Gravitational Waves from First-Order Phase Transitions

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Overview

The Effective Potential

- At colliders: Particles are free in spatial $\pm\infty$
- Early universe: Energetic background plays role

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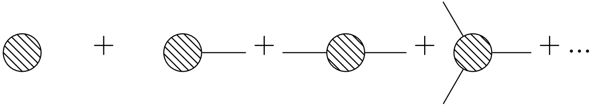
$\rightarrow$ Effective potential

Sum over all one-particle irreducible n-point functions with zero external momentum.

$$V_{\text{eff}}(\phi) = -\sum_n \frac{\phi^n}{n!}\Gamma^{(n)}(p_{\text{ext}} = 0)$$



The Effective Potential for Scalar Fields

Effective potential

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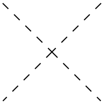
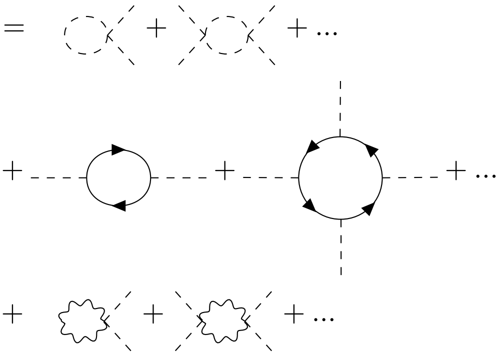
$V_{\text{tree}}(\phi) =$ $+$ 

$$= -\mu^2\phi^2 + \frac{\lambda}{2}\phi^4$$

The Effective Potential for Scalar Fields

$$V_{\text{1-loop}}(\phi) =$$



The Effective Potential for Scalar Fields

$$V_{\text{1-loop}}(\phi) \sim \pm \frac{1}{2} \int \frac{\mathrm{d}^4 k_E}{(2\pi)^4} \log \left[k_E^2 + m^2(\phi) \right]$$

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Finite temperature $\rightarrow$ Matsubara frequency sum

$$\int \frac{\mathrm{d}k^0}{2\pi} f(k^0) \rightarrow \sum_n f(k^0 = \omega_n)$$

$$\begin{aligned} \omega_n &= 2n\pi T && \text{(Bosons)} \\ \omega_n &= (2n+1)\pi T && \text{(Fermions)} \end{aligned}$$

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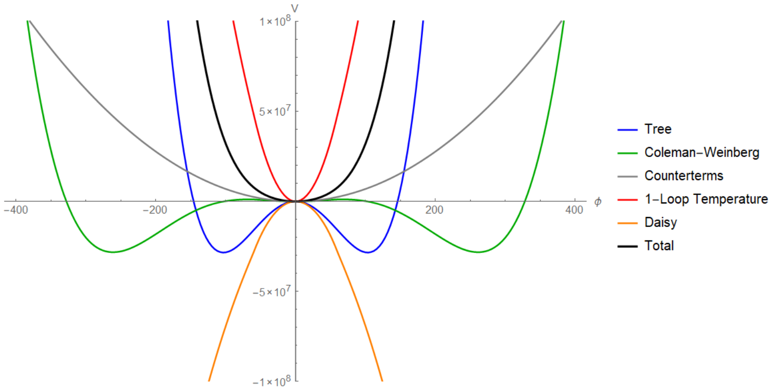
$$\Rightarrow \qquad V^T_{\text{1-loop}}(\phi) = \underbrace{V_{\text{1-loop}}(\phi, T=0)}_{\text{“Coleman-Weinberg”}} + \underbrace{V_{\text{1-loop}}(\phi, T \neq 0)}_{\text{finite temperature}}$$

1-loop Effective Potential

Example at high temperature



1-loop Effective Potential

$T \gg m$ approximation to <u>1st-order</u>:

$$V_{\text{1-loop}}(\phi, T) \approx \left[\frac{1}{24} \sum_{\text{bosons}} g_{\text{b}} m_{\text{b}}^2(\phi) + \frac{1}{48} \sum_{\text{fermions}} g_{\text{f}} m_{\text{f}}^2(\phi) \right] T^2$$
$$\sim \phi^2 T^2$$

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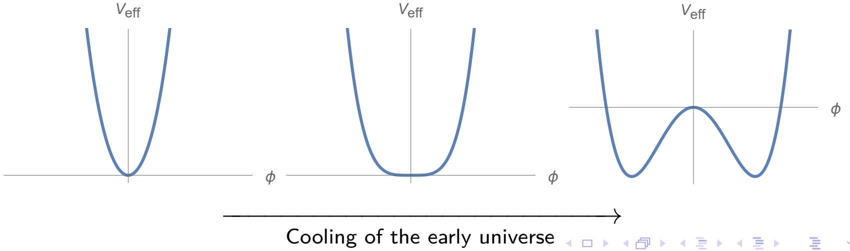

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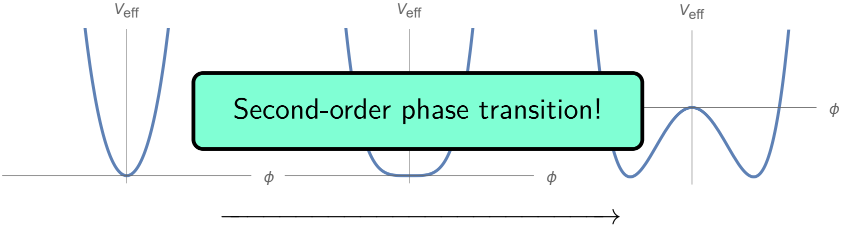

Second-order phase transition!

Cooling of the early universe

1-loop Effective Potential

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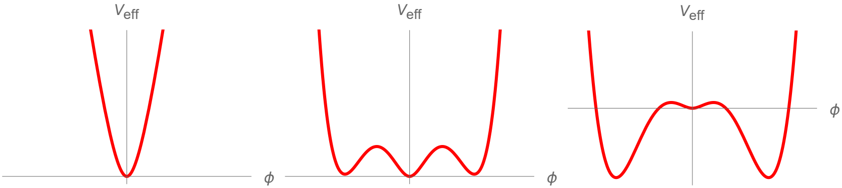

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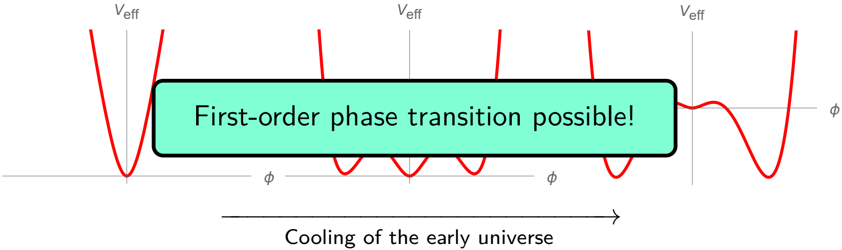

First-Order Phase Transitions

How to get first-order phase transitions?

- In SM: Have Higgs $\lesssim 70\,\mathrm{GeV}$ [hep-lat/9901021]
- Couple additional bosonic d.o.f. to Higgs
- Add ϕ^3 or ϕ^6 terms at tree level
- Vev Flip-Flop with additional scalar [1608.07578]

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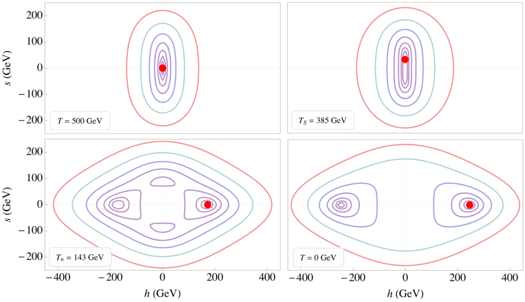

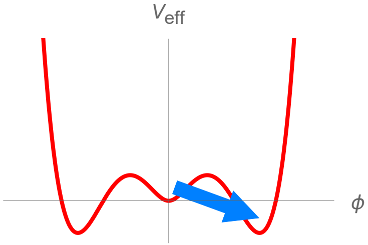

Thermal tunnel rate

- $\Gamma \sim e^{-S_3/T}$

Quantum tunnel rate

- $\Gamma \sim e^{-S_4}$

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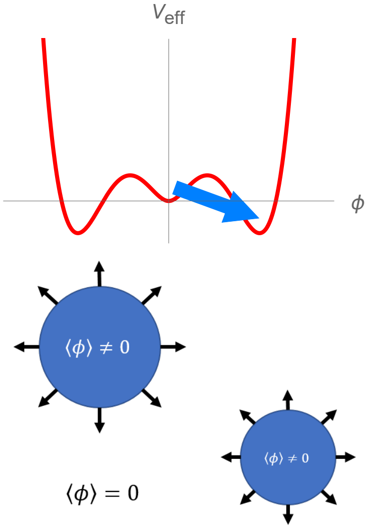
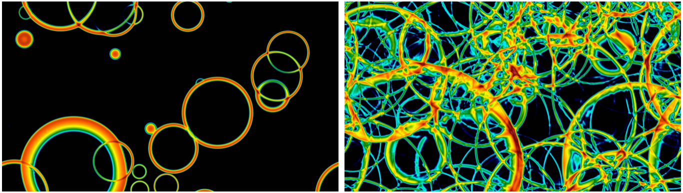
Nucleation criterion

- Probability of creating one bubble of broken phase per Hubble volume $\Gamma H^{-4} \sim 1$



Bubbles collide $\rightarrow$ $SO(3)$ symmetry broken $\rightarrow$ Gravitational waves



[1304.2433]

Gravitational wave sources

- Initial collision of phase transition fronts
- Collision of fluid density waves
- Turbulences after collision

Gravitational Waves

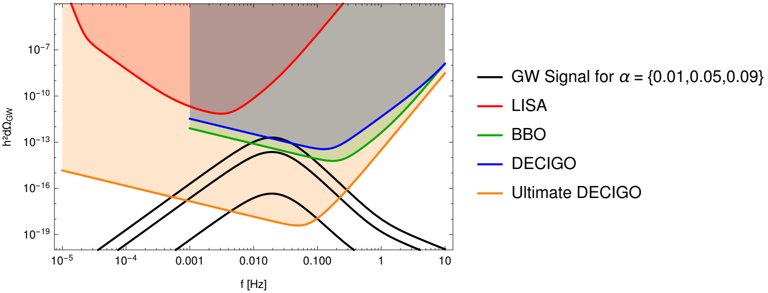
Relevant parameters

Critical temperature	T_c
Nucleation temperature	$T_n < T_c$
Phase transition strength	$\alpha = \frac{\text{latent heat}}{\rho_{\text{rad}*}} = \frac{-\Delta V - T\Delta S}{\rho_{\text{rad}*}}$
Phase transition duration	$\beta^{-1} = \left(\frac{\dot{\Gamma}}{\Gamma}\right)^{-1}$
Bubble wall velocity	v_w

Spectral shape given by simulations as broken power law!

Example: Vev Flip-Flop model

$$T_n \sim 100\,\text{GeV}$$
$$\alpha \sim 0.01 - 0.1$$
$$\beta^{-1} \sim 10^{-3} H^{-1}$$
$$v_w \sim 1 \text{ (assumed)}$$



Recipe

How to calculate GW spectrum for model?

1. Calculate field dependent & thermal masses
2. Write down effective potential
3. Find phases and transitions ($\rightarrow$ *CosmoTransitions*)
4. Extract parameters
5. Plot gravitational wave spectrum

Appendix: Tunnel Action / Bounce Solution

3-/4-dimensional Euclidean action

$$S_3[\phi_3] = \int \mathrm{d}^3x \left[\frac{1}{2}(\nabla\phi_3)^2 + V_{\text{eff}}\left(\phi_3/\sqrt{2}, T\right)\right]$$

$$S_4[\phi_4] = \int \mathrm{d}^4x \left[\frac{1}{2}(\partial_\rho\phi_4)^2 + V_{\text{eff}}\left(\phi_4/\sqrt{2}, T\right)\right] \qquad \text{with } \rho^2 = \tau^2 + r^2$$

Equation of motion (connecting broken and unbroken phase)

$$\partial_r^2\phi_3 + \frac{2}{r}\partial_r = V'(\phi_3, T)$$

$$\partial_\rho^2\phi_4 + \frac{3}{\rho}\partial_\rho = V'(\phi_4, T) \qquad \text{with } \rho^2 = \tau^2 + r^2$$

solved by "bounce solution", with boundary conditions

$$\phi\big|_{r,\rho\to\infty} = 0, \qquad \partial_{r,\rho}\phi\big|_{r,\rho=0} = 0$$