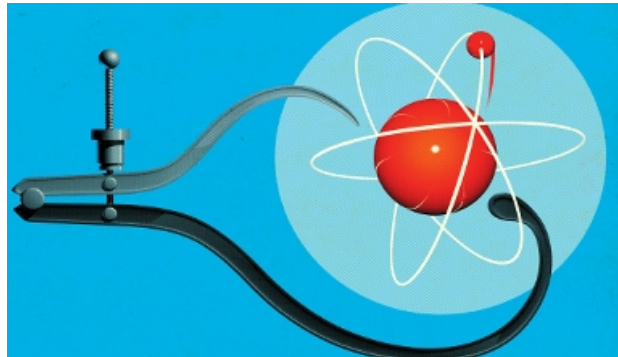


# On the proton radius puzzle



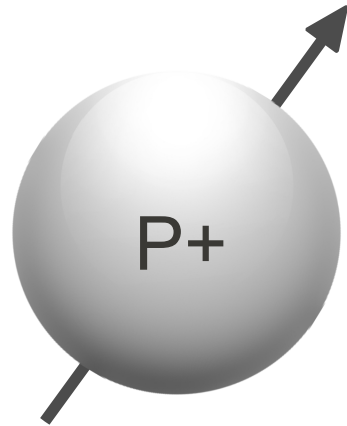
Clara Peset

19th March, 2014

# Outline

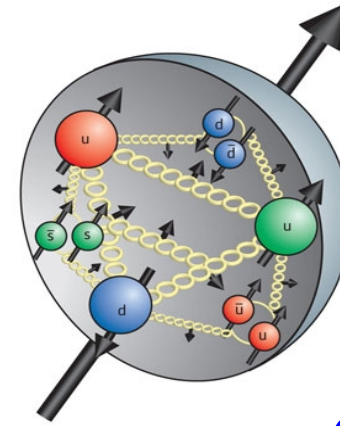
- What is the proton radius?
- The puzzle
- Measurements
  - in hydrogen
  - in muonic hydrogen
- Possible Issues
  - muonic hydrogen experiment
  - theory of the muonic hydrogen
  - theory of the hydrogen levels
  - beyond the Standard Model
- Conclusions

# The proton and its charge radius



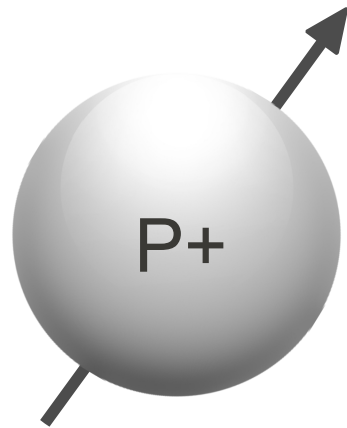
*Rutherford 1917-20*

Stable baryon:  $\tau_p > 10^{34}$  years



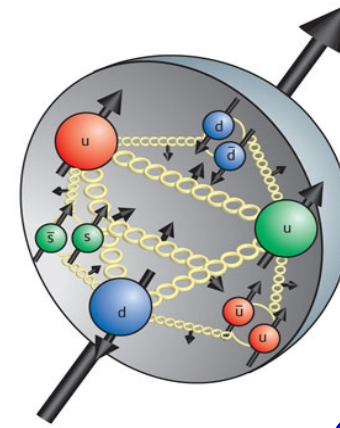
**QCD**

# The proton and its charge radius



*Rutherford 1917-20*

Stable baryon:  $\tau_p > 10^{34}$  years



**QCD**

What is the proton radius?

**EFT (NRQED)**

$$r_p^2 = 6 \left. \frac{dG_{p,E}(q^2)}{dq^2} \right|_{q^2=0}$$

$$c_D^{(p)}(\nu) - c_{D,point}^{(p)}(\nu) \equiv \frac{4}{3} m_p^2 r_p^2$$

**IR divergent!**

# The proton radius puzzle

Value obtained  
from muonic-H:

$$r_p = 0.84184(67) \text{ fm } (\textit{Nature '10})$$

$$r_p = 0.84087(39) \text{ fm } (\textit{Science '13})$$

$$r_p = 0.8433(17) \text{ fm } (\textit{CP, Pineda '14})$$

CODATA value:

$$r_p = 0.8768(32) \text{ fm}$$

5–7  $\sigma$  away!

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CODATA value:

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Probability of the lepton being  
*within* the volume of the proton

$$\left(\frac{r_p}{a_B}\right)^3 = (\alpha m_r r_p)^3$$

$$m_\mu \approx 200 m_e$$

**8 million times larger for muonic hydrogen!**

# What is going on?

Experimental measurements wrong?

Theoretical predictions missing a big contribution?

Interaction of the proton with the muon different from the interaction with the electron?

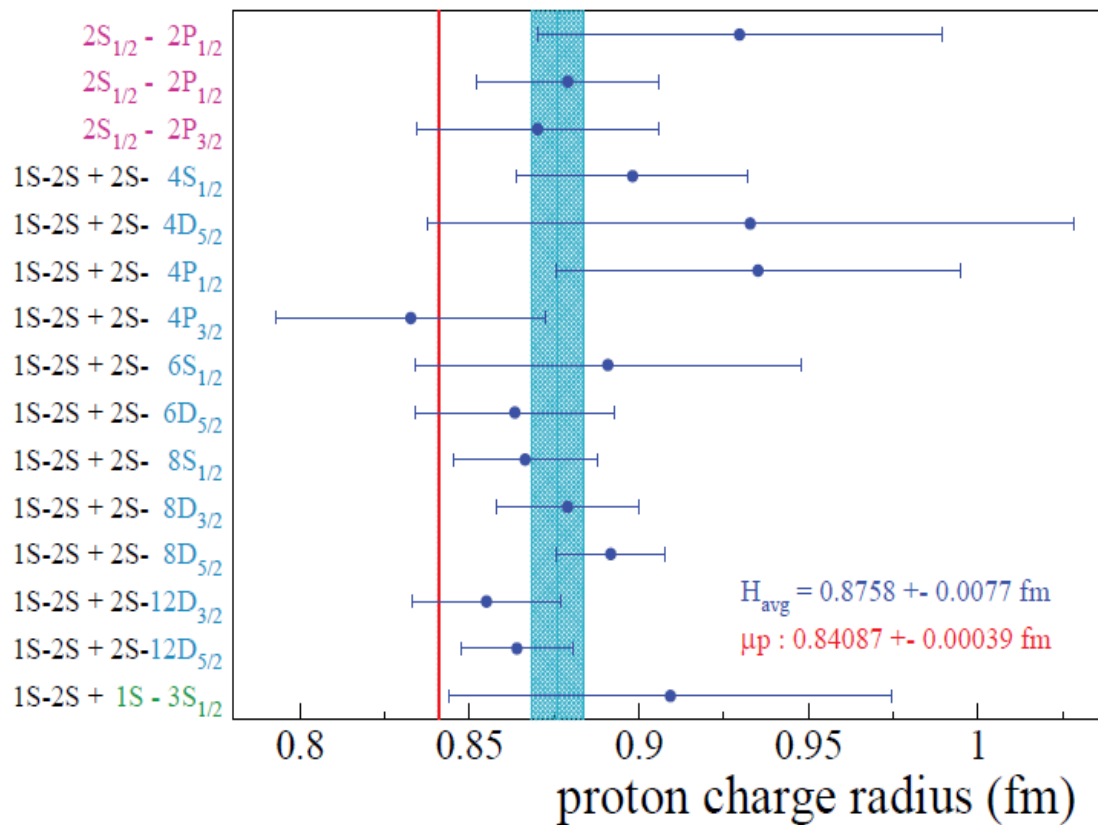
How can we accommodate this within our known theory?

**It's a puzzle!**



# Measurement in hydrogen spectroscopy

Large number of experiments:



Hadronic structure:

$$\text{Lamb shift} \approx r_p^2$$

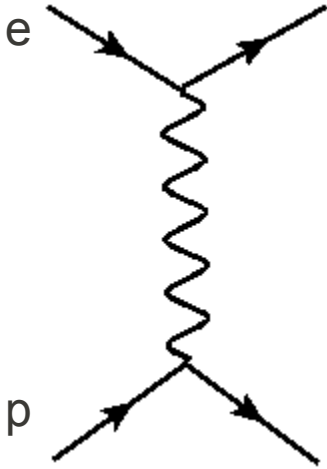
EFT counting:

$$O(m_e \alpha^4 \frac{m_e^2}{m_p^2}) \approx O(m_e \alpha^6)$$

1S-2S transition measured to accuracy  $\frac{4}{10^{15}}$



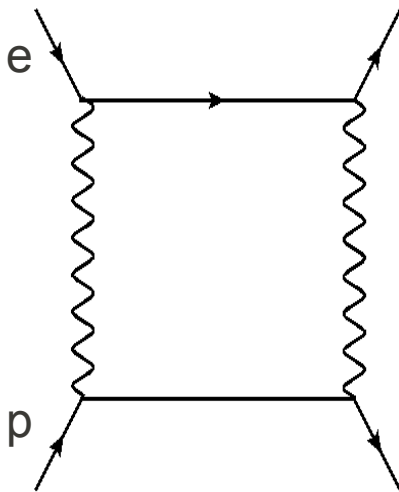
# Measurement in p-e scattering



$$\frac{d\sigma}{d\Omega} \approx \frac{d\sigma}{d\Omega}_{point} \left(1 - \frac{q^2}{4m^2}\right) \left(G_E^2(q^2) - \frac{q^2}{4m^2} G_M^2(q^2)\right)$$

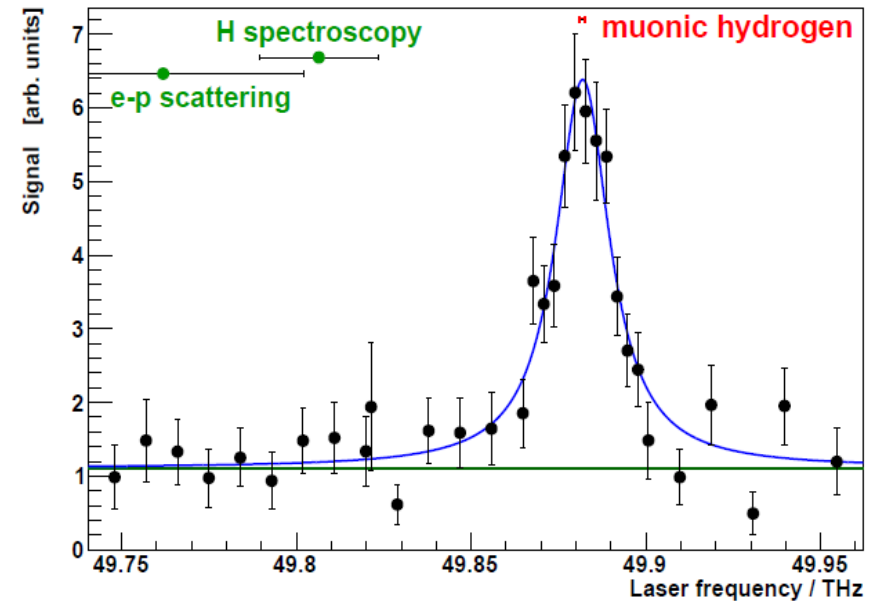
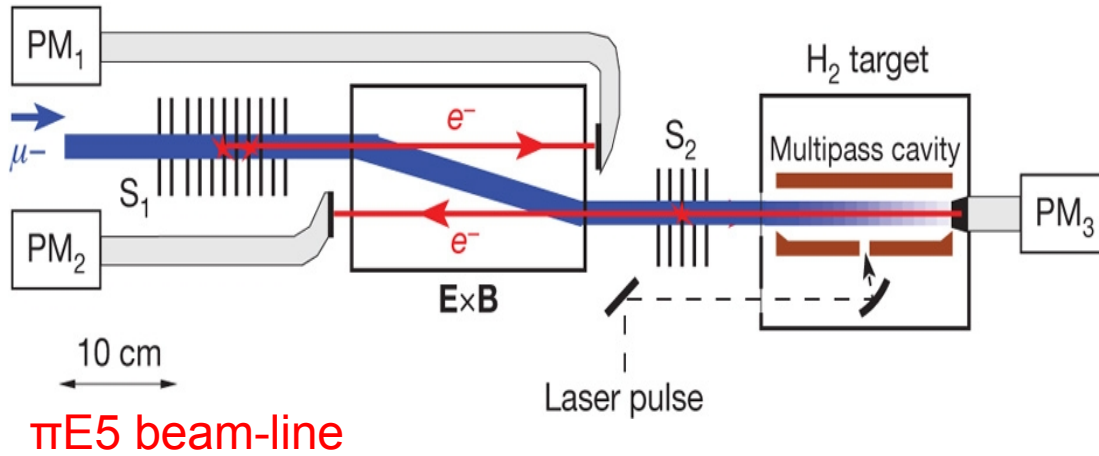
## More complicated measurement:

- Experimental uncertainties
- Fit of absolute normalization
- Radius sensitive to low  $q^2$  data
- Errors truncating:  $G_E^2(q^2) = 1 + q^2 r_p^2 / 6 + q^4 r_p^4 / 120 \dots$
- Higher order effects



off-shell structure of the proton!

# Muonic hydrogen experiment



Distance resonance centre from CODATA  $\sim 4$  times resonance width

Not likely covered by statistical or systematic effects

laser calibration, Doppler shifts...

Presence of an electron:  
ion ( $\mu p e$ ) instead of ( $\mu p$ )

→ ( $\mu p e$ ) is not stable!

# Theory of the muonic hydrogen

$2S_{1/2} \rightarrow 2P_{3/2} \longrightarrow$  LAMB SHIFT+ FINE STRUCTURE + HYPERFINE STRUCTURE

Theoretical equation for the Lamb Shift:

$$\Delta E_L = 206.0243(30) - 5.2271(7) \frac{r_p^2}{\text{fm}^2} + 0.0633(144) + \mathcal{O}(m_r \alpha^6) \text{ meV}$$

pure QED

Hadronic effects

CP, Pineda

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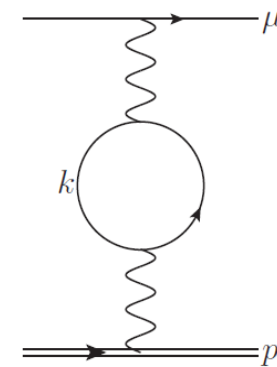
pure QED

Hadronic effects

CP, Pineda

$\mu$ H QED leading contribution: ELECTRON VACUUM POLARIZATION

(vs H where it is electron self energy)



HADRONIC EFFECTS encoded in  $\delta(r)$  potential

EFT

H:  $O(m_e \alpha^6)$

$\mu$ H:  $O(m_\mu \alpha^4)$

# EFT for bound states ( $\mu p$ )

Scales in bound states

Muonic hydrogen  
(Coulomb interaction)

- Hard scale:  $m_r \longrightarrow m_r$
- Soft scale:  $|p| \longrightarrow m_r \alpha$
- Ultrasoft scale:  $E \longrightarrow m_r \alpha^2$

Well separated  
scales!

pNRQED

Scales in  $\mu H$ :

$$m_p \sim m_\rho, \quad m_\mu \sim m_\pi \sim m_r, \quad m_r \alpha \sim m_e$$

Small expansion parameters:  $\frac{m_\pi}{m_p} \sim \frac{m_\mu}{m_p} \approx \frac{1}{9}, \quad \frac{m_e}{m_r} \sim \frac{m_r \alpha}{m_r} \sim \alpha \approx \frac{1}{137}$

Energy levels:

$$E(\mu p) = \frac{-m_r \alpha^2}{2n^2} (1 + c_1 \alpha + c_2 \alpha^2 + c_3 \alpha^3 \dots)$$

$$c_1 \sim c_1 \left( \frac{m_\mu \alpha}{m_e} \right) \text{ pure QED, and } c_n \sim \sum_{j=0}^{\infty} c_n^{(j)} \left( \frac{m_\pi}{m_p} \right)^j; \quad c_n^{(j)} \sim c_n^{(j)} \left( \frac{m_r}{m_\mu}, \frac{m_\mu}{m_\pi} \dots \right)$$

# Hadronic Effects

$$\delta\mathcal{L} = \frac{d_2}{m_p^2} F_{\mu\nu} D^2 F^{\mu\nu} - e \frac{c_D^{(p)}}{m_p^2} N_p^\dagger \nabla \cdot \mathbf{E} N_p + \frac{c_3}{m_p^2} N_p^\dagger N_p \mu^\dagger \mu$$

Hadronic contribution encoded in the delta potential:

$$\delta V^{had} = \frac{D_d^{had}}{m_p^2} \delta^{(3)}(\mathbf{r})$$

Contributes to energy levels as:

$$E_{nl}^{had} = \frac{D_d^{had}}{m_p^2} \frac{(m_r \alpha)^3}{\pi n^3} \delta_{l,0}$$

Dependence on the matching coefficients:

$$D_d^{had} \equiv -c_3^{had} - 16\pi\alpha d_2^{had} + \frac{\pi\alpha}{2} c_D^{had}$$

polarizability  
+ Born effects

Hadronic VP

definition of the proton radius!

# Hadronic vacuum polarization

$$d_2 = d_{2,R} + \frac{m_p^2}{4} \Pi'_{h,\pi}(0) = \frac{m_p^2}{4} \Pi'_h(0)$$

pion vacuum polarization:

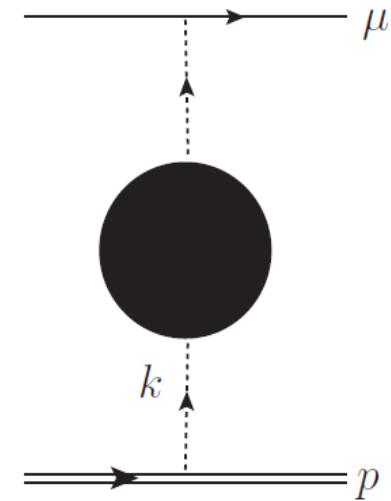
$$\delta E_\pi = \frac{m_r \alpha^5}{24 \pi} \frac{m_r^2}{m_\pi^2} = 0.00121 \text{ meV}$$

Energy shift determined  
from dispersion relations:

$$\delta E = 0.0111(2) \text{ meV} \quad \text{F. Jegerlehner}$$

(large contribution from  $\rho$  even if suppressed by counting)

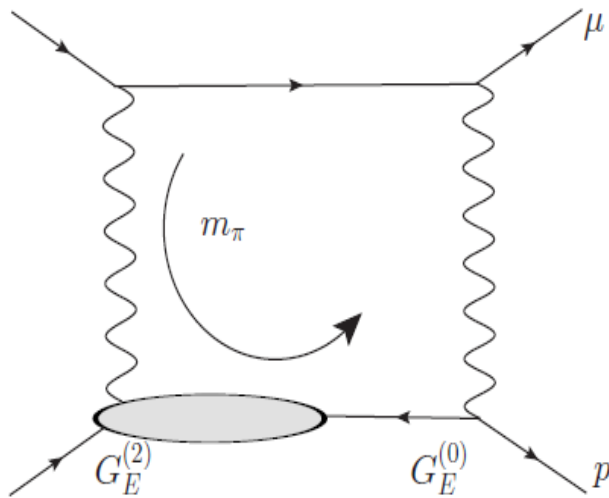
The error small enough so that this computation does not alter general result





# Born effect

$$c_3^{\text{had}} = c_3^{\text{Born}} + c_3^{\text{pol}}$$



$$c_3^{\text{Born}} = \frac{\pi}{2} \alpha^2 m_p^2 m_\mu \langle r_p^3 \rangle$$

Zemach third momentum

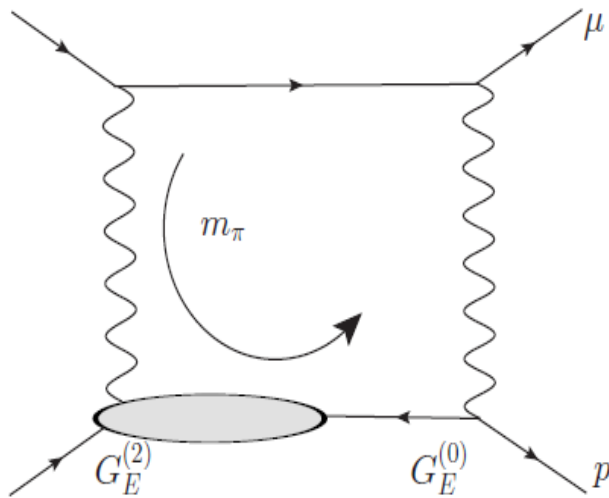
$$\frac{\langle r_p^3 \rangle}{\text{fm}^3} = \frac{96}{\pi} \int d^{D-1}k \frac{1}{\mathbf{k}^6} G_E^{(0)} G_E^{(2)}$$

Large contribution from  $\Delta(1232)$ :  $\delta E = 0.0101 + 0.0056 \dots = 0.0192 \text{ meV}$

- the  $\Delta$  and the proton are degenerate in the large- $N_c$  limit

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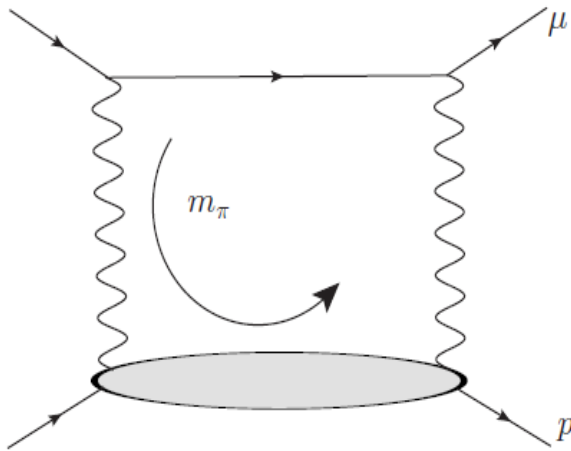
Energy shift determined  
from dispersion relations:

$$\delta E = 0.0255(10) \text{ meV}$$

30% difference in  
compliance with  
higher order effects

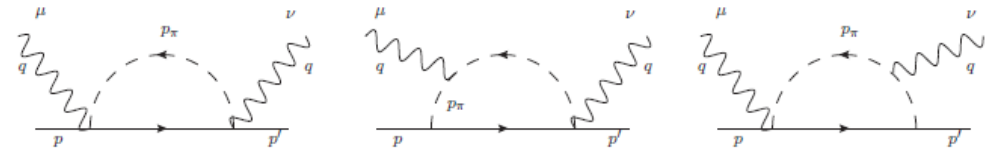
# Proton polarizability

$$c_3^{\text{had}} = c_3^{\text{Born}} + c_3^{\text{pol}}$$



Fit to experiment (model dependent):

$$\begin{aligned} \delta E &= 0.012(2) \text{ meV} && \text{Pachucki} \\ &= 0.015(4) \text{ meV} && \text{Borie} \end{aligned}$$



Chiral +  $\Delta$  computations:

$$\delta E = 0.0185 + 0.0080 = 0.0265 \text{ meV}$$

CP, Pineda

+corrections suppressed by  $\frac{m_\mu}{m_\rho}$



30% uncertainty expected

More accurate SM results can vary this result but not enough to account for the difference

# Theory of hydrogen levels

EFT for the Lamb shift:

Scales in H:  $m_p \sim m_\rho$ ,  $m_r \sim m_e$

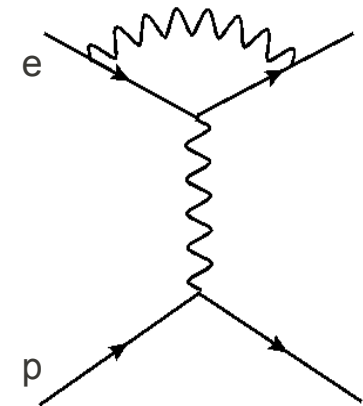
Small expansion parameters:  $\frac{m_e}{m_p} \sim \frac{m_r}{m_p} \approx \frac{1}{2000}$ ,  $\alpha \approx \frac{1}{137}$

Energy levels:  $E_{nl} \sim \sum m_r \alpha^s \left( \frac{m_e}{m_p} \right)^t$

H QED leading contribution: ELECTRON SELF ENERGY

HADRONIC EFFECTS mainly from the finite size

other proton structure effects suppressed by  $\frac{m_e}{m_p}$



LIKELY RIGHT: has been *thoroughly* computed by *many* authors

# Beyond the Standard Model

Are  $\mu p$  and  $e p$  different interactions?

- supported by: the measurement of the **proton radius**  
the **muon g-2** experiment (3- $\sigma$  discrepancy)
- constrained **universality** of lepton interactions

Exciting possibility:

- **new** gauge interaction that differentiates  $\mu$  and  $e$

  
**DARK MATTER CANDIDATE!**

- need for further assumptions and lots of further constrains
- still a feasible possibility!

Need for further experiments

Quantum gravity?

# Future perspectives

## New electron scattering experiments

- higher precision and/or smaller  $q^2$   $\longrightarrow$  technical issues
- Jefferson Lab (2014-2015)

## Elastic muon-proton scattering

- MUSE at PSI (2016)

## Spectroscopy of electronic atoms & ions

- improve measurement of  $R_\infty$  to measure  $r_p$  from 1S-2S transition
- Several projects going on for different processes

## Spectroscopy of exotic atoms

- test pure QED contributions from purely leptonic bound states
- also cross check the measurement with  $\mu\text{He}$  and  $\mu\text{D}$

# Summary

The puzzle: from  $\mu\text{H}$ :  $r_p = 0.8433(17)$  fm  
 CODATA value  
 (from H):  $r_p = 0.8768(32)$  fm

6.4  $\sigma$  away!

Possible sources:

- Electron experiments  $\longrightarrow$  Several already
- Muon experiments  $\longrightarrow$  Cross check
- Hadronic effects of  $\mu\text{H}$  theory  $\longrightarrow$  Not likely much larger
- Beyond SM physics  $\longrightarrow$  Model dependence

On the theoretical side, still possible that we are missing contributions: have to be examined

On the experimental side, more experiments are needed to reinforce the observations



**THANK YOU!**