Behind!

~~Beyond~~ the Standard Model

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Exercise: FCNC and Naturalness

Consider the Fermi theory, and suppose you only know of the existence of the u,d,s quarks. **Estimate*** the contribution to the ΔS=2 coupling $G^{\Delta S=2}$

$$\mathcal{L}^{\Delta S=2} = \frac{G^{\Delta S=2}}{\sqrt{2}}[\bar{d}\gamma^\mu(1-\gamma^5)s][\bar{d}\gamma_\mu(1-\gamma^5)s]$$

that comes at one loop, with hard momentum cutoff Λ, from two insertions of

$$\mathcal{L}^{\Delta S=1} = s_c c_c \frac{G_F}{\sqrt{2}}[\bar{u}\gamma^\mu(1-\gamma^5)s][\bar{d}\gamma_\mu(1-\gamma^5)u]$$

where s_c is the sine of the Cabibbo angle. Assume $m_{u,d,s}=0$

Using the Naturalness criterion, with tuning Δ=1, and $G^{\Delta S=2}_{\rm exp} \sim 3\times 10^{-8} G_F$, estimate the cutoff Λ of the Fermi theory with only u,d and s quarks, and notice that it is not far from the charm quark mass.

Repeat the argument in the presence of the charm quark. Recognise that the GIM mechanism solves the Naturalness problem for FCNC

* estimate here means: ignore the γ-matrix structure of the loop and assume it matches the one of the ΔS=2 operator; do the integral by dim. analysis

Exercise: FCNC and Naturalness

If you like calculations, turn the previous estimate into a calculation of $G^{\Delta S=2}$, using massless u,d,s but massive charm quark.

You will need a one loop (Euclidean) integral: (with $x_i=m_i^2/\Lambda^2$)

$$I_{\mu\nu}=\int^{\Lambda}\frac{d^4k}{(2\pi)^4}\frac{k_\mu k_\nu}{(k^2+m_1^2)(k^2+m_2^2)}=\frac{1}{4}\eta_{\mu\nu}\frac{\Lambda^2}{16\,\pi^2}\frac{x_1(1-x_1\log\frac{1+x_1}{x_1})-1\leftrightarrow 2}{x_1-x_2}$$

And one γ-matrix identity (see Cheng-Li book chapter 12.2):

$$\left[\gamma^\mu\gamma^\lambda\gamma^\nu(1-\gamma^5)\right]_{ab}\left[\gamma_\mu\gamma_\lambda\gamma_\nu(1-\gamma^5)\right]_{cd}=4\left[\gamma^\lambda(1-\gamma^5)\right]_{ab}\left[\gamma_\lambda(1-\gamma^5)\right]_{cd}$$

good luck …

Natural Models

Half a century of thoughts led to only **two mechanisms** that provide a Natural microscopic origin for Higgs mass

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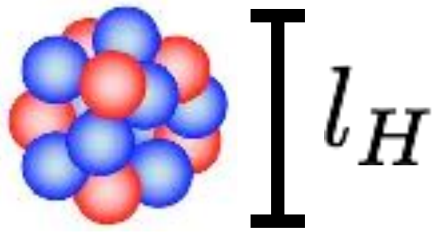
The rest of the course is (mostly) devoted to show how they work

Composite Higgs

Imagine the **Higgs is a bound state of new strong force.**



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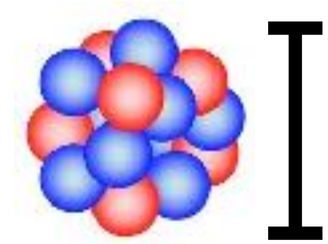

l_H

New sector's confinement scale $m_* = 1/l_H$

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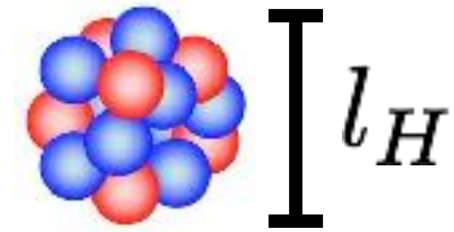

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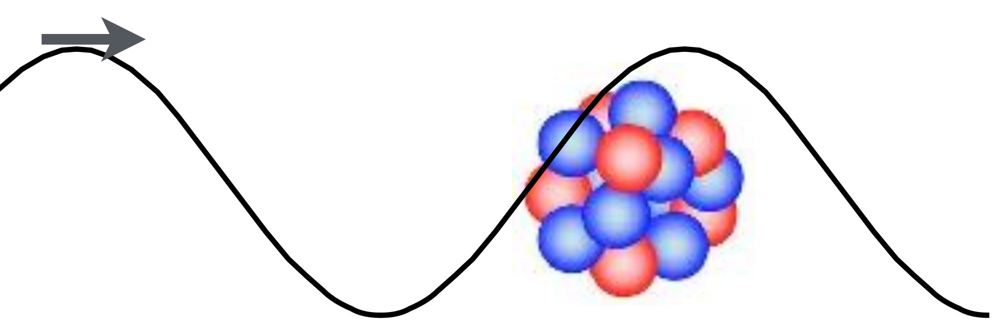
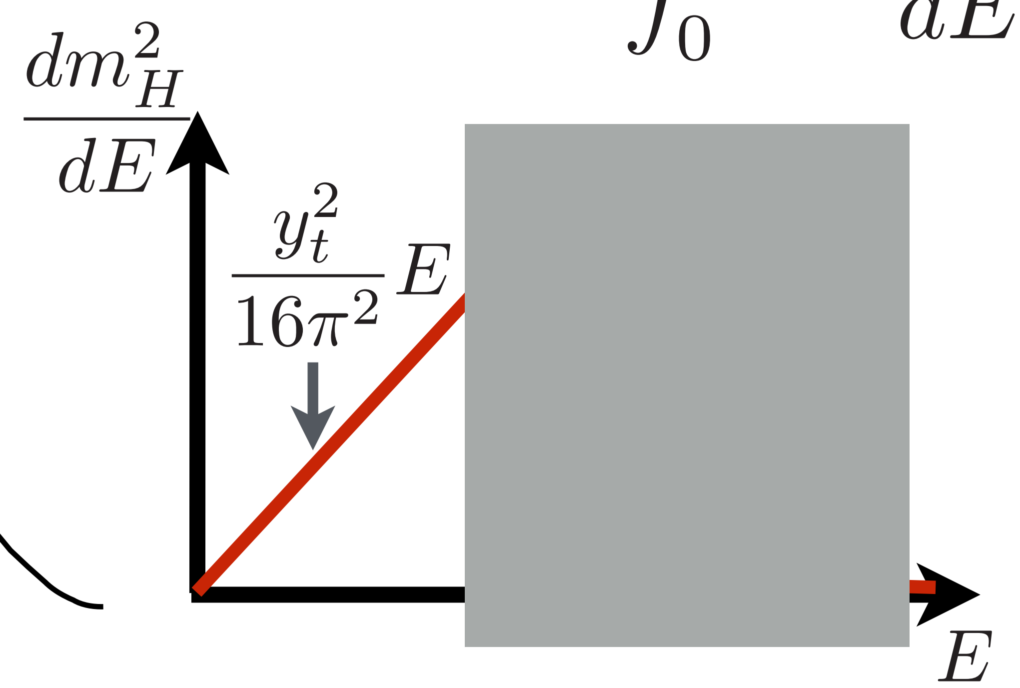
Higgs mass formula: $m_H^2 = \int_0^\infty dE \frac{dm_H^2}{dE}(E; p_{\rm FT})$

Higgs is SM-like
for $E \ll m_*$





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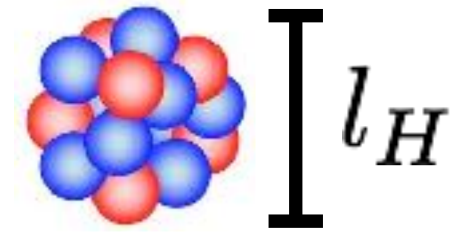

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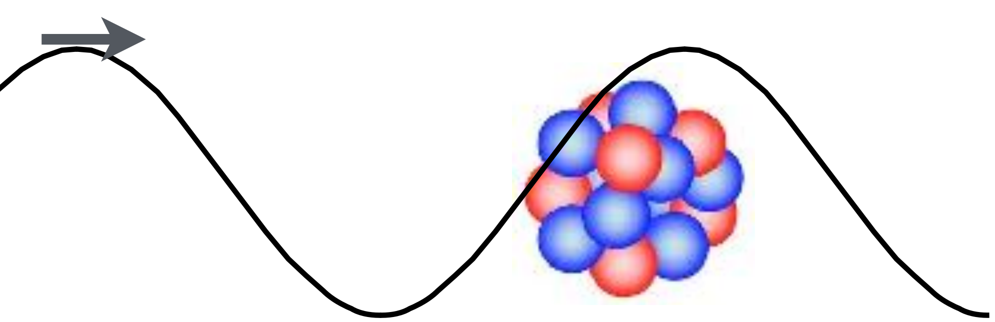
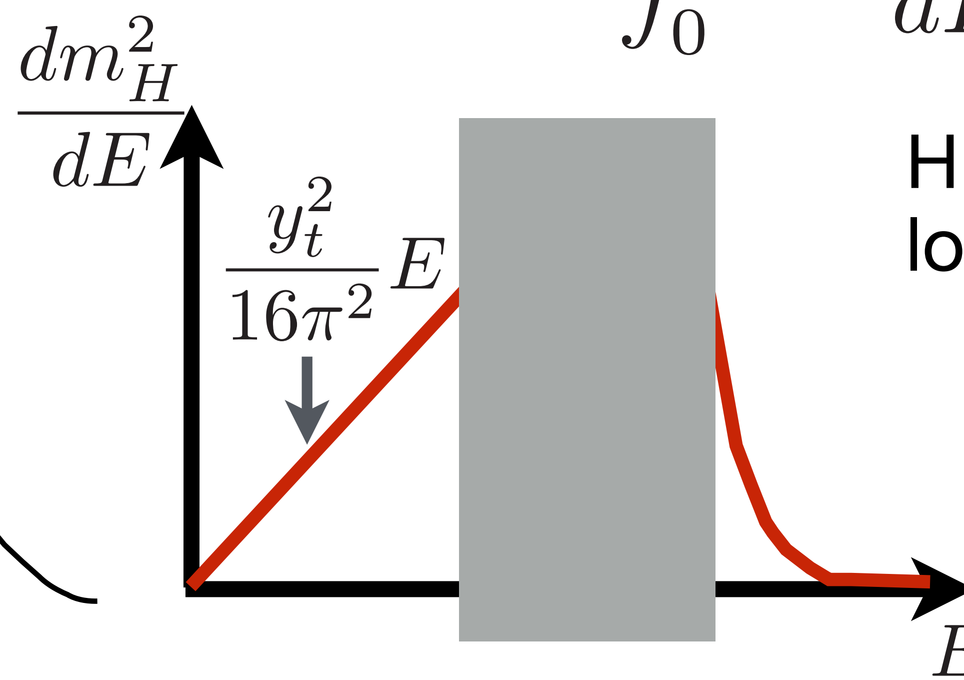
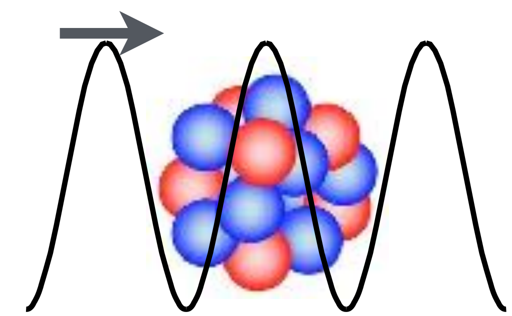




Higgs is **transparent** to low-wavelength modes



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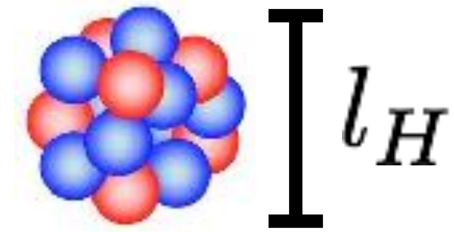

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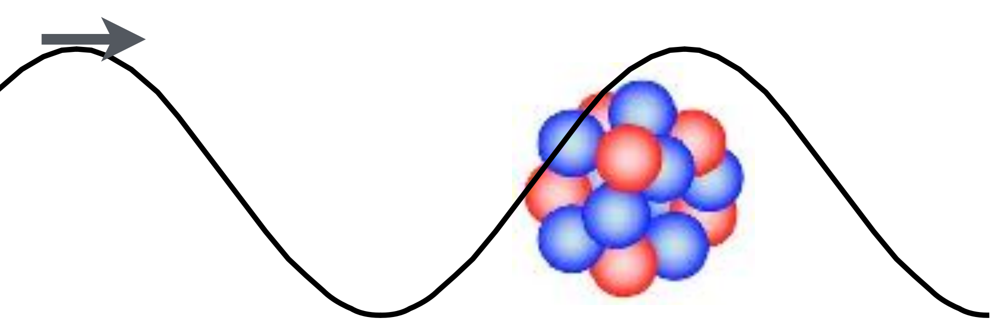

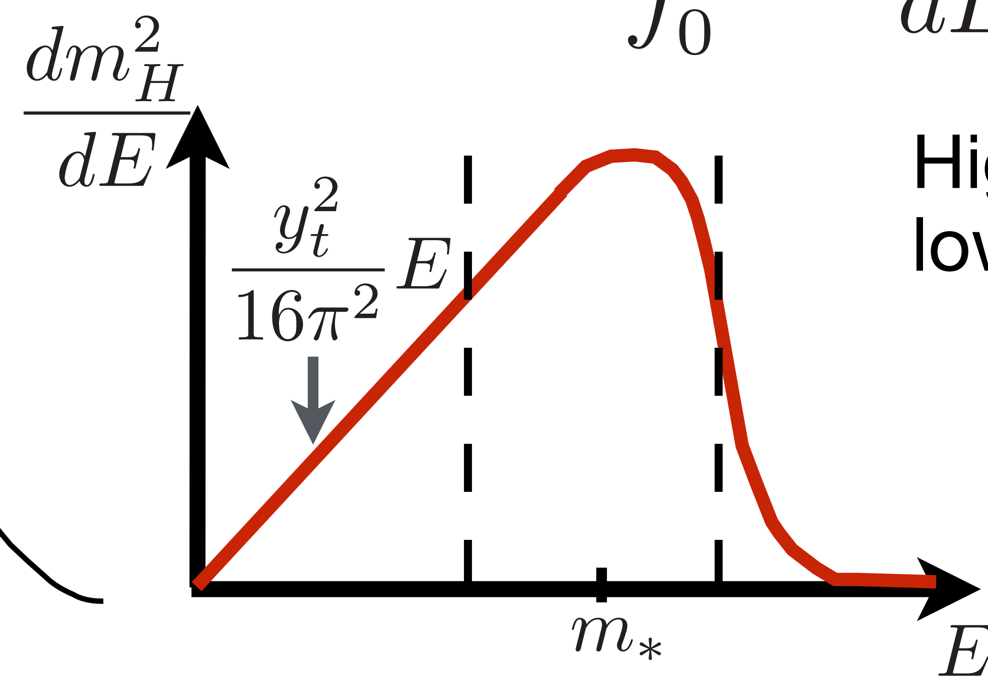

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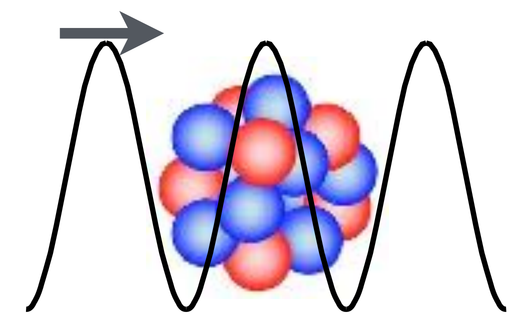
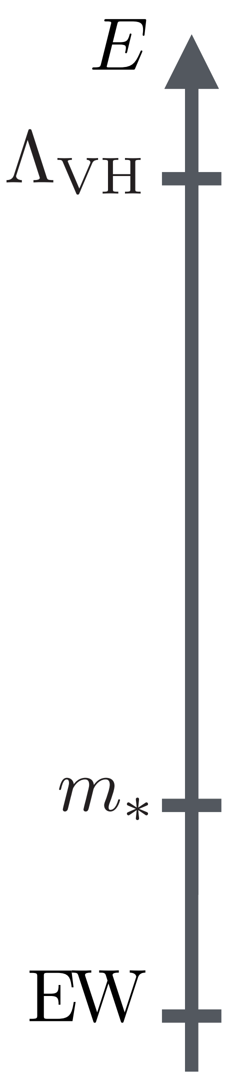


Higgs mass generation **localised** at $E \sim m_*$

Compositeness screens sensitivity to very high energy physics

Composite Higgs

The **Composite Higgs** picture for high energy physics



Two sectors exist at some Very High (>>TeV) scale Λ_{VH}:

Elementary Sector

SM **minus** Higgs

$W_\mu^\alpha, B_\mu, f_{L,R}, \ldots$

Composite Sector

QCD-like confining theory.

No fundamental Higgs.

No Un-Natural d<4 operators.

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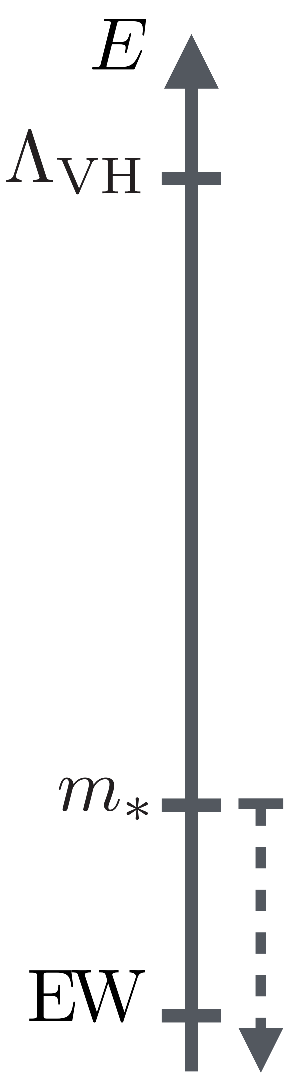

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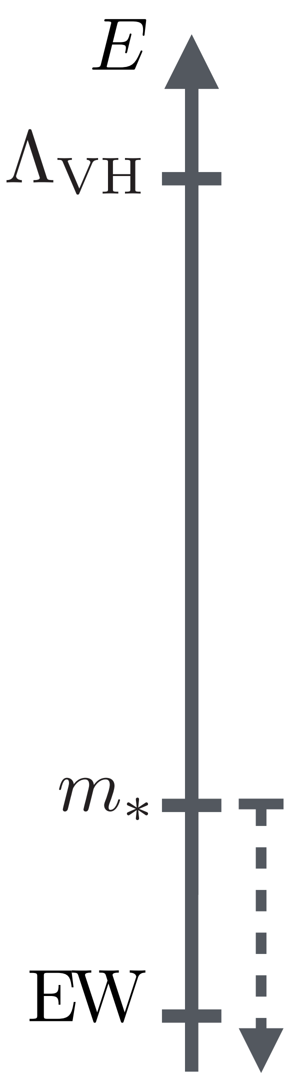

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According to the general tuning formula …

$$\Delta \geq \left(\frac{\Lambda_{\rm SM}}{500\,{\rm GeV}}\right)^2 = \left(\frac{m_*}{500\,{\rm GeV}}\right)^2$$

… Naturalness requires m_* to be below around the TeV.

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For example, pick a random hadron from the PDG list …

LIGHT UNFLAVORED ($S = C = B = 0$)	$I^G(J^{PC})$		$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$	• $\phi(1680)$	$0^-(1^{--})$
• π^0	$1^-(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$
• η	$0^+(0^{-+})$	• $\rho(1700)$	$1^+(1^{--})$
• $f_0(500)$	$0^+(0^{++})$	$a_2(1700)$	$1^-(2^{++})$
• $\rho(770)$	$1^+(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$
• $\omega(782)$	$0^-(1^{--})$	$\eta(1760)$	$0^+(0^{-+})$
• $\eta'(958)$	$0^+(0^{-+})$	• $\pi(1800)$	$1^-(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$	$f_2(1810)$	$0^+(2^{++})$
• $a_0(980)$	$1^-(0^{++})$	$X(1835)$	$?^?(?^{-+})$
• $\phi(1020)$	$0^-(1^{--})$	$X(1840)$	$?^?(?^{??})$
• $h_1(1170)$	$0^-(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$
• $b_1(1235)$	$1^+(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$
• $a_1(1260)$	$1^-(1^{++})$	• $\pi_2(1880)$	$1^-(2^{-+})$
• $f_2(1270)$	$0^+(2^{++})$	$\rho(1900)$	$1^+(1^{--})$
• $f_1(1285)$	$0^+(1^{++})$	$f_2(1910)$	$0^+(2^{++})$

STRANGE ($S = \pm1$, $C = B = 0$)	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K^0_S	$1/2(0^-)$
• K^0_L	$1/2(0^-)$
$K^*_0(800)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K^*_0(1430)$	$1/2(0^+)$
• $K^*_2(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$

CHARMED, STRANGE ($C = S = \pm1$)	$I(J^P)$
• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D^*_{s0}(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}(2573)$	$0(?^?)$
• $D^*_{s1}(2700)^\pm$	$0(1^-)$
$D^*_{sJ}(2860)^\pm$	$0(?^?)$
$D_{sJ}(3040)^\pm$	$0(?^?)$
BOTTOM ($B = \pm1$)	
• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	

$c\bar{c}$	$I^G(J^{PC})$
• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
• $h_c(1P)$	$?^?(1^{+-})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
$X(3823)$	$?^?(?^{?-})$
• $X(3872)$	$0^+(1^{++})$
• $X(3900)^\pm$	$?(1^+)$
$X(3900)^0$	$?(?^?)$
• $\chi_{c0}(2P)$	$0^+(0^{++})$
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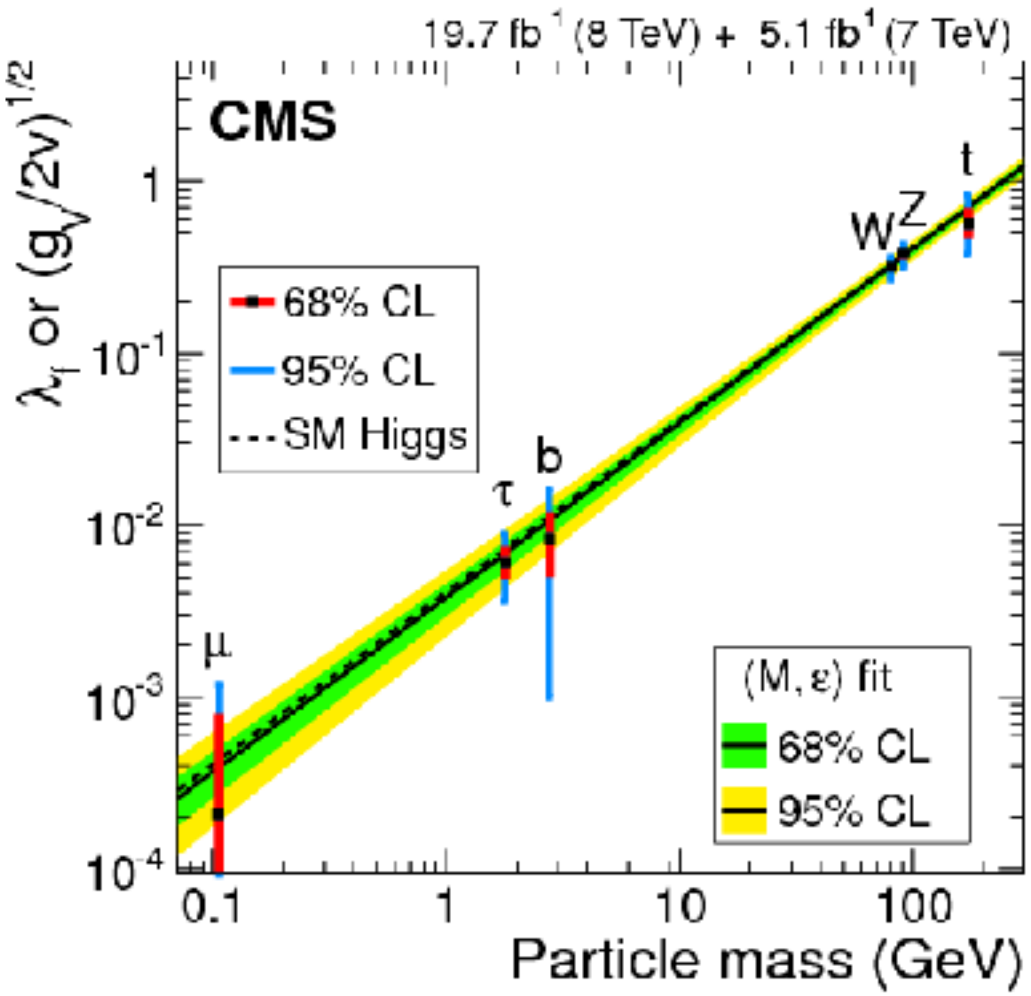

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…spontaneously broken by CS…

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$$\mathcal{G} \quad \longrightarrow \quad \mathcal{H}$$

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One **massless scalar** for each **spontaneously broken** generator.
Small mass from small **explicit** symmetry **breaking. E.g. QCD pions**

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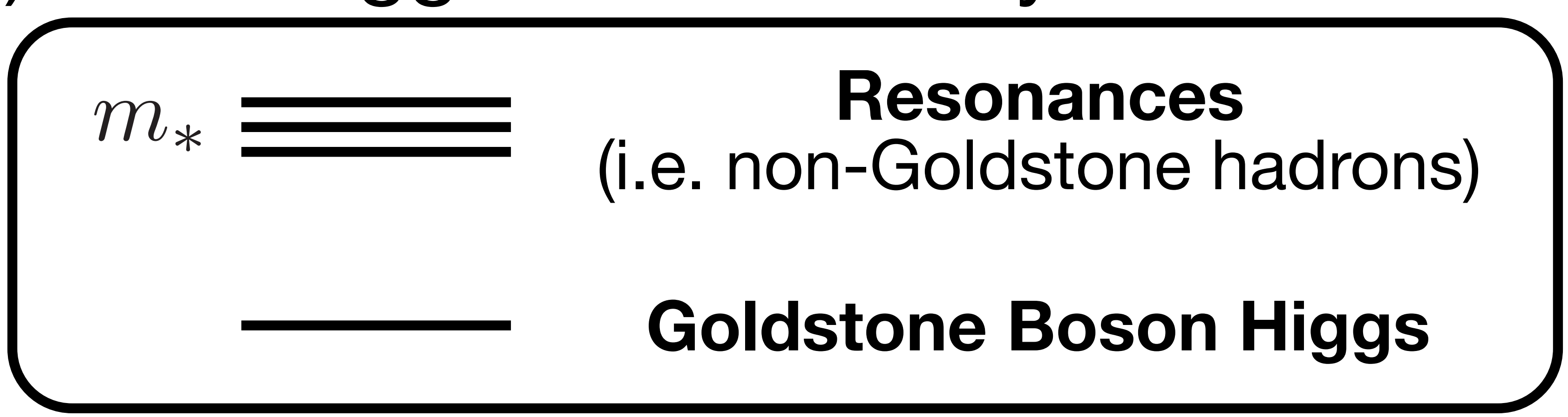
A (pseudo) NGB Higgs automatically addresses issue #1



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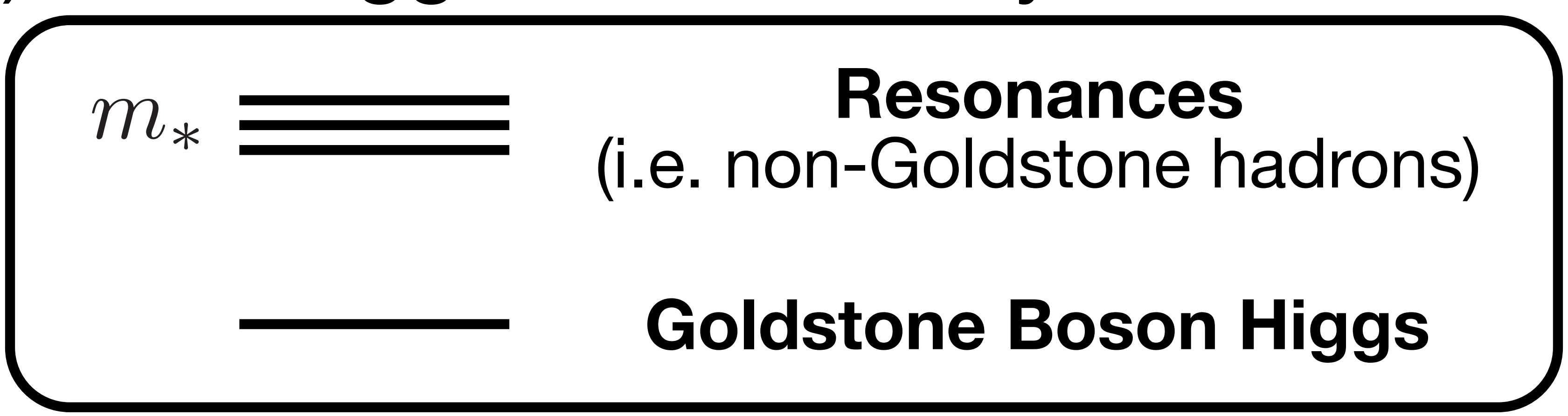

Can also address issue #2, by **“Vacuum Misalignment”**

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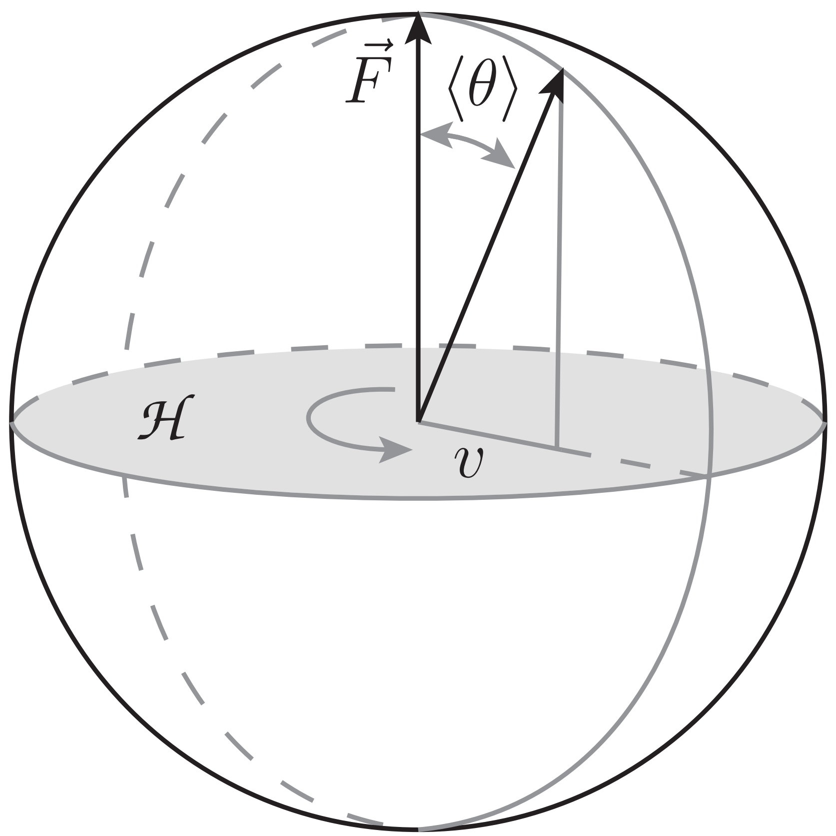

Spontaneous CS breaking: $\mathcal{G} \to \mathcal{H}$

$$\{T^A\} = \{T^a, \widehat{T}^{\hat{a}}\}$$

The **EW group** is in $\mathcal{H}$: $\mathcal{H} \subseteq \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

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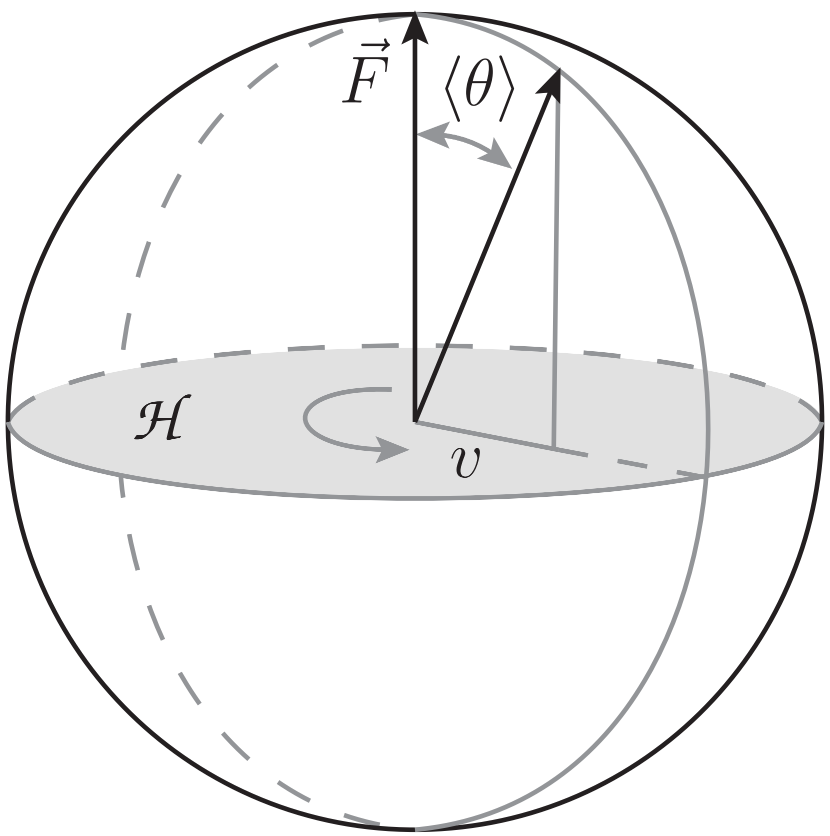

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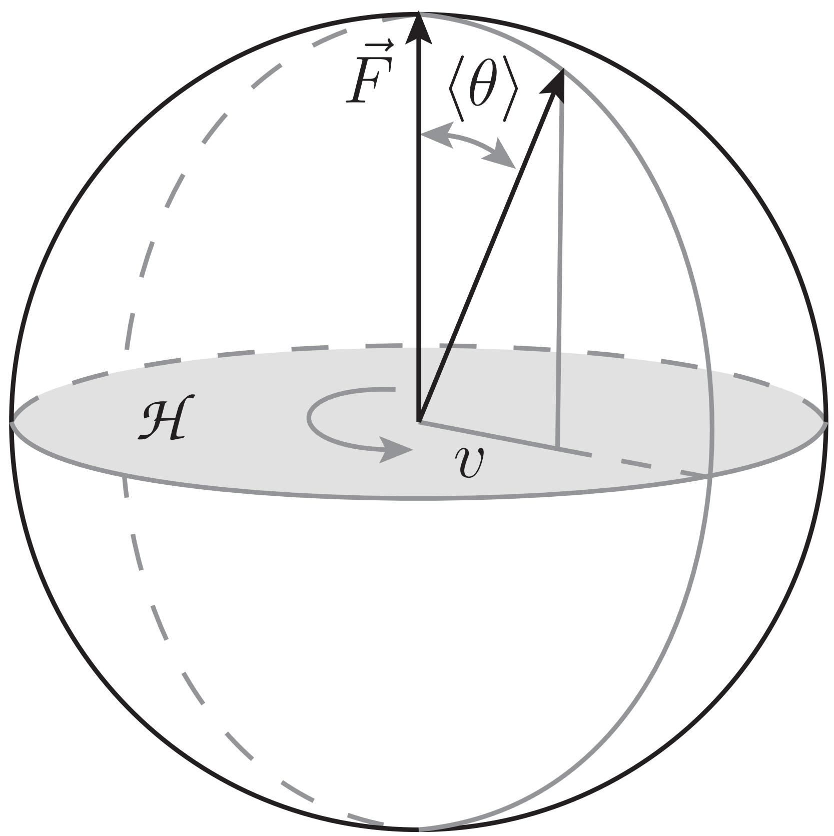

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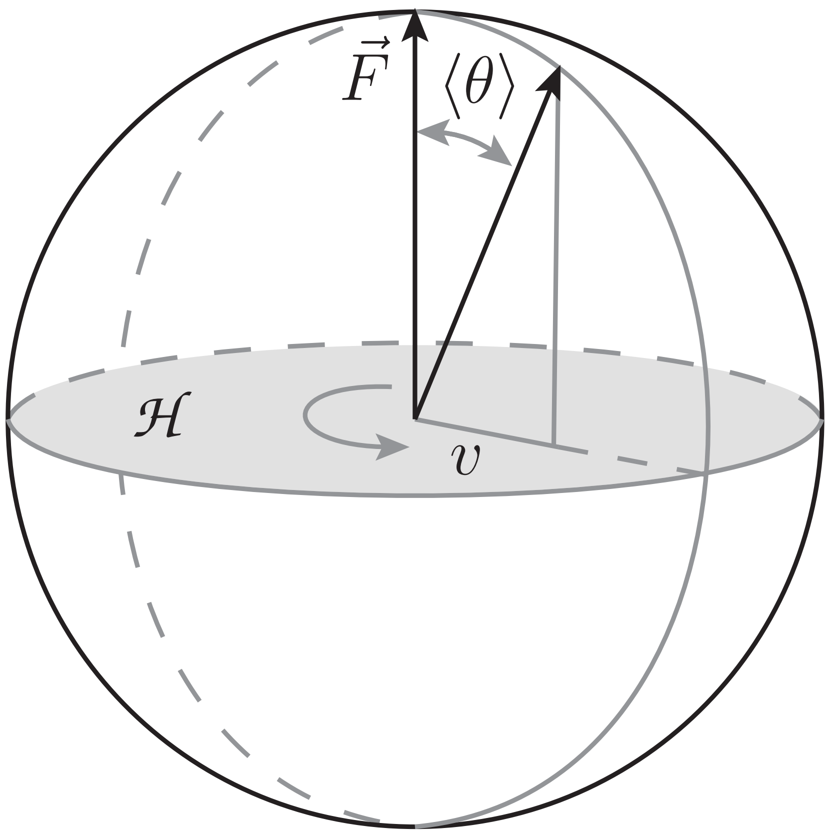

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For $\xi \to 0$ ($f \to \infty$), CS decouples and **the SM is recovered.**

CH Signatures Overview

Composite Higgs **signatures:** (classified by **robustness**)

- **Higgs coupling modifications**
  robustly predicted by symmetries.
  But hard (and long) to improve at LHC
- **Vector resonances**
  reasonable compromise.
- **Top Partners**
  "Naturally" light, but smart (crappy?)
  model-building might make them heavy.
- **Light quarks Partners**
  relevant in some models.

More robust, i.e. more discovery chance or more effective exclusion

Less robust, but maybe easier to make prog.s

The Minimal CH Couplings

The **Minimal Composite Higgs** model:

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10 generators = **6 unbroken + 4 broken**

4 real = 2 complex Higgs components

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$$\{T^A\} = \left\{ T^a = \left\{ \begin{bmatrix} t_L^\alpha & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} t_R^\alpha & 0 \\ 0 & 0 \end{bmatrix} \right\}, \widehat{T}^{\widehat{a}} = \begin{bmatrix} 0 & v^{\widehat{a}} \\ -v^{\widehat{a}} & 0 \end{bmatrix} \right\}$$

$$\begin{array}{c} \mathrm{SO}(4) \simeq \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \\ \cap \\ \mathcal{G}_{\mathrm{EW}} = \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \end{array}$$

$$Y = T_R^3$$

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10 generators **= 6 unbroken + 4 broken**

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$$\begin{array}{l} \mathrm{SO}(4) \simeq \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \\ \cap \\ \mathcal{G}_{\mathrm{EW}} = \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \end{array} \qquad Y = T_R^3$$

Higgs emerges as **fourplet** of $\mathrm{SO}(4)$: $\vec{\Pi} = \{\Pi_1, \Pi_2, \Pi_3, \Pi_4\}$

Converted to **ordinary doublet** by:

$$H = \begin{bmatrix} h_u \\ h_d \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} \Pi^2 + i\Pi^1 \\ \Pi^4 - i\Pi^3 \end{bmatrix}$$

The Minimal CH Couplings

The Composite Higgs picture, at ~TeV energies:

Composite Sector

“Exact” symmetry $\mathrm{SO}(5)$**.**

Spontaneously broken to $\mathrm{SO}(4)$**.**

m_* **Resonances**

pNGB Higgs

The Minimal CH Couplings

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Let us first focus on the **Higgs plus gauge** system.

The Minimal CH Couplings

General theory of Goldstone Bosons (in a nutshell)

NGB definition:

$$\vec{\Phi} = e^{i\theta_{\widehat{a}}\widehat{T}^{\widehat{a}}} \cdot \vec{F}$$

where $\vec{\Phi} \to g \cdot \vec{\Phi}$ under $g \in \mathrm{SO}(5) = \mathcal{G}$

The Minimal CH Couplings

General theory of Goldstone Bosons (in a nutshell)

NGB definition:

$$\vec{\Phi} = e^{i \overbrace{\theta_{\widehat{a}} \widehat{T}^{\widehat{a}} \cdot \vec{F} \equiv e^{i\frac{\sqrt{2}}{f}}}^{\text{just normalisation}} \Pi_{\widehat{a}} \widehat{T}^{\widehat{a}}} \cdot \vec{F}$$

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The Minimal CH Couplings

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important definition

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The **Goldstone Matrix:** (basic object to construct Lagrangians)

$$U[\Pi] = e^{i\frac{\sqrt{2}}{f}\Pi_{\widehat{a}}\widehat{T}^{\widehat{a}}}$$

The Minimal CH Couplings

In the Minimal Model:

$$U\left[\Pi\right] = e^{i\frac{\sqrt{2}}{f}\Pi_i(x)\widehat{T}^i} = \begin{bmatrix} \mathbb{1} - \left(1 - \cos\frac{\Pi}{f}\right)\frac{\vec{\Pi}\,\vec{\Pi}^T}{\Pi^2} & \sin\frac{\Pi}{f}\frac{\vec{\Pi}}{\Pi} \\ -\sin\frac{\Pi}{f}\frac{\vec{\Pi}^T}{\Pi} & \cos\frac{\Pi}{f} \end{bmatrix}$$

Notice the dependence on Π/f.
Makes high order terms vanish for $f\to\infty$ ($\xi\to 0$).

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The Minimal CH Couplings

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(under which Lagrangian is invariant)

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The Minimal CH Couplings

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Easy to understand why …

$$\vec{\Phi} = U[\vec{\Pi}] \cdot \vec{F} \quad \to \quad g \cdot U[\vec{\Pi}] \cdot h^{\dagger} \cdot \vec{F}$$

The Minimal CH Couplings

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$$U[\vec{\Pi}] \to U[\vec{\Pi}^{(g)}] = g \cdot U[\vec{\Pi}] \cdot h^\dagger[g, \vec{\Pi}]$$

Easy to understand why: produces symmetry transformation on $\vec{\Phi}$.

$$\vec{\Phi} = U[\vec{\Pi}] \cdot \vec{F} \quad \to \quad g \cdot U[\vec{\Pi}] \cdot \cancel{h^\dagger} \cdot \vec{F} = g \cdot (U \cdot \vec{F}) = g \cdot \vec{\Phi}$$

The Minimal CH Couplings

The **Higgs-only** low-energy Lagrangian:

$$\mathcal{L}_{\rm Higgs} = \frac{f^2}{2}\left[U^\dagger \partial_\mu U\right]^2_{\widehat{a}}$$

The Minimal CH Couplings

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Straightforward to include **gauge** interactions …

$$\partial_\mu \to D_\mu = \partial_\mu - i g W^\alpha_\mu T^\alpha_L - i g' B_\mu T^3_R$$

… obtaining the **Higgs plus gauge** Lagrangian:

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The Minimal CH Couplings

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Finally, going to the **Unitary Gauge** $H = \begin{bmatrix} 0 \\ \frac{V+h(x)}{\sqrt{2}} \end{bmatrix}$

$$\mathcal{L}_{\text{Higgs+gauge}} = \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2}{4} f^2 \sin^2 \frac{V+h}{f}\left[|W|^2 + \frac{1}{c_w^2} Z^2\right]$$

The Minimal CH Couplings

Let’s see what we got

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1. EW boson masses: (with **custodial** $\rho = 1$ relation)

$$m_W = c_w m_Z = \frac{1}{2} g f \sin\frac{V}{f} = \frac{1}{2} g v \qquad v = f \sin\frac{V}{f} = 246\text{GeV}$$

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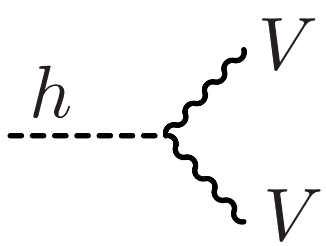
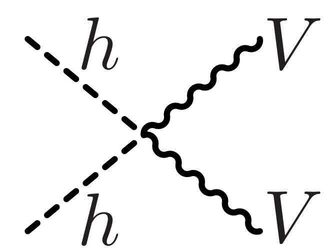
2. SM-like couplings: ($\xi = v^2/f^2 = \sin^2 V/f$)



$$= \sqrt{1-\xi} \times \text{SM}_{hVV}$$



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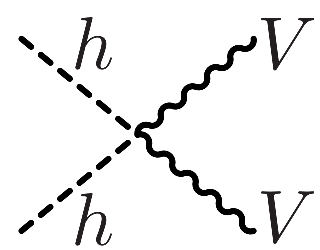

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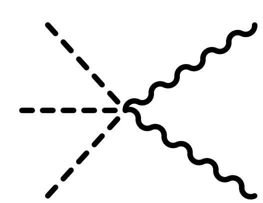
3. Non-SM vertices:



$\sim \xi g^2 / v$ + …

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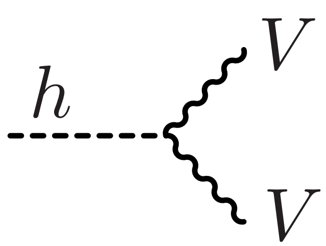

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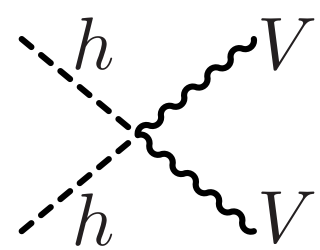

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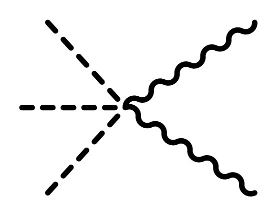
3. Non-SM vertices:  $\sim \xi g^2 / v$ + …

Predicted Higgs coupling deviations, e.g. $\kappa_V = \sqrt{1-\xi}$.

As expected, SM is recovered for $\xi = 0$.