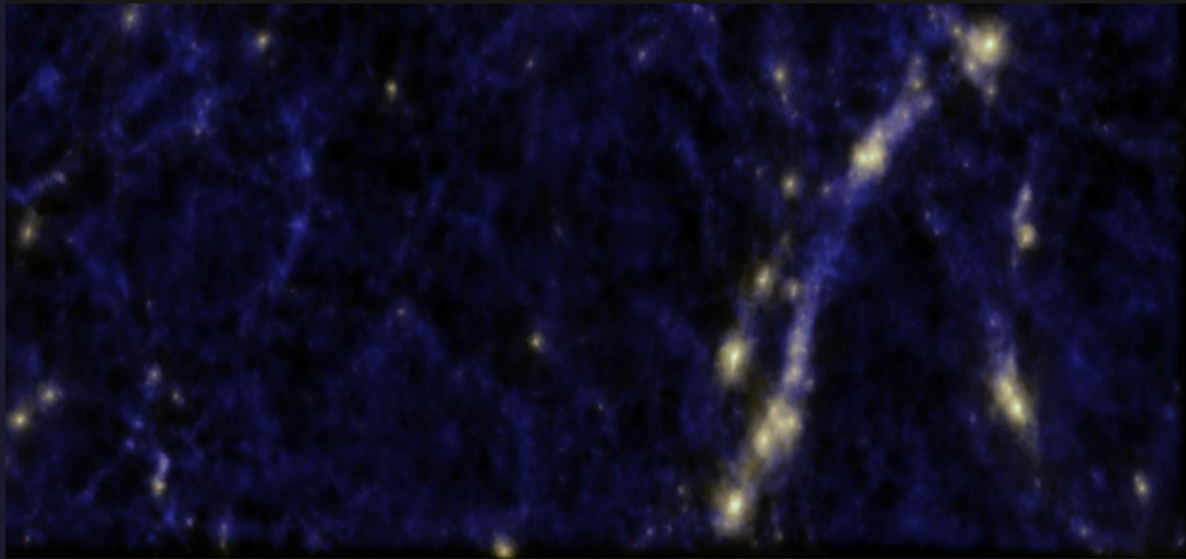
```julia
function factorial(n)
    if n == 0 || n == 1
        return 1
    else
        return n * factorial(n - 1)
    end
end

# Example usage
num = 5
println("The factorial of $num is $(factorial(num))")
```

GWJulia

- Easy to learn
- Using automatic differentation (superior to numerical differentation)
- Features advanced waveforms (e.g. PhenomXHM, PhenomD), all written in Julia
- Very fast, e.g., using XHM 3 detectors ~ 0.5 sec per core)
- Does not rely on external packages (all is written in Julia)

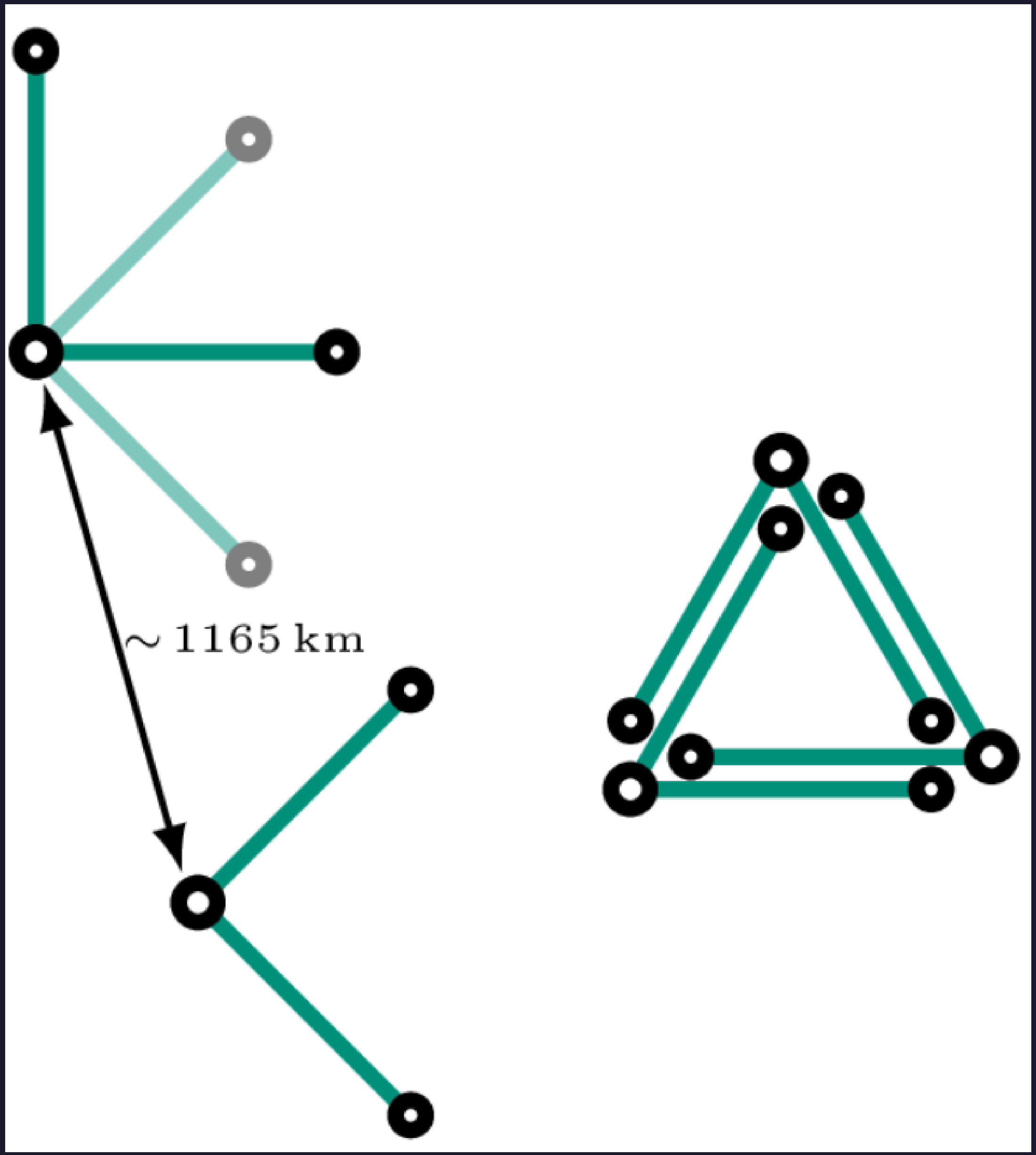


```julia
using Pkg
Pkg.activate("/home/begnoni/GW.jl")
using GW

parameters =
GenerateCatalog(1_000, "BBH");

FisherMatrix(PhenomD(), [CE1Id, CE2NM, ETS], parameters...);
```



Case study – Detector design comparison

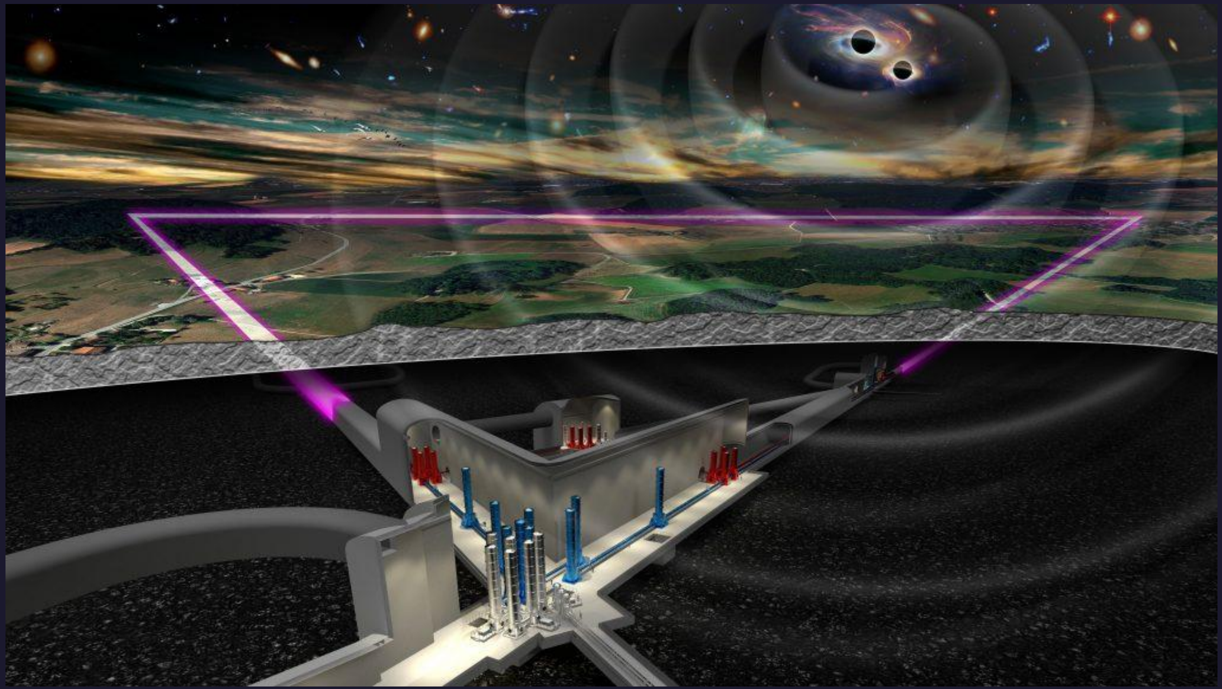
- T: triangular ET with 10 km arms featuring cryogenic technology.
- 2L_0: two aligned 15 km L-shape interferometers one in Sardinia, one in the Meuse–Rhine (MR) Euroregion, both with the cryogenic technology.
- 2L_45: same as 2L_0 with the exception that the orientations lead to β = 45◦.
- 2L_290K_0: same as 2L_0, only one detector features the cryogenic technology.
- 2L_290K_45: same as 2L_45, only one detector features the cryogenic technology.



Credits: [1]

[1] Branchesi et al. 2023
[2] Abac et al, 2025

Detector design comparison - What to expect



What are we interested in?

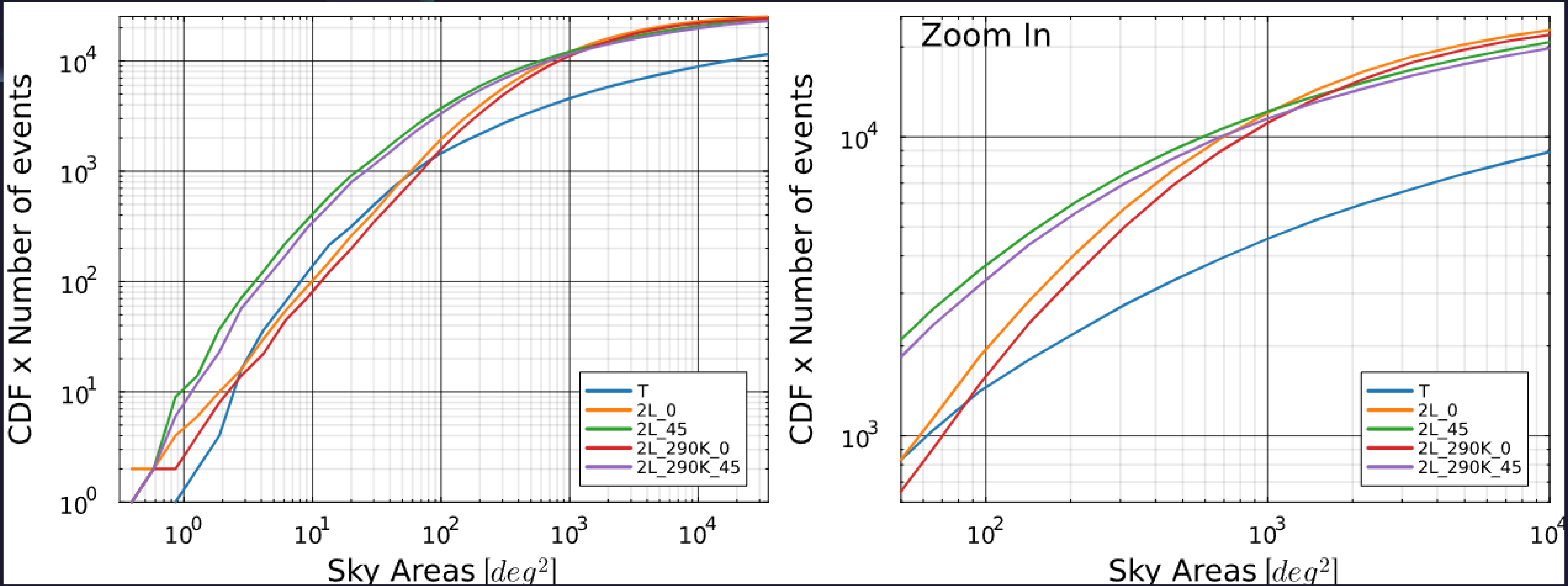
- Cosmology with CBC
- Multimessanger with CBC
- Source properties of CBC
- SGWB

Detector preferences: [1,2]

- CBC : 2L_45 > 2L_0 > T
- SGWB : T ≈ 2L_0 > 2L_45

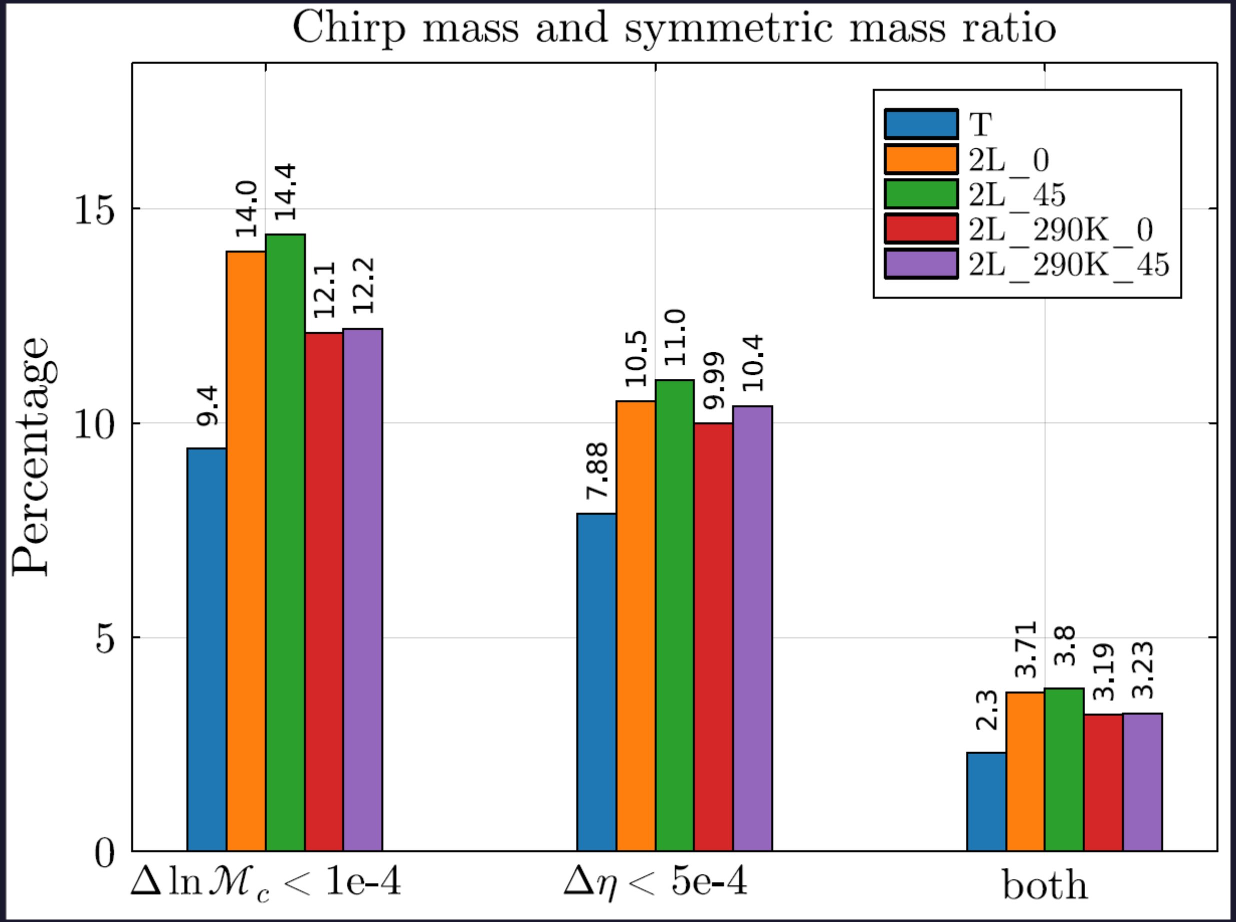
[1] Branchesi et al. 2023
[2] Abac et al. 2025

Case study – Angular Precision



Cumulative density function times the number of events as a function of the 90% sky area. *Right*: zoom in of the left plot

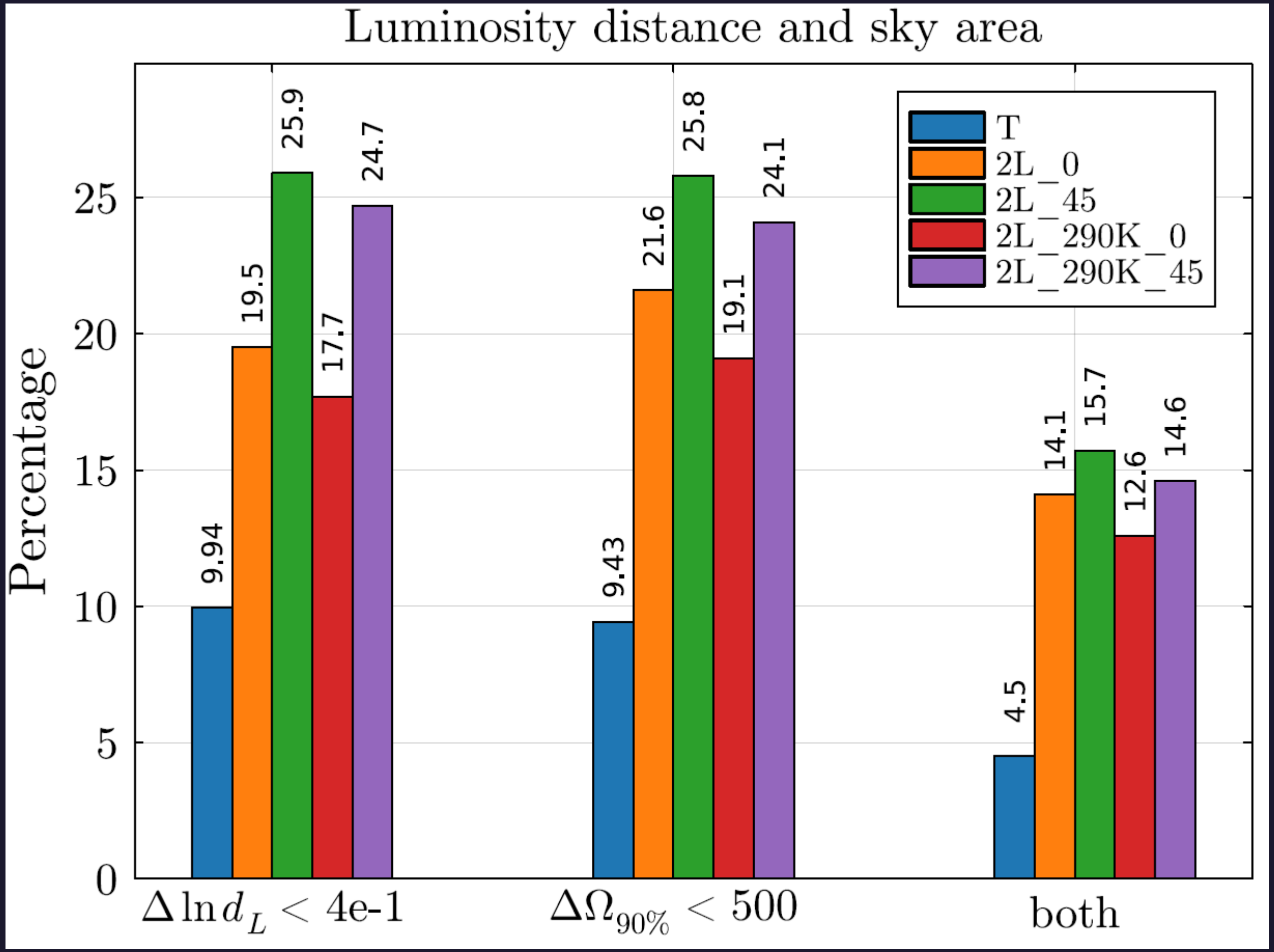
Source properties



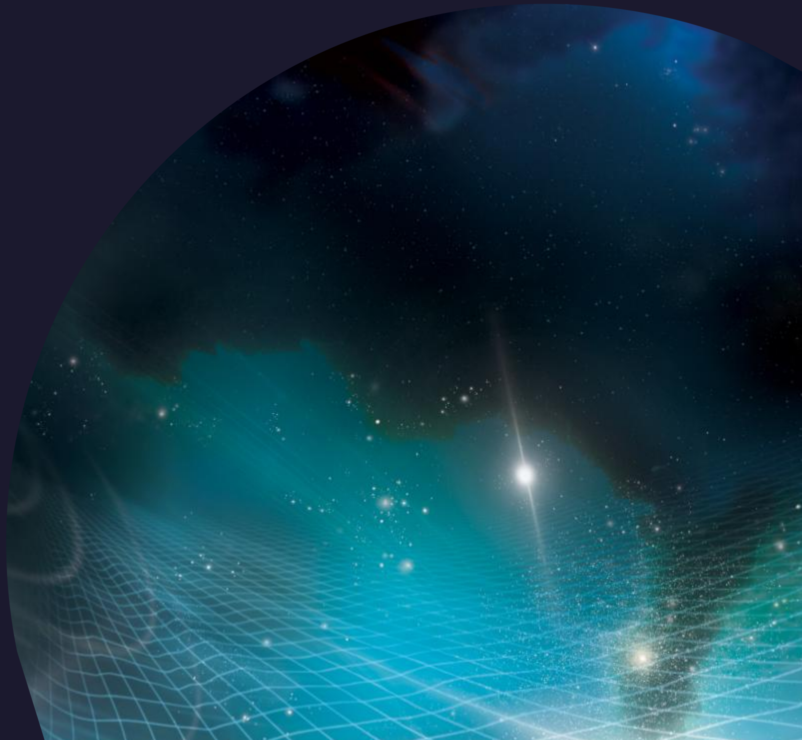
The plot represents the percentage of events of which the 1-sigma error is below 1e-4 for the detector chirp mass and 5e-4 for the symmetric mass ratio. Only $\sim 1/3$ of sources that satisfy the mass ratio requirement satisfy both requirements.

$$\mathcal{M}_{c,source} = \frac{m_1^{3/5} m_2^{3/5}}{(m_1 + m_2)^{1/5}} \qquad \eta = \frac{m_1 m_2}{(m_1 + m_2)^2}$$

Cosmology



The advantage of the 2L_45 over 2L_0 in the luminosity distance and in the sky area is erased when we require both requirements to be satisfied at the same time.



Future plans – Fisher informed HMC

- Bayesian data analysis of CBC is very computationally demanding (e.g., using nested sampling)

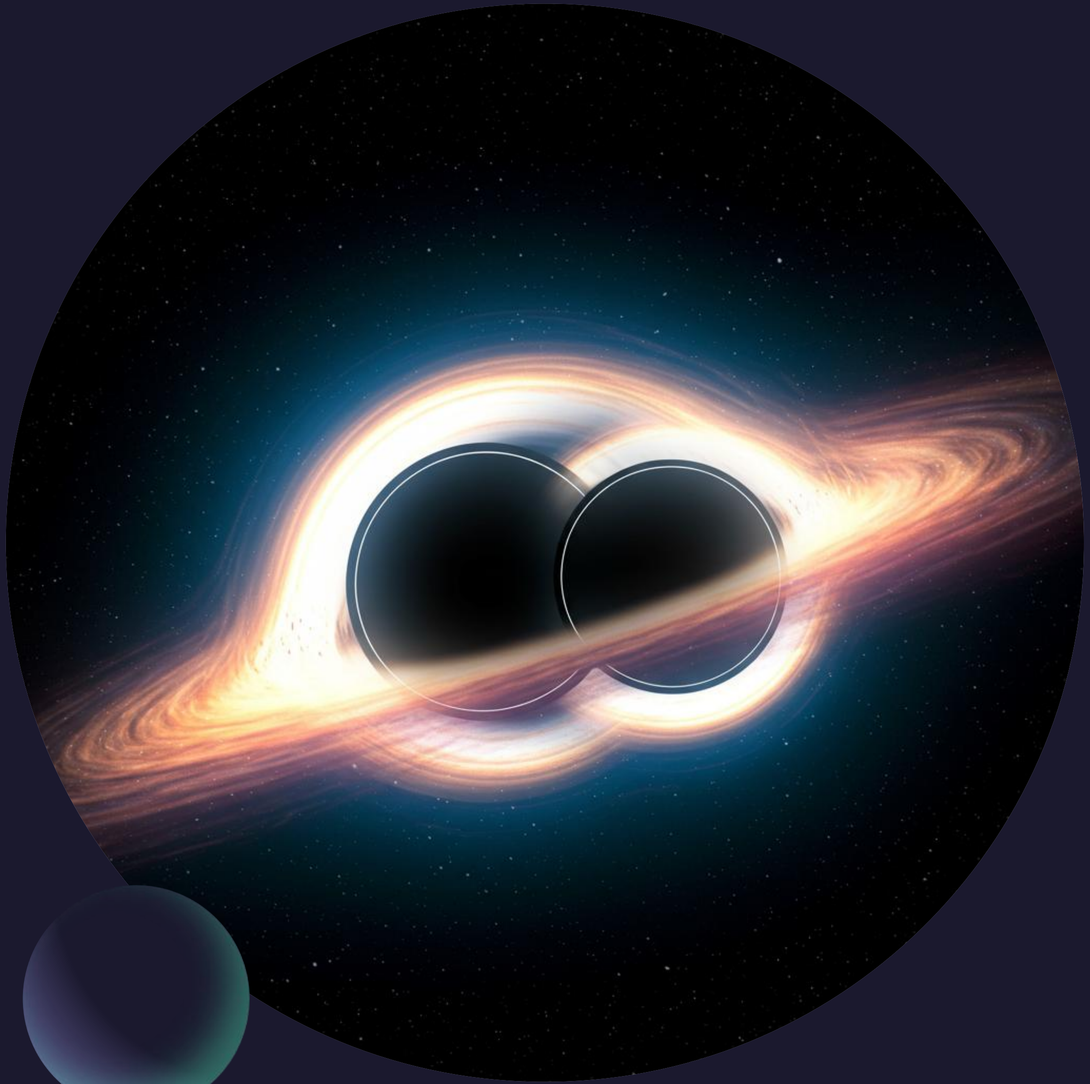
- If we are interested in future observations, we can simplify the inference problem, i.e., we know an approximate location of the maximum posterior

- With automatic differentiation we can perform fast and accurate gradients

- We can perform Hamiltonian Monte Carlo sampling, injecting information from the Fisher Matrix

Beyond GR

- GW from CBC can be important test of GR
- Many possible aspects of GR can be tested, e.g., tidal deviations, spin-induced quadrupole moment
- In this work, we focus on the Post-Newtonian terms



Post-Netwonian expansion

- Approximation valid for weak gravitational fields and low velocities
- Used for the inspiral phase of comparable mass CBC
- Expansion in terms of (v/c)
- Analytical GR predictions for

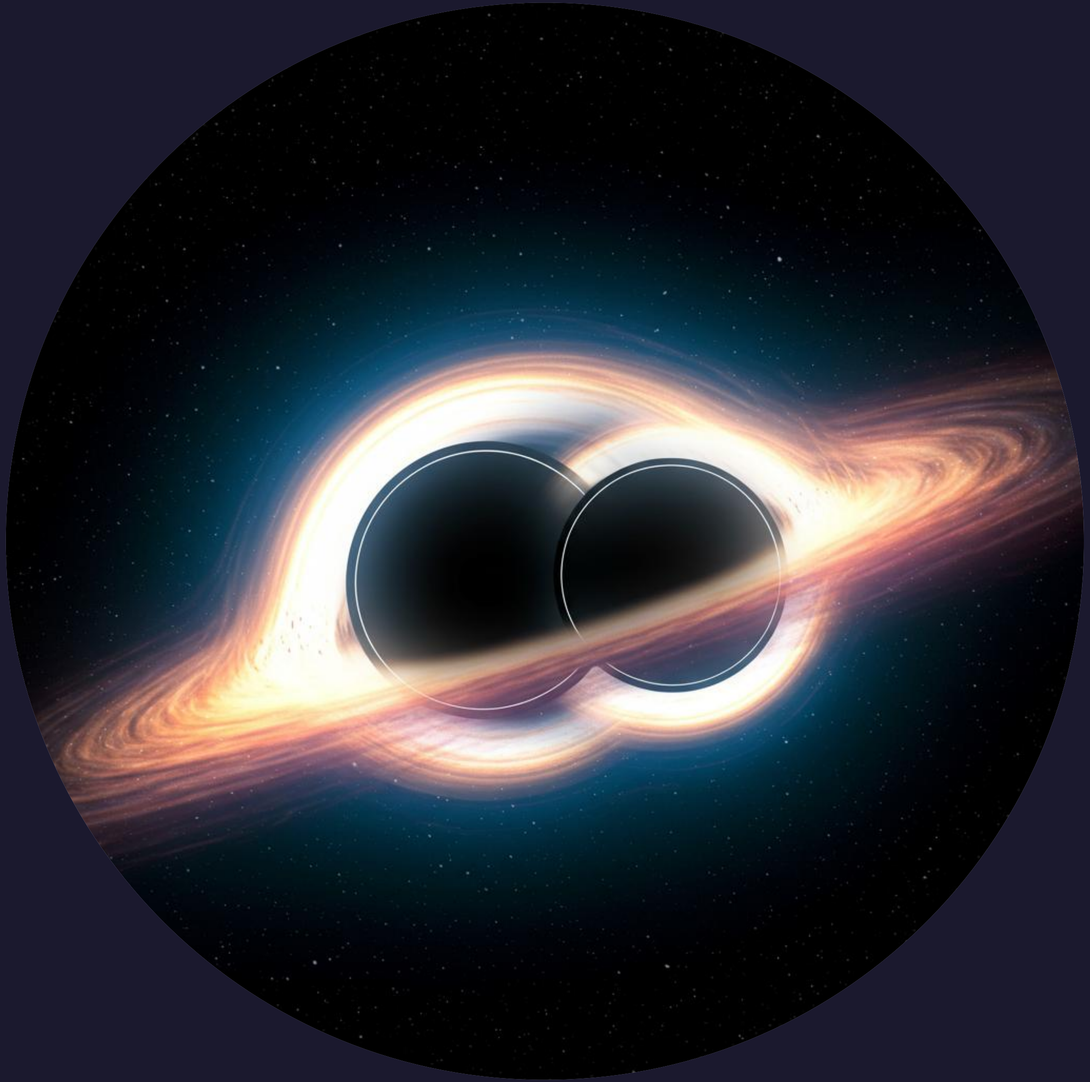
$$\{\varphi_0, \varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_{5l}, \varphi_6, \varphi_{6l}, \varphi_7\}$$

where the index represents two times the PN order

Post-Netwonian terms

- In frequency domain the phase of a GW can be expressed as

$$\Phi(f) \propto \sum_{j=0}^{7} \left[\varphi_j + \varphi_j^{(l)} \ln f\right] f^{(j-5)/3}$$



TIGER framework

In the TIGER framework[9], *one PN term at a time* is modified as

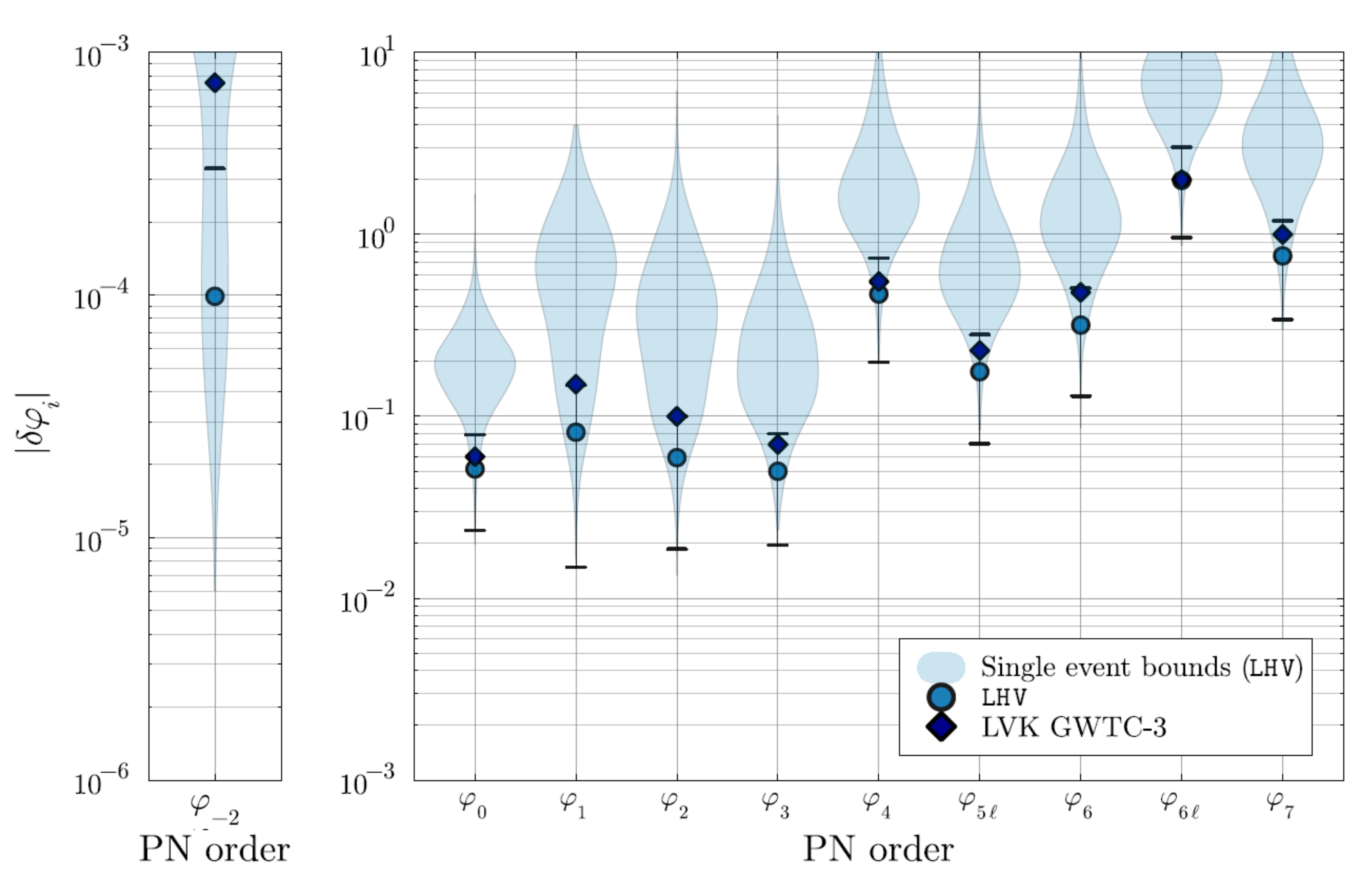
$$\varphi_k^{\mathrm{GR}} \to (1 + \delta\varphi_k)\,\varphi_k^{\mathrm{GR}}$$

We modified accordingly the waveforms PhenomD and PhenomHM, ensuring that they remain C^1.

[9] Agathos et al. 2014

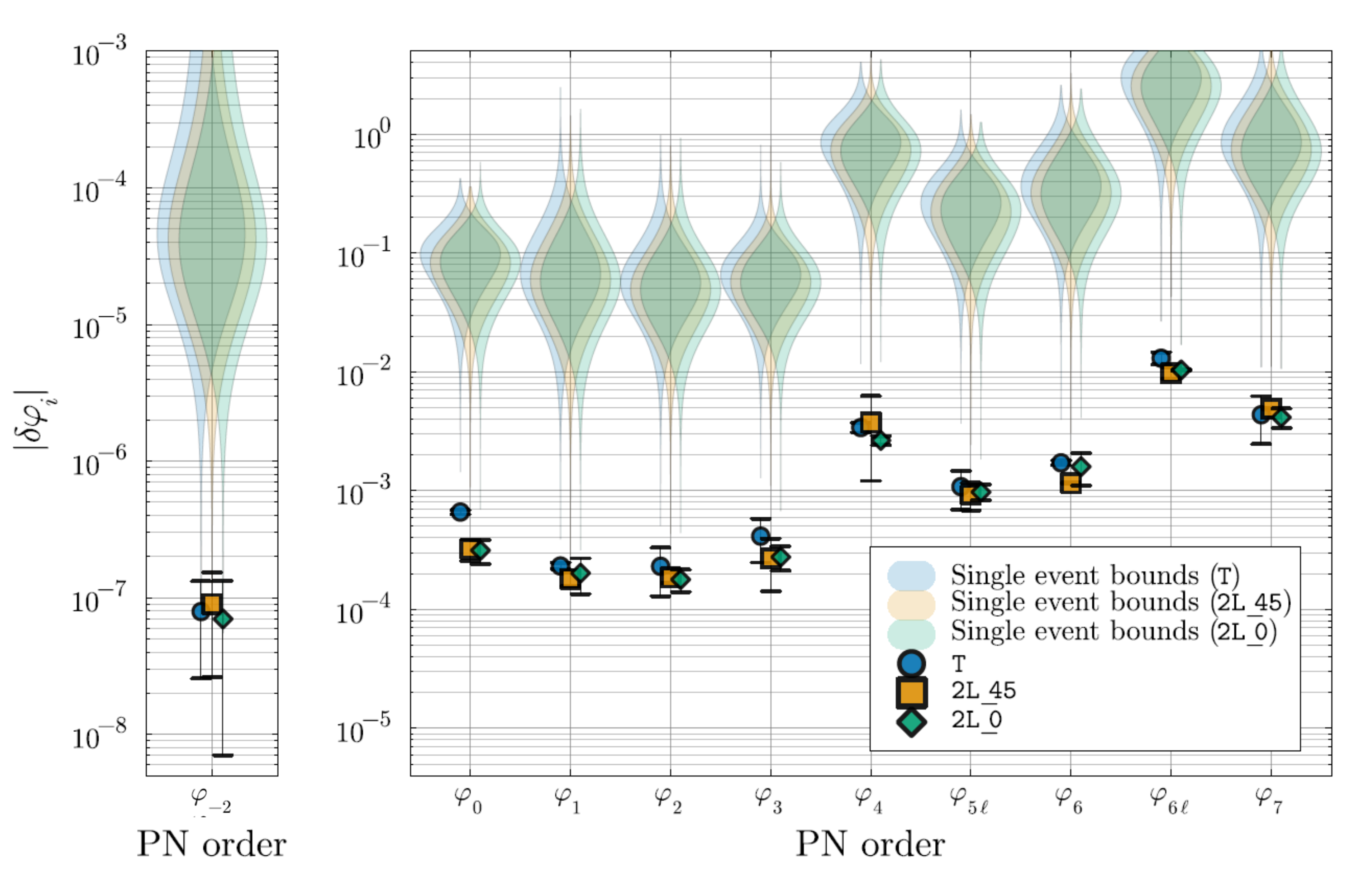
Comparison with LVK

- We test the 90% upper bound from the Fisher analysis with the actual LVK results. The events analyzed are 12 and the different realizations give the error bars. The violin plots are obtained by the single events bounds.
- LHV indicates that the detectors used are Livingston, Hanford and Virgo.
- There is great agreement between the Fisher and the collaboration results



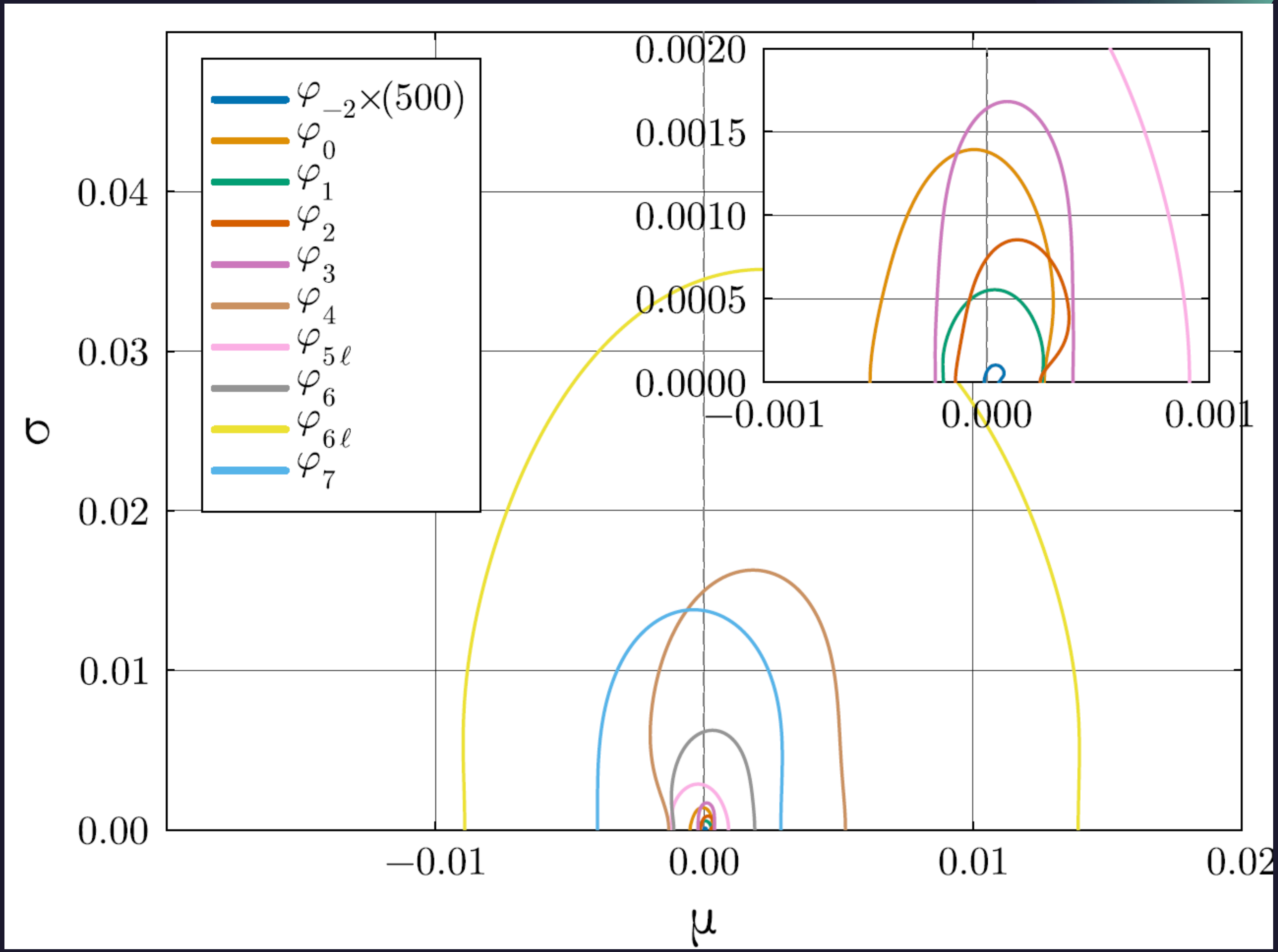
Forecast for ET

- We test 3 detector designs for ET
- All the detectors improve significantly the LVK constraint and the difference is inside the error bars
- These results corresponds to 5/6 months of observations



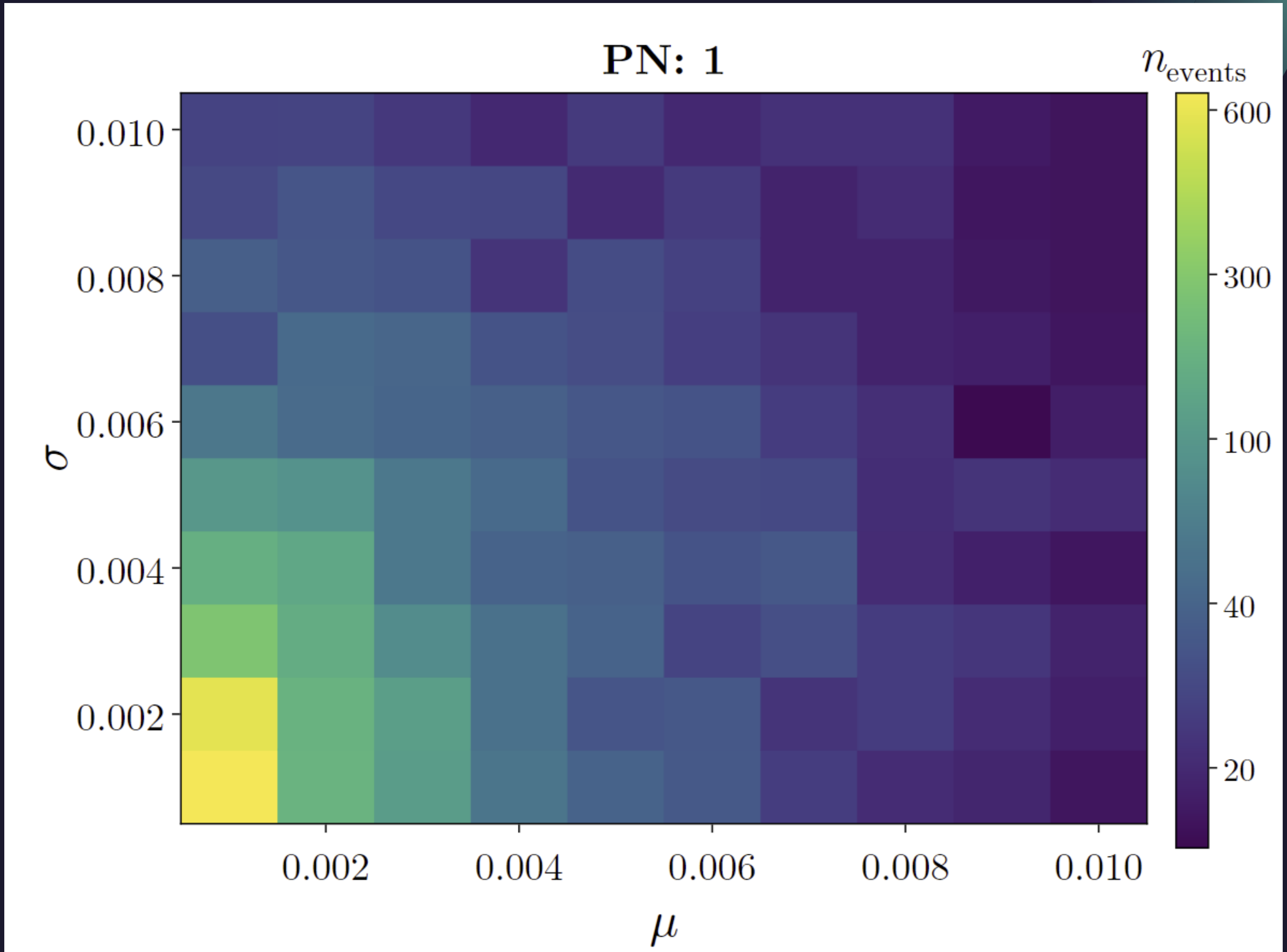
Forecast for ET

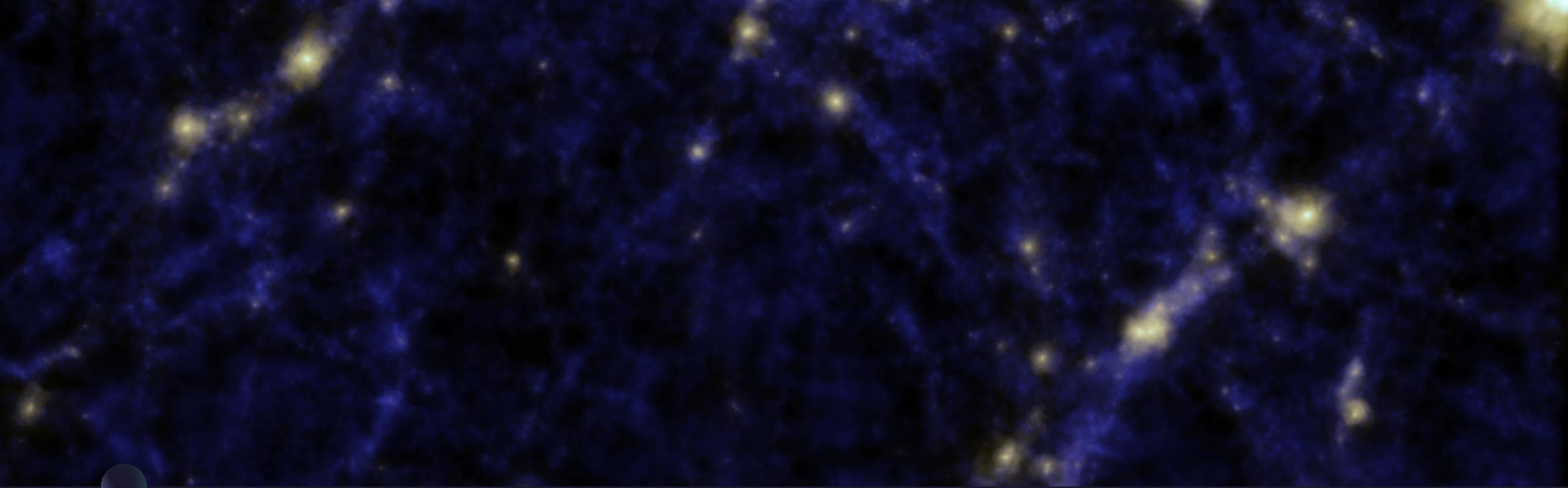
- We model the deviation as a gaussian with mean μ and std σ.
- We inject events with no GR deviations, and we recover the hyperparameters (μ, σ).
- These results corresponds to 4/5 months of observations



Forecast for ET

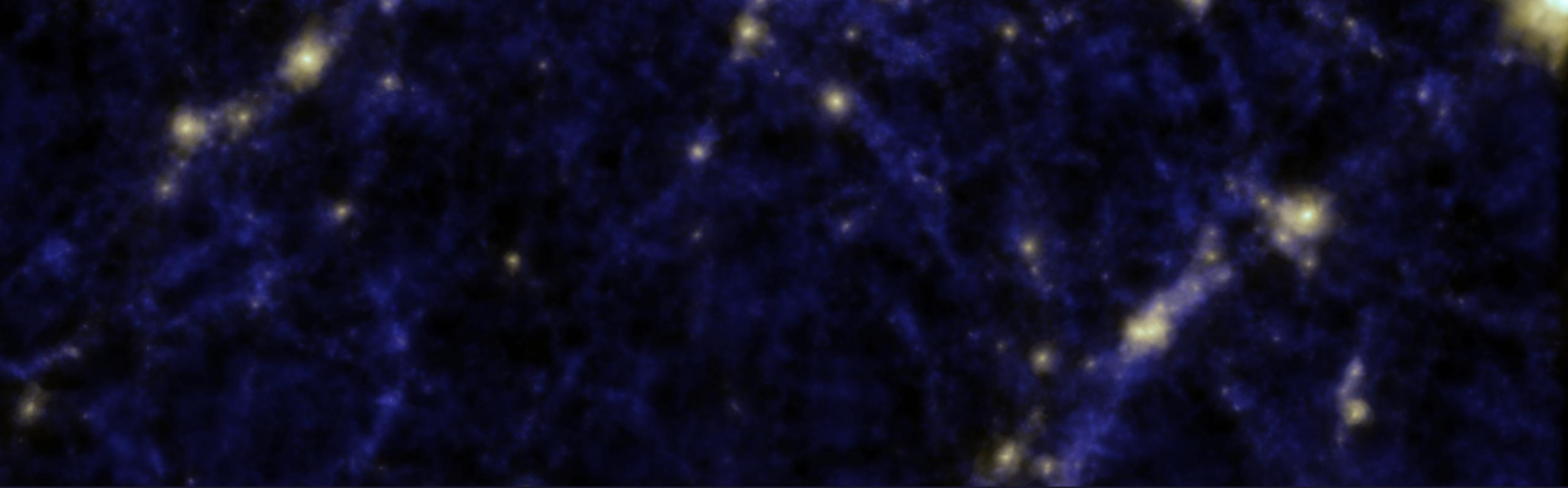
- How many detections are needed to detect a GR deviation at 90% confidence level?
- We inject events with GR deviations drawn from a gaussian with given μ and σ
- These constrains can be achieved in weeks or days of observations
- We used 2L_45 and the PN 1 term





Take home messages

- GWJulia is a fast open-source tool to evaluate Fisher matrices of CBC
- 2L_45 is the best network for CBC with a slight margin over 2L_0, at the cost of losing SGWB
- With ET, BGR test of the PN terms will reach an unprecedented level of precision with just a handful of sources

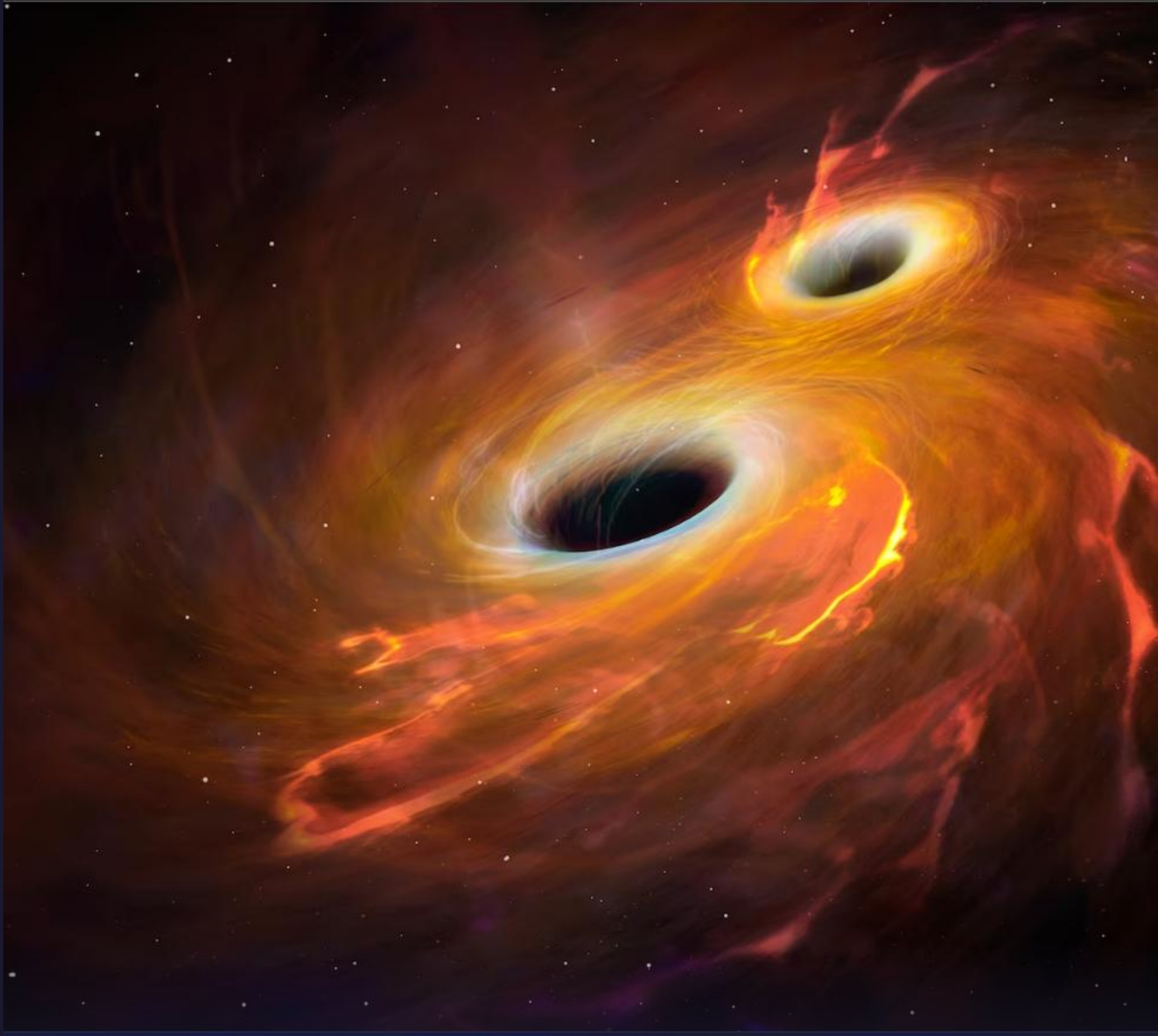


Future prospects

- We aim to do a comprehensive study of the capabilities of ET considering the different comsological science cases

- We plan to use the Fisher Matrix to inform a HMC sampler

Thank you!



Case study – SNR

Network	SNR > 8	SNR > 12	SNR > 20	SNR > 50	SNR > 100
T	87.6 %	71.1 %	43.3 %	9.1 %	1.7 %
2L_0	89.3 %	78.6 %	56.5 %	15.7 %	3.6 %
2L_45	94.1 %	82.9 %	58.1 %	15.6 %	3.5 %
2L_290K_0	87.4 %	75.5 %	52.3 %	13.4 %	2.9 %
2L_290K_45	92.3 %	79.6 %	53.5 %	13.3 %	2.8 %

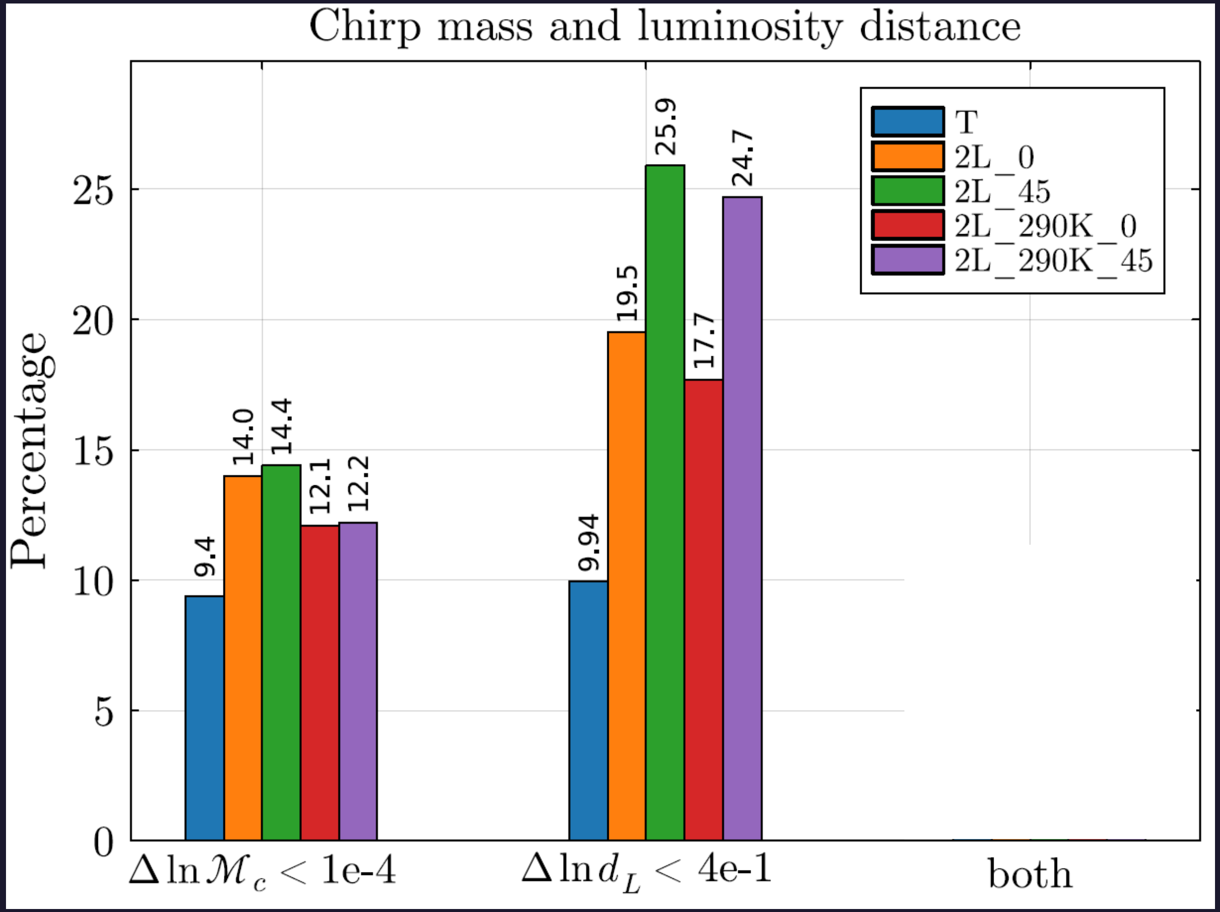
The table represents the percentage of events above the threshold written on top of the column

Case study – Angular Precision

Network	$\Omega < 10\ \text{deg}^2$	$\Omega < 100\ \text{deg}^2$	$\Omega < 1000\ \text{deg}^2$	$\Omega <$ Whole sky
T	0.31 %	3.86 %	12.65 %	34.24 %
2L_0	0.23 %	4.68 %	32.91 %	75.43 %
2L_45	1.01 %	9.63 %	34.04 %	72.22 %
2L_290K_0	0.18 %	3.79 %	30.14 %	72.49 %
2L_290K_45	0.81 %	8.67 %	32.18 %	68.72 %

The table represents the percentage of events of which the 90% sky areas are less than the threshold indicated in each column. The 45° networks outperform the other networks and the T configuration. In particular, the T is comparable when considering the few high-precision sources, however, the performance degrades significantly for sources with precision worse than 1000 deg^2.

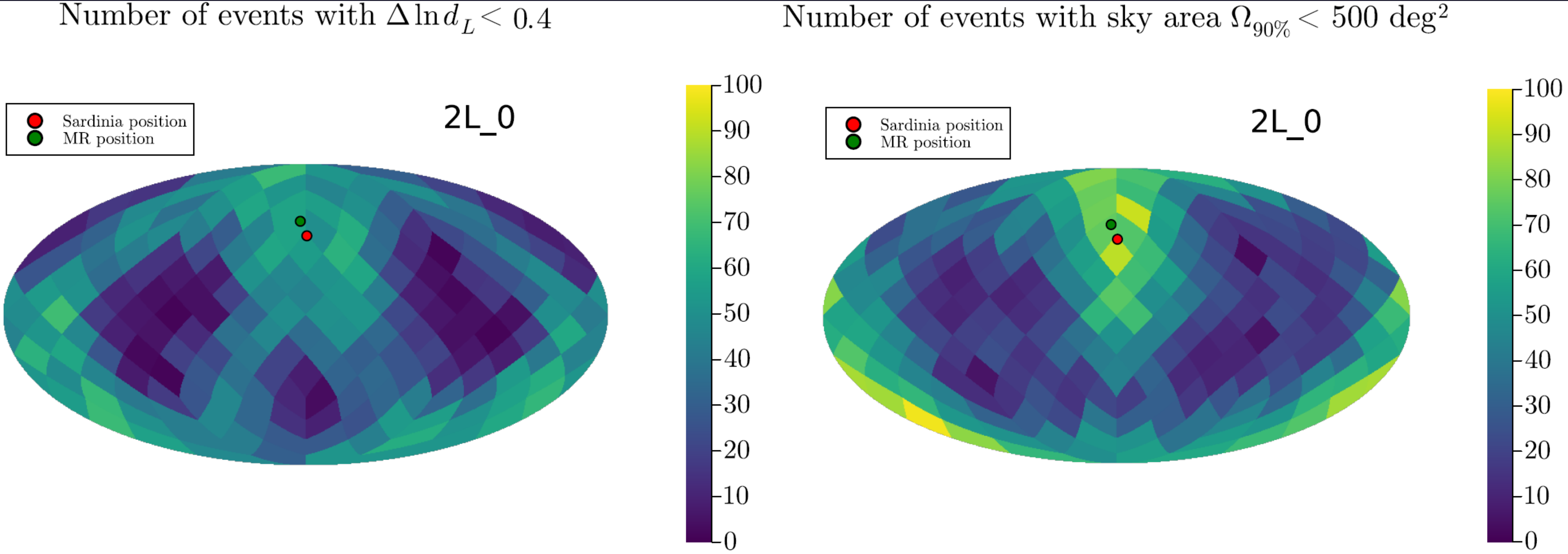
Source properties



The advantage of the 2L_45 over 2L_0 in the luminosity distance is erased when we require also the chirp mass to satisfy the requirement.

$$\mathcal{M}_{c,det} = (1+z)\mathcal{M}_{c,source}$$

Sky area and luminosity distance – 2L_0

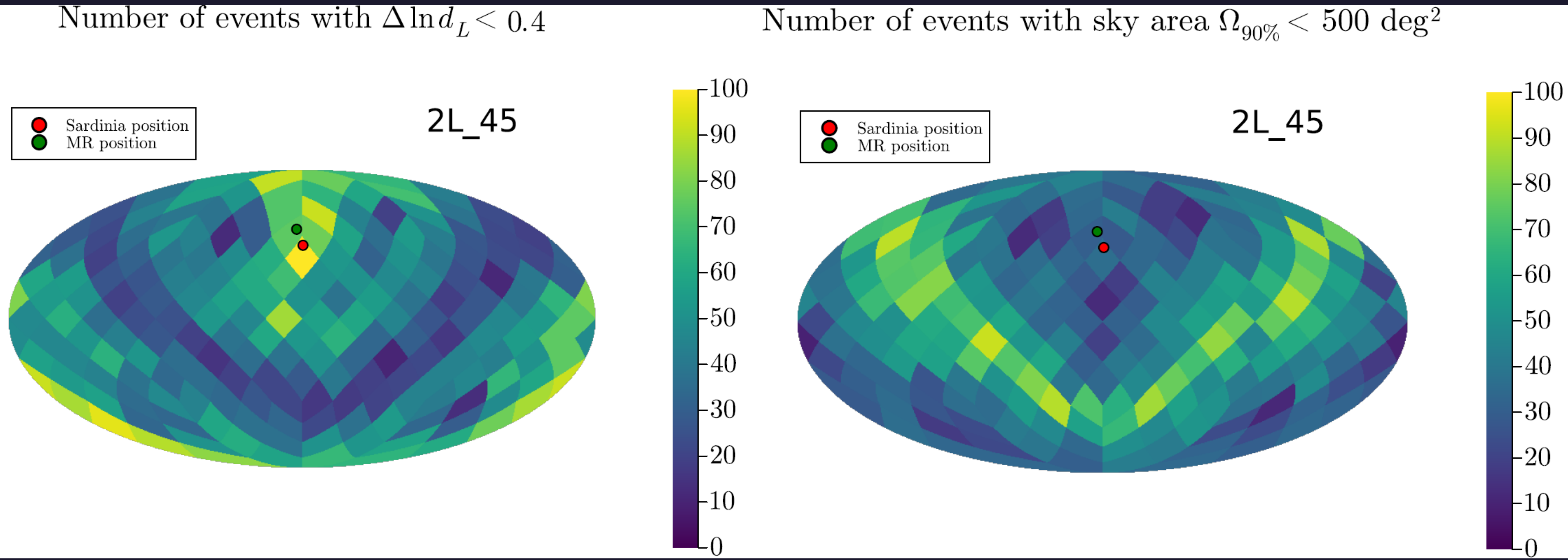


Sky maps in the detector frame for 2L_0.

Left: number of events for which the relative luminosity distance1-sigma error is under 40%

Right: number of events for which the 90% sky area is under 500 sqdeg.

Sky area and luminosity distance – 2L_45



Sky maps in the detector frame for 2L_45.

Left: number of events for which the relative luminosity distance1-sigma error is under 40%

Right: number of events for which the 90% sky area is under 500 sqdeg.