



## Astroparticle physics

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The application of the laws of particle physics to the macroscopic settings of cosmology and astrophysics has provided a detailed picture of how the Universe evolved from a hot and homogeneous initial state into the structures (stars, galaxies, clusters) that we observe nowadays.

Conversely, the Universe is now used as a giant laboratory to test the new models of particle physics in regimes that are out of reach for terrestrial accelerators.

# Outline

## Session I:

- ▶ Survey: extreme energies, extreme densities.
- ▶ Relics from the early Universe: freeze-out.

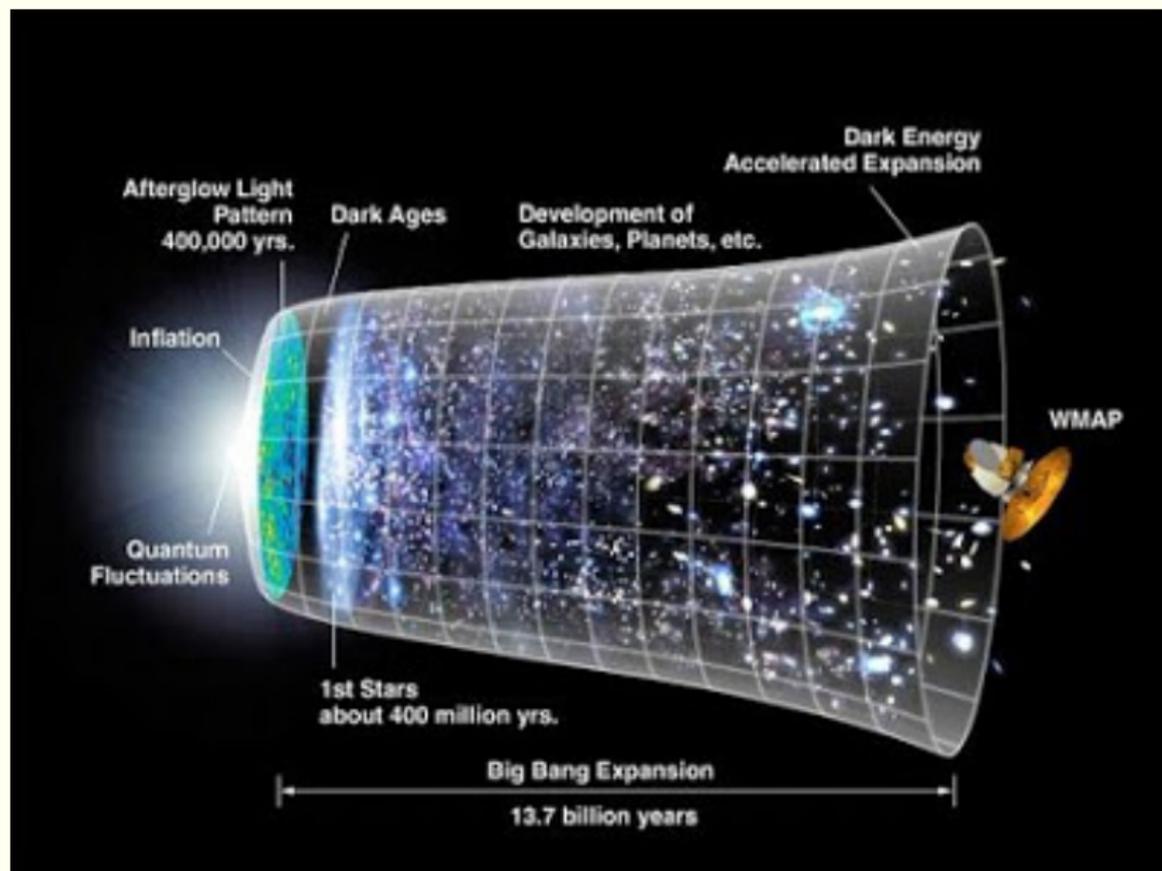
## Session II:

- ▶ The most energetic particles: ultra-high energy cosmic rays.

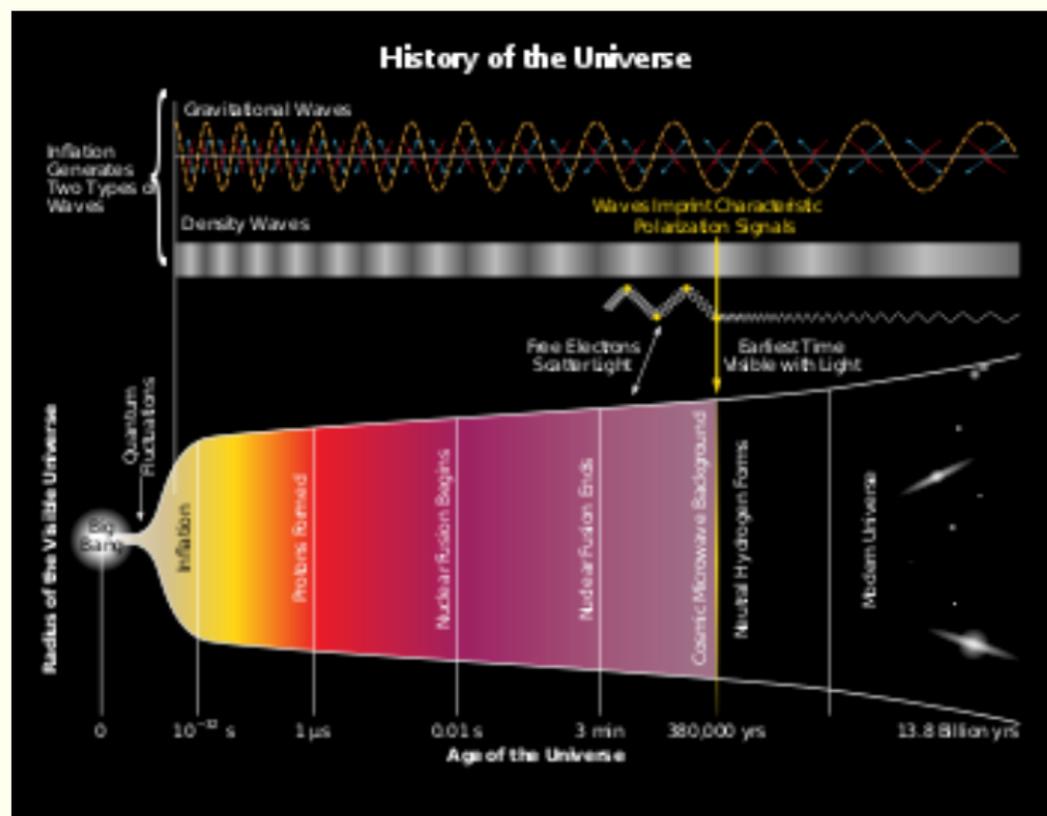
## Session III:

- ▶ Gamma-ray astronomy.
- ▶ Cosmological magnetic fields.

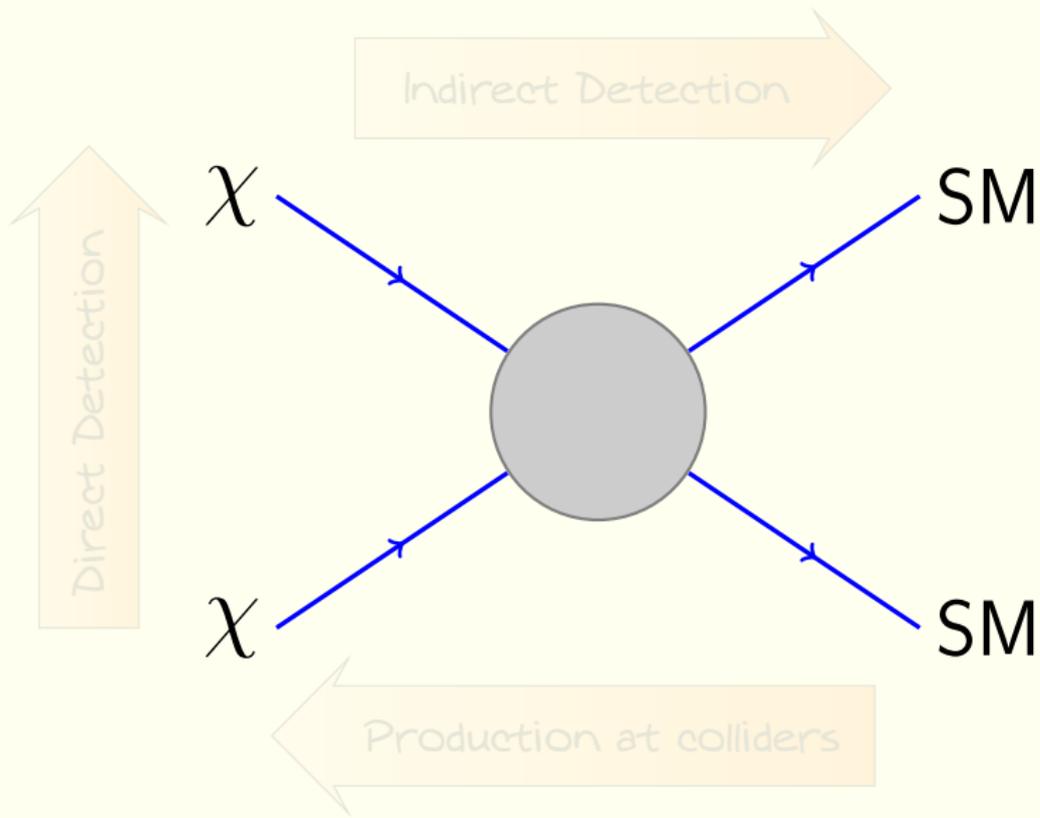
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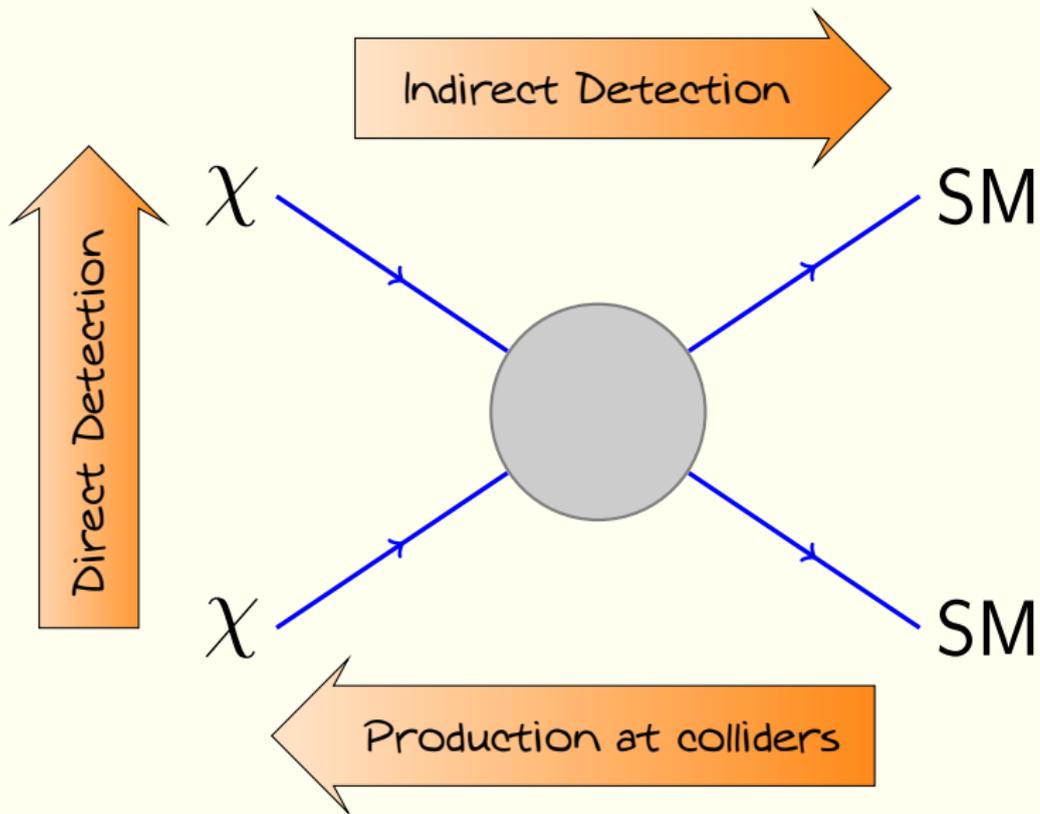


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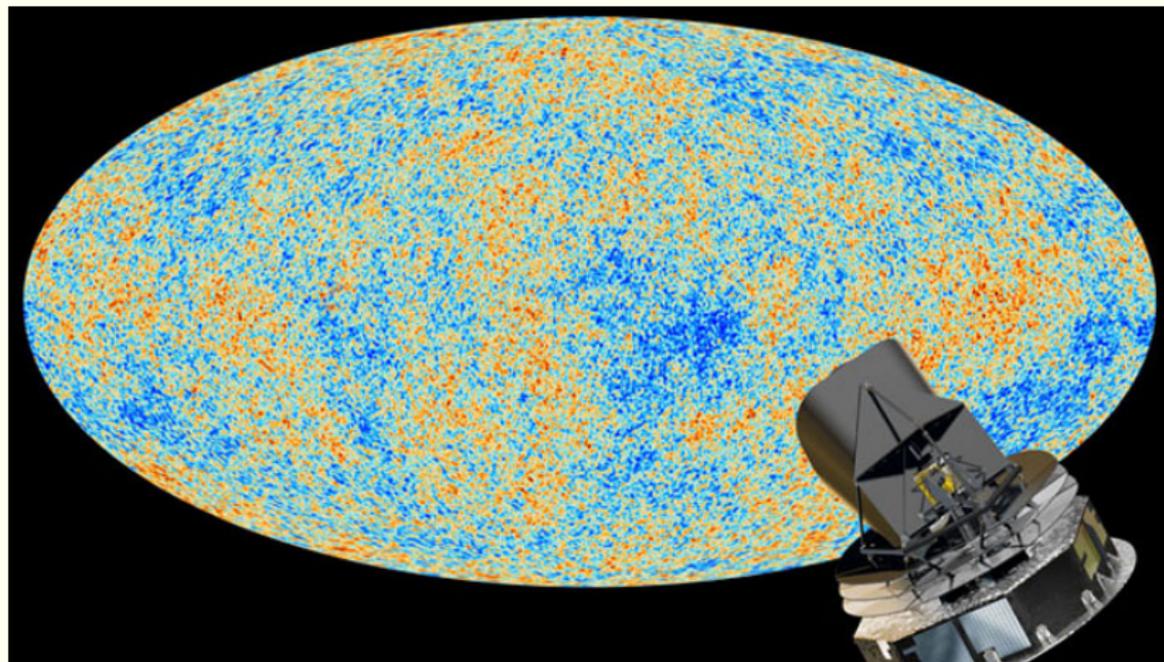


$$T_R \gtrsim 10^{12} \text{ GeV} \text{ (compare 14 TeV at the LHC!)}$$

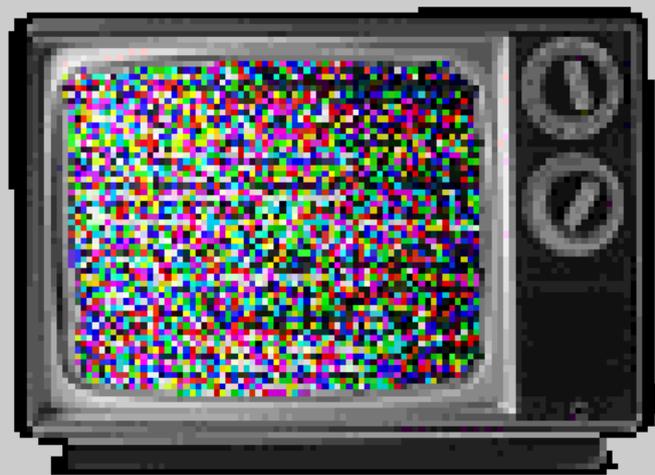




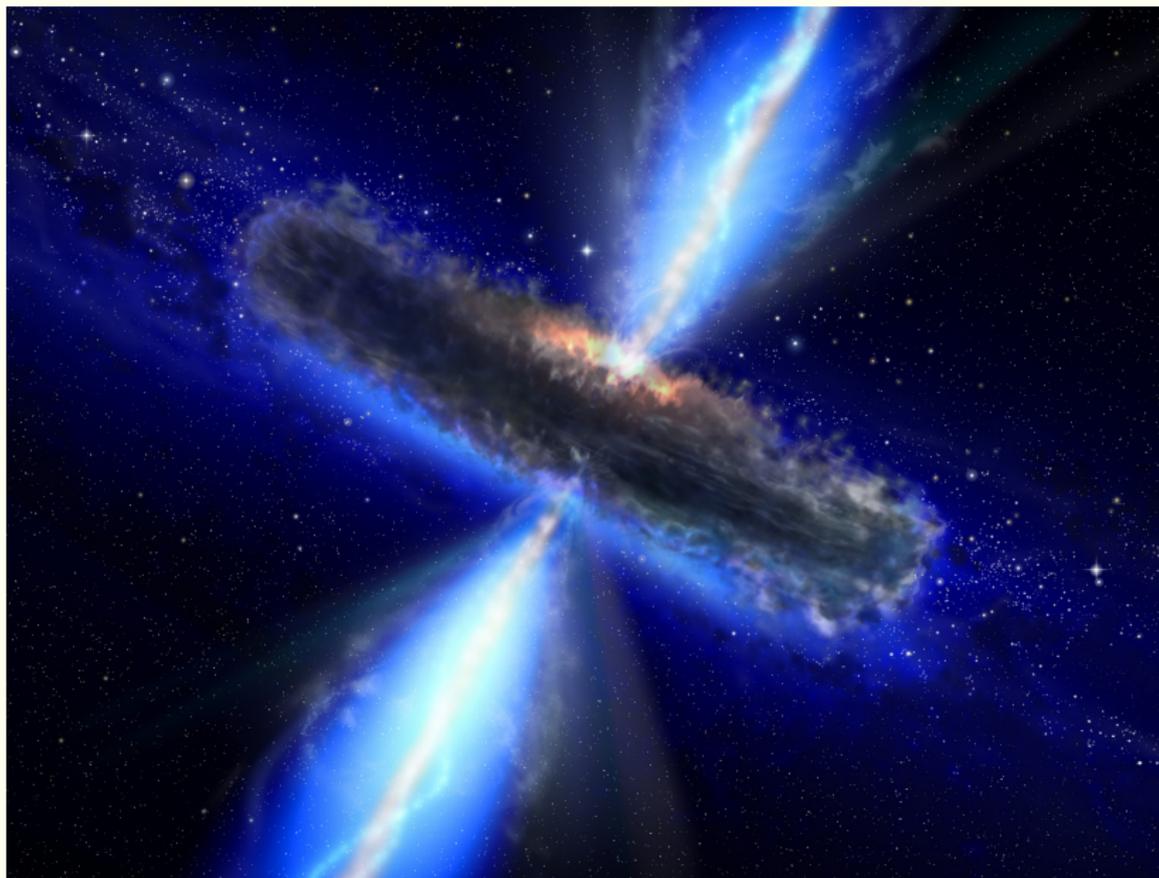
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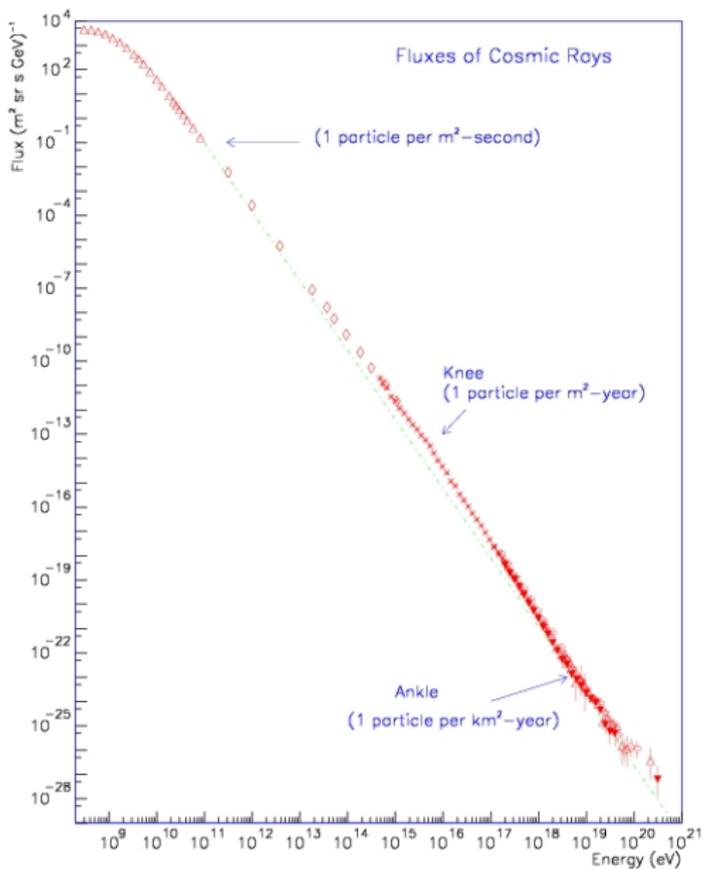


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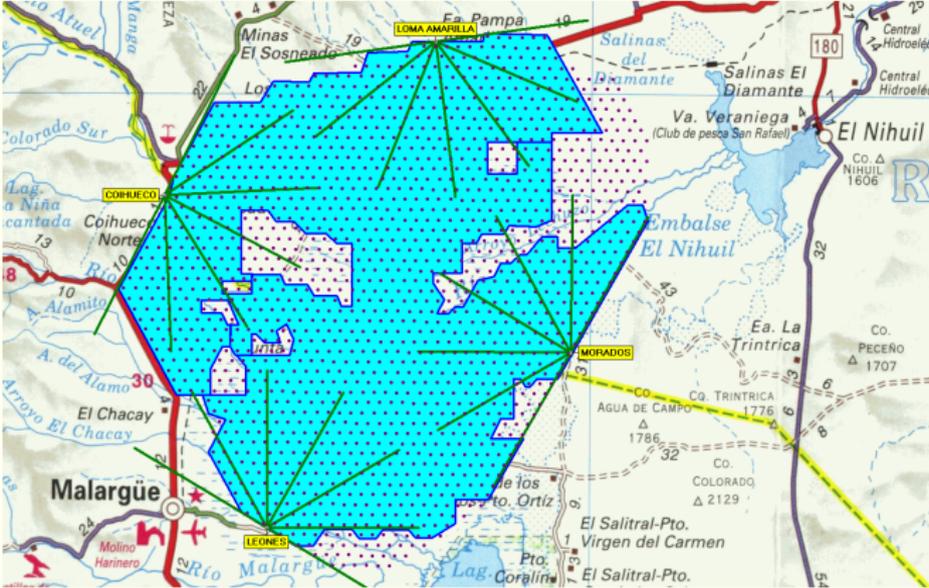


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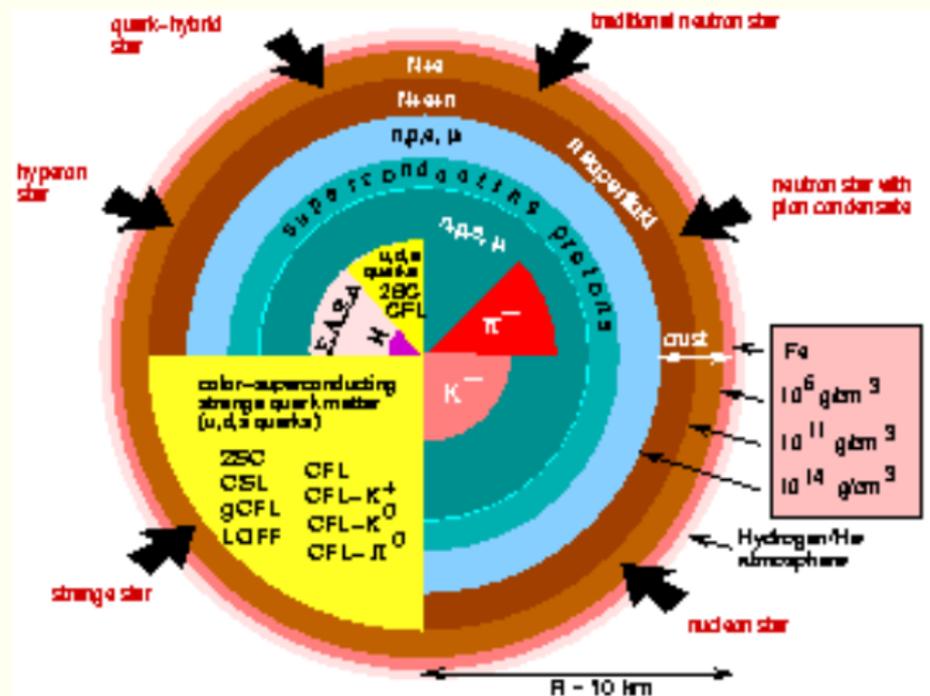


Tobias Winchen

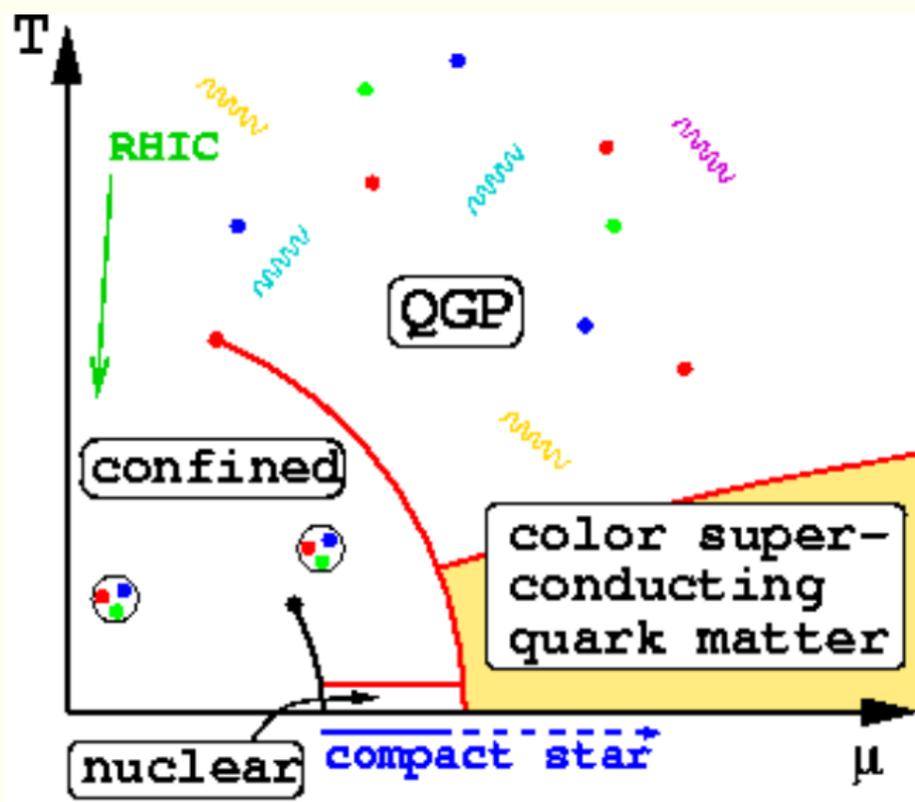
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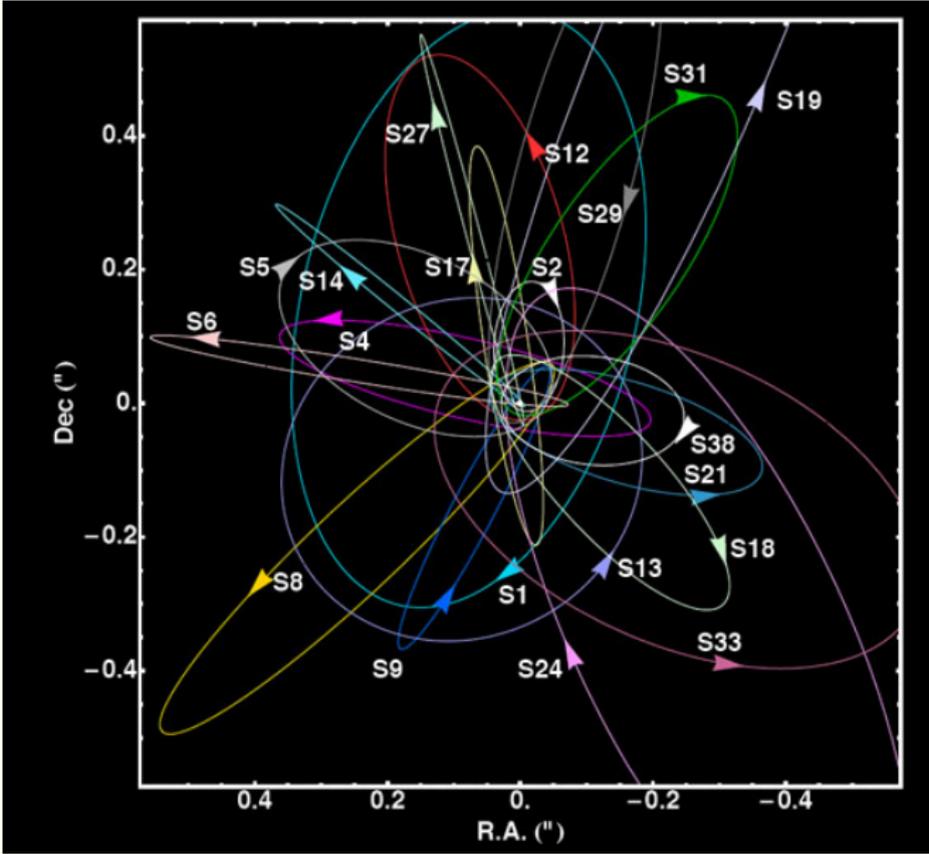
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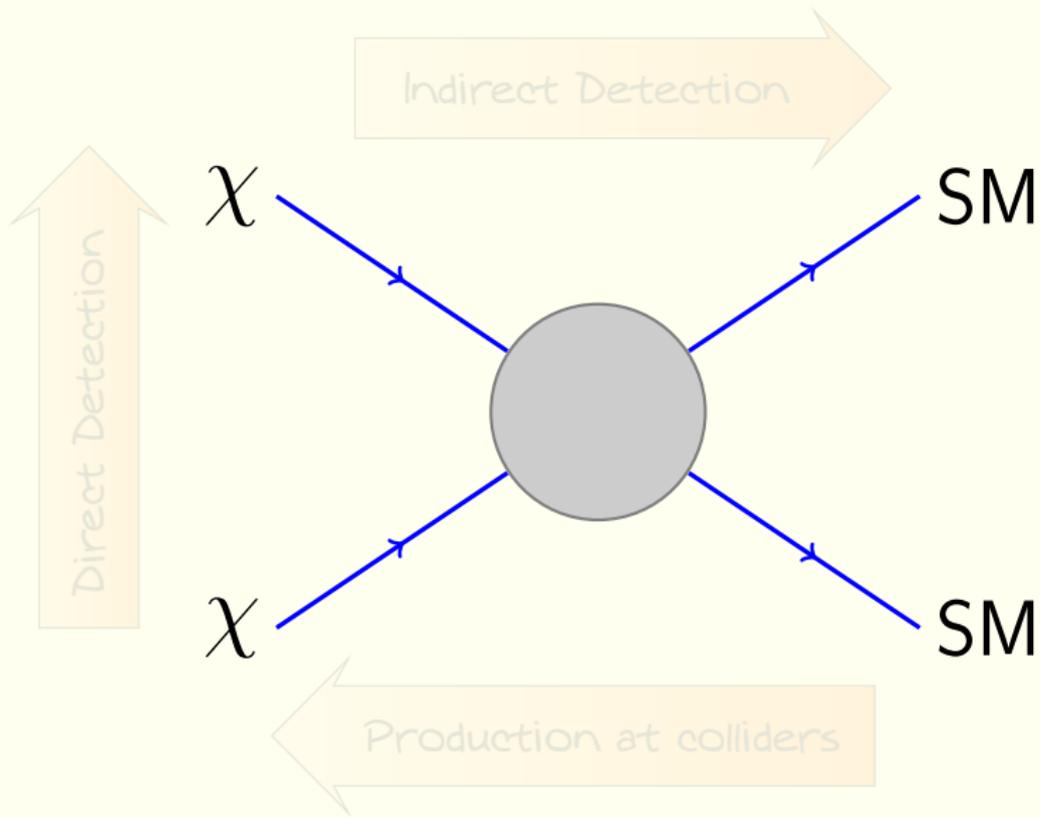


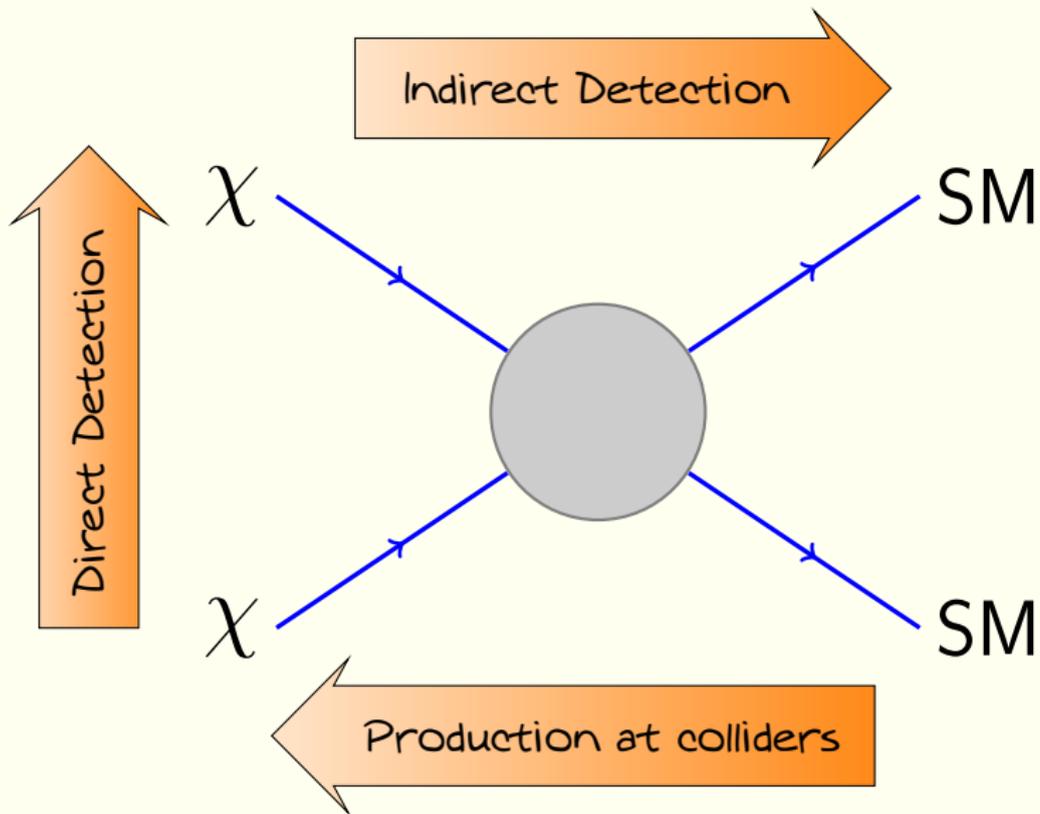
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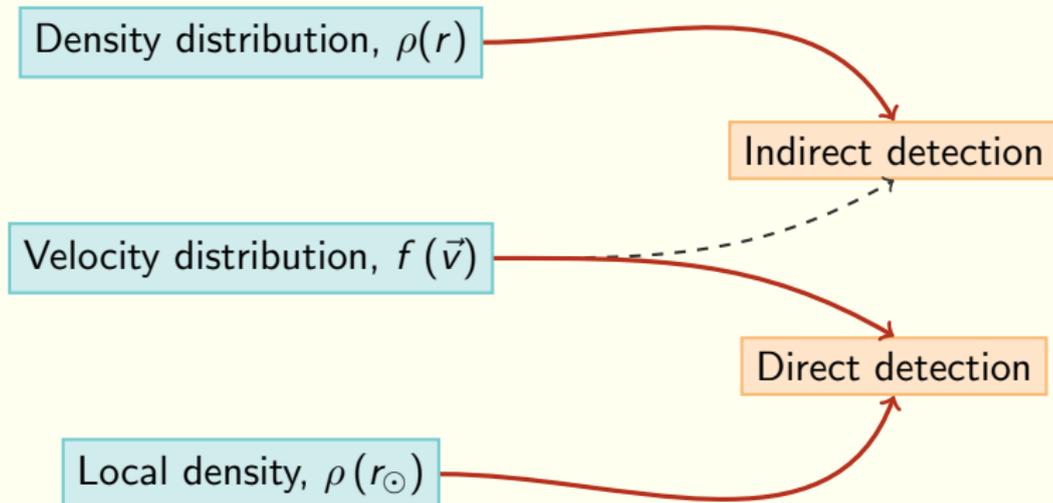
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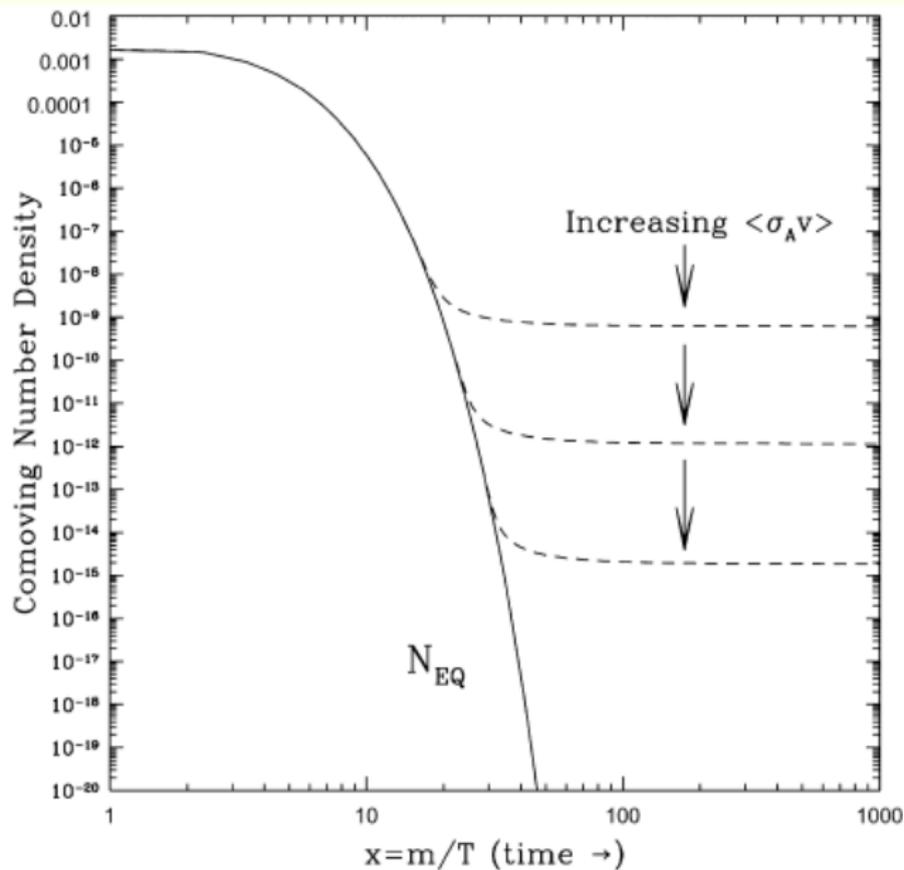




# Astrophysical uncertainties



# Relics from the early Universe



# The distribution function

The information is contained in the phase-space distribution function:

$$f(x, p)$$

$$N^\mu = \int f \frac{p^\mu}{p^0} \frac{d^3 p}{(2\pi)^3} \Rightarrow N^0 \equiv n$$

$$T^{\mu\nu} = \int f \frac{p^\mu p^\nu}{p^0} \frac{d^3 p}{(2\pi)^3} \Rightarrow T^{00} \equiv \rho$$

$$S^\mu = - \int \left[ f \log f \mp (1 \pm f) \log (1 \pm f) \right] \frac{p^\mu}{p^0} \frac{d^3 p}{(2\pi)^3}$$

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This description makes sense as long as

$$\lambda = 1/p < \text{size of the universe.}$$

In the early radiation dominated epoch  $H = 1.66\sqrt{g_*}\frac{T^2}{M_{\text{pl}}}$  and,

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# The Boltzmann equation

Let us follow the variation of  $f(x, p)$  along a world-line  $x(\tau)$ :

$$\frac{d}{d\tau} f(x(\tau), p(\tau)) = \frac{\partial f}{\partial x^\mu} \frac{dx^\mu}{d\tau} + \frac{\partial f}{\partial p^\mu} \frac{dp^\mu}{d\tau}.$$

If the particles only interact gravitationally (between collisions), then they follow geodesics

$$\frac{dp^\mu}{d\tau} = -\Gamma_{\alpha\beta}^{\mu} p^\alpha p^\beta,$$

and the variation of  $f$  is given by  $\hat{L}[f]$  where

$$\hat{L} = p^\mu \frac{\partial}{\partial x^\mu} - \Gamma_{\alpha\beta}^{\mu} p^\alpha p^\beta \frac{\partial}{\partial p^\mu}.$$

The *collisionless* Boltzmann equation states that there is no net variation of  $f$  along the trajectory:

$$\hat{L}[f] = 0$$

## Example: Robertson-Walker

In a homogeneous and isotropic flat RW Universe  $f = f(E, a(t))$ ,  
and

$$\hat{L} = E \frac{\partial}{\partial t} - H |\mathbf{p}|^2 \frac{\partial}{\partial E}$$

The most general solution is *any* function of  $aE$ , e.g.

$$\exp(aE/T_0) = \exp(E/(T_0/a)).$$

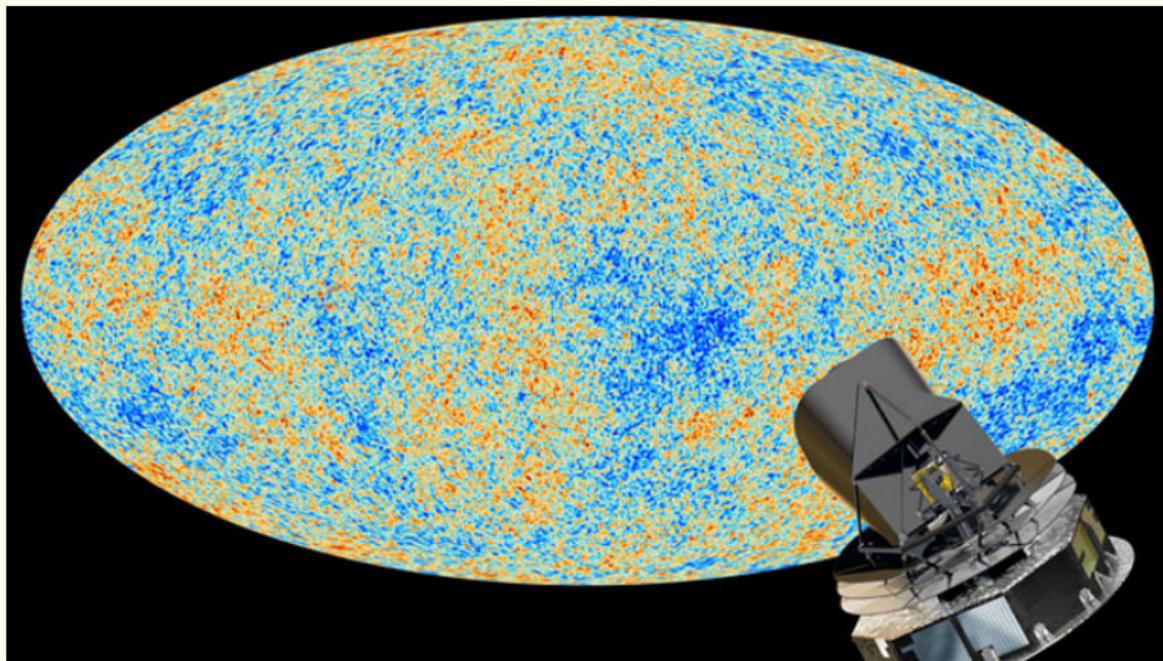
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# Dark matter or stars in a galaxy

We would then use the static, weak field geometry

$$ds^2 = -(1 + 2\Phi) dt^2 + (1 - 2\Phi) d\vec{x}^2.$$

Since the velocities are small,  $p^0 = m$ ,  $p^i = mv^i$ , and the collisionless Boltzmann equation reads

$$\frac{\partial f}{\partial t} + \vec{v} \frac{\partial f}{\partial \vec{x}} - \frac{\partial \Phi}{\partial \vec{x}} \frac{\partial f}{\partial \vec{v}} = 0$$

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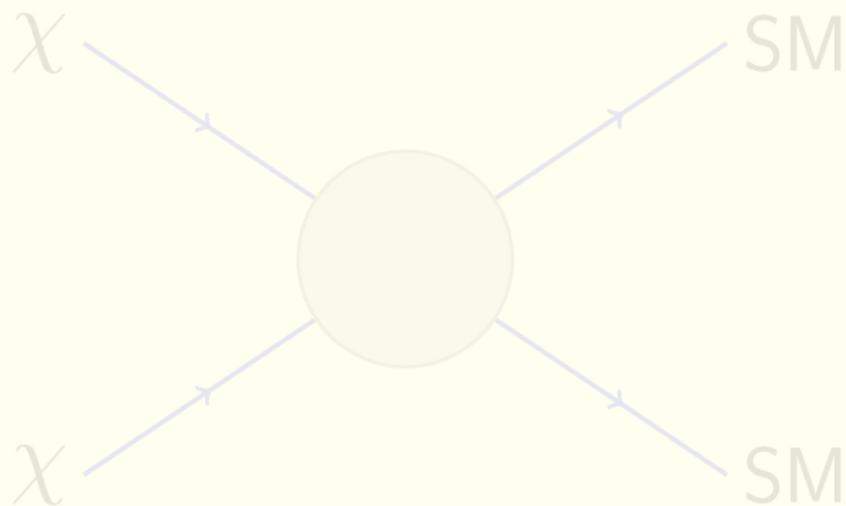
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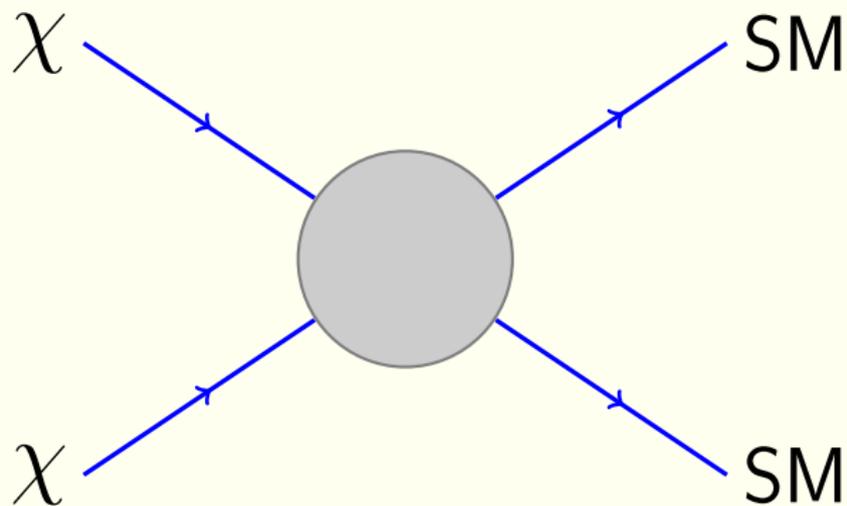
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$$\hat{C}[f] = -\frac{1}{E} \int (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_a - p_b) \\ \times \left[ |\mathcal{M}_{12 \rightarrow ab}|^2 f_1 f_2 (1 \pm f_a)(1 \pm f_b) - |\mathcal{M}_{ab \rightarrow 12}|^2 f_a f_b (1 \pm f_1)(1 \pm f_2) \right]$$

with  $d^3\Pi_i \equiv \frac{d^3 p_i}{(2\pi)^3 2E_i}$ .

Since we are looking for an equation for  $n$ ,

$$\int \hat{L}[f_1] d^3\Pi_1 = \dot{n} + 3Hn.$$

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We can typically assume that the SM annihilation products  $a, b$ , go quickly into equilibrium with the thermal background and replace  $f_{a,b} \rightarrow f_{a,b}^{\text{eq}}$ . Detailed balance allows the replacement  $f_a^{\text{eq}} f_b^{\text{eq}} = f_1^{\text{eq}} f_2^{\text{eq}}$ .

We will also take advantage of the unitarity of the S-matrix:

$$\begin{aligned} & \int \delta^{(4)}(p_1 + p_2 - p_a - p_b) |\mathcal{M}_{12 \rightarrow ab}|^2 d^3\Pi_a d^3\Pi_b \\ &= \int \delta^{(4)}(p_1 + p_2 - p_a - p_b) |\mathcal{M}_{ab \rightarrow 12}|^2 d^3\Pi_a d^3\Pi_b. \end{aligned}$$

Defining the averaged total annihilation cross-section

$$\langle \sigma v_{M\phi l} \rangle = \frac{\int \sigma v_{M\phi l} dn_1^{\text{eq}} dn_2^{\text{eq}}}{\int dn_1^{\text{eq}} dn_2^{\text{eq}}}$$

where the Møller velocity

$$v_{M\phi l} = \sqrt{|\vec{v}_1 - \vec{v}_2|^2 - |\vec{v}_1 \times \vec{v}_2|^2},$$

we recover the familiar result

$$\dot{n} + 3Hn = - \langle \sigma v_{M\phi l} \rangle (n^2 - n_{\text{eq}}^2)$$

One usually works with the yield  $Y \equiv n/s$  as a function of  $x = m/T$ :

$$\frac{dY}{dx} = -\frac{\lambda \langle \sigma v \rangle}{x^2} (Y^2 - Y_{\text{eq}}^2),$$

$$\lambda \equiv \frac{2\pi^2}{45} \frac{M_{\text{pl}} g_{\text{eff}}}{1.66 g_*^{1/2}} m.$$

It can be solved analytically in the two extreme regions

$$\Delta = -\frac{Y_{\text{eq}}'}{2f(x)Y_{\text{eq}}} \quad \text{for } x \ll x_F$$

$$\Delta' = -f(x)\Delta^2 \quad \text{for } x \gg x_F$$

The last equation can be integrated between  $x_F$  and  $\infty$  using  $\Delta_{x_F} \gg \Delta_{\infty}$  to obtain  $Y_{\infty}$ .

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For a heavy particle

$$\langle \sigma v \rangle = a + b \langle v^2 \rangle + \dots \approx a + 6b/x$$

and we obtain the desired result

$$\begin{aligned}\Omega_X h^2 &\approx \frac{10^9 \text{ GeV}^{-1}}{M_{\text{Pl}}} \frac{x_F}{\sqrt{g_*}} \frac{1}{a + 3b/x_F} \\ &\approx \frac{10^{-27} \text{ cm}^3 \text{ s}^{-1}}{a + b/60}.\end{aligned}$$

Unitarity bound:

$$\Omega_X \leq 1 \Rightarrow m \leq 340 \text{ TeV}$$

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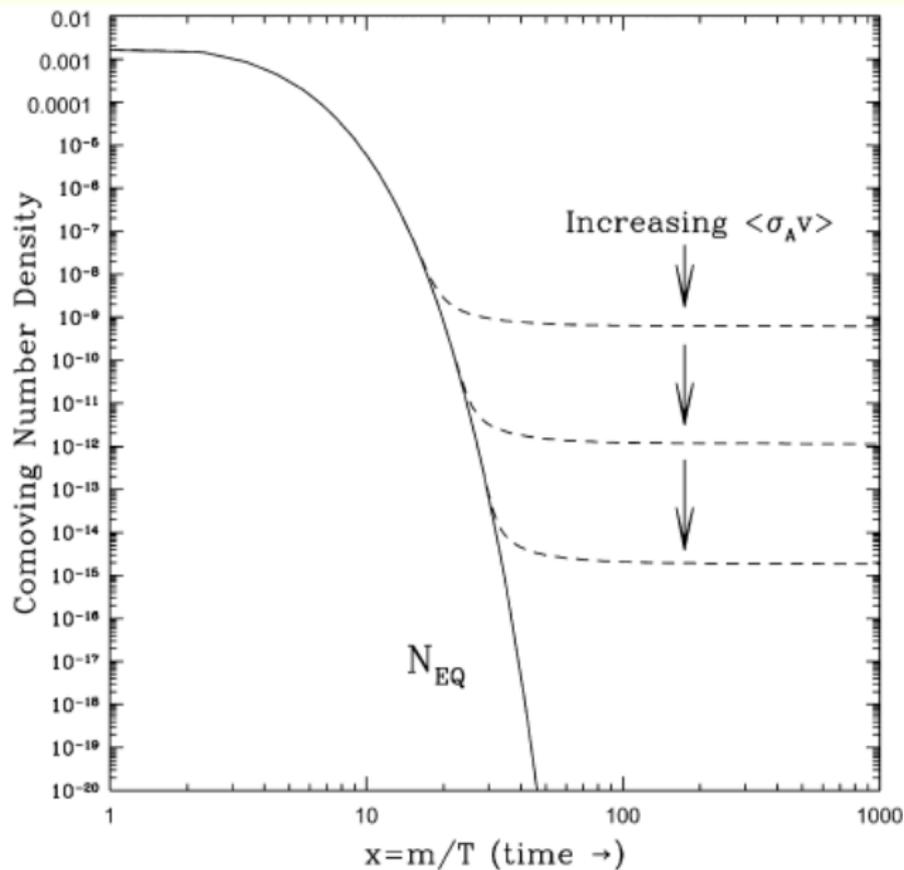
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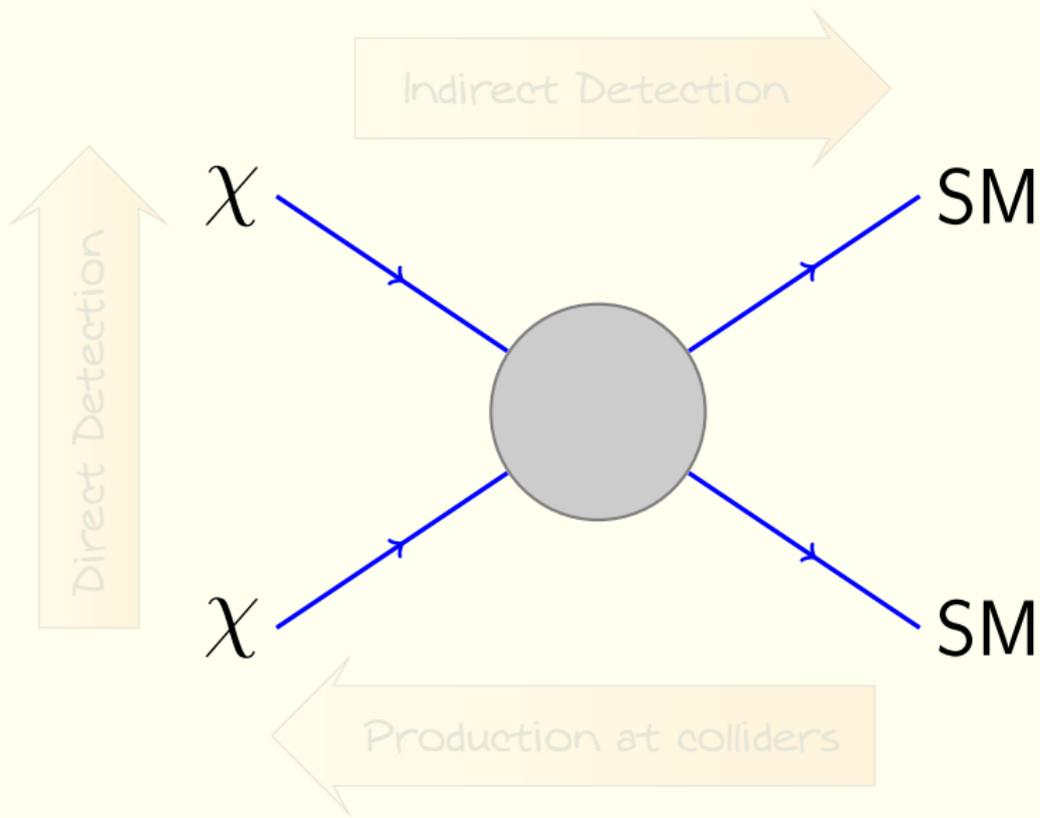
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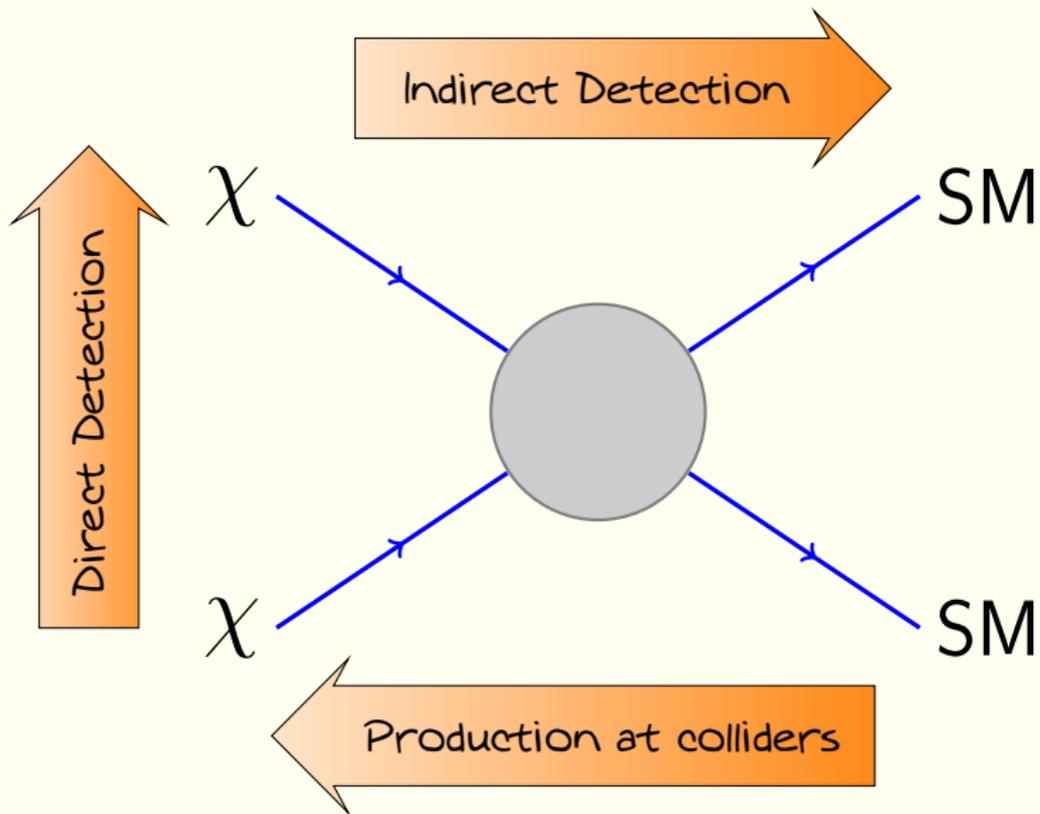
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