1. Consider the neutral mesons time evolution equation:

$$i\frac{\partial}{\partial t}\psi = H\psi,$$

where

$$\psi = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

and

$$H\begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}.$$

   a) Demonstrate the the eigenvalues of H are:

$$\lambda_\pm = M \mp \Re F - \frac{i}{2}(\Gamma \mp 2\Im F)$$

   being $F = \sqrt{(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)}$.

   b) Show that the coefficients that give the eigenstates:

$$|P_H\rangle = p|P^0\rangle + q|\bar{P}^0\rangle$$

$$|P_H\rangle = p|P^0\rangle - q|\bar{P}^0\rangle$$

   obey:

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}$$

   c) Show that the two definitions:

$$\Delta m = 2|M_{12}|$$

   and

$$\Delta m = 2\Re F$$

   are equivalent.

   d) Show also that

$$\Delta\Gamma = 2|\Gamma_{12}|\cos\phi$$

   and

$$\Delta\Gamma = 4\Im F,$$

   where $\phi = \arg(-M_{12}/\Gamma_{12})$, are equivalent.

2. The B-factories are focused into the study of B-meson decays. These accelerators collide electron and positron beams of different energies to produce the $\Upsilon(4S)$ (Upsilon) resonance. These resonances, with a mass of $M_\Upsilon = 10579{,}4$ MeV/c^2 and a width of $\Gamma_\Upsilon = 20{,}25$ MeV, frequently decay into two neutral mesons: $\Upsilon(4S) \to B^0\overline{B}^0$.

   a) Calculate the lifetime of $\Upsilon(4S)$ and determine which interaction mediates it. Considering that the lifetime of B^0 and $\overline{B}^0$ mesons is $\tau_B = 1{,}52 \times 10^{-12}$, what interaction mediates the B^0 decay?

b) Show that for ultrarelativistic electrons and positrons, for which their mass can be neglected ($M_e = 0$), the momenta of e^+ e do e^-, indicated as P_+ and P_-, have to obey, in order to produce the Υ resonance in its mass shell: $4P_+P_- = c^2M_\Upsilon^2$.

c) Consider that the more energetic beam is the electron beam: $P_- > P_+$. Show, again assuming ultrarelativistic electrons and positrons, that in the lab system of reference the maximal momentum of B^0 and $\overline{B}^0$ mesons is:

$$P_B = \frac{(P_- - P_+)}{2} + \frac{(P_- + P_+)}{2}\sqrt{1 - \left(\frac{2M_B}{M_\Upsilon}\right)^2},$$

where $M_B = 5279{,}6$ MeV/c^2 is the mass of the B^0 meson.

d) Show that the mean distance traveled by a B^0 is:

$$\langle x \rangle = \frac{P_B}{M_B}\tau_B.$$

e) At the SLAC PEP-II collider the maximal B^0 mesons traveled a mean distance of 287 μm. Which were the beam momenta of electrons and positrons (P_- and P_+) at PEP-II?