# Formation and evolution of cosmic domain walls

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**Summer Fellowship**
on Beyond the Standard Model Physics
and Cosmology

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# Motivation

Gravitational wave stochastic background → Primordial Universe

Possible sources:

- Binary black hole/neutron star mergers.
- Supernovae.
- Phase transitions (domain walls).

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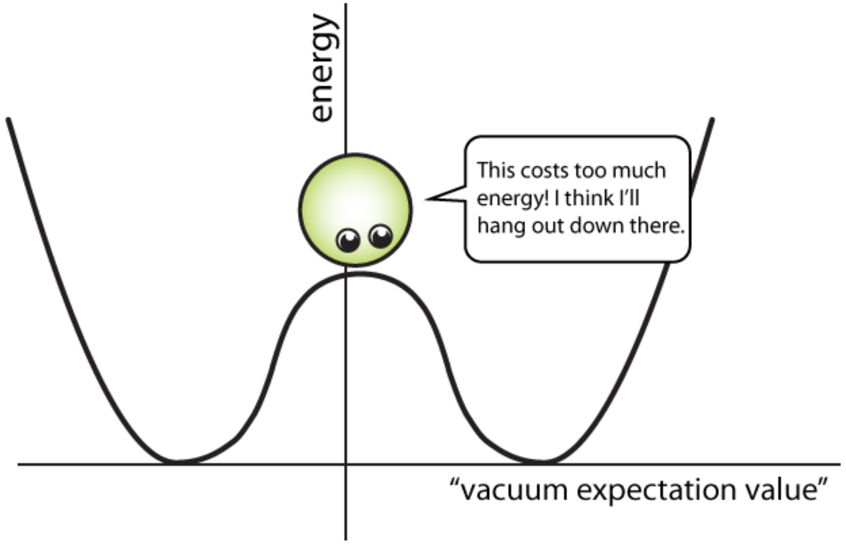
$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2$$




*(Cuentos Cuánticos, 2014)*

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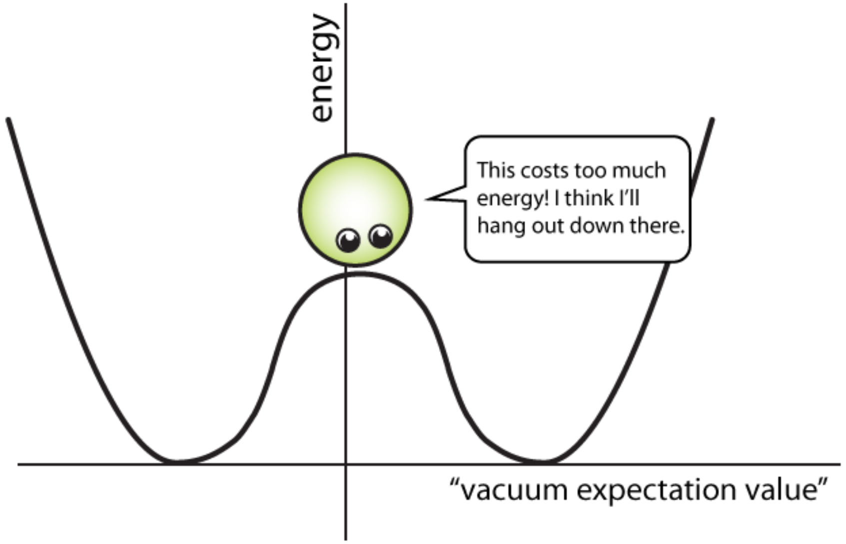


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The thin boundaries between both vacua are precisely the **Domain Walls.**

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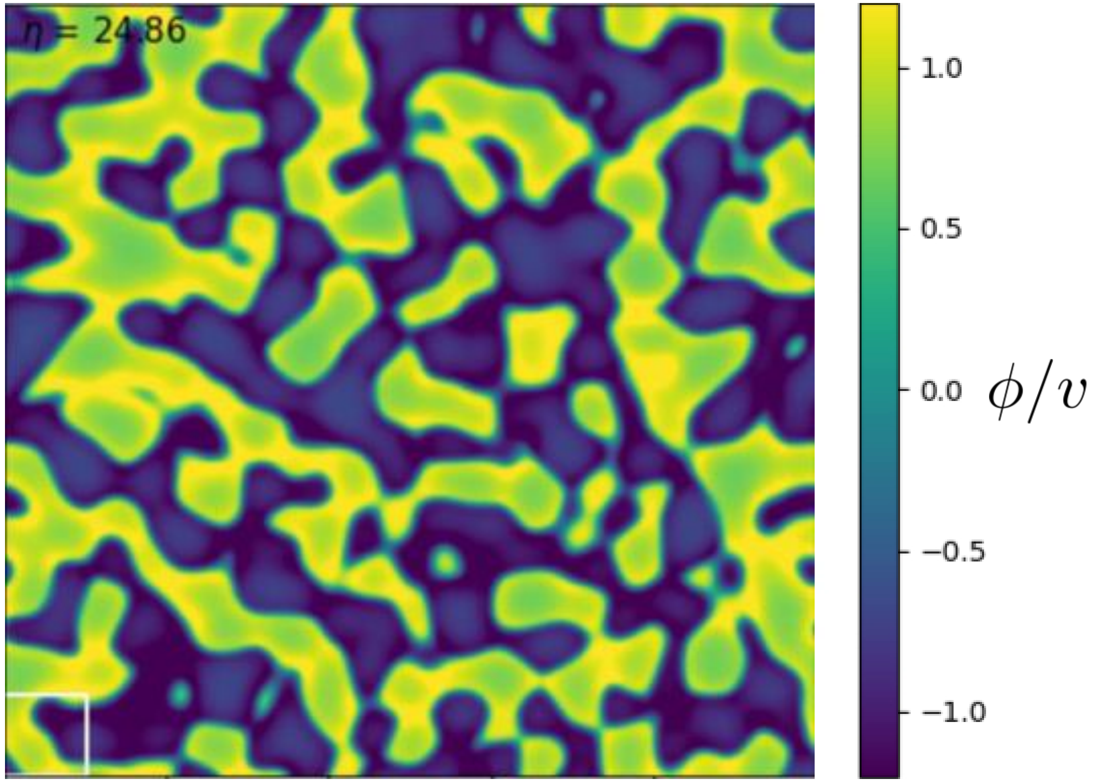


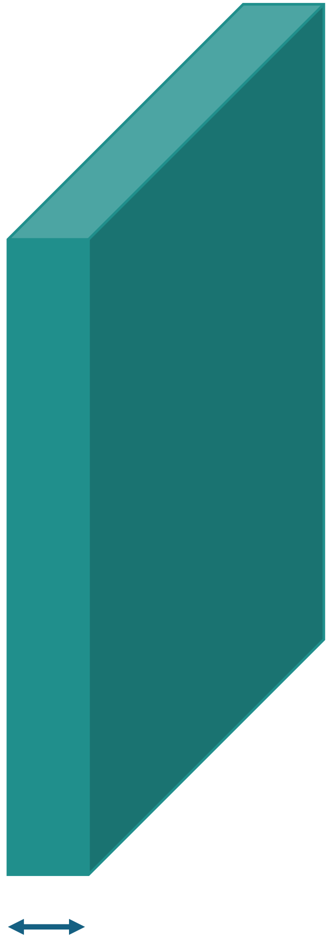

Energy per unit surface:

$$\sigma = 2mv^2/3$$

$$m^{-1} = (2\lambda v^2)^{-1/2}$$ Width of DW.

# Dynamical equations

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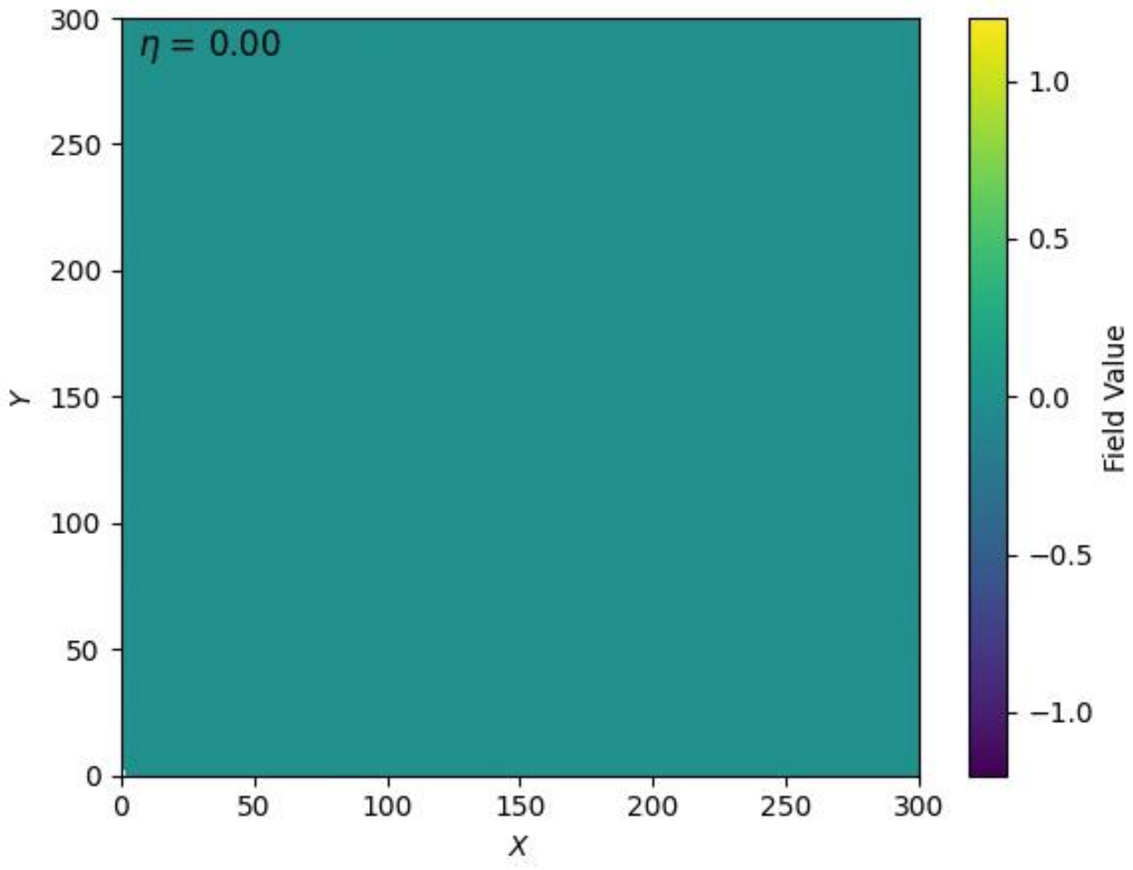
dimension

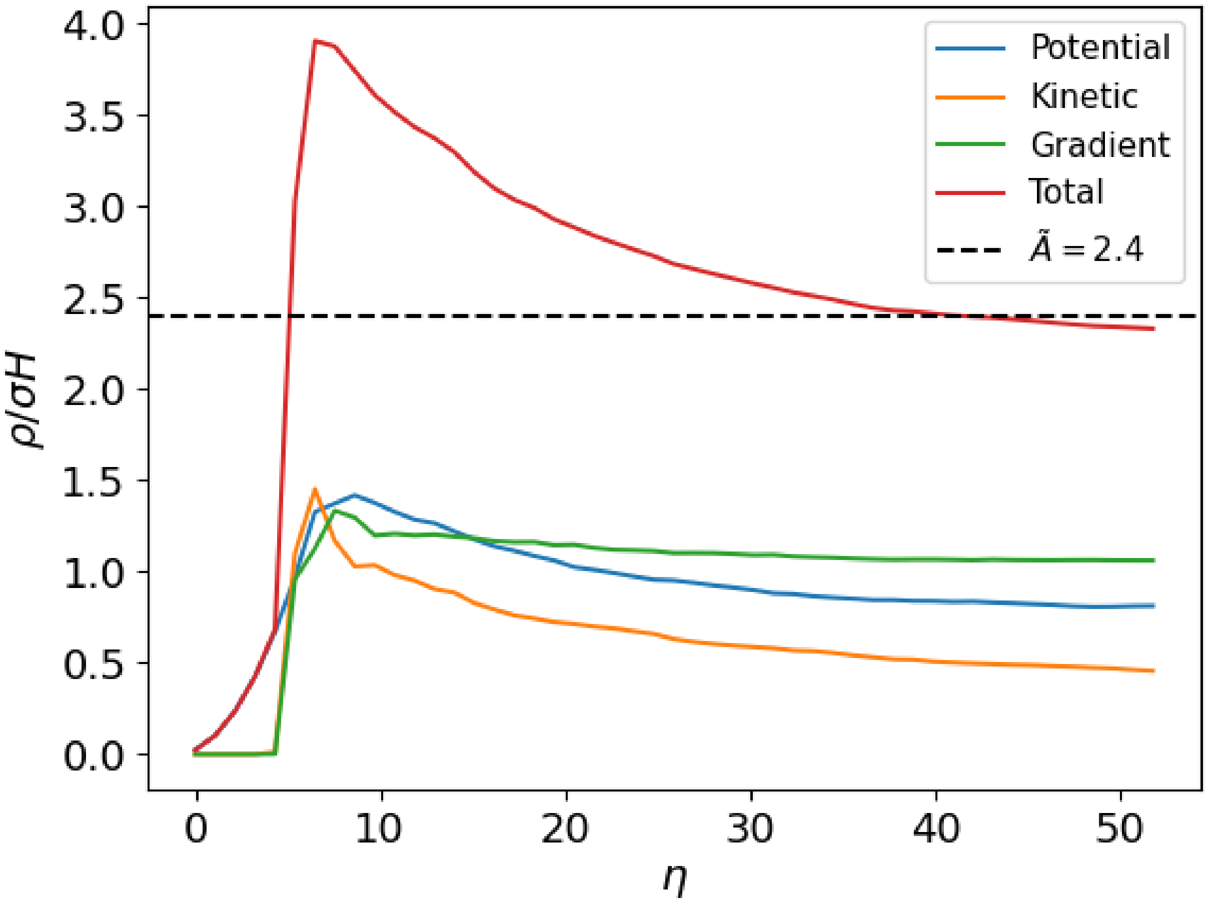
vacuum expectation value

# Evolution of the field

Evolution of $\phi$ for N=1024 and L=300.0. $a(\eta) = (1+\eta)^{1.0}$ fattening



# Numerical results




$$\rho \approx \sigma \frac{\mathrm{A}}{\mathrm{V}} \approx \sigma \frac{H^{-2}}{H^{-3}} \propto a^{-2}$$

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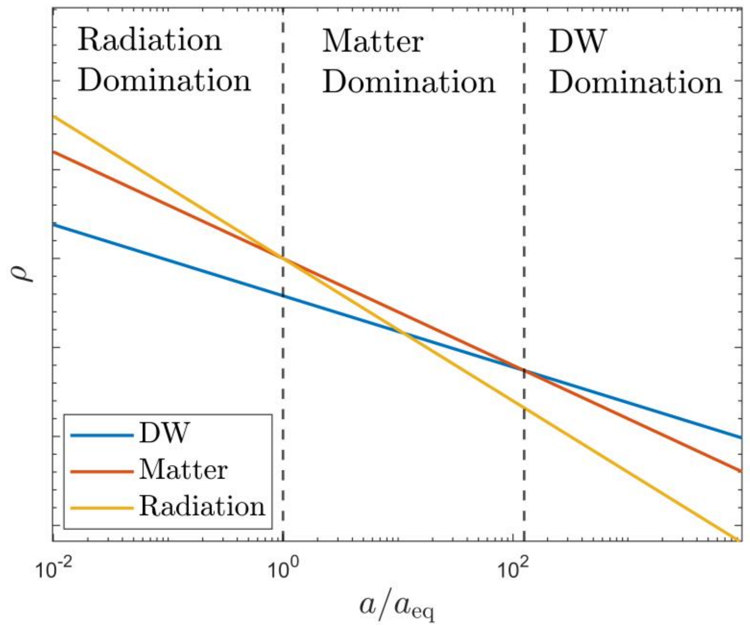


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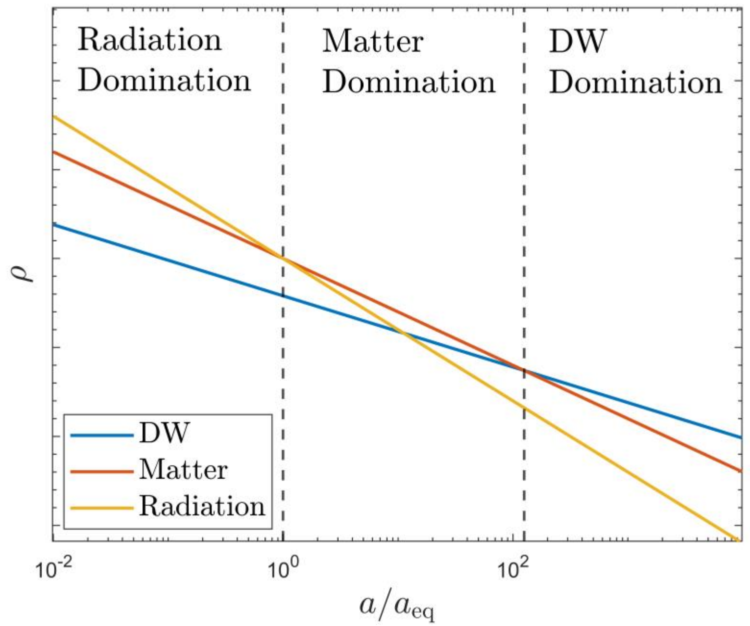


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There will exist a cosmological epoch where domain walls are the dominant substance in the Universe. **Violation of isotropy.**

This is known as the **Domain Wall Problem.**

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There are different types:

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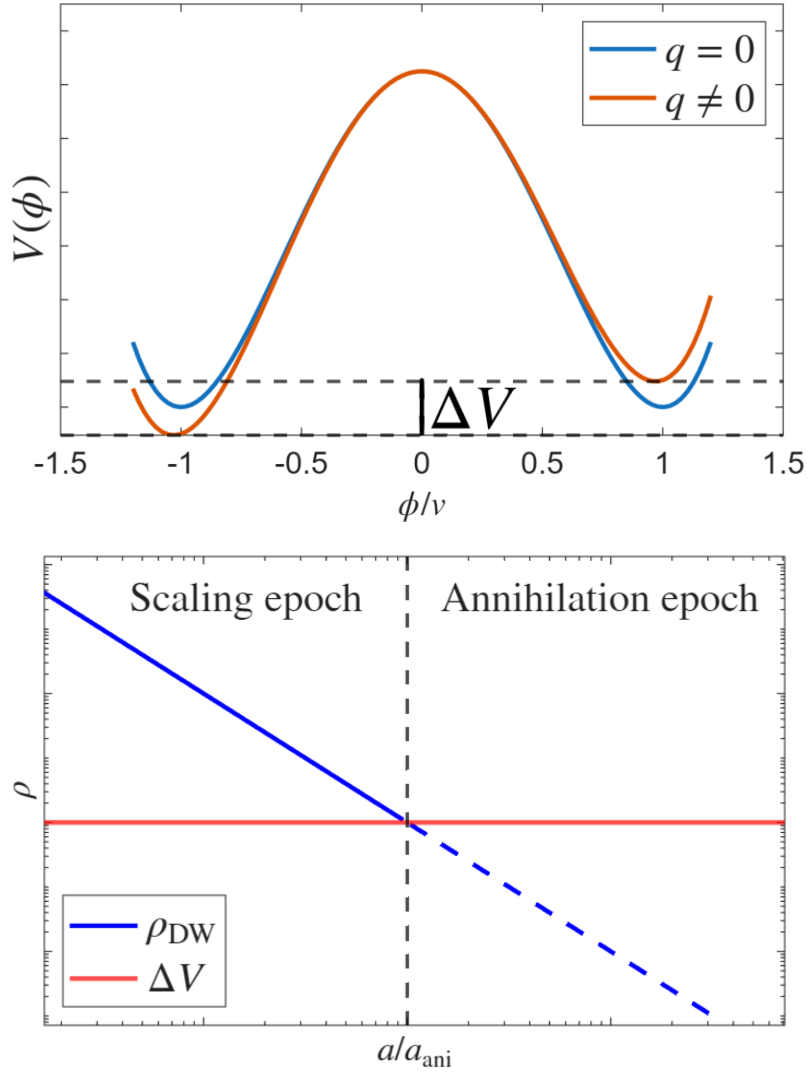



Example: cubic potential

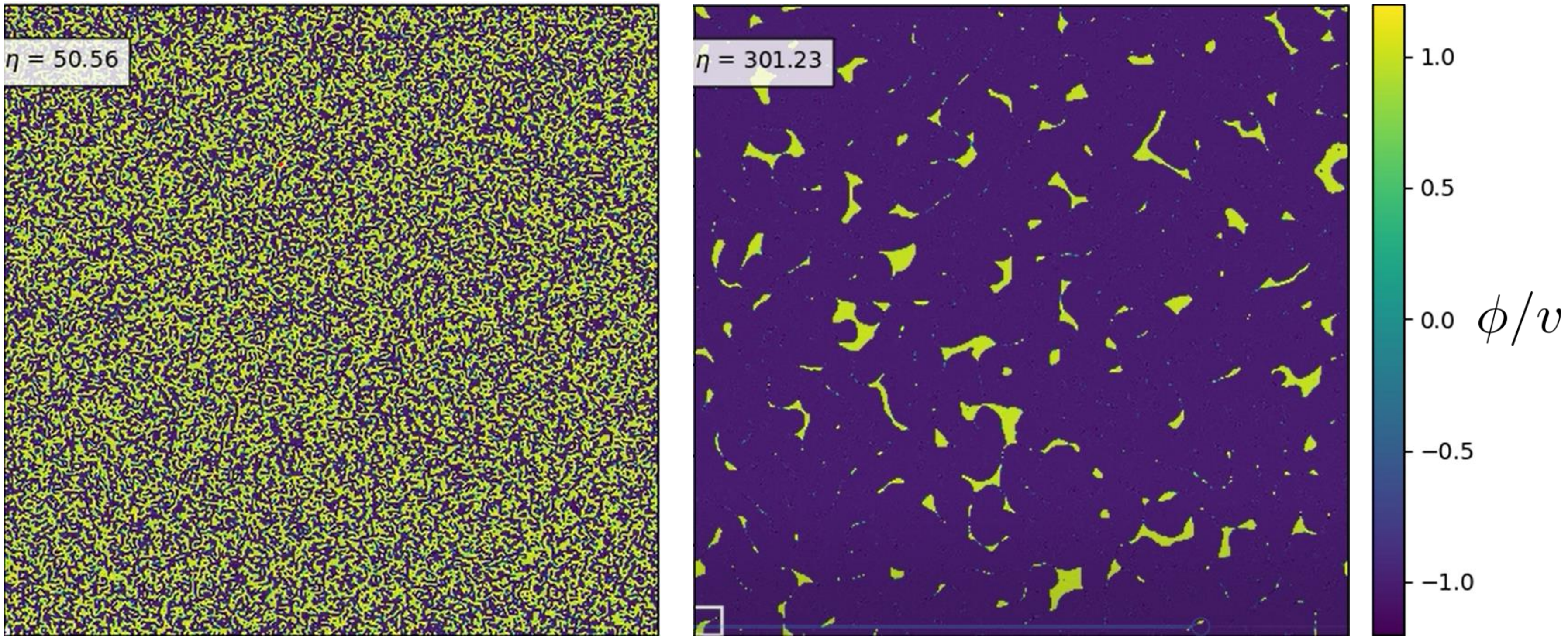
$$V_b = qv\phi^3$$

1. If $a \ll a_{\rm ani} = a(\eta_{\rm ani})$, bias is subdominant. Scaling regime.

2. When $a \geq a_{\rm ani}$, bias dominates. Annihilation.

# Bias

Annihilation mechanisms. There is no scaling regime.

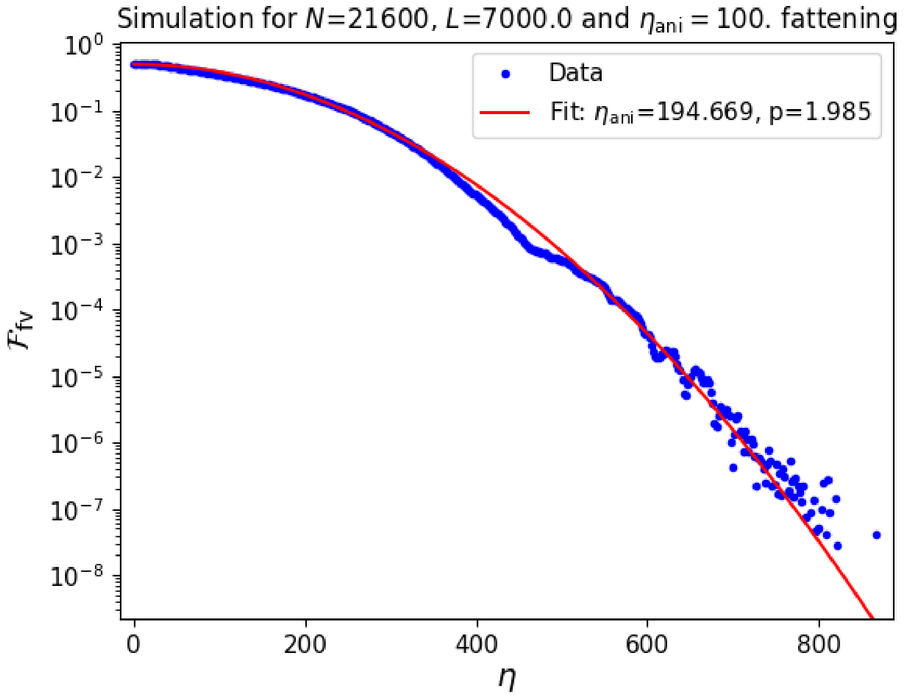


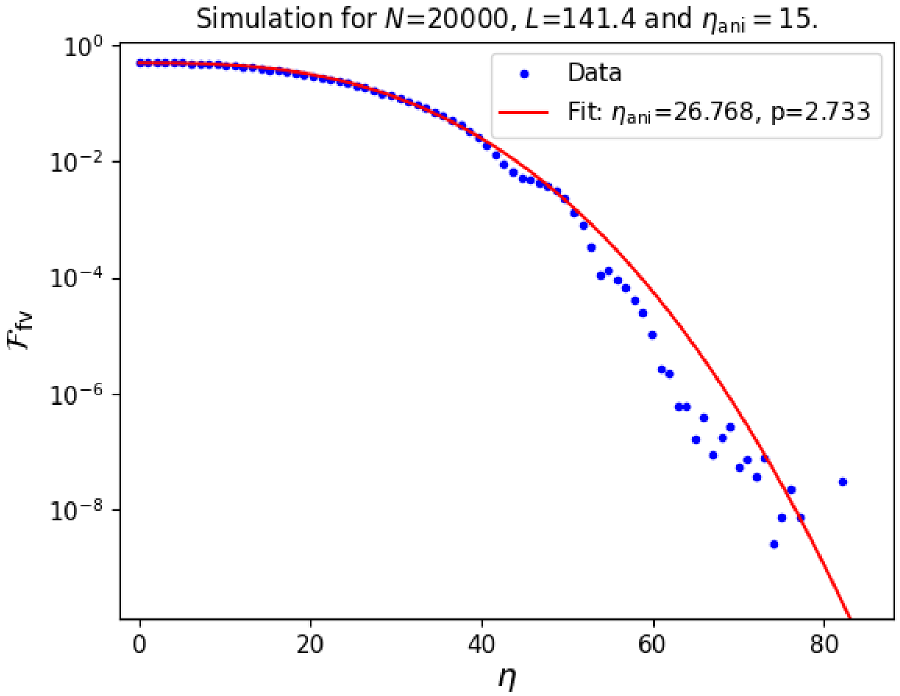

(Simulation for N=21600, L=7000, $\eta_{\mathrm{ani}} \approx 100$)

# False vacuum fraction

The false vacuum fraction can be fitted to $\mathcal{F}_{\rm fv} = \frac{1}{2} e^{-(\eta/\eta_{\rm ani})^p}$ with $p \approx 2-3$.

Cross-check with literature (Correia et. al. 2014).



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- Domain wall problem $\Longrightarrow$ Bias!

- Numerical resolution problem $\Longrightarrow$ Fattening + cluster.

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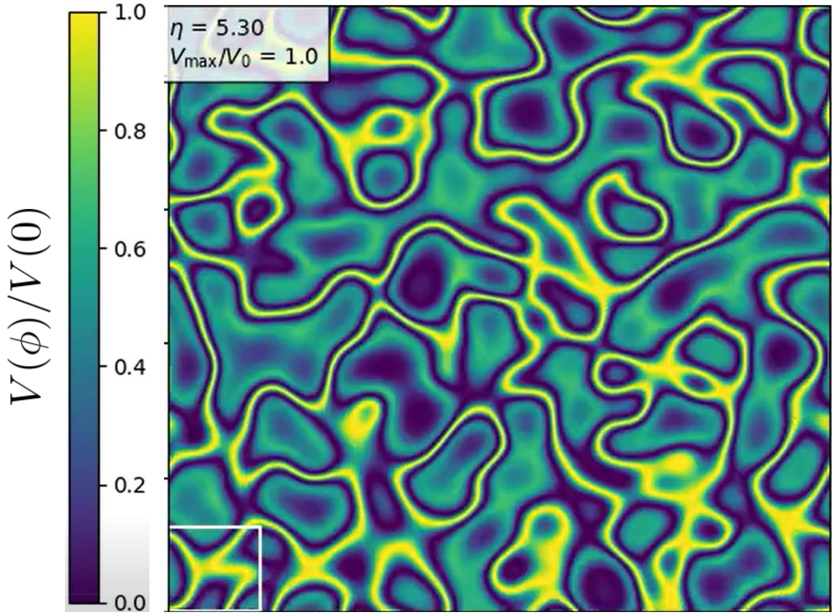

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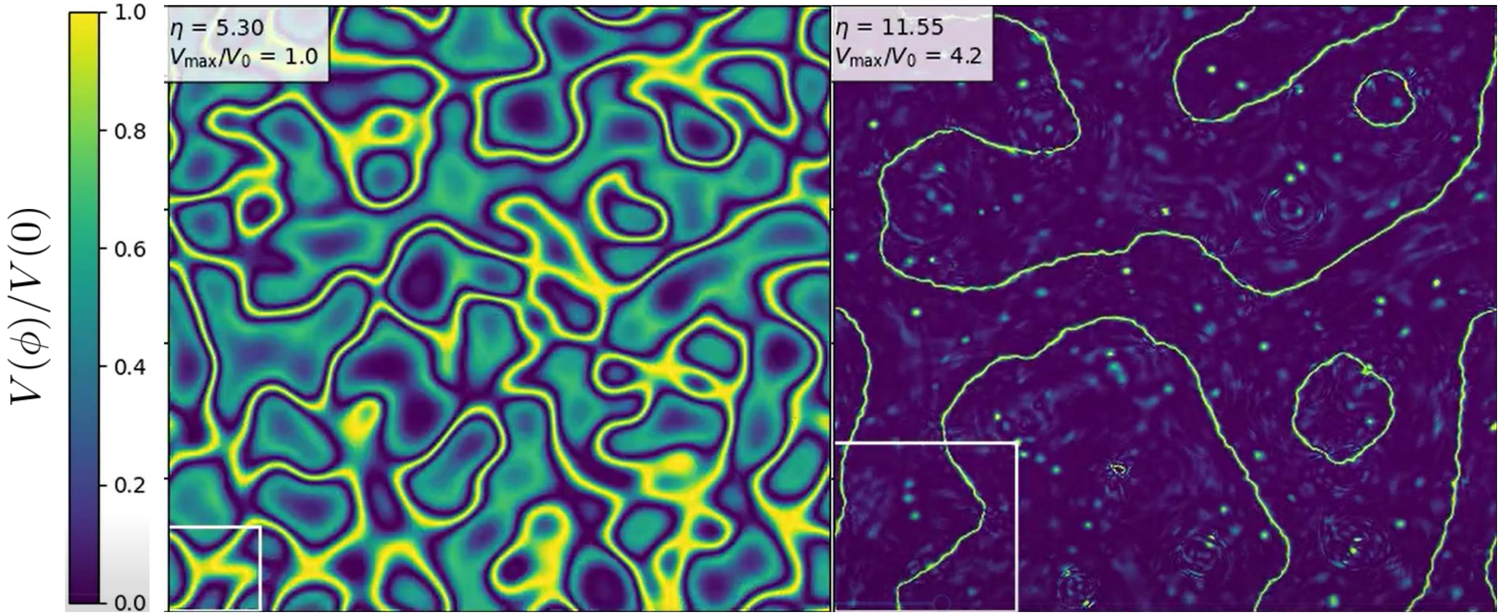

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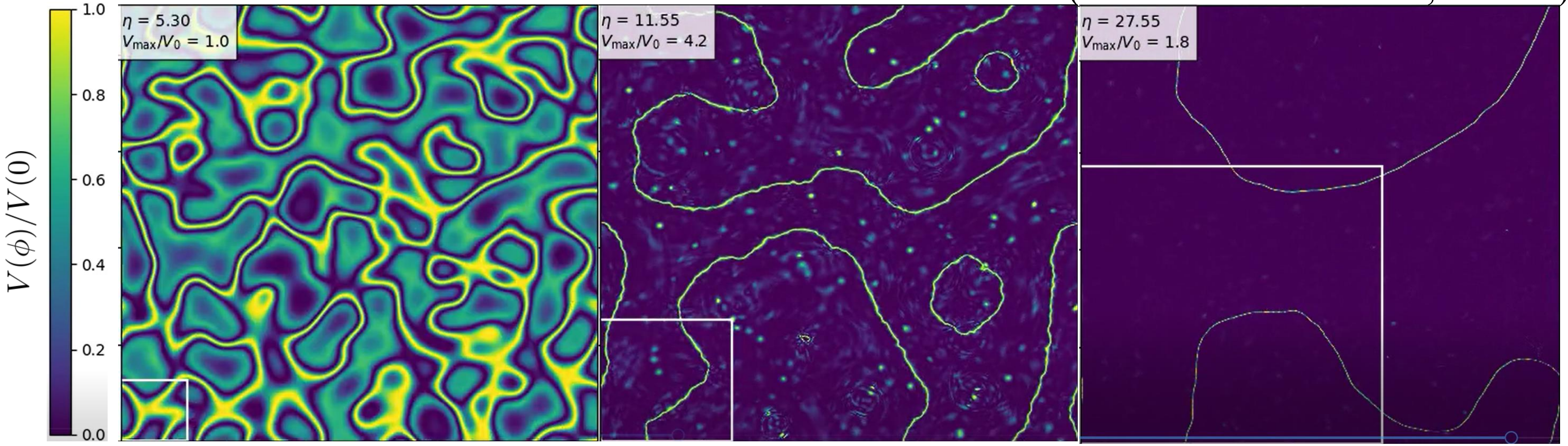

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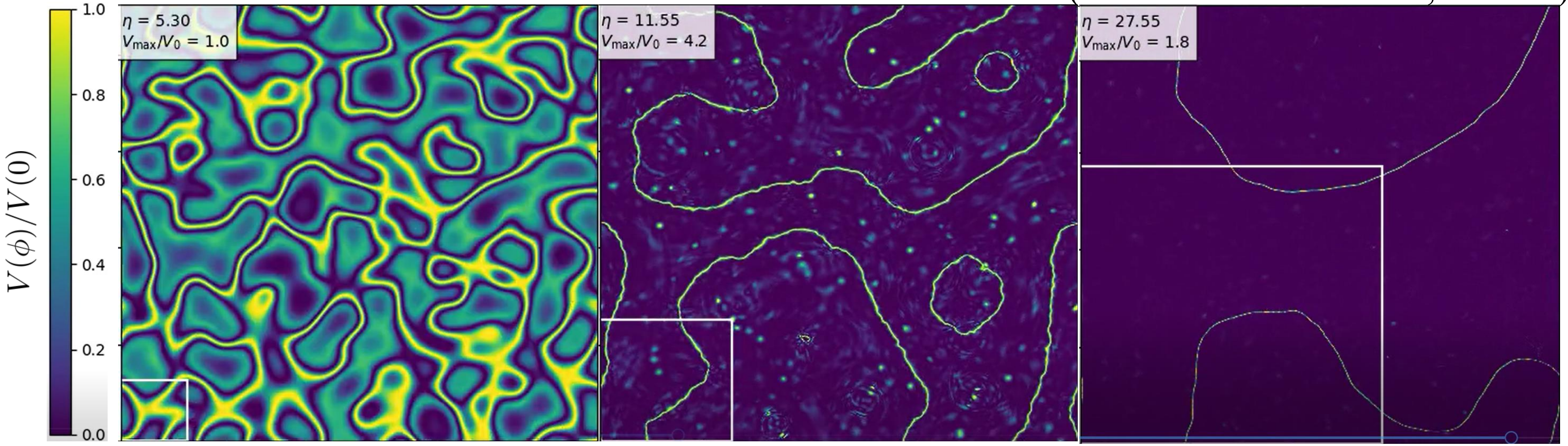


We need a lot of points! $\longrightarrow$ Fattening

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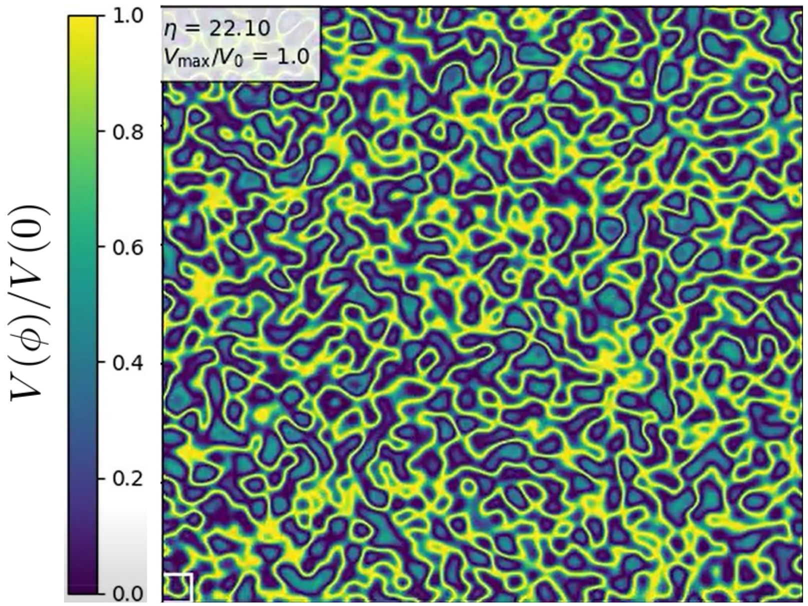

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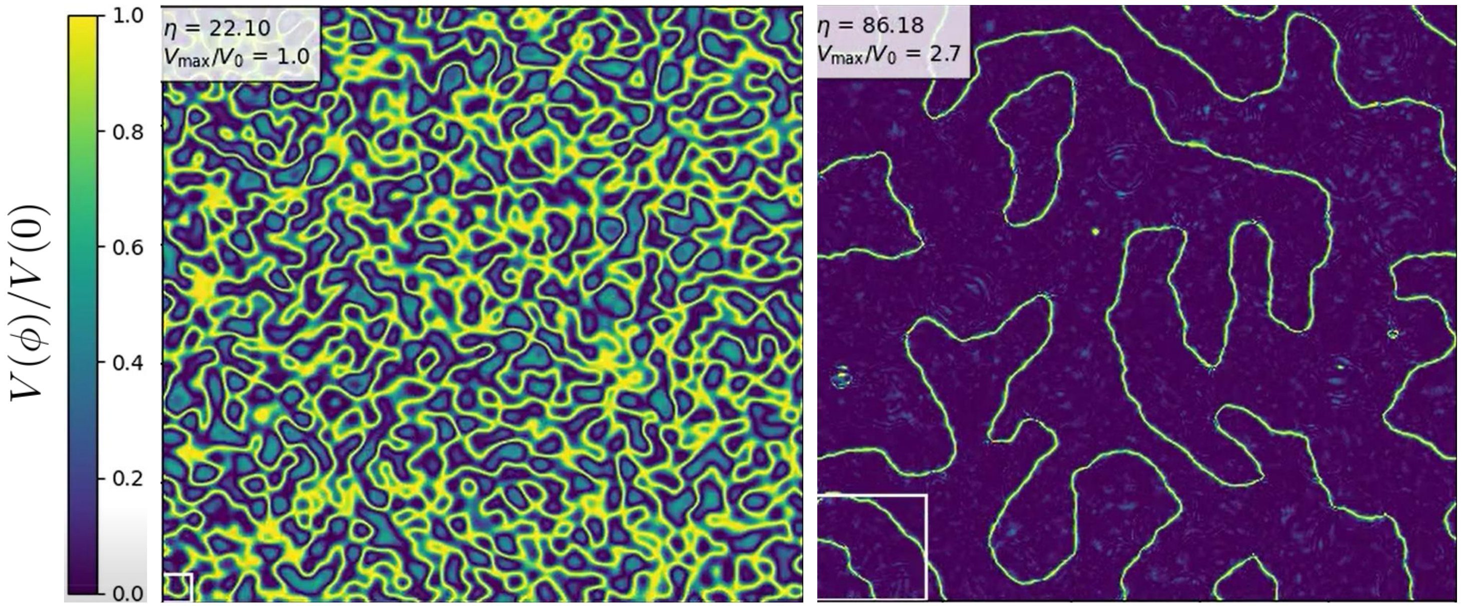

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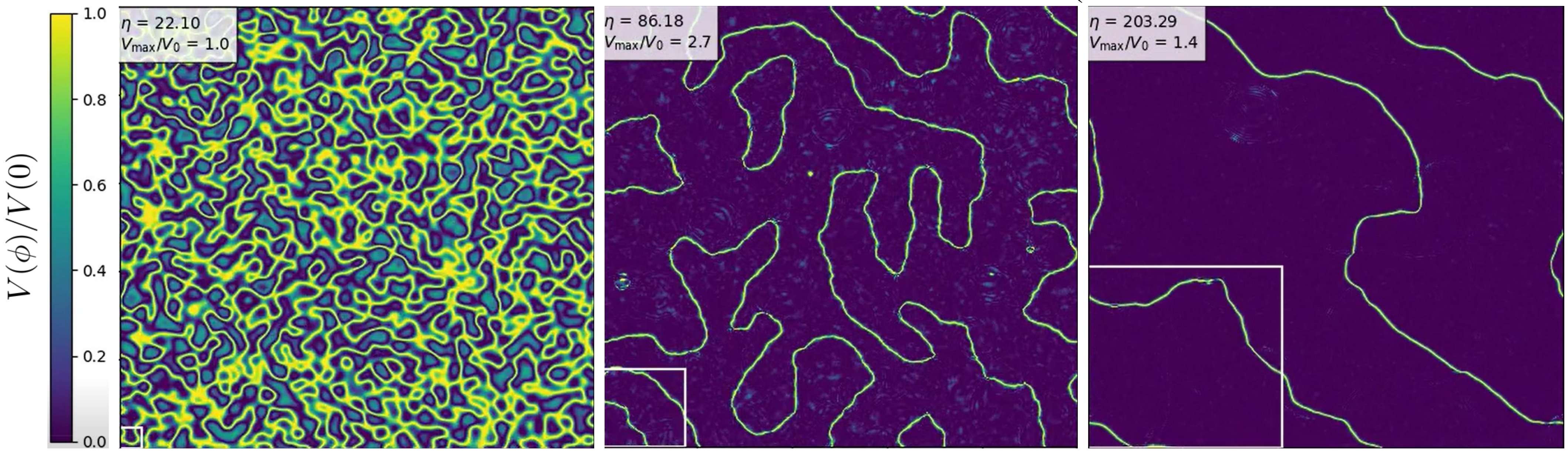
(Simulation for N=2048 and L=500)



# Numerical resolution

We worked in collaboration with PIC to carry out the heaviest simulations.

To take the most of the cluster's potential…

**CLUSTEREASY**
**A Program for Simulating Scalar Field Evolution on Parallel Computers**

Gary Felder
*Department of Physics, Smith College, Northampton, MA 01063, USA*
(Dated: October 24, 2018)

We describe a new, parallel programming version of the scalar field simulation program LATTICEEASY. The new C++ program, CLUSTEREASY, can simulate arbitrary scalar field models on distributed-memory clusters. The speed and memory requirements scale well with the number of processors. As with the serial version of LATTICEEASY, CLUSTEREASY can run simulations in one, two, or three dimensions, with or without expansion of the universe, with customizable parameters and output. The program and its full documentation are available on the LATTICEEASY website at http://www.science.smith.edu/departments/Physics/fstaff/gfelder/latticeeasy/. In this paper we provide a brief overview of what CLUSTEREASY does and the ways in which it does and doesn't differ from the serial version of LATTICEEASY.

# Numerical resolution

We have reached resolution of 2D simulations of $N^2 = (50000)^2 = 2.5 \times 10^9$ points.

Equivalent to a simulation of 25 GB of RAM.

We could cross-checked our results with state-of-the-art results of the literature (Correia et. al. 2014) where N=8000.

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There should exist annihilation mechanisms, whose effects must be observable currently.

How can we possibly measure its effects?

1. Gravitational waves.
2. Primordial black holes.
3. …

# Conclusions

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1. We were able to write codes that simulate the evolution of cosmic domain walls in the Universe.
2. We could overcome the numerical problems related to the resolution of the simulations.
3. We accomplished results comparable to those of the literature.

Thank you!

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