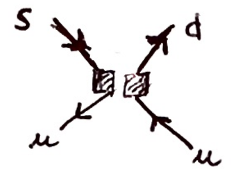
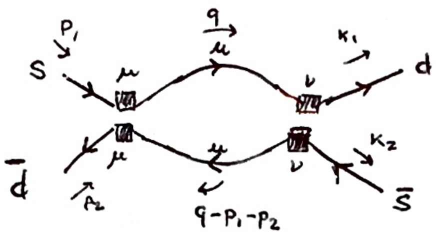
TAE 2017 – Exercise sessions BSM

GIM Mechanism



$$\mathcal{L}^{\Delta S=1} = s_c c_c \frac{4}{\sqrt{2}} G_F \left(\bar{u}_L \gamma^\mu s_L\right)\left(\bar{d}_L \gamma_\mu u_L\right)$$



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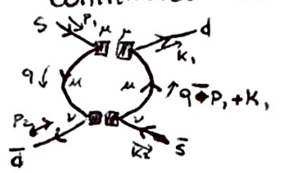


contributes to $\mathcal{L}^{\Delta S=2} = \frac{4}{\sqrt{2}} G^{\Delta S=2} \left(\bar{d}_L \gamma^\mu s_L\right)\left(\bar{d}_L \gamma_\mu s_L\right)$

($K-\bar{K}$ mixing)

$$\mathcal{M}^{loop} = \left(i\, s_c c_c \frac{4}{\sqrt{2}} G_F\right)^2 \int \frac{d^4 q}{(2\pi)^4} \left\{ \left[\bar{u}(k_1)\gamma^\nu P_L \frac{(\not{q})}{q^2} \gamma^\mu P_L u(p_1)\right]\left[\bar{v}(p_2)\, \gamma_\mu P_L \frac{(\not{q}-\not{p}_1-\not{p}_2)}{(q-p_1-p_2)^2} \gamma_\nu P_L v(k_2)\right]\right.$$

$$\left. + \left[\bar{v}(p_2)\gamma^\nu P_L \frac{(\not{q})}{q^2}\gamma^\mu P_L u(p_1)\right]\left[\bar{u}(k_1)\gamma_\mu P_L \frac{(\not{q}-\not{p}_1+\not{k}_1)}{(q-p_1+k_1)^2}\gamma_\nu P_L v(k_2)\right]\right\}$$

- Dimensional analysis: $\int \ldots \sim \frac{\Lambda_F^2}{16\pi^2}$: q powers are $\sim d^4 q \cdot \frac{1}{\not{q}} \cdot \frac{1}{\not{q}} \to \Lambda_F^2$

$$C^{loop,\,\Delta S=2} = \frac{1}{2\pi^2} s_c^2 c_c^2 G_F^2 \Lambda_F^2 \sim \left(2.4\cdot 10^{-3}\, G_F \Lambda_F^2\right) G_F \qquad \left(\begin{matrix} s_c = 0.225 \\ c_c \sim 1\end{matrix}\right)$$

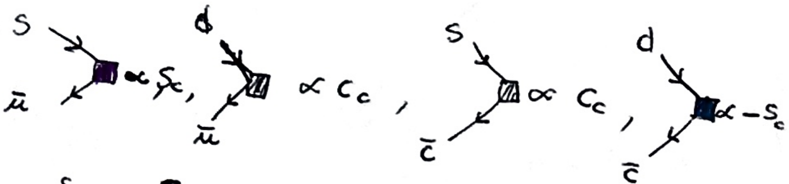
to compare with $G^{\Delta S=2}_{exp} = 3\cdot 10^{-8}\, G_F \implies$ for fine-tuning $\Delta \sim 1$, $G_F = \frac{1}{\sqrt{2}v^2}$

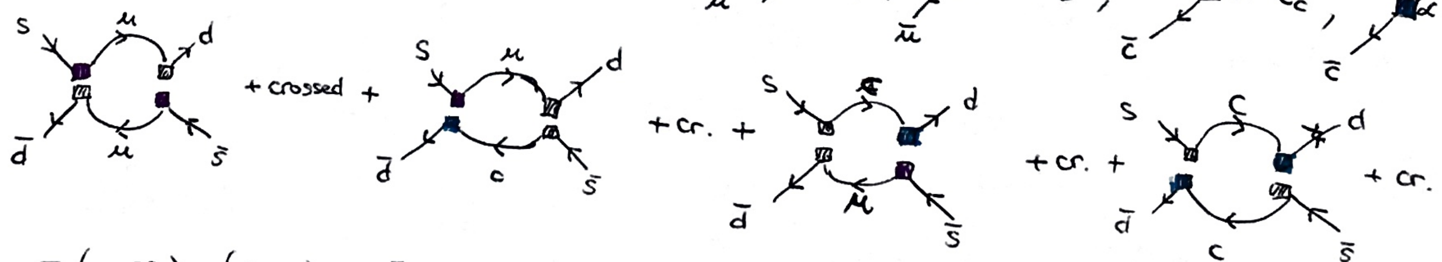
$$\Lambda_F \sim \sqrt{\frac{1}{G_F}}\sqrt{\frac{3\cdot 10^{-8}\cdot \frac{4}{\sqrt{2}}}{2.4\cdot 10^{-3}}} = 3.37\ \text{GeV}$$

(compare with $m_c \sim 1.3$ GeV)

- Including charm quark: now the loop can contain u, c.

$$\mathcal{L}_W \supset \left(\bar{u}_L\ \bar{c}_L\right)\begin{pmatrix} c_c & s_c \\ -s_c & c_c\end{pmatrix}\gamma^\mu \begin{pmatrix} d_L \\ s_L\end{pmatrix} W_\mu \frac{g}{\sqrt{2}}$$



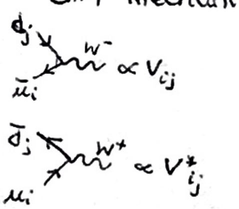


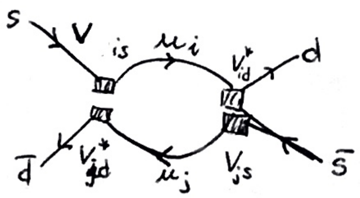
$$= (\text{coeff.})\cdot(\text{loop})\cdot\left[s^2c^2 + (-s^2c^2) + (-s^2c^2) + ((-s)^2c^2)\right] \simeq 0$$

focusing on quadr. divergence, we can ignore m_c contribution

at order 0 in m_c^2

GIM mechanism:





$$\propto \sum_{i,j} \overbrace{V_{is} V^*_{id}}^{\sum_i (V^\dagger)_{di} V_{is} = \delta_{ds} = 0}\ \underbrace{V_{js} V^*_{jd}}_{\sum_j (V^\dagger)_{dj} V_{js} = \delta_{ds} = 0} F\left(m_c^2, m_W^2\right)$$

CALCULATION OF THE LOOP INTEGRAL

$$\sum_{\mu,c} \text{loop} = \left(8\, s_c^2 c_c^2 G_F^2\right)\left[I(0,0) - I(0,m_c) - I(m_c,0) + I(m_c,m_c)\right],$$

m_1 gives only 1 mass flip $\rightarrow$ vanishes

$$I(m_1,m_2) = \int^{\Lambda} \frac{d^4q}{(2\pi)^4}\left[\frac{(\bar d\, \gamma^\nu \gamma_\lambda \gamma^\mu P_L s)(\bar d\, \gamma_\mu \gamma_\rho \gamma_\nu P_L s)}{((q+p_1)^2 - m_1^2)((q-p_2)^2 - m_2^2)}(q+p_1)^\lambda (q-p_2)^\rho + \frac{(\bar d\, \gamma^\nu \gamma_\lambda \gamma^\mu P_L s)(\bar d\, \gamma_\mu \gamma_\rho \gamma_\nu P_L s)}{((q+p_1)^2 - m_1^2)((q+k_1)^2 - m_2^2)}(q+p_1)^\lambda (q+k_1)^\rho\right]$$

We perform the matching in the zero external momentum limit: $p_{1,2}, k_{1,2} \rightarrow 0$.
The integrals bring to an expression $I_{\lambda\rho} \propto \eta_{\lambda\rho}$.
So we already implement this in the bilinears:

$$I(m_1,m_2) = \int^{\Lambda}\frac{d^4q}{(2\pi)^4}\frac{q^\lambda q^\rho}{(q^2-m_1^2)(q^2-m_2^2)}\cdot 2\cdot(\bar d\, \gamma^\nu \gamma_\lambda \gamma^\mu P_L s)(\bar d\, \gamma_\mu \gamma_\rho \gamma_\nu P_L s) =$$ (use the result of the Euclidean integral) $x_i \equiv m_i^2/\Lambda$

$$= \frac{\eta^{\lambda\rho}}{4}\frac{\Lambda^2}{16\pi^2}\,\frac{x_1\left(1 - x_1 \log\frac{1+x_1}{x_1}\right) - 1\leftrightarrow 2}{x_1 - x_2}\cdot 2\,(\bar d\, \gamma^\nu \gamma_\lambda \gamma^\mu P_L s)(\bar d\, \gamma_\mu \gamma_\rho \gamma_\nu P_L s) =$$ (use the formula for γ matr.)

$$= \frac{\Lambda^2}{4\cdot 16\pi^2}\cdot 2\cdot 4\cdot(\bar d_L \gamma^\mu s_L)(\bar d_L \gamma_\mu s_L)\,\frac{x_1\left(1 - x_1 \log\frac{1+x_1}{x_1}\right) - 1\leftrightarrow 2}{x_1 - x_2}$$

We are interested in the expansion at low x, $m_i \ll \Lambda$:

$$x\left(1 - x \ln\frac{1+x}{x}\right) \overset{x\to 0}{\simeq} x\left(1 + x\ln x - x\ln(1+x)\right) \simeq x\left(1 + \underbrace{x\ln x}_{0 \text{ for } x\to 0,\ \forall n\ x^n \ln x \to 0 \text{ for } x \to 0} - x\left(x - \frac{x^2}{2} + \ldots\right)\right) \simeq$$

$$\simeq x + x^2\ln x - x^3 + \frac{x^4}{2} - \frac{x^5}{3} + \ldots$$

$$I(m_1,m_2) \xrightarrow{m_1, m_2 \ll \Lambda} \frac{\Lambda^2}{8\pi^2}(\bar d_L \gamma^\mu s_L)(\bar d_L \gamma_\mu s_L)\left(1 + \frac{x_1^2 \ln x_1 - x_2^2 \ln x_2}{x_1 - x_2} - (x_1^2 + x_2^2 + x_1 x_2) + O(x^3)\right)$$

1: this term cancels in the sum (GIM mechanism)

$$\cancel{\Rightarrow \sum_{c} \text{loop} = \frac{\Lambda^2}{8\pi^2}\cdot 8G_F^2\,(\bar d_L \gamma^\mu s_L)(\bar d_L \gamma_\mu s_L)\, s_c^2 c_c^2\left(-\frac{m_c^4}{\Lambda^4} + O\left(\frac{m_c^6}{\Lambda^6}\right)\right) = \frac{m_c^4}{\Lambda^2 G_F}\frac{s_c^2 c_c^2}{\pi^2}(\bar d_L \gamma^\mu s_L)(\bar d_L \gamma_\mu s_L)}$$

$I(0,0) = 1$

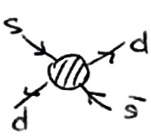
$I(0,m_c) = I(m_c,0) = 1 + x_c \ln x_c - x_c^2$

$$I(m_c,m_c) = 1 - 3x_c^2 + \lim_{x_1,x_2\to x_c}\frac{x_1^2 \ln x_1 - x_2^2 \ln x_2}{x_1 - x_2}$$

$$\lim_{x_1,x_2 \to x_c}\frac{x_1^2 \ln x_1 + (-x_1^2 \ln x_2 + x_1^2 \ln x_2) - x_2^2 \ln x_2}{x_1 - x_2} = \lim_{x_1,x_2\to x_c}\frac{x_1^2 \ln\frac{x_1}{x_2}}{x_1 - x_2} + \underbrace{\lim_{x_1,x_2\to x_c}(x_1+x_2)\ln x_2}_{=2x_c \ln x_c} =$$

$(x_1 = \lambda x_2)$

$$= 2x_c \ln x_c + \underbrace{\lim_{\lambda\to 1}\lambda^2\frac{\ln\lambda}{\lambda - 1}}_{=1}\cdot x_c = x_c + 2x_c \ln x_c$$



$$= \frac{\Lambda^2}{16\pi^2}G_F^2\,(\bar d_L \gamma^\mu s_L)(\bar d_L \gamma_\mu s_L)\cdot s_c^2 c_c^2\cdot\left[\cancel{(1-1-1+1)} + (x_c + \cancel{(-1-1+2)\,x_c \ln x_c} - (-1-1+3)x_c^2\right.$$

$$= \frac{G_F^2}{16\pi^2}(\bar d_L \gamma^\mu s_L)(\bar d_L \gamma_\mu s_L)\cdot s_c^2\cdot c_c^2\left[\boxed{m_c^2} - \frac{m_c^4}{\Lambda^2} + O\left(\frac{m_c^4}{\Lambda^4}\right)\right]$$