Gravitational wave forest from string axiverse

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J.Soda & Y.U. Euro. Phys. J. C 78, 9, 779 (2018)

Kitajima, Soda & Y.U. JCAP 10, 008 (2018)

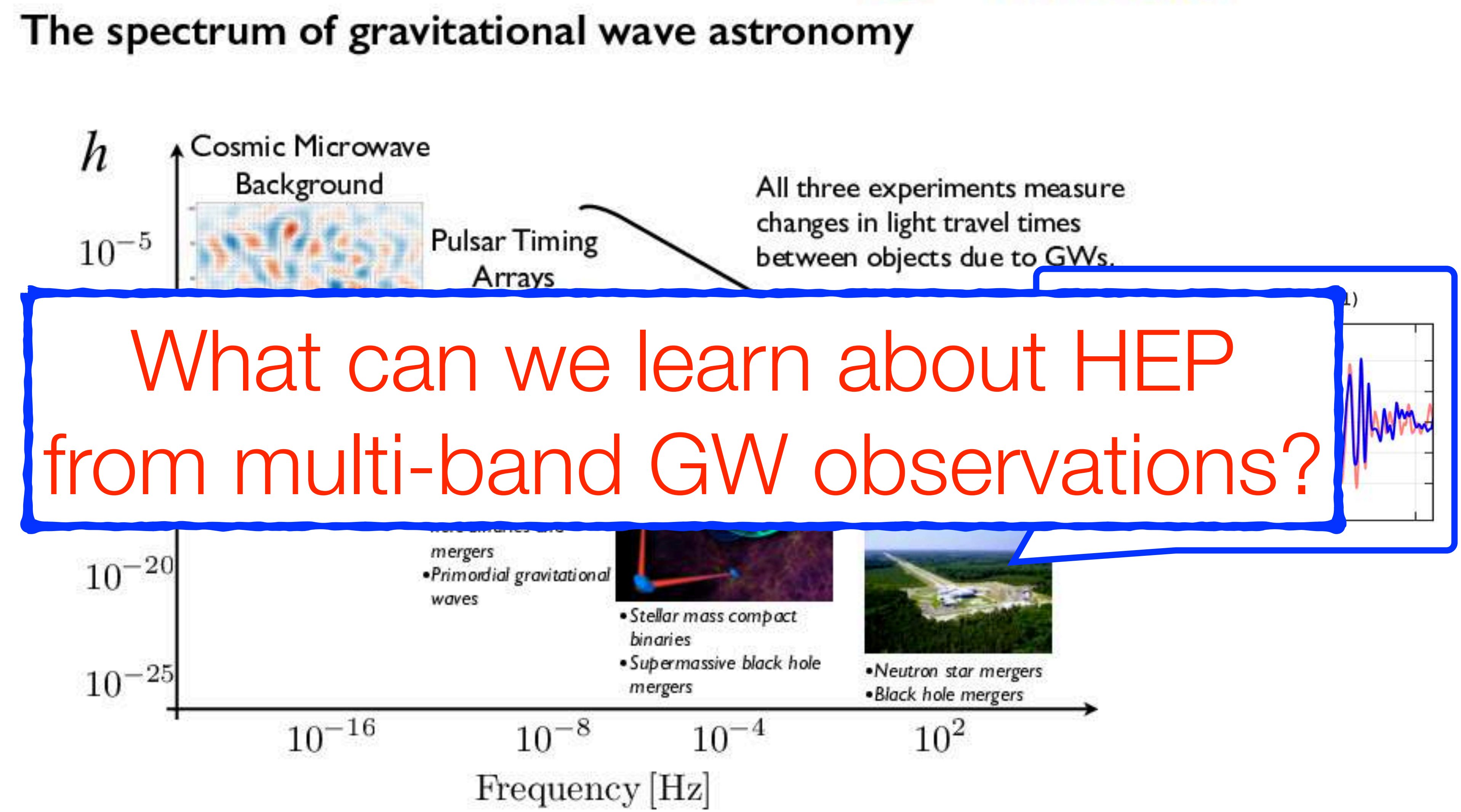
Kitajima, Soda & Y.U. in progress

Fukunaga, Kitajima & Y.U. in progress

w/ Hayato Fukunaga, Naoya Kitajima (Nagoya U.), Jiro Soda (Kobe U.)

Multi-wavelength GW era

from NANOGrav



What can we learn about HEP from multi-band GW observations?

String axiverse

Arvanitaki et al. (10), $\cdots$

Superstring theory in compact 6D

↓

4D low energy EFT + Axions + Moduli ….

$$m_a^2 \sim \frac{\mu^4}{f_a^2} e^{-\#\sigma_i}$$

Wide mass ranges → Probe of exDim

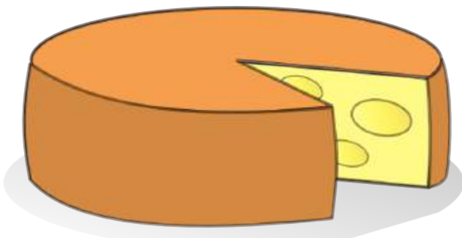
↓

Inflaton, DM candidate (Fuzzy DM)

Hu et al.(00), $\cdots$

ex. Large Volume Scenario *Conlon et al. (05)*

Thraxions *Hebecker + (18)*



→ Predicts extremely light axions

Axion as dark matter

$m_a << H$

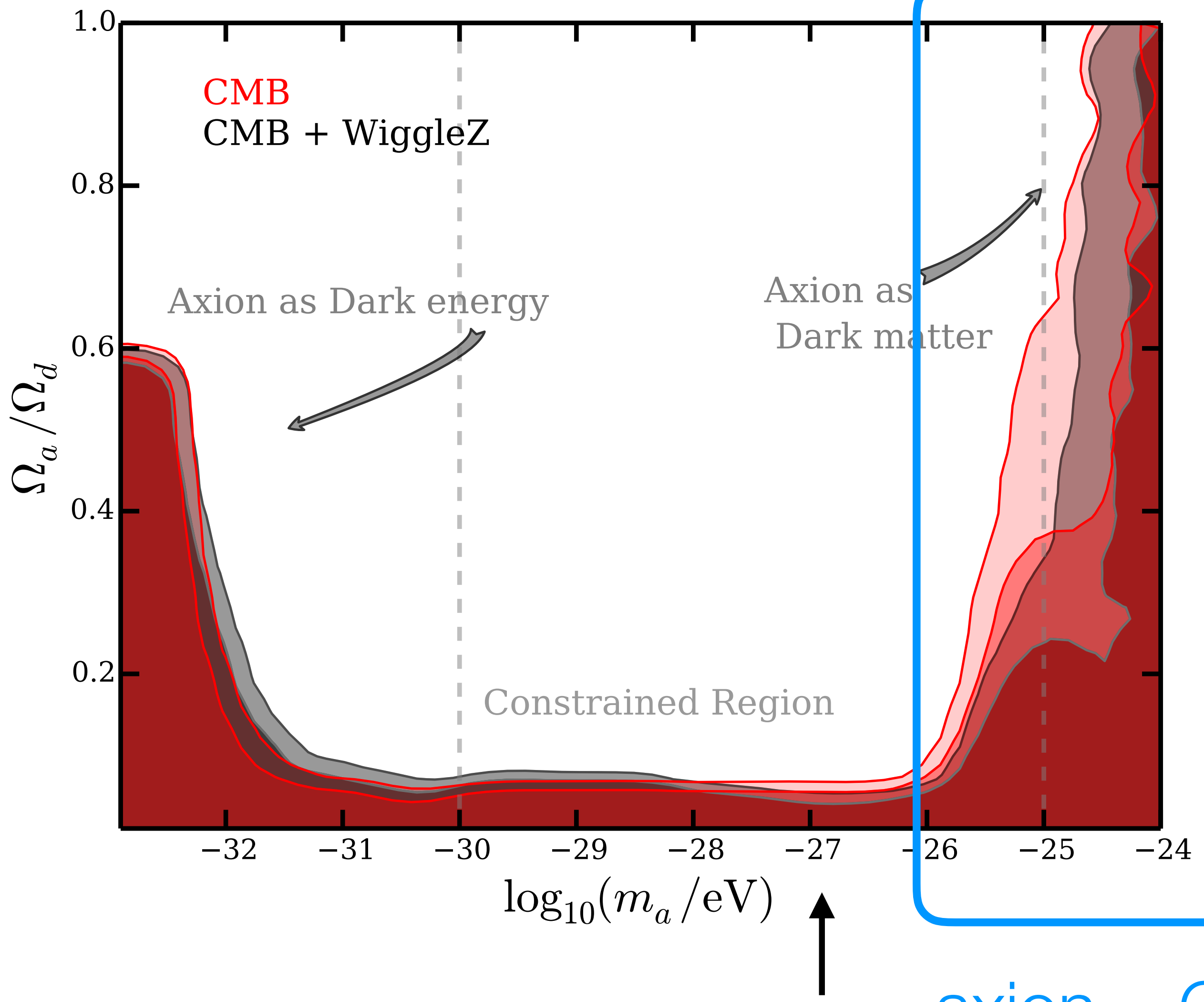
axion → Λ

$m_a >> H$

axion → DM

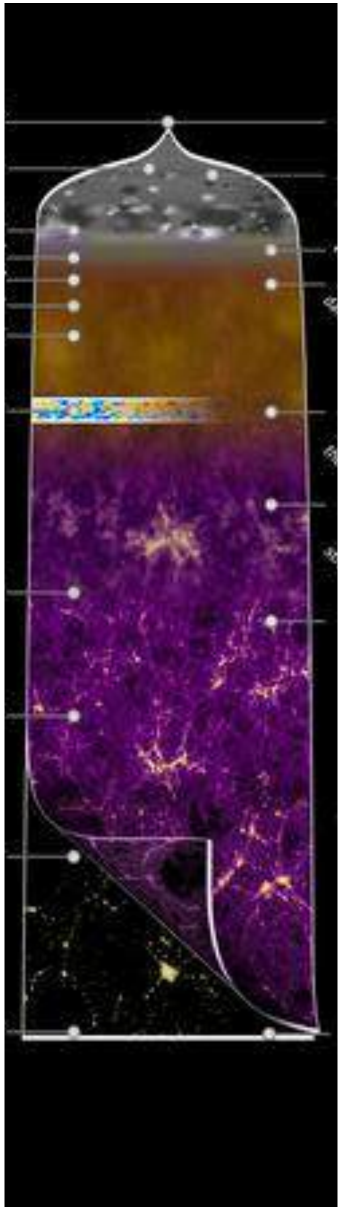
$$\Omega_d = \Omega_a + \Omega_c$$

Marsh (15)

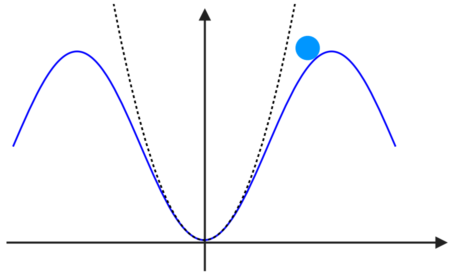


axion ~ CDM?

Axion search from GWs



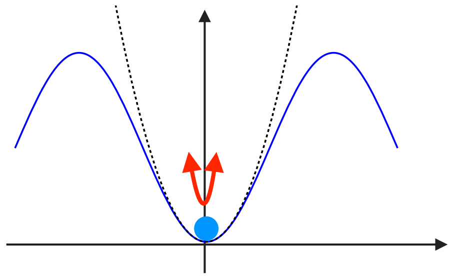
Onset of oscillation



~ Λ (w=-1)

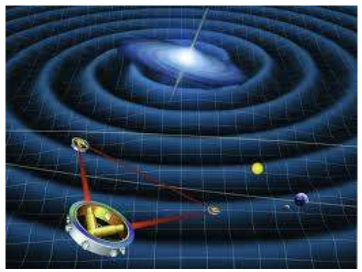
Transition

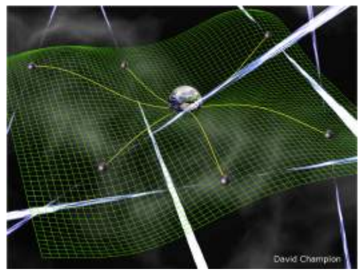
Resonance inst.



~ dust (w=0)

(b)GW



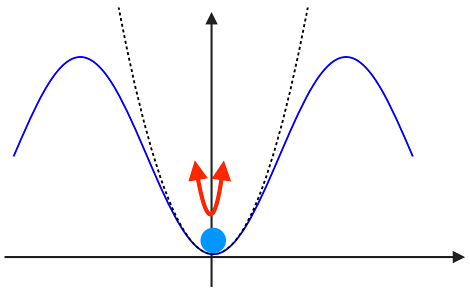


Bottom-line story

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{1}{a^2}\nabla^2 h_{ij} = \frac{2}{M_{\rm Pl}^2}\Pi_{ij}^{\rm TT} \qquad \partial_i h_{ij} = 0 \text{ and } h_{ii} = 0$$

h: +, -

homogeneous axion



$\phi F\tilde{F}$

$\delta\phi$ → $h_{\rm ij,\,h}$

$$\Pi_{ij}^{\rm TT} = \frac{1}{a^2}P_{ij}^{lm}\partial_l\phi\partial_m\phi$$

A_+, A_- → $h_{\rm ij,\,+} \neq h_{\rm ij,\,-}$

$$\Pi_{ij}^{\rm TT} = \frac{1}{a^2}P_{ij}^{lm}F_{l\mu}F_m^{\ \mu}$$

$$\epsilon_{ij}^{(\pm)} \sim \epsilon_i^{(\pm)}\epsilon_j^{(\pm)}$$

resonance instability

non-linear

Contents

1. Trigger: Revisit of parametric resonance

2. Outcome: GW emissions

3. Prospect

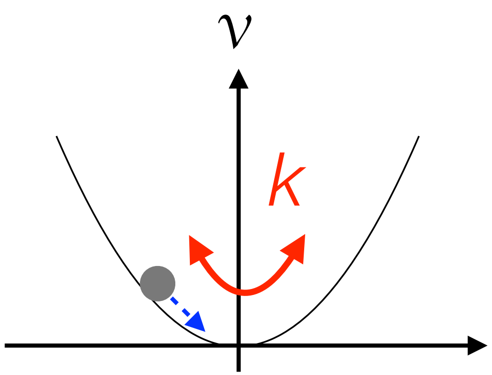
Parametric resonance in cosmology

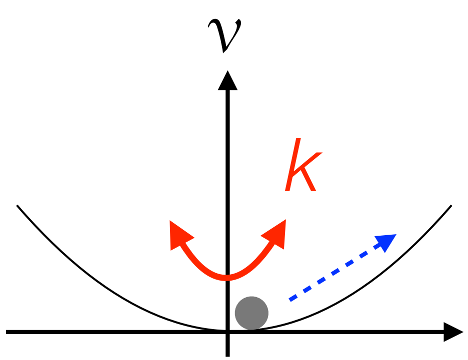
- Reheating: inflaton → SM   *e.g., Kofman, Linde, Starobinsky (97)*
- Axion DM

e.g. periodic mass $m(t+T) = m(t)$

$$\omega_k^2(t) = k^2 + m^2(t) \quad \longrightarrow \quad \omega_k^2(t) = \left(\frac{k}{a(t)}\right)^2 + m^2(t)$$

Redshift disturbs.





potential opens

only k whose phases match to $m(t)$ grow

(parametric resonance)

Mathieu equation

$$\frac{d^2 y_k}{dt^2} + \omega_k^2(t) y_k(t) = 0 \qquad\qquad \omega_k^2(t) = A_k - 2q\cos(mt)$$

e.g., bg k=0 mode $\phi(t) \propto \cos(mt)$

q >>1: Broad resonance

- Significant violation of adiabatic condition
- No peak, Rapid exponential growth for $k < k_*$

q <<1: Narrow resonance

- No violation of adiabatic condition
- Narrow peak(s)

Issues to be discussed

Q1. Parametric resonance for general Hill eq.?

Q. What characterizes
- Growth rate Re[μ]?
- Shape of the spectrum?
- Duration?

e.g. Self-interaction for axion, CS coupling $\phi F\tilde{F}$

New instability w/AC and peaky spectrum → GW emission

Kitajima, Soda & Y.U. (18)

Q2. Impact of cosmic expansion?

Does H ≠ 0 always disturb parametric resonance?

Normalization

Neglect back-reaction on geometry

⟶ Axion's dynamics is independent of (m, f)

$$\partial_t^2\phi + 3H\partial_t\phi - \frac{\partial^2}{a^2}\phi + V_{,\phi} = 0$$

$$\tilde{x}^\mu \equiv m x^\mu \qquad \tilde{\phi} \equiv \frac{\phi}{f}$$

$$V(\phi) = (mf)^2\,\tilde{V}(\tilde{\phi})\,, \qquad \tilde{\phi} \equiv \frac{\phi}{f}$$

e.g. $\tilde{V}(\tilde{\phi}) = 1 - \cos\tilde{\phi}$

↓

$$\partial_{\tilde{t}}^2\tilde{\phi} + 3\frac{H}{m}\partial_{\tilde{t}}\tilde{\phi} - \frac{\partial_{\tilde{\mathbf{x}}}^2}{a^2}\tilde{\phi} + \tilde{V}_{,\tilde{\phi}} = 0$$

Delayed onset of oscillation

if initially $\left|\frac{\tilde{V}_{,\tilde{\phi}}}{\tilde{\phi}}\right|$ <<1, $\frac{H_{\rm osc}}{m} \sim \sqrt{\left|\frac{\tilde{V}_{\tilde{\phi}}}{\tilde{\phi}}\right|} \ll 1$ delayed oscillation

e.g. $\tilde{V}(\tilde{\phi}) = 1 - \cos\tilde{\phi}$ with $\tilde{\phi}_i \sim \pi$

(Time scale of cosmic exp.) (Time scale of V driven motion)

1/H $\sqrt{|V_\phi/\phi|} = \sqrt{|\tilde{V}_{\tilde{\phi}}/\tilde{\phi}|}/m$

Slow-roll <<

recall $\partial_t^2\phi + V_{,\phi} = 0$

Onset ~

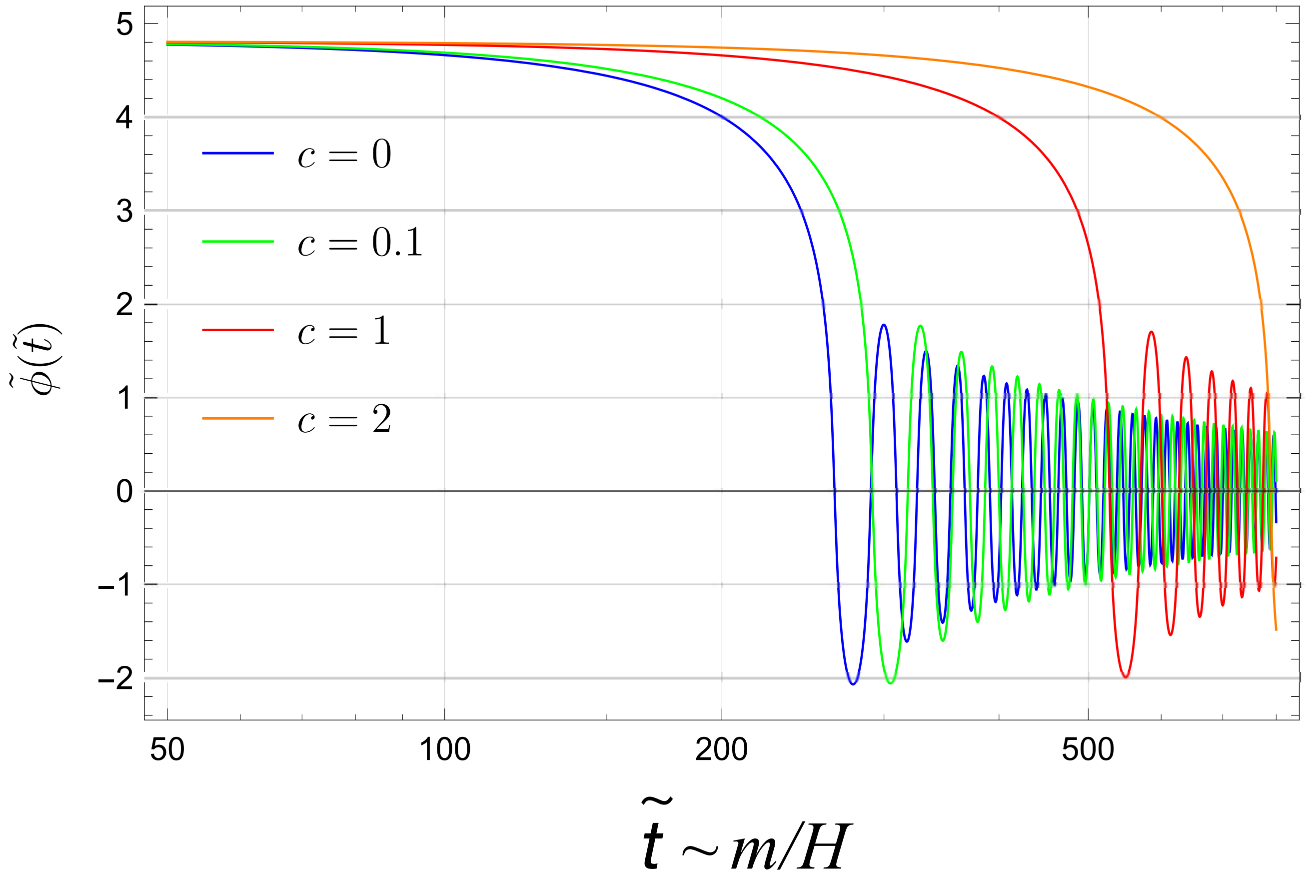
Oscillation >>

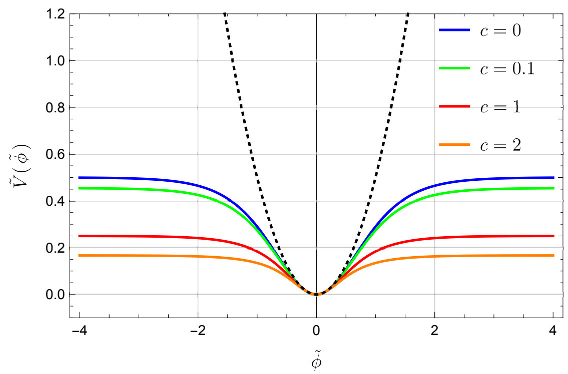
Evolution of k=0 mode

α-attractor

$$\tilde{V}(\tilde{\phi}) = \frac{1}{2}\frac{(\tanh\tilde{\phi})^2}{1+c(\tanh\tilde{\phi})^2}$$

RD





Kallosh & Linde + (13, 14,⋯)

Soda & Y.U.(17)

Onset of oscillation is not $m \sim H$, but delayed!

Scalar potential of axion

continuous shift sym.

$\phi \to \phi + c$

NP effects
e.g. instanton effects

$\phi \to \phi + 2\pi n/f$

$n \in \mathbf{Z}$

$V(\phi) \sim \Lambda^4 \cos\phi/f$

Potential can be more flatten than $\cos\phi/f$

i) ~~Dilute instanton gas approximation~~

see. implications for axion=inflaton, *Nomura + (17, 18)* $V(\phi) = M^4\left[1 - \frac{1}{(1+(\phi/F)^2)^p}\right]$

ii) Non-min. coupling w/gravity, Non-canonical kinetic term

Recall α attractor model for Re[T] *Kallosh & Linde + (13, 14, ⋯)*

iii) Superposition of multiple cosine terms

Setup: Scalar potential

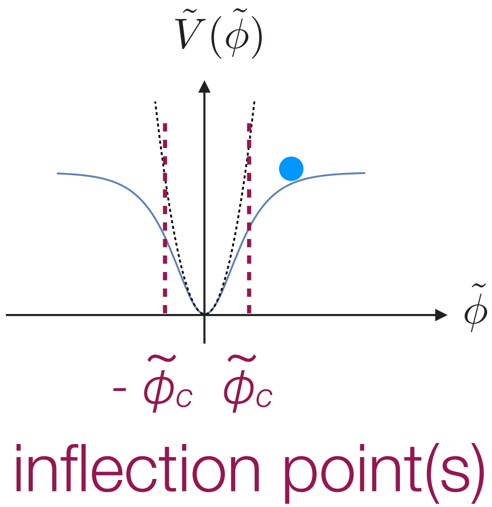
$$V(\phi) = (mf)^2\,\tilde{V}(\tilde{\phi})\,, \qquad \tilde{\phi} \equiv \frac{\phi}{f}$$

where $\tilde{V}(\tilde{\phi})$ satisfies

(0) Z_2 symmetry

(1) $\tilde{V}(\tilde{\phi}) \to \tilde{\phi}^2/2$ in the limit $\tilde{\phi} \to 0$

(2) Initially $\left|\frac{\tilde{V}_{,\tilde{\phi}}}{\tilde{\phi}}\right| \ll 1$ for $|\tilde{\phi}| > 1$



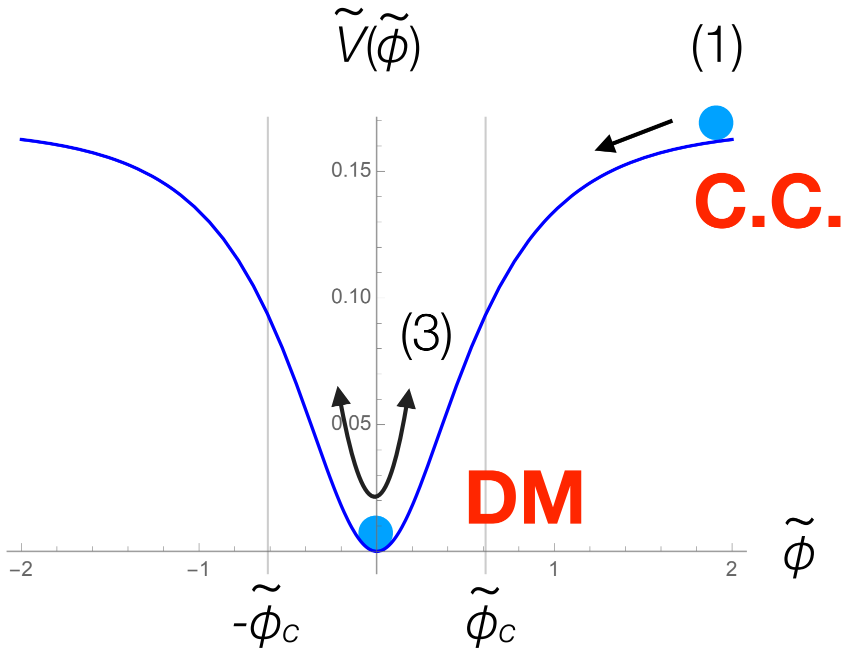
Timeline

$mt \sim \frac{m}{H}$

(1) Slowly rolling down in $\tilde{V}_{\tilde{\phi}\tilde{\phi}} < 0$

Tachyonic inst. $\frac{k}{am} \leq \sqrt{|\tilde{V}_{\tilde{\phi}\tilde{\phi}}|}$

mt_{osc}



$\tilde{V}_{\tilde{\phi}\tilde{\phi}} < 0$ $\tilde{V}_{\tilde{\phi}\tilde{\phi}} > 0$ $\tilde{V}_{\tilde{\phi}\tilde{\phi}} < 0$

(2) Highly anharmonic osci.

$\tilde{V}_{\tilde{\phi}\tilde{\phi}} < 0 \longleftrightarrow \tilde{V}_{\tilde{\phi}\tilde{\phi}} > 0$

Focus! *Soda & Y.U.(17), Kitajima, Soda, & Y.U. (18)*

(3) Slightly anharmonic osci.

$$\tilde{V} = \frac{\tilde{\phi}^2}{2} + \frac{\lambda}{4!}\tilde{\phi}^4 + O(\tilde{\phi}^4)$$

small corrections

(4) Harmonic osci.

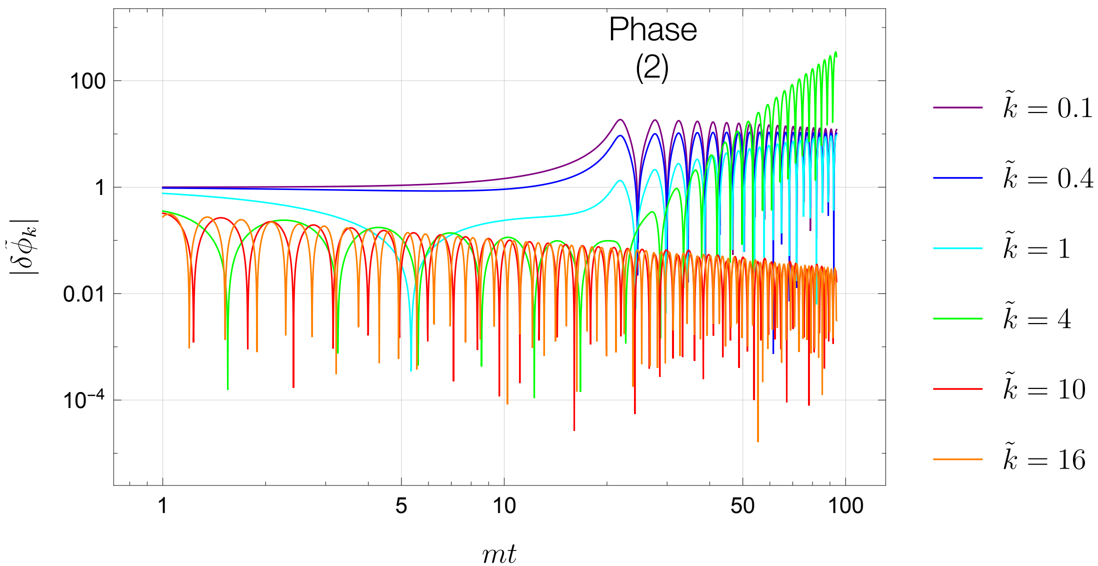
~CDM (at large scales)

Overall evolution in linear regime

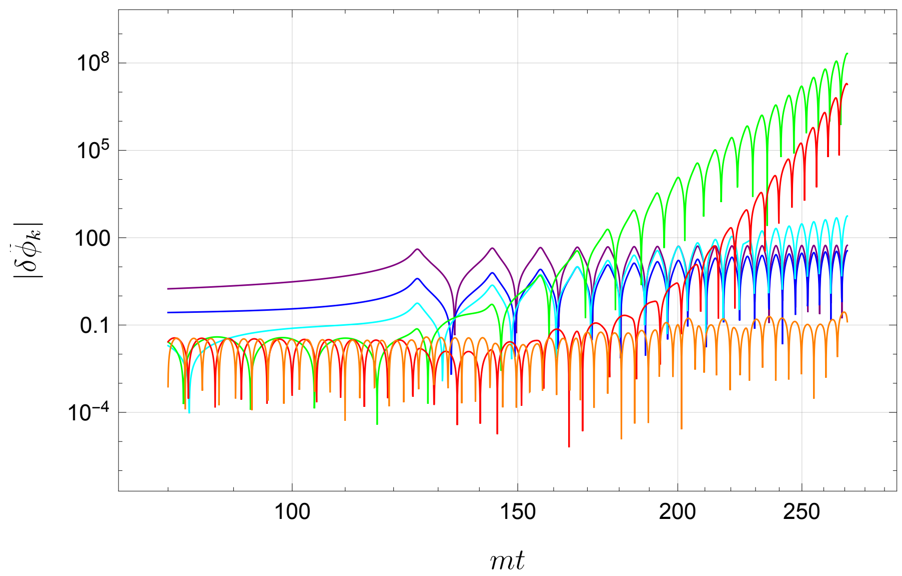
$$\tilde{V}(\tilde{\phi}) = \frac{1}{2}\frac{(\tanh\tilde{\phi})^2}{1+c(\tanh\tilde{\phi})^2}$$

Kitajima, Soda & Y.U. (18)

V"<0 | | Phase (3)



V"<0 | Phase (2)



Narrow res. dominant

- Smaller growth rate
- Narrower peak

- Larger growth rate
- broader peak
- $|d\omega_k/d\tilde{t}/\omega_k^2| > 1$

No disturbance due to H ≠ 0

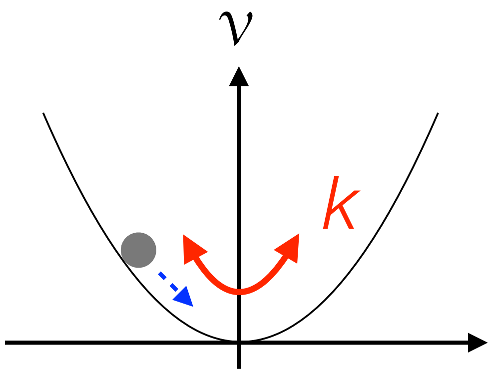
?

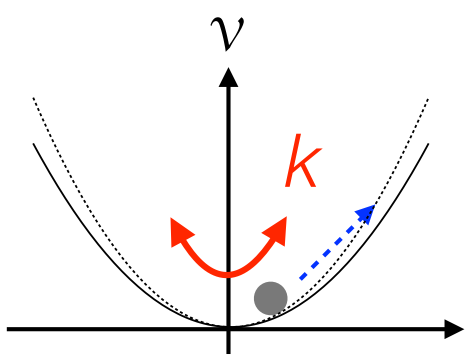
Self-resonance

$$\omega_k^2(t) = \left(\frac{k}{a(t)m}\right)^2 + \tilde{V}_{\tilde{\phi}\tilde{\phi}}(\tilde{t})$$

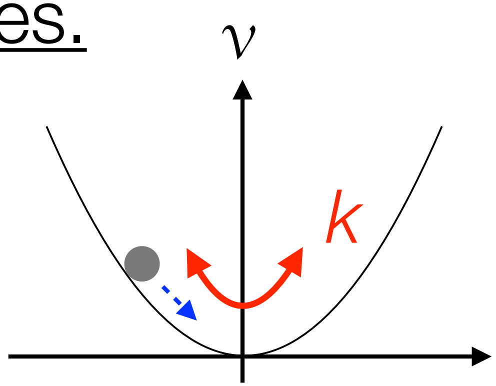
potential opens

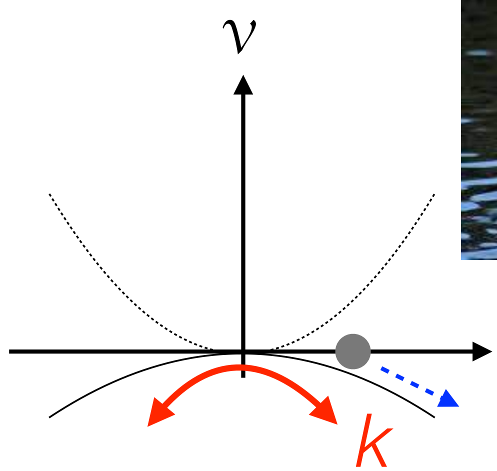
Narrow res.





Flapping res.





Floquet analysis

$$\frac{d^2 y_k(\tilde{t}\,)}{d\tilde{t}^2} + \omega_k^2(\tilde{t}\,) y_k(\tilde{t}\,) = 0 \qquad\qquad y_k(\tilde{t}) \equiv a^{3/2}\delta\phi_k(\tilde{t})$$

$$\omega_k^2(t) = \left(\frac{k}{a(t)m}\right)^2 + \tilde{V}_{\tilde{\phi}\tilde{\phi}}(\tilde{t}) = A_k - 2q\psi(\tilde{t}\,)$$

quasi-periodic ($H_{osc}/m << 1$)

Mathieu eq.

What characterizes
- Growth rate Re[μ]? — q
- Shape of the spectrum? — position A_k, width q
- Duration?

Fukunaga, Kitajima, & Y.U. (in progress)

Floquet analysis

$$\tilde{V}(\tilde{\phi}) = \frac{1}{2}\tilde{\phi}^2(1+\tilde{\phi}^2)^c$$

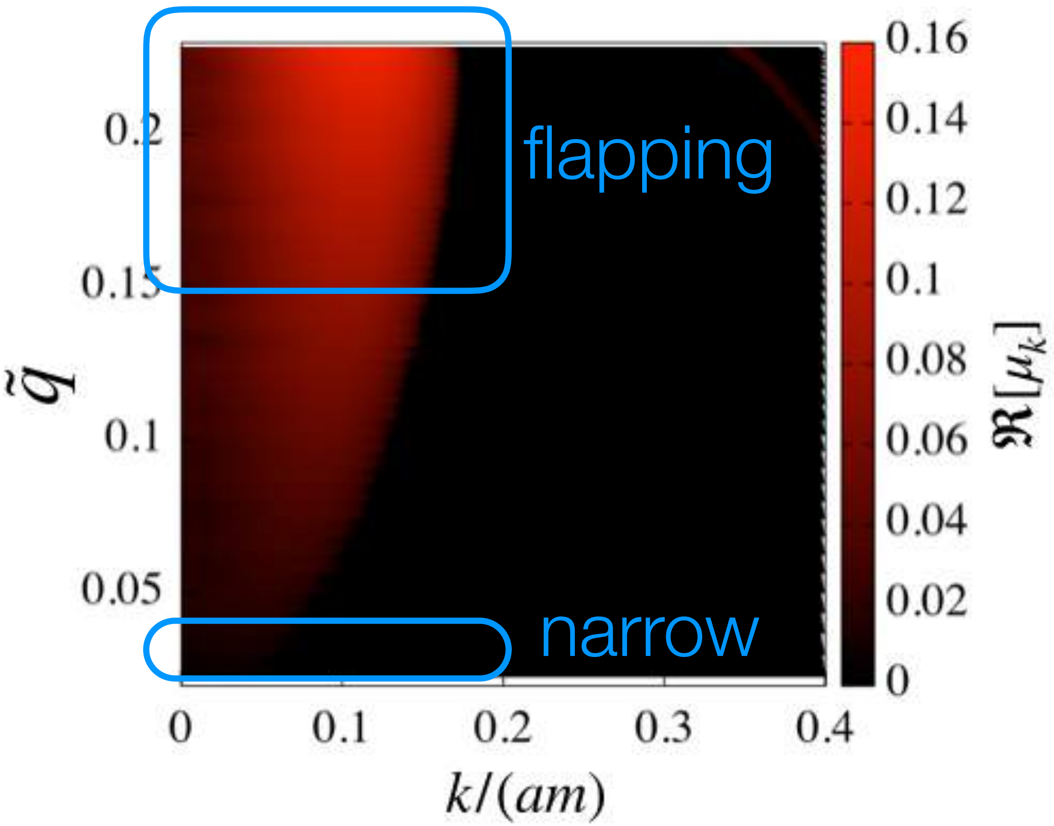
$$\tilde{q} \equiv \sqrt{\frac{\langle(\omega_k^2 - \langle\omega_k^2\rangle)^2\rangle}{2}}$$

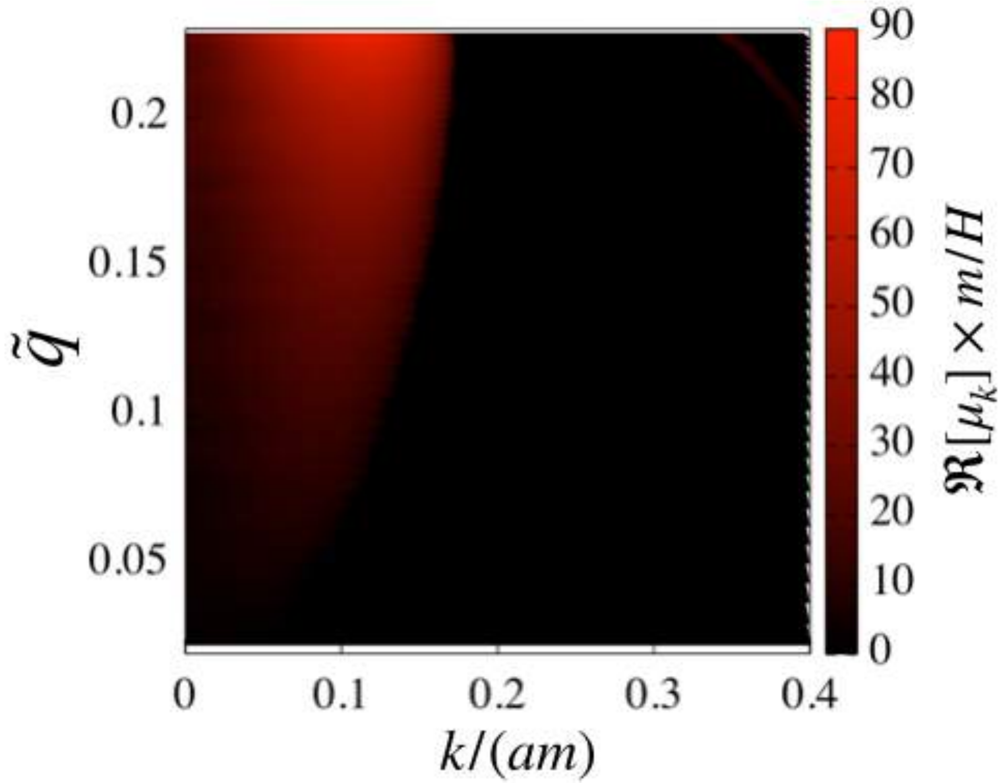
$$\langle F(\tilde{t})\rangle \equiv \frac{1}{T}\int_{\tilde{t}-\frac{T}{2}}^{\tilde{t}+\frac{T}{2}} d\tilde{t}' F(\tilde{t}')$$

- Amplitude of the osci. contribution

- $\tilde{q} \to q_{\mathrm{Mathieu}}$ if $\psi(\tilde{t}) = \cos\tilde{t}$

N=20





Classification of self-resonance

$$\tilde{q} = \sqrt{\frac{\langle(\tilde{V}_{\tilde{\phi}\tilde{\phi}} - \langle\tilde{V}_{\tilde{\phi}\tilde{\phi}}\rangle)^2\rangle}{2}}$$

Fukunaga, Kitajima, & Y.U. (in progress)

1. $\tilde{q} \gg 1$: Broad resonance
2. $\tilde{q} = \mathcal{O}(1)$: Intermediate resonance $\supset$ Flapping resonance
3. $\tilde{q} \ll 1$: Narrow resonance

Re[μ]

*1 Characterization by $\tilde{q}$ not adiabatic condition.

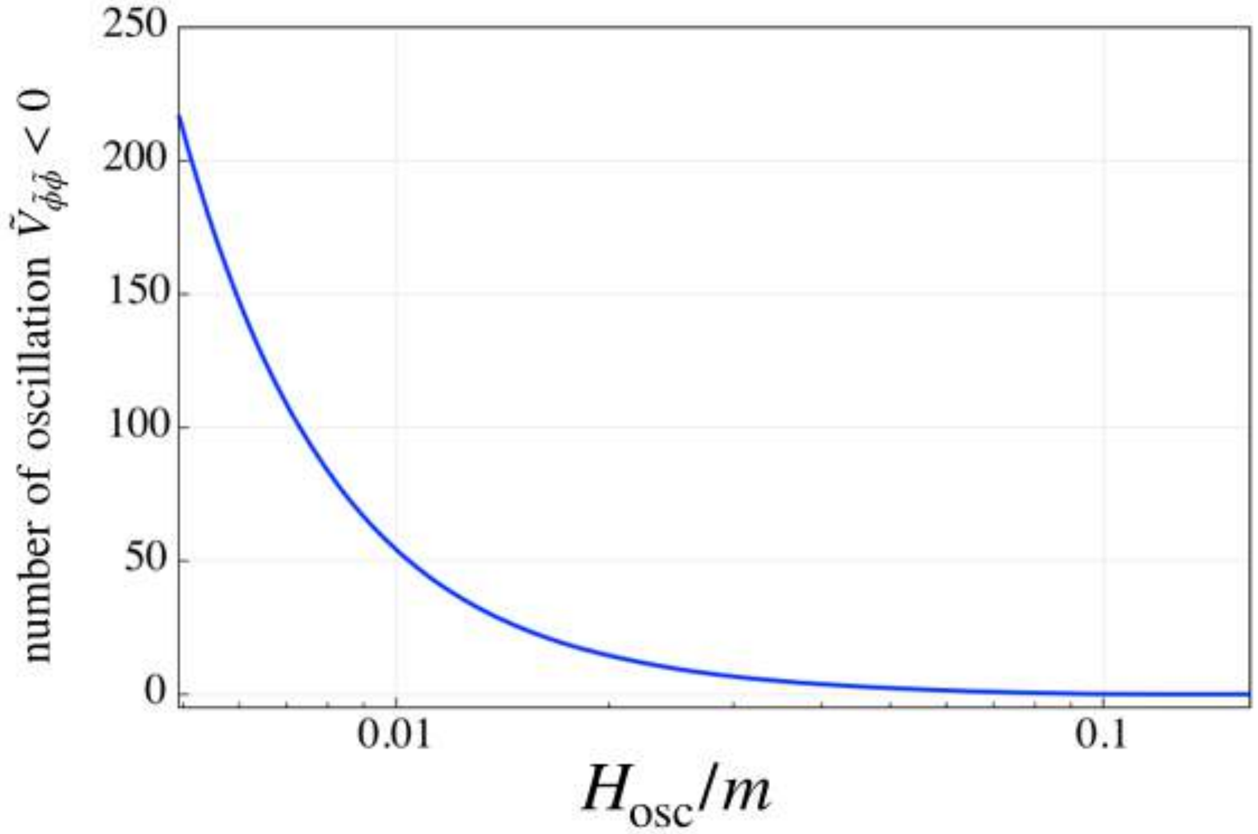
*2 Broad res. is unlikely for self-resonance.

Flapping res. is the most efficient. (Empirical)

Duration of flapping res.

Fukunaga, Kitajima, & Y.U. (in progress)

Strong correlation w/ H_{osc}/m

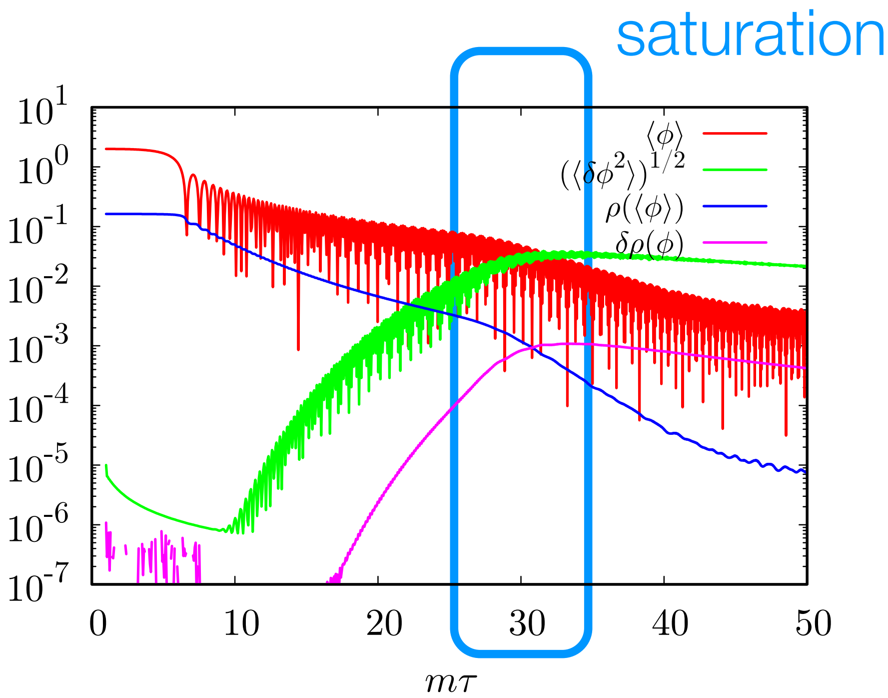


N.B. Flapping res. does not persist for cosine potential.

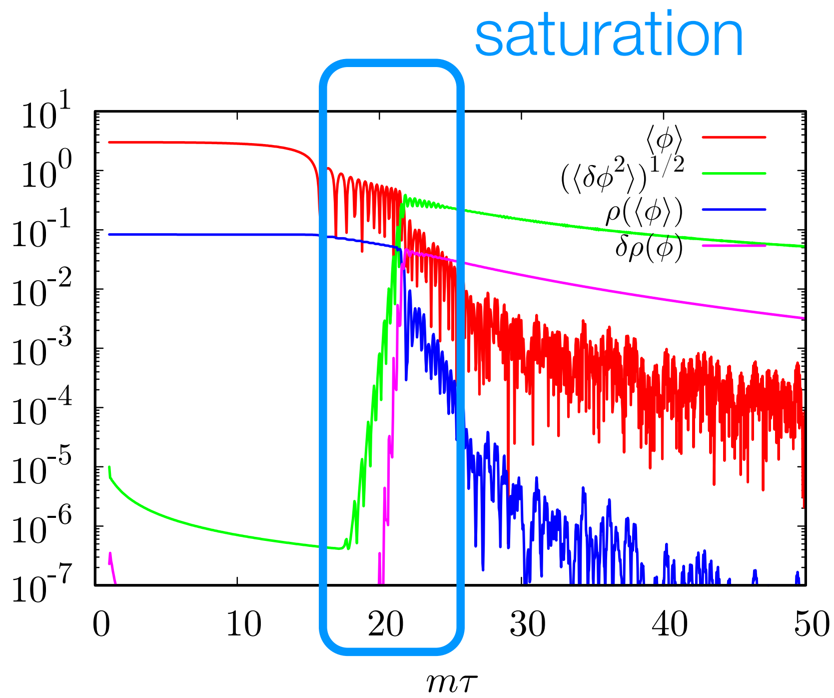
$$H_{\rm osc}/m \sim 1/\ln|\dot{\tilde{\phi}}_i|^{-1}$$

Lattice simulation

Kitajima, Soda & Y.U. (18)



(b) $c = 2, \phi_i = 2f$



(a) $c = 5, \phi_i = 3f$

Narrow res. dominant

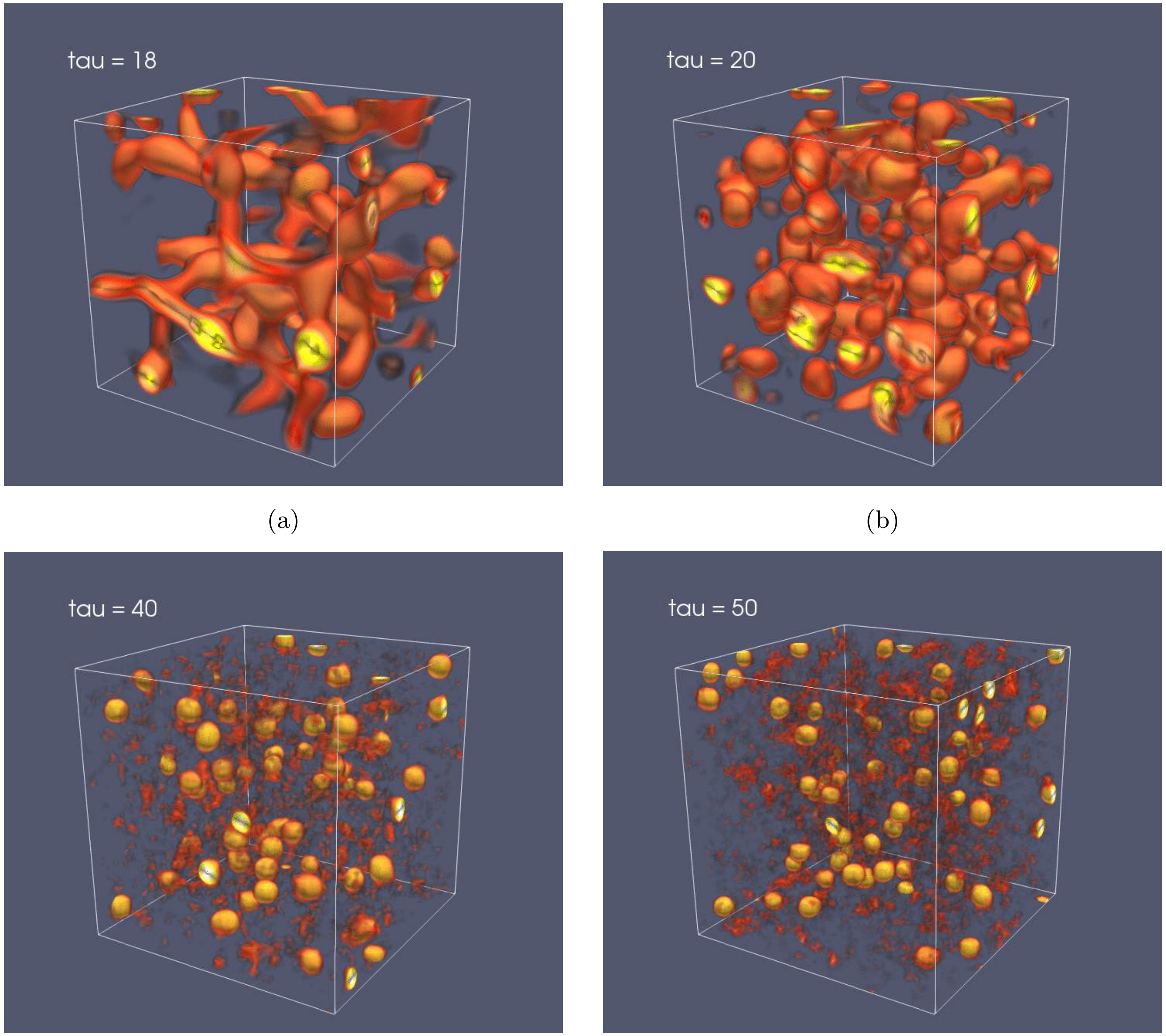
Flapping res. dominant

Cosmic exp. does not stop growth, but backreaction does.

Lattice simulation $N_{grid}=(256)^3$

Oscillon formation

Kitajima, Soda & Y.U. (18)



(a) (b) (c) (d)

Kasuya et al.(03)
Amin & Shirokoff(10)
Amin et al.(2010)
Amin et al.(2014),⋯

$N_{grid}=(256)^3$

$c = 5$ and $\phi_i = 2f$. The red, yellow and white region correspond to $\rho/\bar{\rho} > 2$, 4 and 10

Resonant production of gauge fields

$$\mathcal{L}_{int} = \frac{\alpha}{4}\frac{\phi}{f} F\tilde{F}$$

Coulomb gauge (A_0=0, $\partial_i A_i$=0), Linear analysis

$$\frac{d^2 \mathcal{A}_h}{d\tilde{t}^2} + \frac{H}{m}\frac{d\mathcal{A}_h}{d\tilde{t}} + \omega_h^2 \mathcal{A}_h = 0 \qquad \omega_h^2 \equiv \left(\frac{k}{am}\right)^2 - h\alpha \frac{d\tilde{\phi}}{d\tilde{t}}\frac{k}{am}$$

h: +, -

for m/H_{osc}>>1

α >> 1 Broad resonance

α ~O(1) Flapping resonance

α << 1 Narrow resonance

persistent

$\tilde{q} \propto \alpha$

Phase difference → circular polarization

Contents

1. Trigger: Resonance instabilities

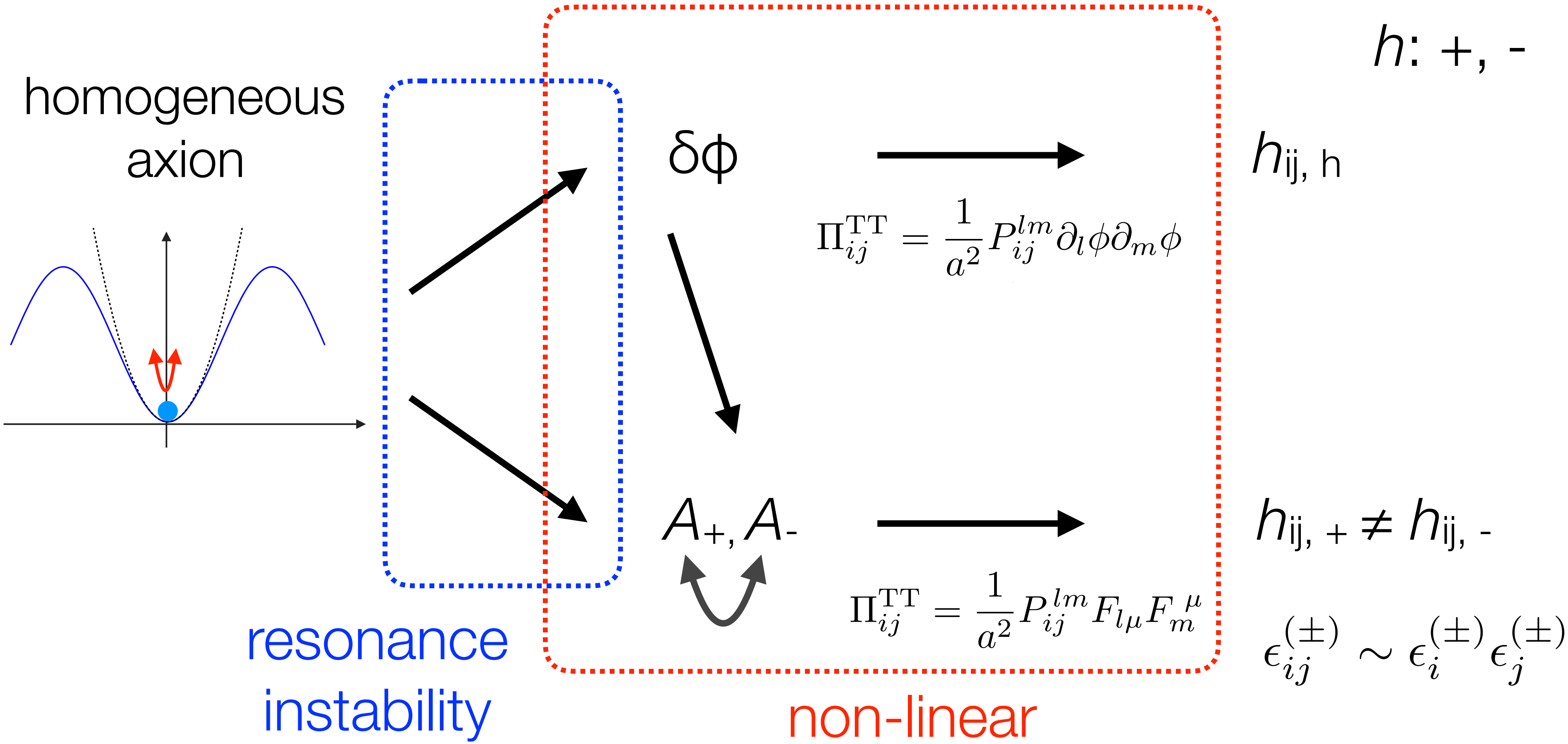
More systematic analysis, see Hayato's talk

2. Outcome: GW emissions

3. Prospect

Resonant GW production from axions

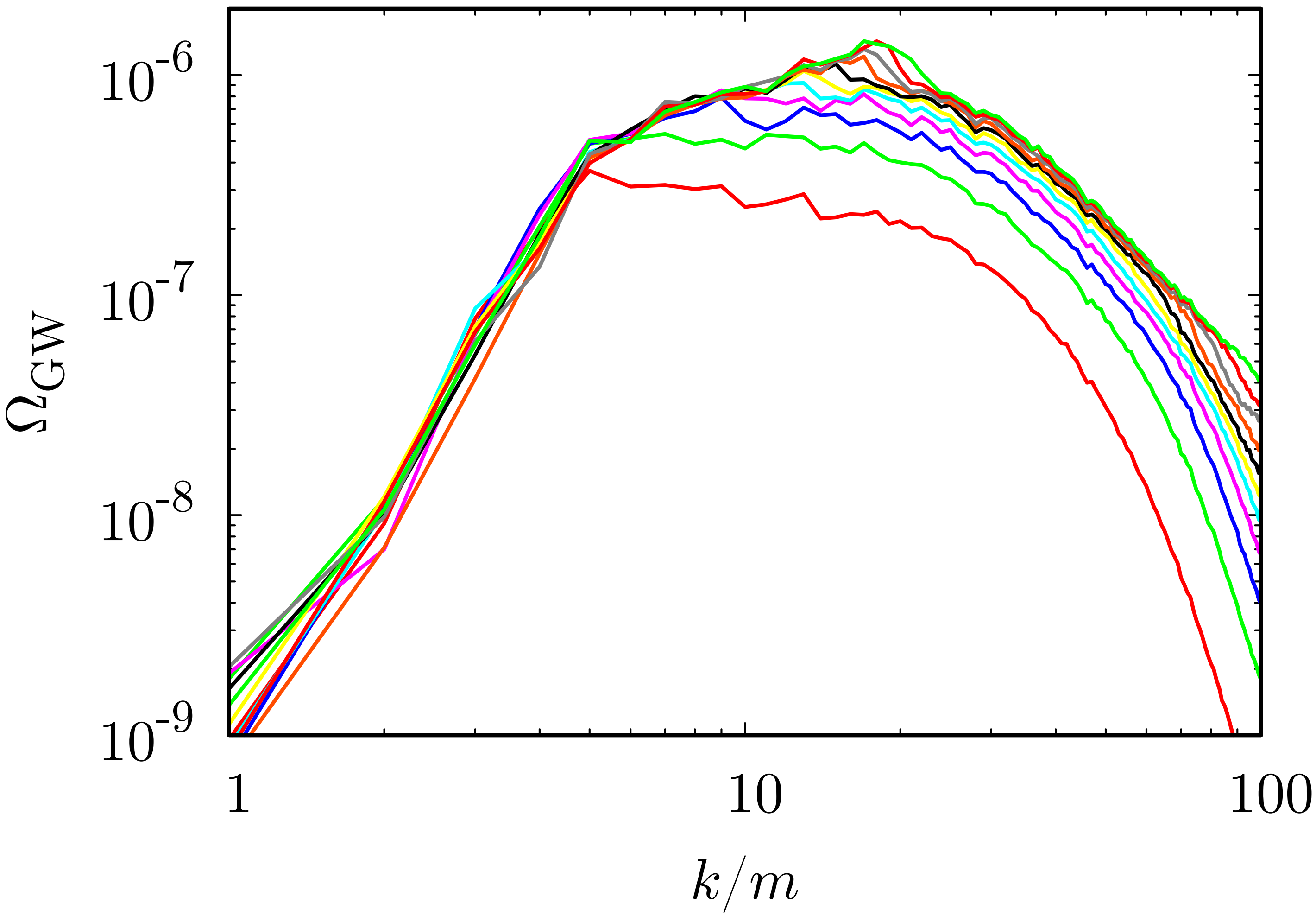
$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{1}{a^2}\nabla^2 h_{ij} = \frac{2}{M_{\rm Pl}^2}\Pi_{ij}^{\rm TT} \qquad \partial_i h_{ij} = 0 \text{ and } h_{ii} = 0$$



α=0

GW spectrum

Kitajima, Soda & Y.U. (18)



$f = 10^{16}$ GeV, $c = 5$ and $\phi_i = 3f$

to evaluate the present value, x Ω_r

Contents

1. Trigger: Resonance instabilities

More systematic analysis, see Hayato's talk

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GW's frequency

Kitajima, Soda & Y.U. (18)

Redshifted frequency

$$\nu_0 = \frac{\kappa m}{2\pi}\left(\frac{a_{\rm em}}{a_0}\right)$$

$$\kappa \equiv \frac{\omega_{\rm phys}}{m} = \frac{k^{\rm em}_{\rm peak}}{m\, a_{\rm em}} = \frac{k^{(res)}_{\rm peak}}{m\, a_{\rm res}} \times \frac{k^{(em)}_{\rm peak}/a_{\rm em}}{k^{(res)}_{\rm peak}/a_{\rm res}}$$

momentum flow due to turbulence

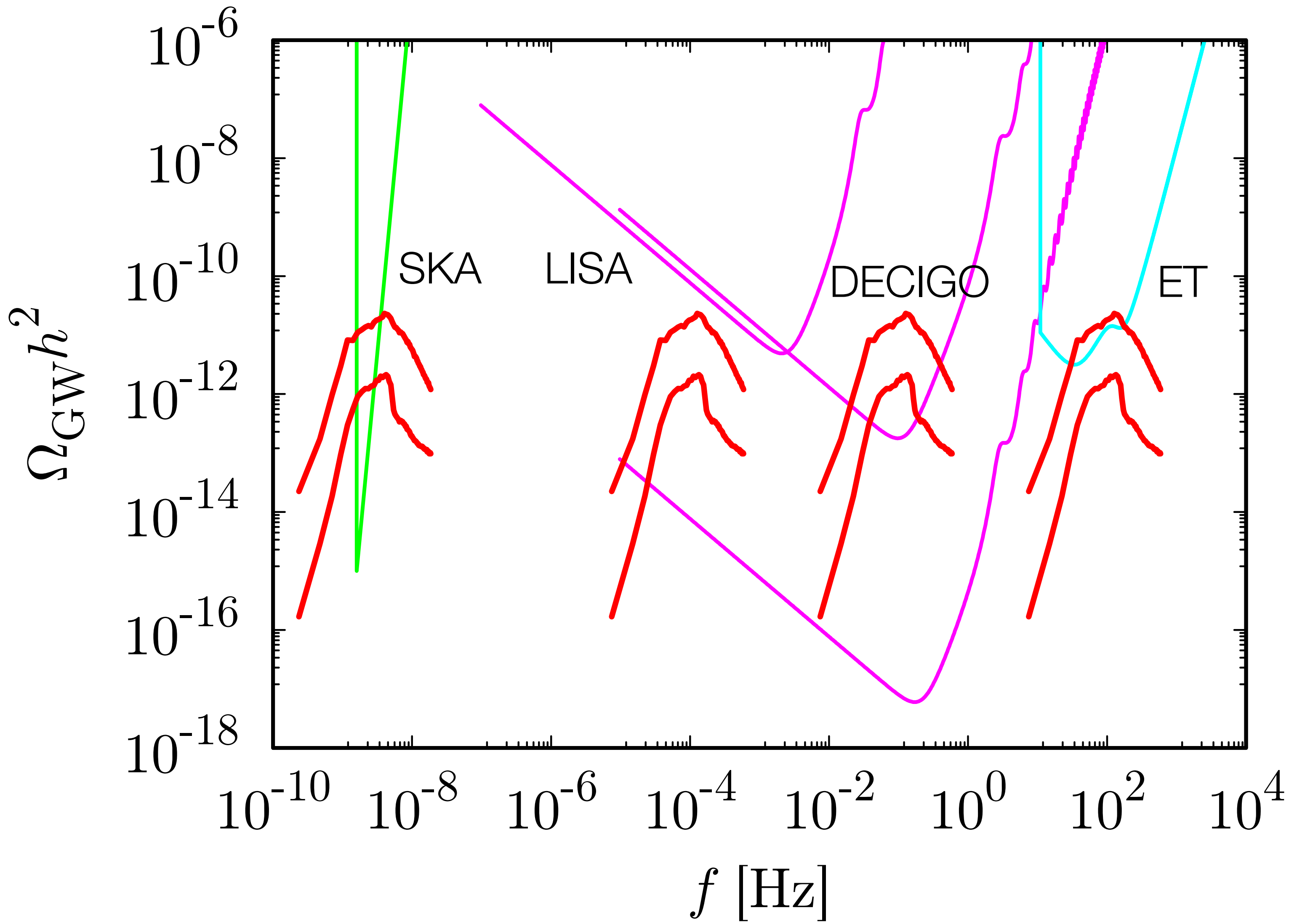
e.g. GWs emitted during radiation domination

$$\nu_0 = \frac{\kappa m}{2\pi} \times \left(\frac{\rho_{\rm r,0}}{\rho_{\rm r,\,em}}\right)^{1/4} \simeq 0.78{\rm nHz}\,\kappa\left(\frac{m}{H_{\rm em}}\right)^{1/2}\left(\frac{m}{10^{-12}{\rm eV}}\right)^{1/2}$$

$\alpha=0$

GW forest

Kitajima, Soda & Y.U. (18)



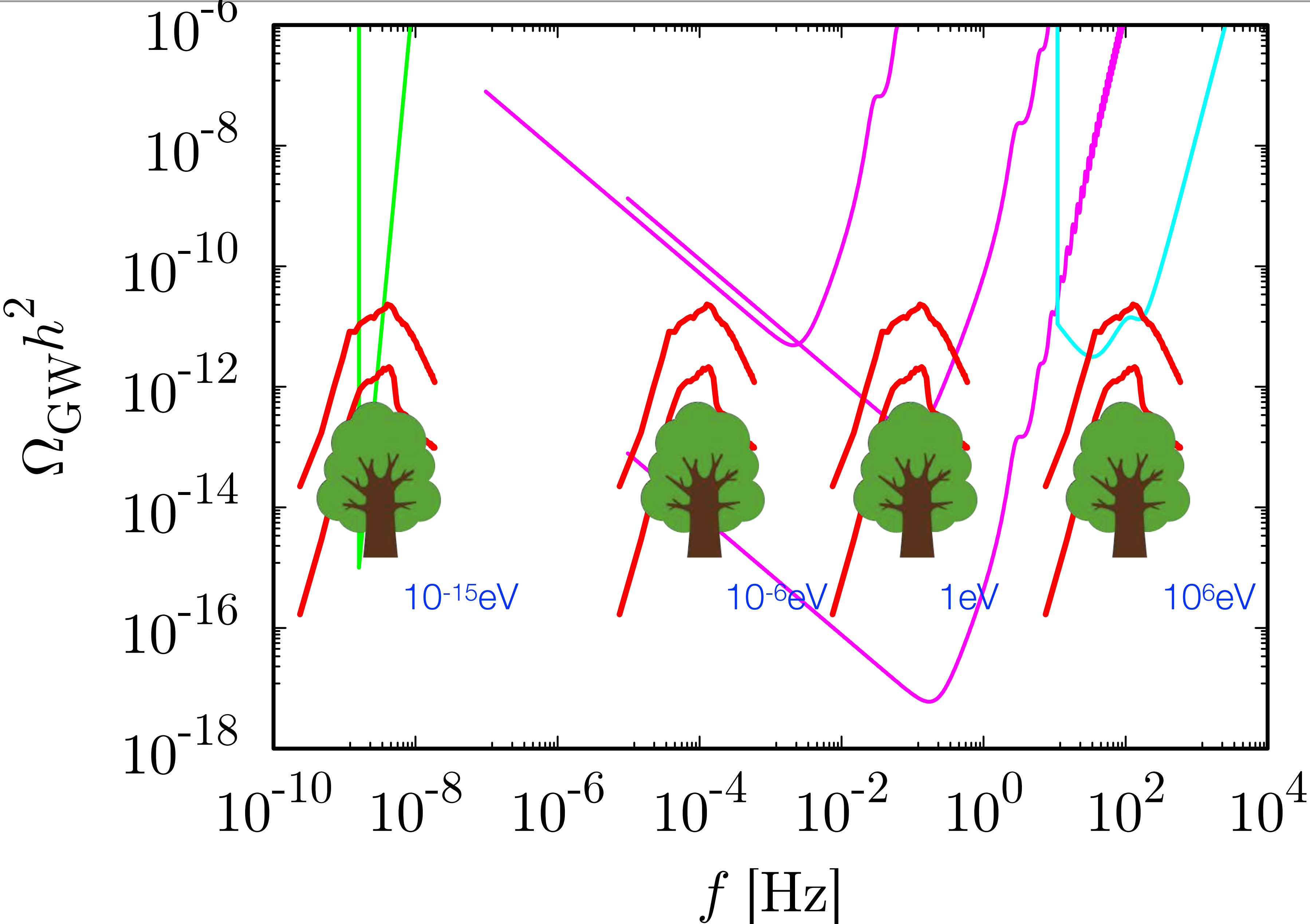
Axions from string theory $f \sim 10^{16}$ GeV

e.g., Svrcek & Witten (06)

α=0

GW forest

Kitajima, Soda & Y.U. (18)



$\alpha=0$

GWs from axion DM

Kitajima, Soda, Y.U. (18)

freq. of GW ν_0 ⟶ mass m

+

abundance of axion ⟶ decay const. f x mass m

<u>Crude Order estimation</u>

using $\varphi(t, x) \sim f\,(a_{\rm osc}/a)^{3/2}$

Δ : Sym. suppression (< 1)

$$\Omega_{\rm GW} h^2 \simeq 0.8 \times 10^{-18} \kappa^4 \Delta^2 \left(\frac{\rm nHz}{\nu_0}\right)^2 (\Omega_\phi h^2)^2$$

for κ=10 $\Omega_{\rm GW} h^2 \simeq 10^{-16}$ at $\nu_0 = $nHz

or lower frequency btwn CMB & PTAs?

Prospects on polarized GW forest

$\alpha \neq 0$

Kitajima, Soda & Y.U. (in prep.)

What about $\alpha \neq 0$?

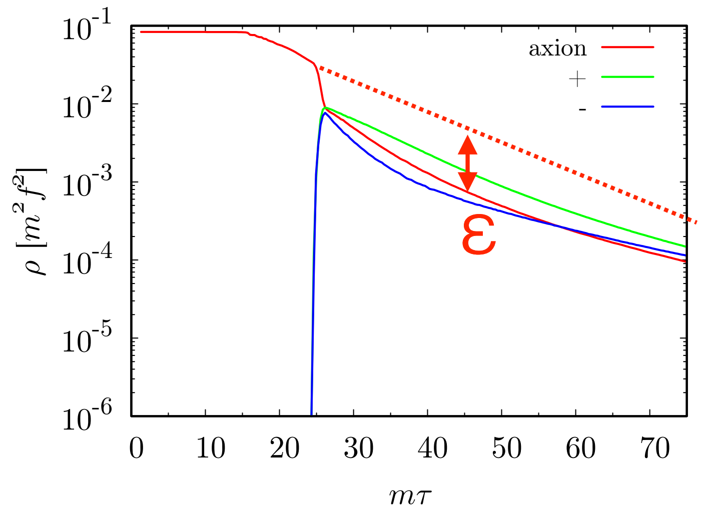
- GW circularly polarized

  see also Adshead +(18)

- More prominent GW emission

  - Larger Δ (Less symmetric)

  - Weaker abundance restriction



GW from axion DM

$$\Omega_{\rm GW} h^2 \simeq 0.8 \times 10^{-18} \kappa^4 \Delta^2 \left(\frac{\rm nHz}{\nu_0}\right)^2 (\Omega_\phi h^2)^2 \quad \times \frac{1}{\varepsilon^2}$$

Summary

New window of axions in plateau

Theory side

Resonant instabilities lead to copious emission of GWs

Keys: Delayed oscillation

Phenomenology side

Predicts bGWs at various frequencies, multi-band obs.?

- Peaky spectrum
- Circularly polarized GWs

GWs from axion DM: sweet spot is btwn CMB till PTAs.