Behind!

~~Beyond~~ the Standard Model

Andrea Wulzer

Università degli Studi di Padova

CERN

École Polytechnique Fédérale de Lausanne

CH Signatures Overview

Composite Higgs **signatures:** (classified by **robustness**)

More robust, i.e. more discovery chance or more effective exclusion

- **Higgs coupling modifications**
  robustly predicted by symmetries.
  But hard (and long) to improve at LHC
- **Vector resonances**
  reasonable compromise.
- **Top Partners**
  "Naturally" light, but smart (crappy?) model-building might make them heavy.
- **Light quarks Partners**
  relevant in some models.

Less robust, but maybe easier to make prog.s

The Minimal CH Couplings

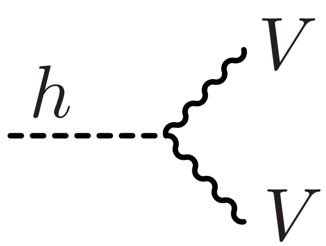
Let's see what we got

$$\mathcal{L}_{\text{Higgs+gauge}} = \frac{1}{2}(\partial_\mu h)^2 + \frac{g^2}{4} f^2 \sin^2 \frac{V+h}{f} \left[|W|^2 + \frac{1}{c_w^2} Z^2 \right]$$

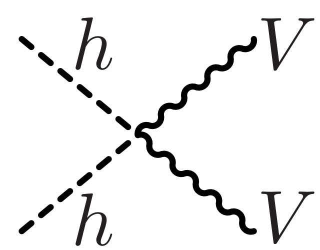
1. EW boson masses: (with **custodial** $\rho = 1$ relation)

$$m_W = c_w m_Z = \frac{1}{2} g f \sin\frac{V}{f} = \frac{1}{2} g v \qquad v = f \sin\frac{V}{f} = 246\text{GeV}$$

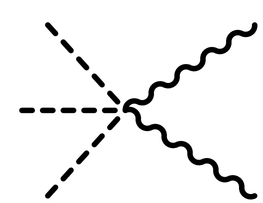
2. SM-like couplings: $(\xi = v^2/f^2 = \sin^2 V/f)$



$= \sqrt{1-\xi} \times \text{SM}_{hVV}$



$= (1-2\xi) \times \text{SM}_{hhVV}$

3. Non-SM vertices:  $\sim \xi g^2/v$ + …

Predicted Higgs coupling deviations, e.g. $\kappa_V = \sqrt{1-\xi}$.

As expected, SM is recovered for $\xi = 0$.

The Minimal CH Couplings

We now turn to **Higgs plus fermions**

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Composite Sector

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Spontaneously broken to $\mathrm{SO}(4)$.

m_* **Resonances**

pNGB Higgs

$\mathcal{L}^g_{\rm int} = g W_\mu J^\mu$

Elementary Sector

SM **gauge fields:** W^α_μ, B_μ.

Coupled by **gauging.**

SM **fermions:** $\{t_L, b_L\}$, $t_R, \ldots$

Coupled by **??.**

The Minimal CH Couplings

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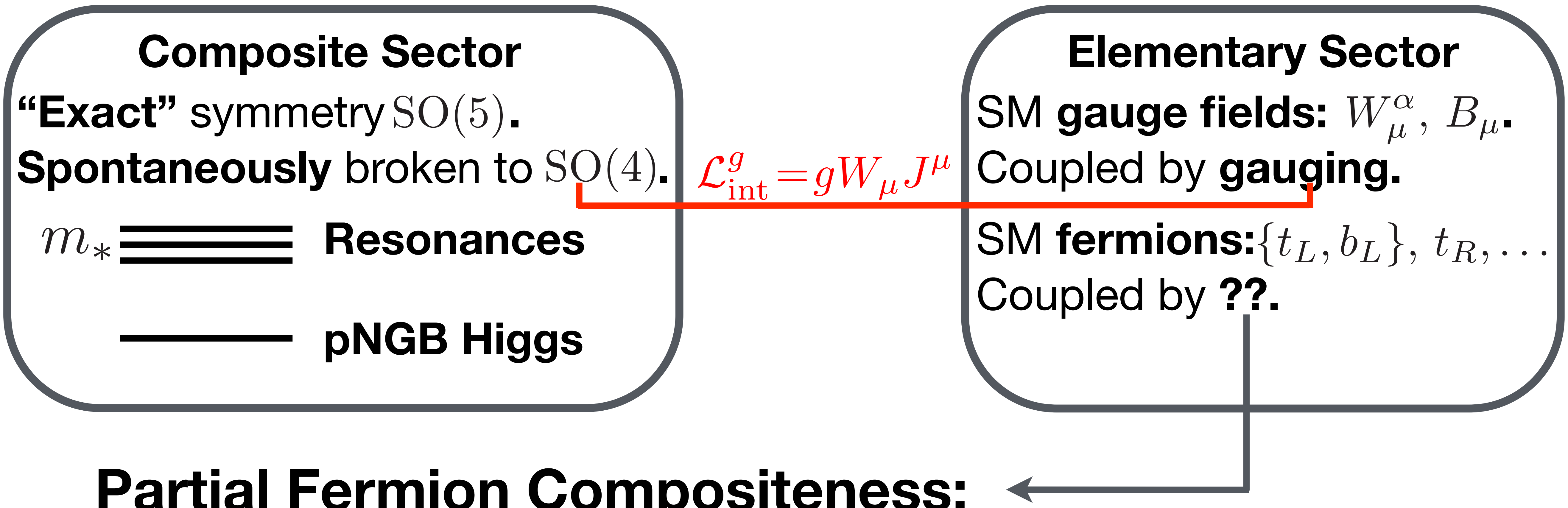

Partial Fermion Compositeness:

$$\mathcal{L}^f_{\text{int}} = \lambda \overline{f} \mathcal{O}$$

with $\mathcal{O}$ a **Composite Sector fermionic operator.**

The Minimal CH Couplings

More precisely (focusing on the top quark sector)

$$\mathcal{L}^f_{\rm int} = \lambda_R \bar{t}_R \mathcal{O}_L + \lambda_L \overline{q}_L \mathcal{O}_R$$

with $\mathcal{O}_{L,R}$ (being CS op.s) in some $\mathrm{SO}(5)$ representation.

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$$\mathcal{L}^f_{\rm int} = \lambda_R \overline{T}^I_R \mathcal{O}^I_L + \lambda_L \overline{Q}^I_L \mathcal{O}^I_R$$

"Embeddings" T_R and Q_L pick up the right components:

$$T_R = \{0,\, 0,\, 0,\, 0,\, t_R\} \qquad\qquad Q_L = \frac{1}{\sqrt{2}}\{-i\, b_L,\, -b_L,\, -i\, t_L,\, t_L,\, 0\}$$

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$$U[\vec{\Pi}] \to \vec{U}[\vec{\Pi}^{(g)}] = g \cdot U[\vec{\Pi}] \cdot h^\dagger[g, \vec{\Pi}]$$

$$h = \begin{bmatrix} h_{4\times 4} & 0 \\ 0 & 1 \end{bmatrix} \in \mathrm{SO}(4) = \mathcal{H}$$

The Minimal CH Couplings

Result:

$$\mathcal{L}_{\text{Higgs+Top}} = \quad (\quad \ldots \quad) \quad (\overline{Q}_L)^I (U)_I^5 \, (U^\dagger)_5^I (T_R)_I$$

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Under the symmetry …

$$(\overline{Q}_L)^I (U)_I^5 \xrightarrow{g} (\overline{Q}_L)^J (g^\dagger)_J{}^I g_I{}^J (U)_I{}^k h_k{}^5$$

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$$(\overline{Q}_L)^I (U)_I^5 \xrightarrow{g} (\overline{Q}_L)^J \cancel{(g^\dagger)_J{}^I g_I{}^J} (U)_I{}^k h_k{}^5 = (\overline{Q}_L)^I (U)_I{}^5$$

$$h_k{}^5 = \delta_k^5$$

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Result: (in the Unitary Gauge)

$$\mathcal{L}_{\text{Higgs+Top}} = \left(\quad \ldots \quad \right) \quad (\overline{Q}_L)^I (U)_I^5 \, (U^\dagger)_5^I (T_R)_I$$

$$= \left(\quad \ldots \quad \right) \quad \frac{1}{2\sqrt{2}} \sin \frac{2(V + h(x))}{f} \bar{t}t$$

We get:

1. Top mass

The Minimal CH Couplings

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$$\mathcal{L}_{\mathrm{Higgs+Top}} = -\frac{\sqrt{2}m_t}{\sqrt{\xi(1-\xi)}}(\overline{Q}_L)^I (U)_I^5 \, (U^\dagger)_5^I (T_R)_I$$
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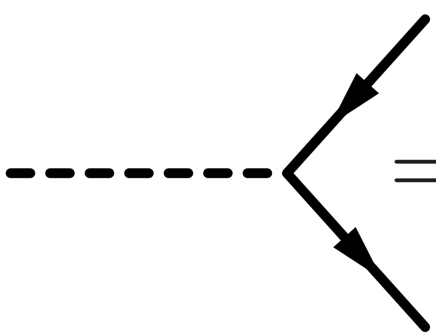
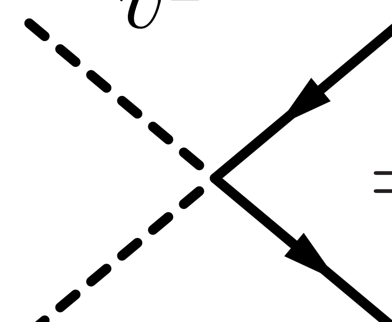
We get:

1. Top mass, sets normalisation $(\xi = v^2/f^2 = \sin^2 V/f)$
2. Couplings: $\mathcal{L} = -m_t\bar{t}t - k_t\frac{m_t}{v}h\,\bar{t}t - c_2\frac{m_t}{v^2}h^2\bar{t}t + \dots$



$$= \frac{1-2\xi}{\sqrt{1-\xi}} \times \text{SM}_{htt}$$



$$= -\xi\frac{4m_t}{v^2}$$

SM-Like: $\kappa_t = (1-2\xi)/\sqrt{1-\xi}$

Non-SM-Like:
(visible in HH prod?)

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Same result for the **bottom:** (if again in the **5**)

$$\kappa_b = \kappa_t = \kappa_F = \frac{\sqrt{1-2\xi}}{\sqrt{1-\xi}}$$

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Different results for **different representations:**

$$\mathrm{MCHM}_{\mathbf{5}\oplus\mathbf{5}} \Longrightarrow k_F = \frac{1-2\xi}{\sqrt{1-\xi}}$$

$$\mathrm{MCHM}_{\mathbf{4}\oplus\mathbf{4}} \Longrightarrow k_F = \sqrt{1-\xi}$$

$$\mathrm{MCHM}_{\mathbf{14}\oplus\mathbf{1}} \Longrightarrow k_F = \frac{1-2\xi}{\sqrt{1-\xi}}$$

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For the **vectors** the **result is universal:**

$$\kappa_V = \sqrt{1-\xi}$$

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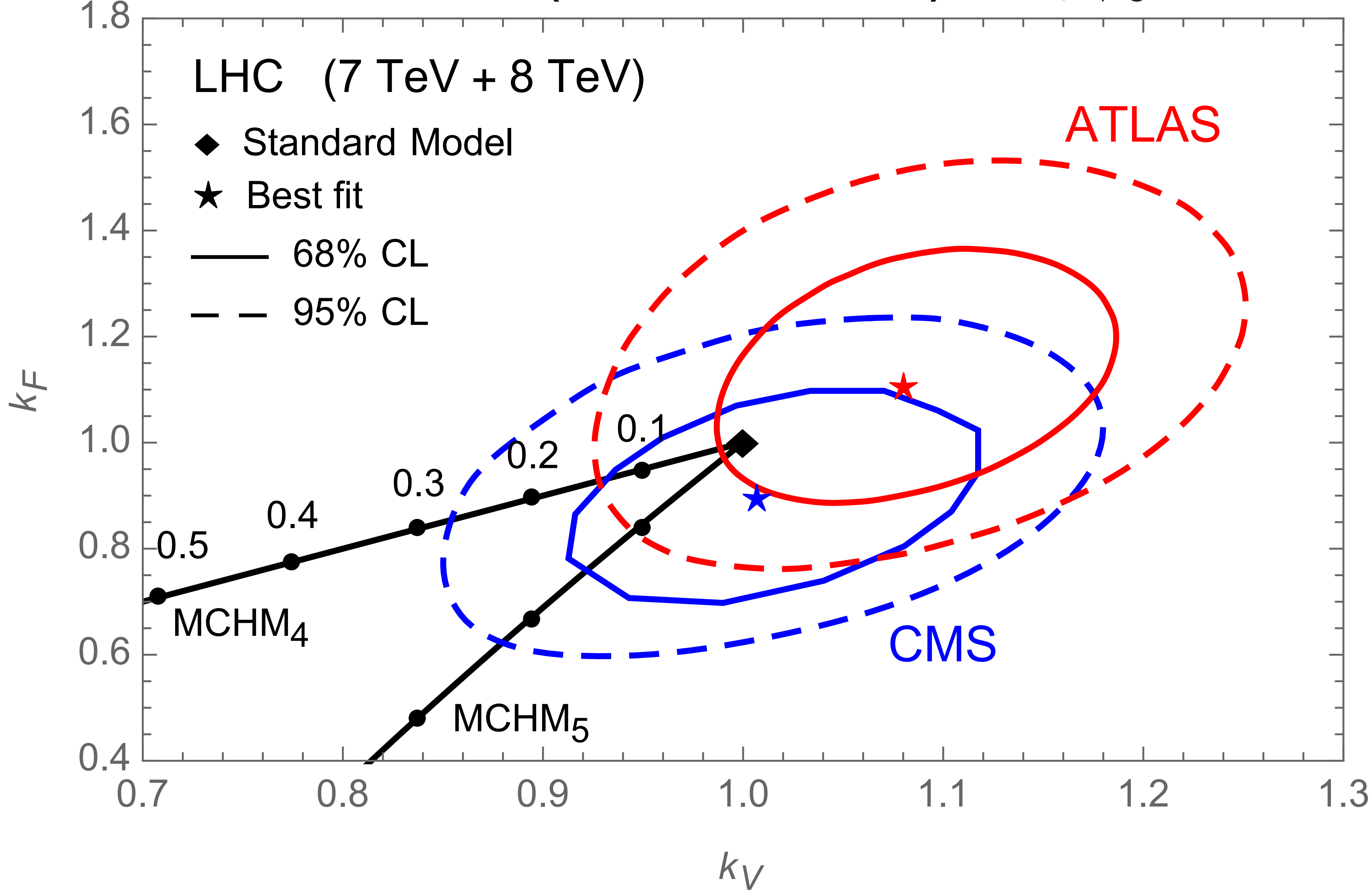
Current bound (from ATLAS) is $\xi \lesssim 0.15$.



Expected LHC-300 reach (with SM central value): $\xi < 0.1$.

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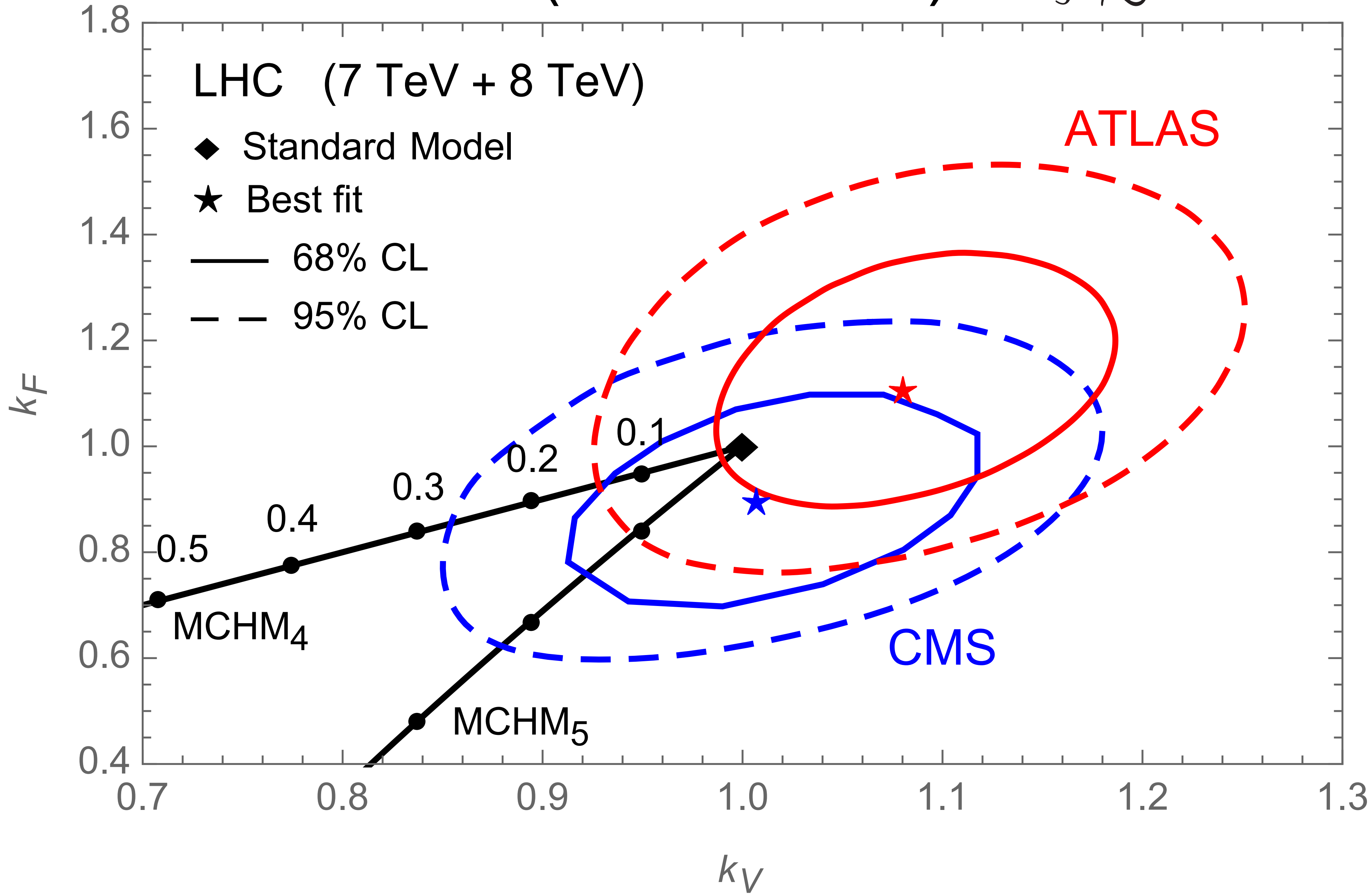

CAVEATS:

1) Easy to encounter $\kappa_t \neq \kappa_b$
2) Easy to find **extra Goldstone scalars** that contribute by mixing

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“Exact” symmetry $\mathrm{SO}(5)$**.**

Spontaneously broken to $\mathrm{SO}(4)$**.**

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SM **gauge fields:** $W^\alpha_\mu,\ B_\mu$**.**

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… we expect **two** (sets of) resonance multiplets:

CH Vectors $\longleftrightarrow$ J

CH Top Partners $\longleftrightarrow$ $\mathcal{O}_F$

CH Vectors

Must be at least one **triplet** and one **singlet** (SM currents)

Triplet has the most interesting phenomenology:

1. $M_{\pm} \simeq M_0$ (essentially degenerate), $\sigma_{\pm} \simeq 2\sigma_0$(from PDF)
2. Couplings to quarks potentially small (suppressed prod., lept. decay)
3. $\Gamma[V_0 \to W^+W^-] \simeq \Gamma[V_0 \to Zh] \simeq \Gamma[V_{\pm} \to W^{\pm}Z] \simeq \Gamma[V_{\pm} \to W^{\pm}h]$
4. Dilepton or diboson final state cover different regions of par. space

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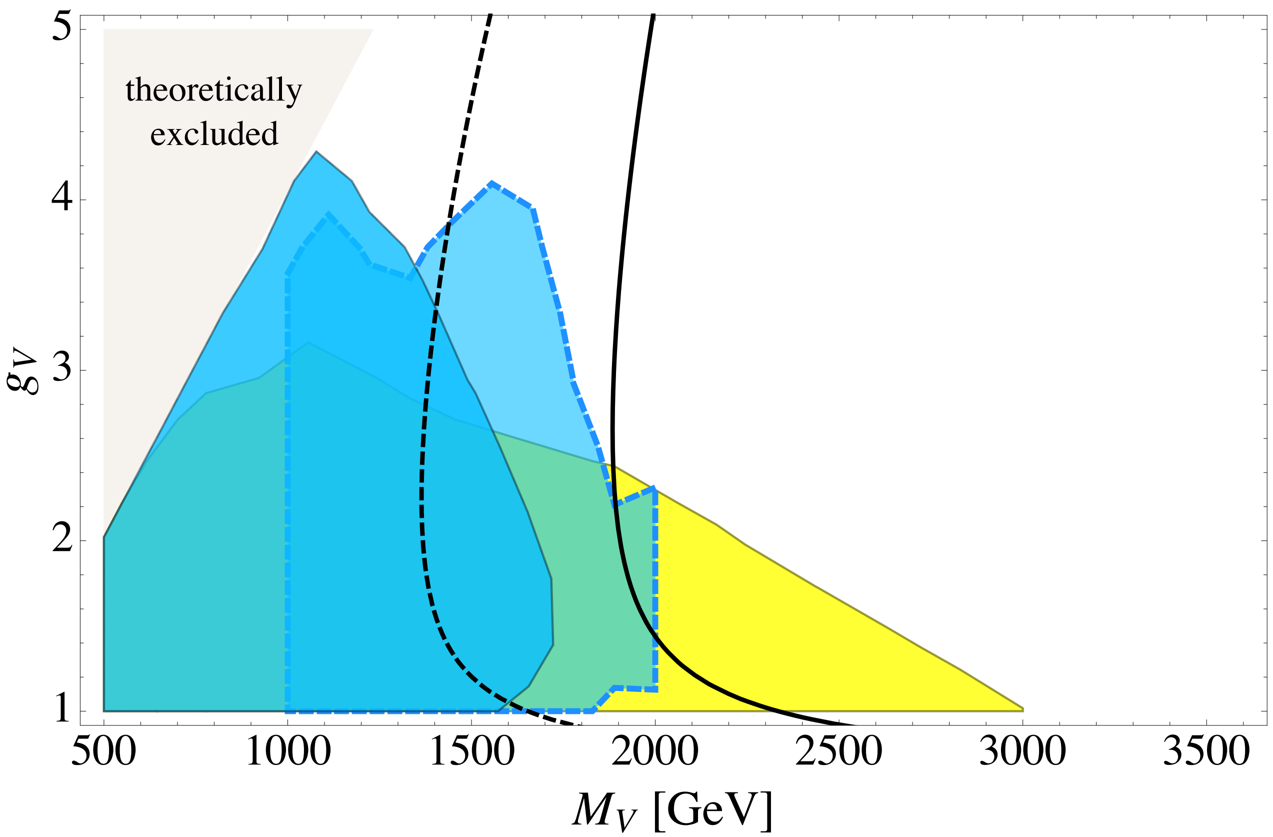

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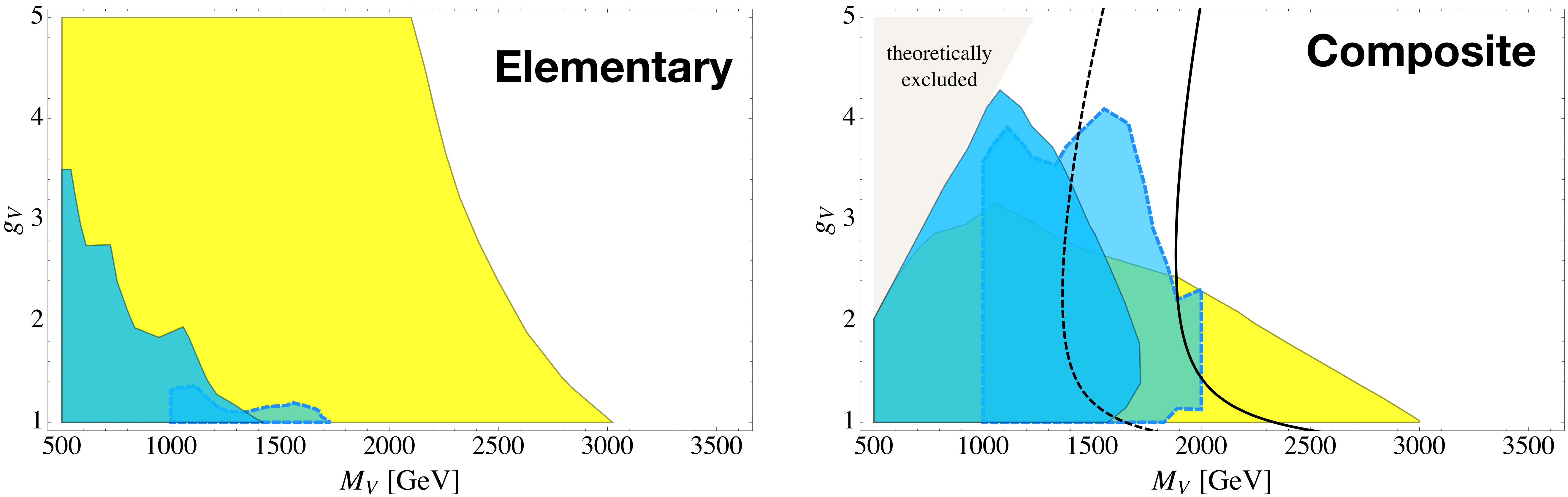
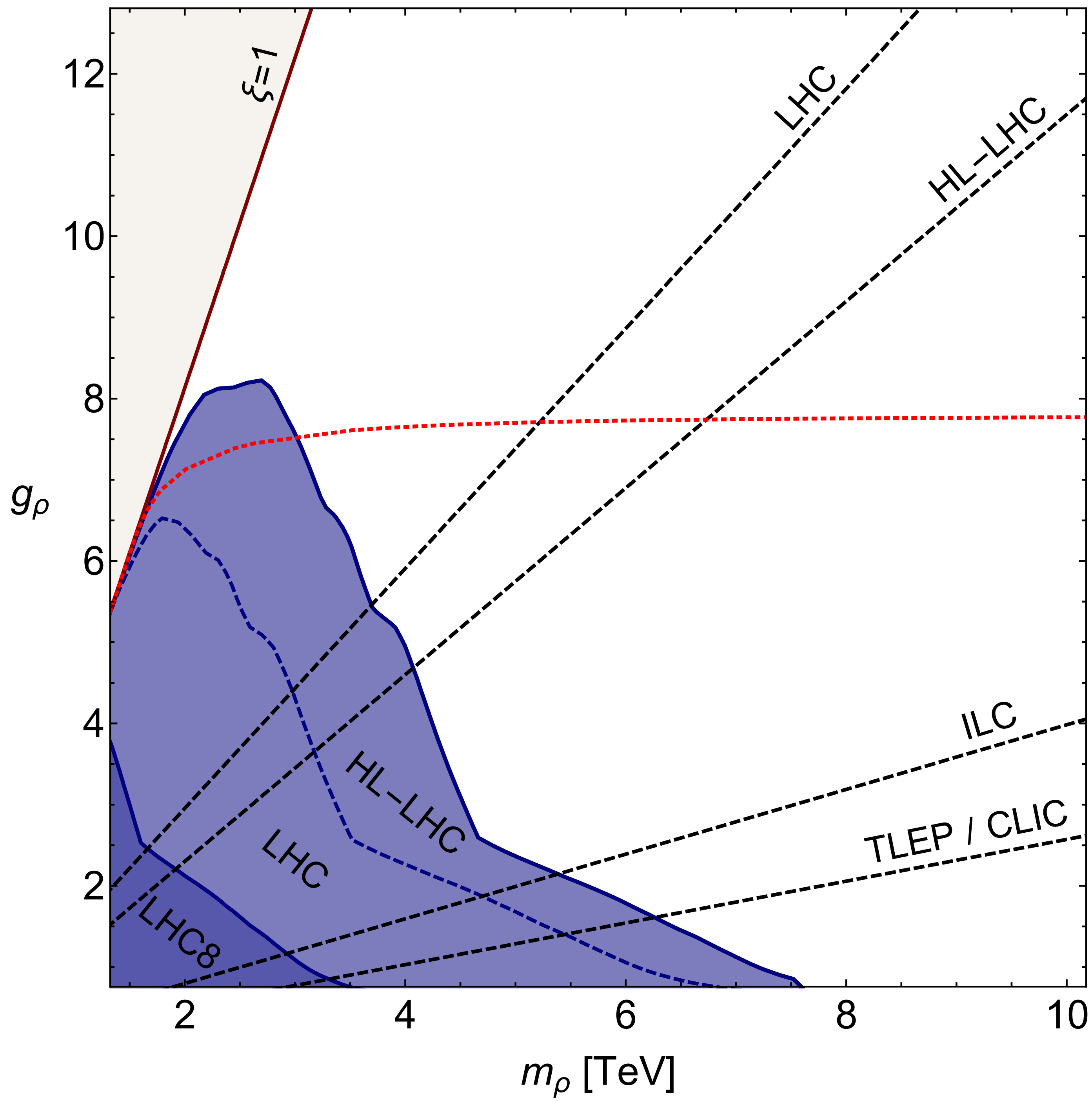


Very different from Elementary Z’/W’!

CH Vectors

LHC projections:



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Typical EW Quantum Numbers:

Two SM doublets:

$$\left[\begin{pmatrix} T \\ B \end{pmatrix} \begin{pmatrix} X_{5/3} \\ X_{2/3} \end{pmatrix}\right]$$

Nearly mass-degenerate,
since part of fourplet.

One SM singlet:

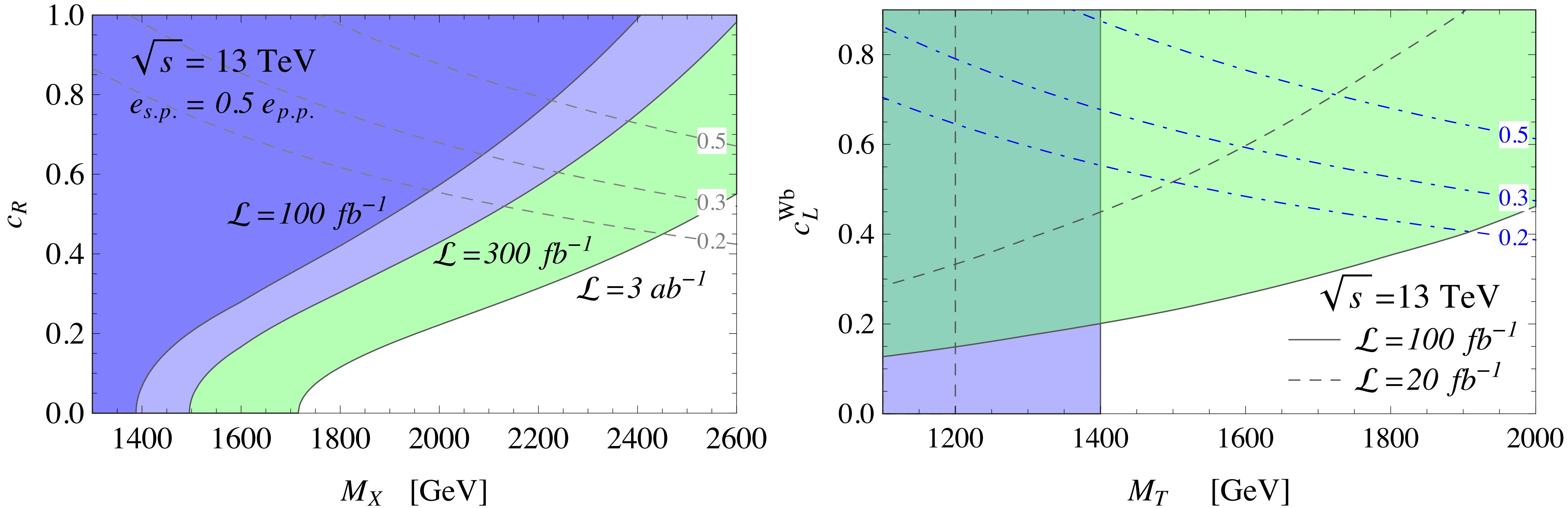
$$\widetilde{T}$$

only one charge-2/3 state

Typical Branching Ratios:

$\widetilde{T}$	Wb	Zt	ht
$X_{5/3}, B$	Wt		
$\widetilde{X}_{2/3}, T$	Zt	ht	

CH Top Partners



Very Important: Top Partners mass **directly connected** with the level of tuning in the theory:

$$\delta m_H^2 \sim \frac{\lambda_{L,R}^2}{8\pi^2} M_{\rm TP}^2 = \lambda_{L,R}^2 \left(\frac{M_{\rm TP}}{500\ {\rm GeV}}\right)^2 (125\ {\rm GeV})^2$$

Somewhat like stops in SUSY

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Comprehensive LHC search program is currently being developed. **Room for big improvements** with 13 TeV run.

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Natural Models

Half a century of thoughts led to only **two mechanisms** that provide a Natural microscopic origin for Higgs mass

Compositeness **Supersymmetry**

The rest of the course is (mostly) devoted to show how they work

SUSY theory

Supersymmetry is the **ultimate symmetry**

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In a **relativistic QFT** (with mass), the most general content of symmetry generators (conserved charges) looks like this:

Poincaré

$$[P_\mu, P_\nu] = 0$$

$$[P_\mu, M_{\rho\sigma}] = \mathrm{i}(\eta_{\mu\rho}P_\sigma - \eta_{\mu\sigma}P_\rho)$$

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Internal

$$[B_r, B_s] = i c_{rs}{}^t B_t$$

$$[B_r, P_\mu] = [B_r, M_{\mu\nu}] = 0$$

(**scalar** charges)

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SUSY

$$[Q_{\alpha i}, P_\mu] = [\bar{Q}^i{}_{\dot\alpha}, P_\mu] = 0$$

$$[Q_{\alpha i}, M_{\mu\nu}] = (\sigma_{\mu\nu})_\alpha{}^\beta Q_{\beta i}$$

$$[\bar{Q}^i{}_{\dot\alpha}, M_{\mu\nu}] = -\bar{Q}^i{}_{\dot\beta}(\bar\sigma_{\mu\nu})^{\dot\beta}{}_{\dot\alpha}$$

SUSY charges are **Spinors**

$$[Q_{\alpha i}, B_r] = (b_r)_i{}^j Q_{\alpha j}$$

$$[\bar{Q}^i{}_{\dot\alpha}, B_r] = -\bar{Q}^j{}_{\dot\alpha}(b_r)_j{}^i$$

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$$[P_\mu, P_\nu] = 0$$

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$$[M_{\mu\nu}, M_{\rho\sigma}] = \mathrm{i}(\eta_{\nu\rho}M_{\mu\sigma} - \eta_{\nu\sigma}M_{\mu\rho} - \eta_{\mu\rho}M_{\nu\sigma} + \eta_{\mu\sigma}M_{\nu\rho})$$

Internal

$$[B_r, B_s] = \mathrm{i}c_{rs}{}^t B_t$$

$$[B_r, P_\mu] = [B_r, M_{\mu\nu}] = 0$$

(**scalar** charges)

SUSY

$$[Q_{\alpha i}, P_\mu] = [\bar{Q}^i{}_{\dot\alpha}, P_\mu] = 0$$

$$[Q_{\alpha i}, M_{\mu\nu}] = (\sigma_{\mu\nu})_\alpha{}^\beta Q_{\beta i}$$

$$[\bar{Q}^i{}_{\dot\alpha}, M_{\mu\nu}] = -\bar{Q}^i{}_{\dot\beta}(\bar\sigma_{\mu\nu})^{\dot\beta}{}_{\dot\alpha}$$

SUSY charges are **Spinors**

$$[Q_{\alpha i}, B_r] = (b_r)_i{}^j Q_{\alpha j}$$

$$[\bar{Q}^i{}_{\dot\alpha}, B_r] = -\bar{Q}^j{}_{\dot\alpha}(b_r)_j{}^i$$

More SUSY

$$\{Q_{\alpha i}, \bar{Q}^j{}_{\dot\beta}\} = 2\delta_i^j(\sigma^\mu)_{\alpha\dot\beta}P_\mu$$

$$\{Q_{\alpha i}, Q_{\beta j}\} = 2\varepsilon_{\alpha\beta}Z_{ij}$$

$$\{\bar{Q}^i{}_{\dot\alpha}, \bar{Q}^j{}_{\dot\beta}\} = -2\varepsilon_{\dot\alpha\dot\beta}Z^{ij}$$

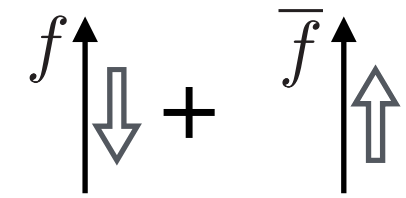
SUSY charges **Anticommute**

SUSY theory

Basic Notation: **Weyl spinors**

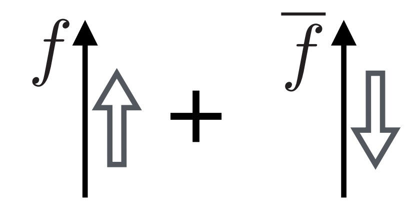
Dirac $= \Psi = \begin{bmatrix} (\psi_\mathrm{L})_\alpha \\ \\ (\psi_\mathrm{R})^{\dot\alpha} \end{bmatrix} =$

Weyl Left $\underset{(m=0)}{\equiv}$



+

Weyl Right $\underset{(m=0)}{\equiv}$



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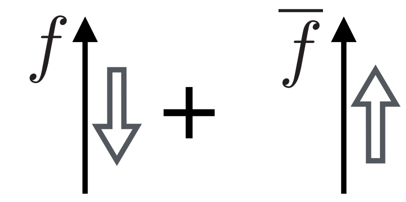

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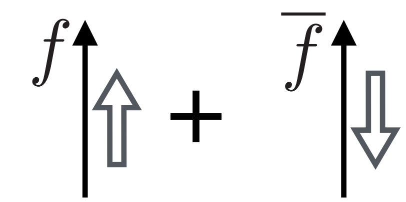

Weyl spinors $\psi_{\rm L,R}$ are distinct Lorentz representations

$$\gamma^\mu = \begin{bmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{bmatrix} \qquad \begin{aligned} \sigma^\mu &= \{\mathbb{1}, \vec{\sigma}\} \\ \overline{\sigma}^\mu &= \{\mathbb{1}, -\vec{\sigma}\} \end{aligned}$$

$$\Sigma^{\mu\nu} = \frac{i}{4}\left[\gamma^\mu, \gamma^\nu\right] = \begin{bmatrix} \frac{i}{4}\sigma^{[\mu}\overline{\sigma}^{\nu]} & 0 \\ 0 & \frac{i}{4}\overline{\sigma}^{[\mu}\sigma^{\nu]} \end{bmatrix} \equiv \begin{bmatrix} (\sigma^{\mu\nu})_\alpha{}^\beta & 0 \\ 0 & (\overline{\sigma}^{\mu\nu})^{\dot\alpha}{}_{\dot\beta} \end{bmatrix}$$

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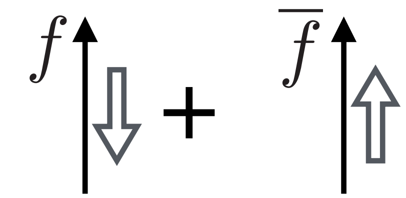

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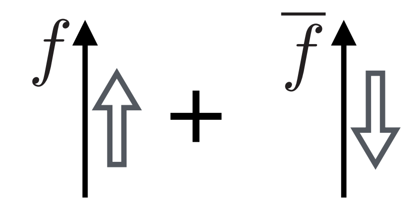

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Lorentz on Left

Lorentz on Right

SUSY theory

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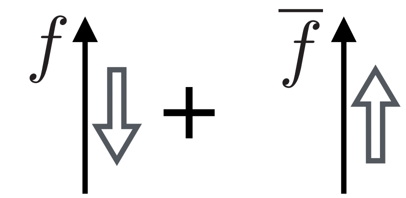
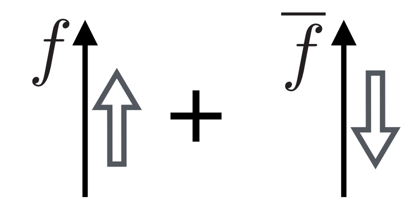
Weyl Left $\underset{(m=0)}{\equiv}$ 

Weyl Right $\underset{(m=0)}{\equiv}$ 

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Transform **independently:**

$$\begin{cases} (\delta\psi_{\rm L})_\alpha = -\dfrac{i}{2}\omega_{\mu\nu}\left(\sigma^{\mu\nu}\right)_\alpha{}^\beta (\psi_{\rm L})_\beta \\ (\delta\psi_{\rm R})^{\dot\alpha} = -\dfrac{i}{2}\omega_{\mu\nu}\left(\overline{\sigma}^{\mu\nu}\right)^{\dot\alpha}{}_{\dot\beta} (\psi_{\rm R})^{\dot\beta} \end{cases}$$

SUSY theory

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Weyl Left $\underset{(m=0)}{\equiv} f\uparrow\Downarrow + \overline{f}\uparrow\Uparrow$

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Different, but **conjugate:**

$$\left[\left(\sigma^{\mu\nu}\right)_\alpha{}^\beta\right]^* = \epsilon_{\dot\alpha\dot\gamma}\epsilon^{\dot\beta\dot\delta}\left(\overline{\sigma}^{\mu\nu}\right)^{\dot\gamma}{}_{\dot\delta}$$

$\overline{\psi}_{\mathrm{L}}^{\dot\alpha} \equiv \epsilon^{\dot\alpha\dot\beta}\left[(\psi_{\mathrm{L}})_\beta\right]^\dagger$ transforms like ψ_{R}.

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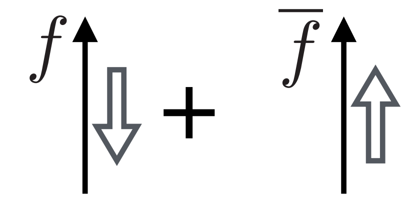

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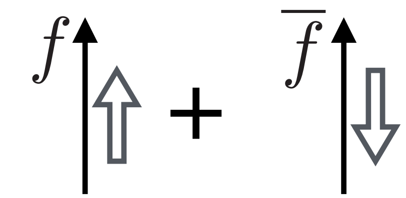

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Trade all Right for Left.
In SUSY, only use Weyl Left.

Raising and lowering:

$$\epsilon^{1\,2} = \epsilon^{\dot 1\,\dot 2} = -\epsilon_{1\,2} = -\epsilon_{\dot 1\,\dot 2} = +1$$

$$\psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta \qquad \overline{\psi}_{\dot\alpha} = (\psi_\alpha)^\dagger$$

SUSY theory

Simplest SUSY: 2 complex (4 real) charges $Q_\alpha = (\overline{Q}_{\dot\alpha})^\dagger$
with the (N=1) Algebra …

$$[Q_\alpha, P_\mu] = 0 \qquad \{Q_\alpha, Q_\beta\} = 0$$

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SUSY refrain: **Bosons = Fermions**. Let's see why.

Take one particle moving along z-axis, with helicity λ …

$$h = S^3 = M^{1\,2}$$

Helicity operator

$$h|\phi_\lambda\rangle = M^{1\,2}|\phi_\lambda\rangle = \lambda|\phi_\lambda\rangle$$

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End up with particles of **same mass** …

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but **different helicity.**

$$M^{1\,2}\,[Q_\alpha|\phi_\lambda\rangle] = (Q_\alpha M^{1\,2} + [M^{1\,2}, Q_\alpha])|\phi_\lambda\rangle = \begin{bmatrix} \lambda - \frac{1}{2} & 0 \\ 0 & \lambda + \frac{1}{2} \end{bmatrix}_\alpha{}^\beta Q_\beta |\phi_\lambda\rangle$$

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$h = \lambda$ **SUSY** $h = \lambda \pm 1/2$

Boson/Fermion ⟷ **Fermion/Boson**

Degenerate multiplets with Fermions and Bosons

SUSY theory

Famous SUSY multiplets:

Chiral Multiplet

Fermions: Bosons:

1 Weyl 1 comp. scalar

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leptons and **s**leptons
Higgs**ino** and Higgs

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Famous SUSY multiplets:

Vector Multiplet

Bosons: Fermions:

1 gauge 1 Weyl

gauge (e.g. γ, g) and gaug**ino**

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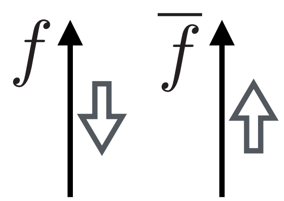

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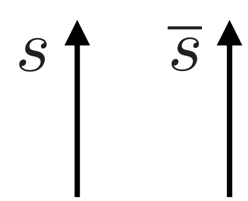

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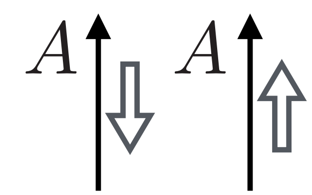

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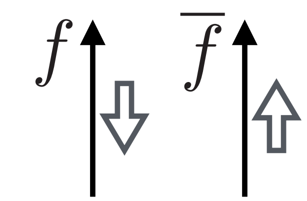

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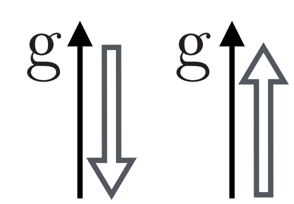
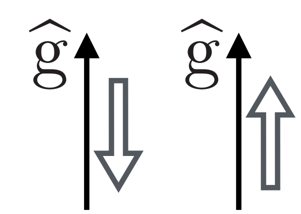
Gravity Multiplet

Bosons:



graviton

Fermions:



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$$\begin{cases} \delta\phi = 2\zeta\psi \\ \delta\psi = -\zeta F - i\partial_\mu\phi\sigma^\mu\bar{\zeta} \\ \delta F = -2i\partial_\mu\psi\sigma^\mu\bar{\zeta} \end{cases}$$

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A simple SUSY Lagrangian:

$$\mathcal{L} = i\,\overline{\psi}\overline{\sigma}^\mu\partial_\mu\psi - \frac{m}{2}(\psi\,\psi + \overline{\psi\psi}) + \partial_\mu\phi^\dagger\partial^\mu\phi - m(\phi F + \phi^\dagger F^\dagger) + F^\dagger F$$

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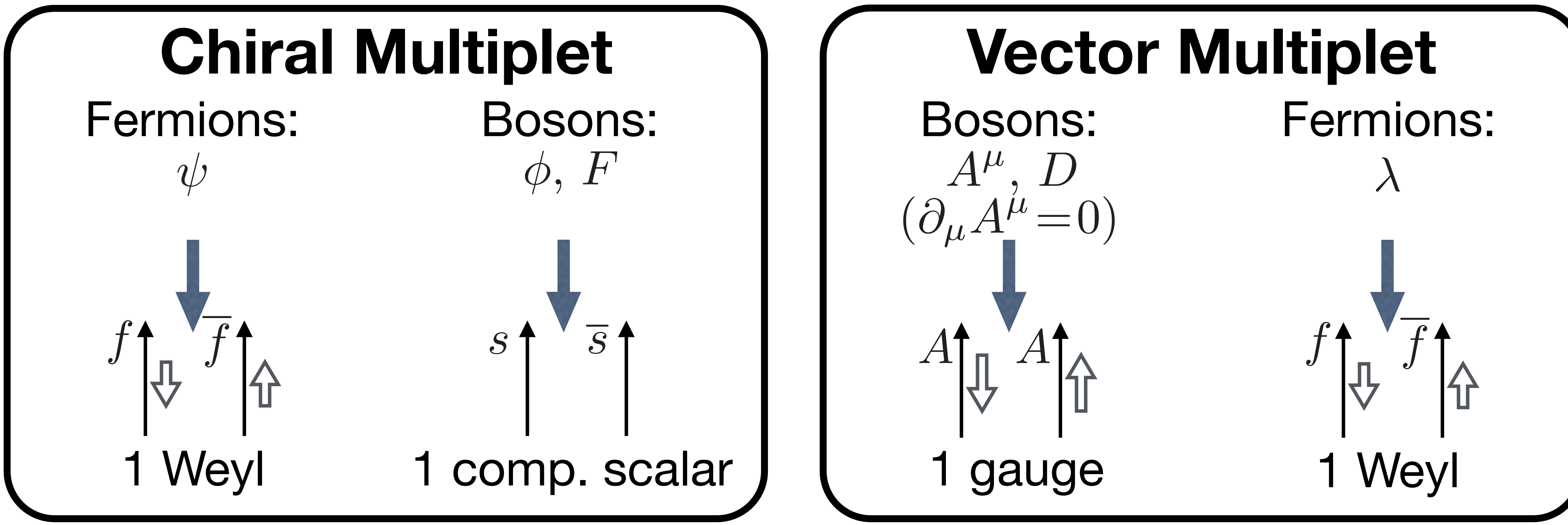
Solving EOM: $\mathcal{L} \to i\,\overline{\psi}\overline{\sigma}^\mu\partial_\mu\psi - \frac{m}{2}(\psi\psi + \overline{\psi}\,\overline{\psi}) + \partial_\mu\phi^\dagger\partial^\mu\phi - m^2\phi^\dagger\phi$

SUSY theory

Famous SUSY multiplets of fields:



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SuperFields: a tool to write SUSY Lagrangians

Basic idea:

ordinary space-time

coordinates x^μ

translations

$$P_\mu = -i\frac{\partial}{\partial x^\mu}$$

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translations

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→

super space-time

coordinates $\{x^\mu, \theta^\alpha, \overline{\theta}_{\dot\alpha}\}$

translations

$$P_\mu = -i\frac{\partial}{\partial x^\mu} \quad Q_\alpha \sim -i\frac{\partial}{\partial \theta^\alpha} \quad \overline{Q}^{\dot\alpha} \sim -i\frac{\partial}{\partial \overline{\theta}_{\dot\alpha}}$$

SUSY theory

SuperFields: a tool to write SUSY Lagrangians

Basic idea:

ordinary space-time → **super** space-time

coordinates x^μ → coordinates $\{x^\mu, \theta^\alpha, \overline{\theta}_{\dot{\alpha}}\}$

translations $P_\mu = -i\frac{\partial}{\partial x^\mu}$ → translations $P_\mu = -i\frac{\partial}{\partial x^\mu}$ $Q_\alpha \sim -i\frac{\partial}{\partial \theta^\alpha}$ $\overline{Q}^{\dot{\alpha}} \sim -i\frac{\partial}{\partial \overline{\theta}_{\dot{\alpha}}}$

ordinary field $F(x)$ → **super** field $\mathcal{F}(x, \theta, \overline{\theta})$

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Important details:

$$Q_\alpha = -i\left(\frac{\partial}{\partial \theta^\alpha} + i(\sigma^\mu)_{\alpha\dot{\alpha}}\overline{\theta}^{\dot{\alpha}}\partial_\mu\right) \quad \overline{Q}^{\dot{\alpha}} = -i\left(\frac{\partial}{\partial \overline{\theta}^{\alpha}} + i(\overline{\sigma}^\mu)^{\dot{\alpha}\alpha}\theta_\alpha\partial_\mu\right)$$

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$$\mathcal{F}(x,\theta,\overline{\theta}) = a(x) + \theta b(x) + \overline{\theta}c(x) + \theta\theta d(x) + \overline{\theta}\overline{\theta}e(x) + \theta\sigma^\mu\overline{\theta}f_\mu(x) + \theta\theta\overline{\theta}g(x) + \overline{\theta}\overline{\theta}\theta h(x) + \overline{\theta}\overline{\theta}\theta\theta i(x)$$

General SF is a polynomial in $\theta, \overline{\theta}$. Coefficients are ordinary B and F fields

SUSY theory

Chiral SuperField:

$$\Phi = \phi(y) + \sqrt{2}\theta\psi(y) - \theta\theta F(y), \quad \text{with} \;\; y^\mu = x^\mu - i\theta\sigma^\mu\overline{\theta}$$

Vector SuperField: (Wess-Zumino gauge)

$$V = \theta\sigma^\mu\overline{\theta}A_\mu(x) + i\theta\theta\overline{\theta\lambda}(x) - i\overline{\theta\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\overline{\theta\theta}D(x)$$

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Rules:

- SF can be summed and multiplied, giving other SFs
- Functions of Chiral SF (no conjugate) are Chiral SF
- Covariant derivatives, $D_\alpha, \overline{D}_{\dot{\alpha}}$, can be defined on SF
- SUSY invariants: $\theta\theta\overline{\theta\theta}$ comp. of any SF, or $\theta\theta$ of ChSF

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The ChSF Lagrangian: (one or many ChSF)

$$[\Phi^\dagger\Phi]_{\theta\theta\overline{\theta\theta}} = i\overline{\psi}\overline{\sigma}^\mu\partial_\mu\psi + \partial_\mu\phi^\dagger\partial^\mu\phi + F^\dagger F$$

$$[W(\Phi)]_{\theta\theta} + \text{h.c.} = \frac{\partial W}{\partial\Phi}|_\phi F - \frac{1}{2}\frac{\partial^2 W}{\partial\Phi\partial\Phi}|_\phi\psi\psi + \text{h.c.}$$

$$W(\Phi) = \textbf{superpotential} = a\Phi + \frac{1}{2}m\Phi^2 + \frac{1}{3}\lambda\Phi^3$$

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Eliminating auxiliary fields:

$$F^\dagger = -\frac{\partial W}{\partial\Phi}|_\phi \quad \Longrightarrow \quad V_F[\phi] = \left|\frac{\partial W}{\partial\Phi}|_\phi\right|^2$$ **= F-term potential**

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The VeSF Lagrangian: (SUSY gauge theory)

no need to know what $\mathcal{W}^\alpha$ is

$$\frac{1}{4}\left[\mathcal{W}^\alpha\mathcal{W}_\alpha\right]_{\theta\theta} + \text{h.c.} = i\overline{\lambda}\overline{\sigma}^\mu\partial_\mu\lambda - \frac{1}{4}A_{\mu\nu}A^{\mu\nu} + \frac{1}{2}D^2$$

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→ **"SUSY-gauge" interactions**

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→ **"SUSY-gauge" interactions**

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$$D = qg\,|\phi|^2 \quad \Longrightarrow \quad V_D[\phi] = \frac{1}{2}q^2g^2\,|\phi|^4$$ **= D-term potential**

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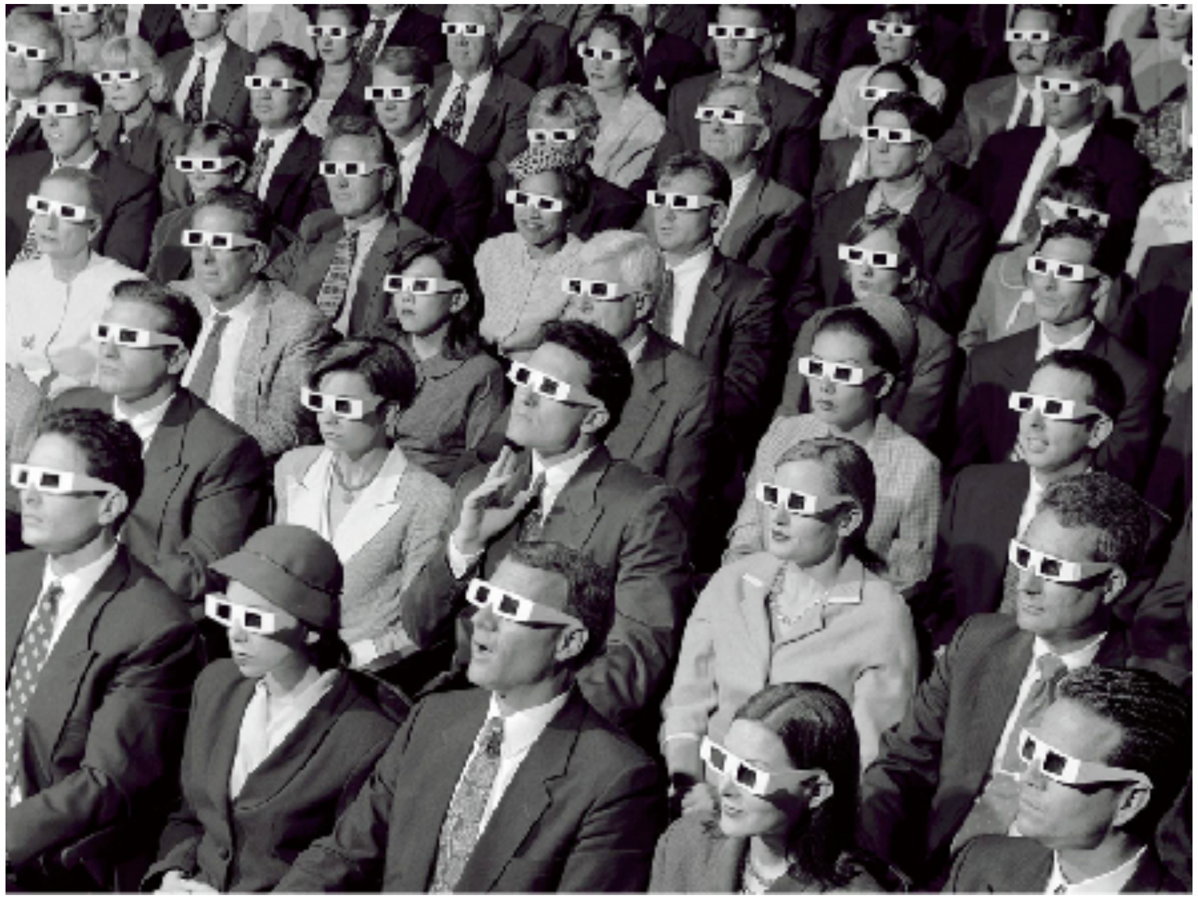

Theorists
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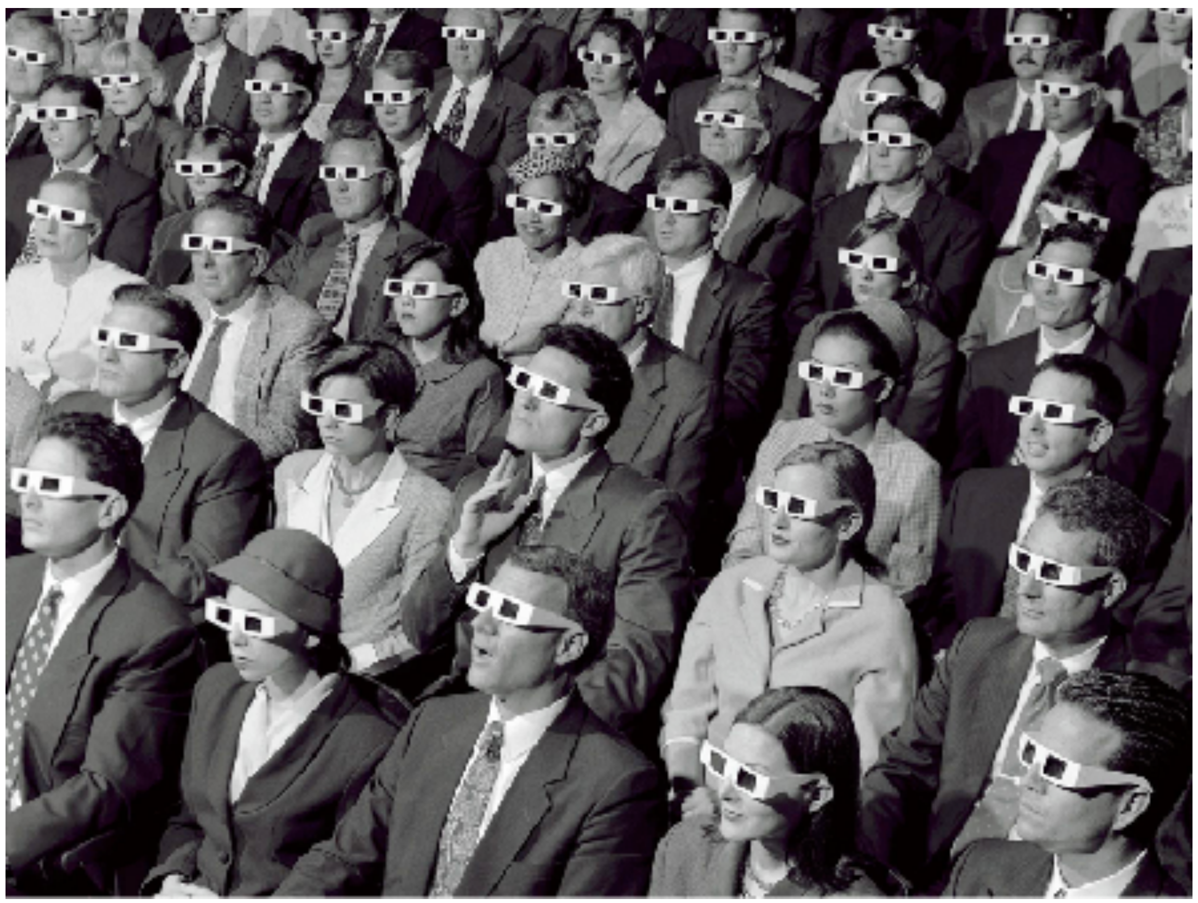

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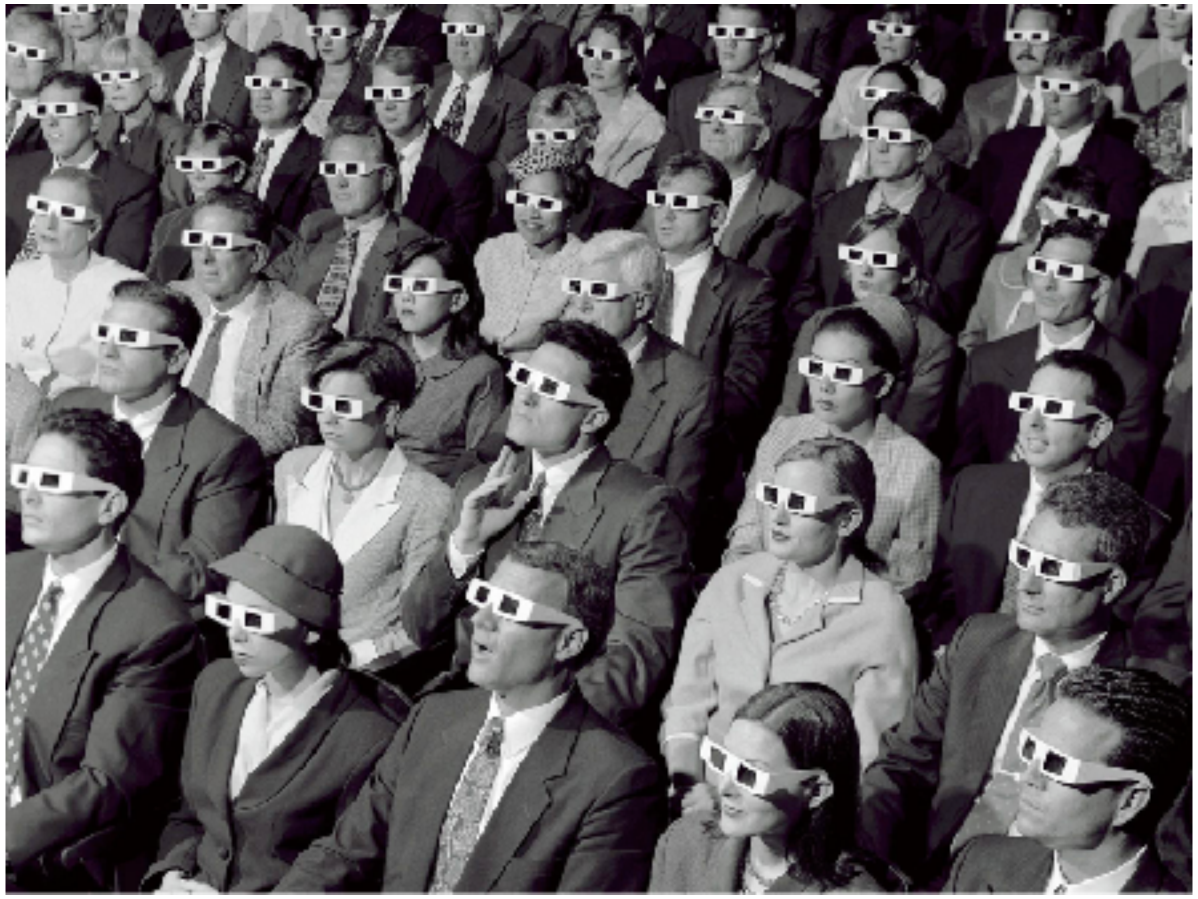

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Try to make everybody happy by **“SUSY and Naturalness”**