

*CP* violation in  $\eta$  muonic decays  
based on JHEP 1901 (2019) 031

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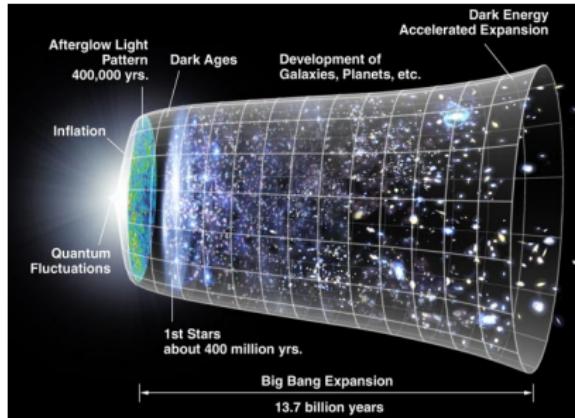


BIST  
Barcelona Institute of  
Science and Technology



## Motivations (I)

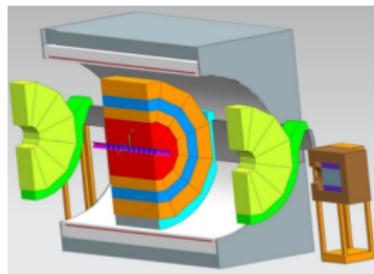
- Not enough CP-violation for matter-antimatter asymmetry
- Expected additional (non-negligible) CP-violation sources
- The  $\eta$  meson is a flavor singlet: CP eigenstate and SM “background-free”



## Motivations (II)

Provide MCs and estimates for proposed experiments sensitivities

- REDTOP: an  $\eta$  factory ( $10^{13}$   $\eta$  mesons)
- Proposed @FLab (queuing), then @CERN
- Aimed to measure CP-violation BSM
- Measures the polarization of muons



Provide MC amplitudes and asses existing bounds (nEDMs)

## — Outline —

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1. New physics: SMEFT and hadronization details
2. CP asymmetries and sensitivities
  - 2.1  $\eta \rightarrow \mu^+ \mu^-$
  - 2.2  $\eta \rightarrow \gamma \mu^+ \mu^-$
  - 2.3  $\eta \rightarrow \mu^+ \mu^- e^+ e^-$
3. nEDM bounds
4.  $D_s \rightarrow \ell \nu$  bounds
5. Conclusions and outlook

## Section 1

New physics: SMEFT and hadronization details

## SMEFT: CP-violating operators of interest (I)

Assume heavy physics: SMEFT + low energies:  $\gamma, G, \ell$  (no,  $W, \varphi$ )

$X^3$		$\varphi^6$ and $\varphi^4 D^2$		$\psi^2 \varphi^3$	
$Q_G$	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_\varphi$	$(\varphi^\dagger \varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger \varphi)(\bar{l}_p e_r \varphi)$
$Q_{\bar{G}}$	$f^{ABC} \tilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi \square}$	$(\varphi^\dagger \varphi) \square (\varphi^\dagger \varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p u_r \tilde{\varphi})$
$Q_W$	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$(\varphi^\dagger D^\mu \varphi)^*$ $(\varphi^\dagger D_\mu \varphi)$	$Q_{d\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p d_r \varphi)$
$Q_{\bar{W}}$	$\varepsilon^{IJK} \tilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2 \varphi^2$		$\psi^2 X \varphi$		$\psi^2 \varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger \varphi G_{\mu\nu}^A G^{A\mu\nu}$	$Q_{eW}$	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{l}_p \gamma^\mu l_r)$
$Q_{\varphi \bar{G}}$	$\varphi^\dagger \varphi \tilde{G}_{\mu\nu}^A G^{A\mu\nu}$	$Q_{eB}$	$(\bar{l}_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger \varphi W_{\mu\nu}^I W^{I\mu\nu}$	$Q_{uG}$	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{\varphi} G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r)$
$Q_{\varphi \bar{W}}$	$\varphi^\dagger \varphi \tilde{W}_{\mu\nu}^I W^{I\mu\nu}$	$Q_{uW}$	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{\varphi} W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{q}_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger \varphi B_{\mu\nu} B^{\mu\nu}$	$Q_{uB}$	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tilde{\varphi} B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{q}_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi \bar{B}}$	$\varphi^\dagger \varphi \tilde{B}_{\mu\nu} B^{\mu\nu}$	$Q_{dG}$	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) \varphi G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{u}_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi W_{\mu\nu}^I B^{\mu\nu}$	$Q_{dW}$	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{d}_p \gamma^\mu d_r)$
$Q_{\varphi \bar{W}B}$	$\varphi^\dagger \tau^I \varphi \tilde{W}_{\mu\nu}^I B^{\mu\nu}$	$Q_{dB}$	$(\bar{q}_p \sigma^{\mu\nu} d_r) \varphi B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\tilde{\varphi}^\dagger D_\mu \varphi)(\bar{u}_p \gamma^\mu d_r)$

CP-V@hadronic level

CP-V@leptonic-quark level

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## SMEFT: CP-violating operators of interest (I)

Assume heavy physics: SMEFT + low energies:  $\gamma, G, \ell$  (no,  $W, \varphi$ )

$X^3$		$\varphi^6$ and $\varphi^4 D^2$		$\psi^2 \varphi^3$	
$Q_G$	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_\varphi$	$(\varphi^\dagger \varphi)^3$	$Q_{u\varphi}$	electron EDM
$Q_{\bar{G}}$	$f^{ABC} \tilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi \square}$	$(\varphi^\dagger \varphi) \square (\varphi^\dagger \varphi)$	$Q_{d\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p u_r \tilde{\varphi})$
$Q_W$	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$(\varphi^\dagger D^\mu \varphi)^*$ $(\varphi^\dagger D_\mu \varphi)$	$Q_{d\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p d_r \varphi)$
$Q_{\widetilde{W}}$	$\varepsilon^{IJK} \tilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2 \varphi^2$		$\psi^2 X \varphi$		$\psi^2 \varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger \varphi G_{\mu\nu}^A G^{A\mu\nu}$	$Q_{eW}$ $(l_p \sigma^{\mu\nu} e_r) l_\nu W_{\mu\nu}^I$	muon EDM	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{l}_p \gamma^\mu l_r)$
$Q_{\varphi \bar{G}}$	$\varphi^\dagger \varphi \tilde{G}_{\mu\nu}^A G^{A\mu\nu}$	$Q_{eB}$ $(l_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$		$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger \varphi W_{\mu\nu}^I W^{I\mu\nu}$	$Q_{uG}$ $(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{\varphi} G_{\mu\nu}^A$		$Q_{\varphi e}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r)$
$Q_{\varphi \widetilde{W}}$	$\varphi^\dagger \varphi \tilde{W}_{\mu\nu}^I W^{I\mu\nu}$	$Q_{uW}$ $(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{\varphi} W_{\mu\nu}^I$		$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{q}_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger \varphi B_{\mu\nu} B^{\mu\nu}$	$Q_{uB}$ $(\bar{q}_p \sigma^{\mu\nu} u_r) \tilde{\varphi} B_{\mu\nu}$		$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi)(\bar{q}_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi \bar{B}}$	$\varphi^\dagger \varphi \tilde{B}_{\mu\nu} B^{\mu\nu}$	$Q_{dG}$ $(\bar{q}_p \sigma^{\mu\nu} T^A d_r) \varphi G_{\mu\nu}^A$		$Q_{\varphi u}$	$(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi)(\bar{u}_p \gamma^\mu u_r)$
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$Q_{\varphi \widetilde{W}B}$	$\varphi^\dagger \tau^I \varphi \tilde{W}_{\mu\nu}^I B^{\mu\nu}$	$Q_{dB}$ $(\bar{q}_p \sigma^{\mu\nu} d_r) \varphi B_{\mu\nu}$		$Q_{\varphi ud}$	$i(\tilde{\varphi}^\dagger D_\mu \varphi)(\bar{u}_p \gamma^\mu d_r)$

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Assume heavy physics: SMEFT + low energies:  $\gamma, G, \ell$  (no,  $W, \varphi$ )

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
$Q_{ll}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$	$Q_{ee}$	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$	$Q_{le}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$	$Q_{uu}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$	$Q_{lu}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	$Q_{dd}$	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{ld}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$	$Q_{eu}$	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$	$Q_{qe}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	$Q_{ed}$	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{d}_s \gamma^\mu T^A d_t)$
$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating			
$Q_{ledq}$	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	$Q_{duq}$	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} \left[ (d_p^\alpha)^T C u_r^\beta \right] \left[ (q_s^j)^T C l_t^k \right]$		
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r) \varepsilon_{jk} (\bar{q}_s^k d_t)$	$Q_{qqu}$	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} \left[ (q_p^{\alpha j})^T C q_r^{\beta k} \right] \left[ (u_s^\gamma)^T C e_t \right]$		
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r) \varepsilon_{jk} (\bar{q}_s^k T^A d_t)$	$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} \varepsilon_{mn} \left[ (q_p^{\alpha j})^T C q_r^{\beta k} \right] \left[ (q_s^m)^T C l_t^n \right]$		
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r) \varepsilon_{jk} (q_s^k u_t)$	$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma} (\tau^I \varepsilon)_{jk} (\tau^I \varepsilon)_{mn} \left[ (q_p^{\alpha j})^T C q_r^{\beta k} \right] \left[ (q_s^m)^T C l_t^n \right]$		
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r) \varepsilon_{jk} (q_s^k \sigma^{\mu\nu} u_t)$	$Q_{duu}$	$\varepsilon^{\alpha\beta\gamma} \left[ (d_p^\alpha)^T C u_r^\beta \right] \left[ (u_s^\gamma)^T C e_t \right]$		

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## SMEFT: Hadronization details

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- Hadronic CP-violating ( $\mathcal{CP}_H$ ) terms: additional CP-violating  $\eta \rightarrow \gamma^* \gamma^*$  TFFs

$$\begin{aligned} i\mathcal{M}^{\mu\nu} = ie^2 & \left( \epsilon^{\mu\nu\rho\sigma} q_{1\rho} q_{2\sigma} F_{\eta\gamma^*\gamma^*}(q_1^2, q_2^2) + [g^{\mu\nu}(q_1 \cdot q_2) - q_2^\mu q_1^\nu] F_{\eta\gamma^*\gamma^*}^{\mathcal{CP}1}(q_1^2, q_2^2) \right. \\ & \left. + [g^{\mu\nu} q_1^2 q_2^2 - q_1^2 q_2^\mu q_2^\nu - q_2^2 q_1^\mu q_1^\nu + (q_1 \cdot q_2) q_1^\mu q_2^\nu] F_{\eta\gamma^*\gamma^*}^{\mathcal{CP}2}(q_1^2, q_2^2) \right), \end{aligned}$$

with  $F_{\eta\gamma^*\gamma^*}(q_1^2, q_2^2)$  known and  $F_{\eta\gamma^*\gamma^*}^{\mathcal{CP}1,2}(q_1^2, q_2^2) \simeq \epsilon_{1,2} F_{\eta\gamma^*\gamma^*}(q_1^2, q_2^2)$  taken

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with  $F_{\eta\gamma^*\gamma^*}(q_1^2, q_2^2)$  known and  $F_{\eta\gamma^*\gamma^*}^{\mathcal{CP}1,2}(q_1^2, q_2^2) \simeq \epsilon_{1,2} F_{\eta\gamma^*\gamma^*}(q_1^2, q_2^2)$  taken

- Lepton-quark CP-violating ( $\mathcal{CP}_{HL}$ ) terms ( $\mathcal{O}_{lequ}^{(1)}, \mathcal{O}_{ledq}; [\mathcal{O}_{lequ}^{(3)}]$ )

$$\mathcal{O}_{lequ}^{(1)} = \frac{c_{\ell equ}^{(1)prst}}{v^2} (\bar{\ell}_p^j e_r) \epsilon_{jk} (\bar{q}_s^k u_t) \rightarrow -\frac{|\text{Im } c_{\ell equ}^{(1)prst}|}{2v^2} [(\bar{e} i \gamma^5 e)(\bar{u} u) + (\bar{e} e)(\bar{u} i \gamma^5 u)] \\ \mathcal{O}_{ledq} = \frac{c_{\ell edq}^{prst}}{v^2} (\bar{\ell}_p^j e_r) (\bar{d}_s q_t^i) \rightarrow \frac{|\text{Im } c_{\ell edq}^{prst}|}{2v^2} [(\bar{e} i \gamma^5 e)(\bar{d} d) - (\bar{e} e)(\bar{d} i \gamma^5 d)]$$

## SMEFT: Hadronization details

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- Hadronic CP-violating ( $\mathcal{CP}_H$ ) terms: additional CP-violating  $\eta \rightarrow \gamma^* \gamma^*$  TFFs

$$i\mathcal{M}^{\mu\nu} = ie^2 \left( \epsilon^{\mu\nu\rho\sigma} q_{1\rho} q_{2\sigma} F_{\eta\gamma^*\gamma^*}(q_1^2, q_2^2) + [g^{\mu\nu}(q_1 \cdot q_2) - q_2^\mu q_1^\nu] F_{\eta\gamma^*\gamma^*}^{\mathcal{CP}1}(q_1^2, q_2^2) \right. \\ \left. + [g^{\mu\nu} q_1^2 q_2^2 - q_1^2 q_2^\mu q_2^\nu - q_2^2 q_1^\mu q_1^\nu + (q_1 \cdot q_2) q_1^\mu q_2^\nu] F_{\eta\gamma^*\gamma^*}^{\mathcal{CP}2}(q_1^2, q_2^2) \right),$$

with  $F_{\eta\gamma^*\gamma^*}(q_1^2, q_2^2)$  known and  $F_{\eta\gamma^*\gamma^*}^{\mathcal{CP}1,2}(q_1^2, q_2^2) \simeq \epsilon_{1,2} F_{\eta\gamma^*\gamma^*}(q_1^2, q_2^2)$  taken

- Lepton-quark CP-violating ( $\mathcal{CP}_{HL}$ ) terms ( $\mathcal{O}_{lequ}^{(1)}, \mathcal{O}_{ledq}; [\mathcal{O}_{lequ}^{(3)}]$ )

$$\mathcal{L} = -\mathcal{C}(\eta \bar{e}^\rho e^\tau); \quad \mathcal{C} \equiv \frac{\text{Im } c_{\mathcal{O}}}{2\nu^2} \left[ \langle 0 | \bar{q}^a i\gamma^5 q^a | \eta \rangle \rightarrow \frac{F_\eta^q m_\pi^2 \text{tr}(a\lambda^q) + F_\eta^s (2m_K^2 - m_\pi^2) \text{tr}(a\lambda^s)}{2m_a} \right] \sim 10^{-6}$$

## SMEFT: Hadronization details

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- Hadronic CP-violating ( $\mathcal{CP}_H$ ) terms: additional CP-violating  $\eta \rightarrow \gamma^* \gamma^*$  TFFs

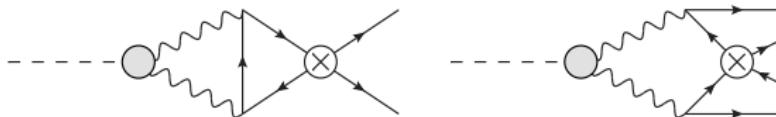
$$i\mathcal{M}^{\mu\nu} = ie^2 \left( \epsilon^{\mu\nu\rho\sigma} q_{1\rho} q_{2\sigma} F_{\eta\gamma^*\gamma^*}(q_1^2, q_2^2) + [g^{\mu\nu}(q_1 \cdot q_2) - q_2^\mu q_1^\nu] F_{\eta\gamma^*\gamma^*}^{\mathcal{CP}1}(q_1^2, q_2^2) \right. \\ \left. + [g^{\mu\nu} q_1^2 q_2^2 - q_1^2 q_2^\mu q_2^\nu - q_2^2 q_1^\mu q_1^\nu + (q_1 \cdot q_2) q_1^\mu q_2^\nu] F_{\eta\gamma^*\gamma^*}^{\mathcal{CP}2}(q_1^2, q_2^2) \right),$$

with  $F_{\eta\gamma^*\gamma^*}(q_1^2, q_2^2)$  known and  $F_{\eta\gamma^*\gamma^*}^{\mathcal{CP}1,2}(q_1^2, q_2^2) \simeq \epsilon_{1,2} F_{\eta\gamma^*\gamma^*}(q_1^2, q_2^2)$  taken

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- electron EDMs set strong bounds for  $\mathcal{O}_{\ell e} \rightarrow$  assumed irrelevant



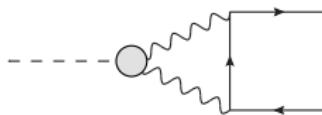
## Section 2

CP asymmetries and sensitivities

$$\eta \rightarrow \mu^+ \mu^-$$


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- In the SM it is  $\alpha^2$ , helicity-flip, and loop suppressed (interesting!)



- The most general matrix element (CP-even and odd, resp.)

$$i\mathcal{M} = i [g_P(\bar{u}i\gamma^5 v) + g_S(\bar{u}v)],$$

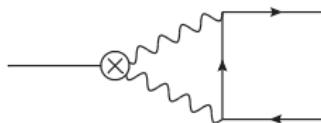
- No asymmetry without considering polarizations (only for muons)

$$|\mathcal{M}(\lambda \mathbf{n}, \bar{\lambda} \bar{\mathbf{n}})|^2 = \frac{m_\eta^2}{2} \left[ |g_P|^2 (1 - \lambda \bar{\lambda} [\mathbf{n} \cdot \bar{\mathbf{n}}]) + |g_S|^2 \beta_\mu^2 (1 - \lambda \bar{\lambda} [\mathbf{n}_z \bar{\mathbf{n}}_z - \mathbf{n}_T \cdot \bar{\mathbf{n}}_T]) \right. \\ \left. + 2 [\lambda \bar{\lambda} \operatorname{Re}(g_P g_S^*) (\bar{\mathbf{n}} \times \mathbf{n}) \cdot \beta_\mu + \operatorname{Im}(g_P g_S^*) \beta_\mu \cdot (\lambda \mathbf{n} - \bar{\lambda} \bar{\mathbf{n}})] \right],$$

- Needs to compute  $g_S$ :

$$CP_H \sim \epsilon_i \times \text{SM}$$

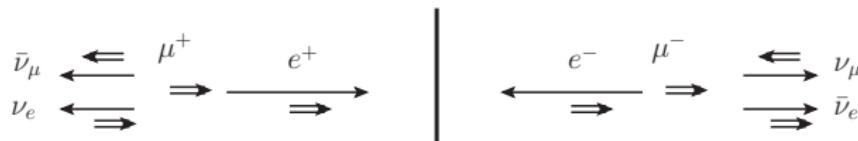
$$CP_{HL} = -\mathcal{C} \text{ from } \mathcal{L} = -\mathcal{C}(\eta \bar{\mu} \mu)$$



—  $\eta \rightarrow \mu^+ \mu^-$  —

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- Enough for MC (Geant4), but realistic estimate: how to measure polarization?



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$$\text{BR}(\mu^\pm, \lambda \mathbf{n}) = \frac{d\Omega}{4\pi} n(x) [1 \mp \lambda b(x, x_0) \boldsymbol{\beta} \cdot \mathbf{n}] dx,$$

with  $x = 2E_e/m_\mu \in (0, 1)$ ,  $n(x) = 2x^2(3 - 2x)$  and  $b(x) = (1 - 2x)/(3 - 2x)$ .

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- Enough for MC (Geant4), but realistic estimate: how to measure polarization?

$$\text{BR}(\mu^\pm, \lambda \mathbf{n}) = \frac{d\Omega}{4\pi} n(x) [1 \mp \lambda b(x, x_0) \boldsymbol{\beta} \cdot \mathbf{n}] dx,$$

with  $x = 2E_e/m_\mu \in (0, 1)$ ,  $n(x) = 2x^2(3 - 2x)$  and  $b(x) = (1 - 2x)/(3 - 2x)$ .

- The resulting decay width

$$\begin{aligned} |\mathcal{M}(\lambda \mathbf{n}, \bar{\lambda} \bar{\mathbf{n}})|^2 &= \frac{m_\eta^2}{2} \left[ |g_P|^2 (1 - \lambda \bar{\lambda} [\mathbf{n} \cdot \bar{\mathbf{n}}]) + |g_S|^2 \beta_\mu^2 (1 - \lambda \bar{\lambda} [n_z \bar{n}_z - \mathbf{n}_T \cdot \bar{\mathbf{n}}_T]) \right. \\ &\quad \left. + 2 [\lambda \bar{\lambda} \operatorname{Re}(g_P g_S^*) (\bar{\mathbf{n}} \times \mathbf{n}) \cdot \boldsymbol{\beta}_\mu + \operatorname{Im}(g_P g_S^*) \boldsymbol{\beta}_\mu \cdot (\lambda \mathbf{n} - \bar{\lambda} \bar{\mathbf{n}})] \right], \end{aligned}$$

—  $\eta \rightarrow \mu^+ \mu^-$  —————

- Enough for MC (Geant4), but realistic estimate: how to measure polarization?

$$\text{BR}(\mu^\pm, \lambda \mathbf{n}) = \frac{d\Omega}{4\pi} n(x) [1 \mp \lambda b(x, x_0) \boldsymbol{\beta} \cdot \mathbf{n}] dx,$$

with  $x = 2E_e/m_\mu \in (0, 1)$ ,  $n(x) = 2x^2(3 - 2x)$  and  $b(x) = (1 - 2x)/(3 - 2x)$ .

- The resulting decay width

$$\begin{aligned} \frac{d\Gamma}{\Gamma_{\gamma\gamma}} &= 2\beta_\mu \left( \frac{\alpha m_\mu}{\pi m_\eta} \right)^2 \left[ |\mathcal{A}|^2 (1 + b\bar{b} \{ \boldsymbol{\beta} \cdot \bar{\boldsymbol{\beta}} \}) + 2\beta_\mu \tilde{g}_S \{ b\bar{b}[(\boldsymbol{\beta} \times \bar{\boldsymbol{\beta}}) \cdot \hat{z}] \text{Re } \mathcal{A} \right. \\ &\quad \left. - (b\beta_z + \bar{b}\bar{\beta}_z) \text{Im } \mathcal{A} \} + \tilde{g}_S^2 (1 + b\bar{b} \{ \beta_z \bar{\beta}_z - \boldsymbol{\beta}_T \cdot \bar{\boldsymbol{\beta}}_T \}) \right] d_{e^\pm}, \end{aligned}$$

—  $\eta \rightarrow \mu^+ \mu^-$  —

---

- Enough for MC (Geant4), but realistic estimate: how to measure polarization?

$$\text{BR}(\mu^\pm, \lambda \mathbf{n}) = \frac{d\Omega}{4\pi} n(x) [1 \mp \lambda b(x, x_0) \boldsymbol{\beta} \cdot \mathbf{n}] dx,$$

with  $x = 2E_e/m_\mu \in (0, 1)$ ,  $n(x) = 2x^2(3 - 2x)$  and  $b(x) = (1 - 2x)/(3 - 2x)$ .

- The resulting decay width

$$\begin{aligned} \frac{d\Gamma}{\Gamma_{\gamma\gamma}} = 2\beta_\mu \left( \frac{\alpha m_\mu}{\pi m_\eta} \right)^2 & \left[ |\mathcal{A}|^2 (1 + b\bar{b}\{\boldsymbol{\beta} \cdot \bar{\boldsymbol{\beta}}\}) + 2\beta_\mu \tilde{g}_S \{b\bar{b}[(\boldsymbol{\beta} \times \bar{\boldsymbol{\beta}}) \cdot \hat{z}] \text{Re } \mathcal{A} \right. \\ & \left. - (b\beta_z + \bar{b}\bar{\beta}_z) \text{Im } \mathcal{A} \} + \tilde{g}_S^2 (1 + b\bar{b}\{\beta_z \bar{\beta}_z - \boldsymbol{\beta}_T \cdot \bar{\boldsymbol{\beta}}_T\}) \right] d_{e^\pm}, \end{aligned}$$

- How to measure polarization? Asymmetries!

$$A_L \equiv \frac{N(c_\theta > 0) - N(c_\theta < 0)}{N(\text{all})} = \bar{A}_L = \frac{\beta_\mu}{3} \frac{\text{Im } \mathcal{A} \tilde{g}_S}{|\mathcal{A}|^2}, \quad \tilde{g}_S = \frac{-gs}{2m_\mu \alpha^2 F_{\eta\gamma\gamma}}$$

$$A_T \equiv \frac{N(s_{\phi-\bar{\phi}} > 0) - N(s_{\phi-\bar{\phi}} < 0)}{N(\text{all})} = \frac{\pi \beta_\mu}{36} \frac{\text{Re } \mathcal{A} \tilde{g}_S}{|\mathcal{A}|^2},$$

$\eta \rightarrow \mu^+ \mu^-$ 

- Enough for MC (Geant4), but realistic estimate: how to measure polarization?

$$\text{BR}(\mu^\pm, \lambda \mathbf{n}) = \frac{d\Omega}{4\pi} n(x) [1 \mp \lambda b(x, x_0) \boldsymbol{\beta} \cdot \mathbf{n}] dx,$$

with  $x = 2E_e/m_\mu \in (0, 1)$ ,  $n(x) = 2x^2(3 - 2x)$  and  $b(x) = (1 - 2x)/(3 - 2x)$ .

- The resulting decay width

$$\begin{aligned} \frac{d\Gamma}{\Gamma_{\gamma\gamma}} = 2\beta_\mu \left( \frac{\alpha m_\mu}{\pi m_\eta} \right)^2 & \left[ |\mathcal{A}|^2 (1 + b\bar{b}\{\boldsymbol{\beta} \cdot \bar{\boldsymbol{\beta}}\}) + 2\beta_\mu \tilde{g}_S \{b\bar{b}[(\boldsymbol{\beta} \times \bar{\boldsymbol{\beta}}) \cdot \hat{z}] \text{Re } \mathcal{A} \right. \\ & \left. - (b\beta_z + \bar{b}\bar{\beta}_z) \text{Im } \mathcal{A} \} + \tilde{g}_S^2 (1 + b\bar{b}\{\beta_z \bar{\beta}_z - \boldsymbol{\beta}_T \cdot \bar{\boldsymbol{\beta}}_T\}) \right] d_{e^\pm}, \end{aligned}$$

- How to measure polarization? Asymmetries!

$$\begin{aligned} A_L^H &= 0.11\epsilon_1 - 0.04\epsilon_2, & A_L^L &= -|\text{Im}(2.7(c_{\ell equ}^{(1)2211} + c_{\ell edq}^{2211}) - 4.1c_{\ell edq}^{2222}) \times 10^{-2}, \\ A_T^H &= -0.07\epsilon_1 - 0.002\epsilon_2, & A_T^L &= -|\text{Im}(1.6(c_{\ell equ}^{(1)2211} + c_{\ell edq}^{2222}) - 2.5c_{\ell edq}^{2222}) \times 10^{-3}, \end{aligned}$$

$\eta \rightarrow \mu^+ \mu^-$ 

- Enough for MC (Geant4), but realistic estimate: how to measure polarization?

$$\text{BR}(\mu^\pm, \lambda \mathbf{n}) = \frac{d\Omega}{4\pi} n(x) [1 \mp \lambda b(x, x_0) \boldsymbol{\beta} \cdot \mathbf{n}] dx,$$

with  $x = 2E_e/m_\mu \in (0, 1)$ ,  $n(x) = 2x^2(3 - 2x)$  and  $b(x) = (1 - 2x)/(3 - 2x)$ .

- The resulting decay width

$$\begin{aligned} \frac{d\Gamma}{\Gamma_{\gamma\gamma}} = 2\beta_\mu \left( \frac{\alpha m_\mu}{\pi m_\eta} \right)^2 & \left[ |\mathcal{A}|^2 (1 + b\bar{b}\{\boldsymbol{\beta} \cdot \bar{\boldsymbol{\beta}}\}) + 2\beta_\mu \tilde{g}_S \{b\bar{b}[(\boldsymbol{\beta} \times \bar{\boldsymbol{\beta}}) \cdot \hat{z}]\} \text{Re } \mathcal{A} \right. \\ & \left. - (b\beta_z + \bar{b}\bar{\beta}_z) \text{Im } \mathcal{A} \} + \tilde{g}_S^2 (1 + b\bar{b}\{\beta_z \bar{\beta}_z - \boldsymbol{\beta}_T \cdot \bar{\boldsymbol{\beta}}_T\}) \right] d_{e^\pm}, \end{aligned}$$

- How to measure polarization? Asymmetries!

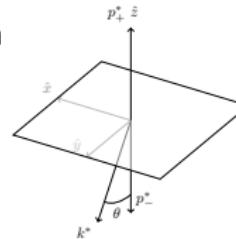
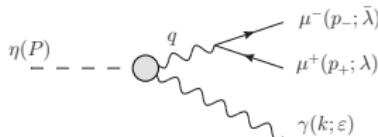
$$\begin{aligned} A_L^H &= 0.11\epsilon_1 - 0.04\epsilon_2, & A_L^L &= -|\text{Im}(2.7(c_{\ell equ}^{(1)2211} + c_{\ell edq}^{2211}) - 4.1c_{\ell edq}^{2222})| \times 10^{-2}, \\ A_T^H &= -0.07\epsilon_1 - 0.002\epsilon_2, & A_T^L &= -|\text{Im}(1.6(c_{\ell equ}^{(1)2211} + c_{\ell edq}^{2222}) - 2.5c_{\ell edq}^{2222})| \times 10^{-3}, \end{aligned}$$

- RETOP:  $10^7 \eta \rightarrow \mu^+ \mu^-$ ; Noise:  $3 \times 10^{-4}$ ;  $\epsilon_{1(2)} \sim 10^{-3(2)}$  and  $c_{\mathcal{O}}^{22st} \sim 10^{-2}$

$$\eta \rightarrow \mu^+ \mu^- \gamma$$


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- For simplicity SM@LO and interference term



- Along the same steps one finds

$$\frac{d\Gamma_{CP_H}}{\Gamma_{\gamma\gamma}} = \frac{\alpha}{\pi} \frac{\text{Im } \tilde{F}_{\eta\gamma\gamma^*}(s) \tilde{F}_{\eta\gamma\gamma^*}^{CP1*}(s)}{s} (1 - x_\mu)^3 ds dy d_{e^\pm} \left[ \sqrt{1 - \beta_\mu^2} \sin \theta (b\beta_y - \bar{b}\bar{\beta}_y) - \cos \theta (b\beta_z - \bar{b}\bar{\beta}_z) \right],$$

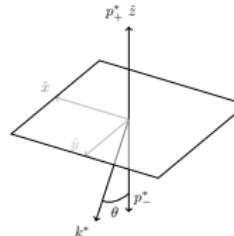
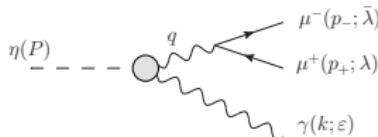
$$\frac{d\Gamma_{CP_{HL}}}{\Gamma_{\gamma\gamma}} = \frac{\alpha}{\pi} \frac{(1 - x_\mu)}{s(1 - y^2)} \frac{2C ds dy d_{e^\pm}}{e^2 m_\eta F_{\eta\gamma\gamma}} \left[ \tilde{\alpha}_R \text{Re } \tilde{F}_{\eta\gamma\gamma^*}(s) + \tilde{\alpha}_I \text{Im } \tilde{F}_{\eta\gamma\gamma^*}(s) \right],$$

- Richer final state  $\rightarrow$  additional asymmetries

$$A_{L\gamma} \equiv \frac{N(s_\phi > 0) - N(s_\phi < 0)}{N(\text{all})}, \quad A_{TL} \equiv \frac{N(c_\phi c_{\bar{\theta}} > 0) - N(c_\phi c_{\bar{\theta}} < 0)}{N(\text{all})}.$$

$$\eta \rightarrow \mu^+ \mu^- \gamma$$

- For simplicity SM@LO and interference term



- Along the same steps one finds

$$\frac{d\Gamma_{\mathcal{CP}_H}}{\Gamma_{\gamma\gamma}} = \frac{\alpha}{\pi} \frac{\text{Im } \tilde{F}_{\eta\gamma\gamma^*}(s) \tilde{F}_{\eta\gamma\gamma^*}^{CP1*}(s)}{s} (1 - x_\mu)^3 ds dy d\theta \pm \left[ \sqrt{1 - \beta_\mu^2} \sin \theta (b\beta_y - \bar{b}\bar{\beta}_y) - \cos \theta (b\beta_z - \bar{b}\bar{\beta}_z) \right],$$

$$\frac{d\Gamma_{\mathcal{CP}_{HL}}}{\Gamma_{\gamma\gamma}} = \frac{\alpha}{\pi} \frac{(1 - x_\mu)}{s(1 - y^2)} \frac{2C ds dy d\theta \pm}{e^2 m_\eta F_{\eta\gamma\gamma}} \left[ \tilde{\alpha}_R \text{Re } \tilde{F}_{\eta\gamma\gamma^*}(s) + \tilde{\alpha}_I \text{Im } \tilde{F}_{\eta\gamma\gamma^*}(s) \right],$$

- Richer final state  $\rightarrow$  additional asymmetries

$$A_L^H = 0 \quad A_L^{HL} = -4 \text{Im} (1.1(c_{\ell equ}^{(1)221} + c_{\ell edq}^{221}) - 1.7 c_{\ell edq}^{2222}) \times 10^{-7},$$

$$A_{L\gamma}^H = -0.002 \epsilon_1 \quad A_{L\gamma}^{HL} = 5 \text{Im} (1.1(c_{\ell equ}^{(1)2211} + c_{\ell edq}^{2211}) - 1.7 c_{\ell edq}^{2222}) \times 10^{-6},$$

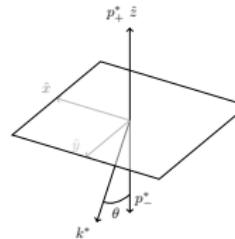
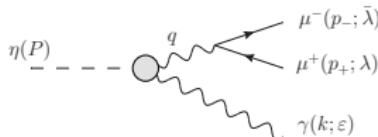
$$A_{TL}^H = 0 \quad A_{TL}^{HL} = 2 \text{Im} (1.1(c_{\ell equ}^{(1)221} + c_{\ell edq}^{221}) - 1.7 c_{\ell edq}^{2222}) \times 10^{-5},$$

$$A_T^H = 0 \quad A_T^{HL} = -5 \text{Im} (1.1(c_{\ell equ}^{(1)221} + c_{\ell edq}^{221}) - 1.7 c_{\ell edq}^{2222}) \times 10^{-6}.$$

$$\eta \rightarrow \mu^+ \mu^- \gamma$$


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- For simplicity SM@LO and interference term



- Along the same steps one finds

$$\frac{d\Gamma_{CP_H}}{\Gamma_{\gamma\gamma}} = \frac{\alpha}{\pi} \frac{\text{Im } \tilde{F}_{\eta\gamma\gamma^*}(s) \tilde{F}_{\eta\gamma\gamma^*}^{CP1*}(s)}{s} (1 - x_\mu)^3 ds dy d\epsilon^\pm \left[ \sqrt{1 - \beta_\mu^2} \sin \theta (b\beta_y - \bar{b}\bar{\beta}_y) - \cos \theta (b\beta_z - \bar{b}\bar{\beta}_z) \right],$$

$$\frac{d\Gamma_{CP_{HL}}}{\Gamma_{\gamma\gamma}} = \frac{\alpha}{\pi} \frac{(1 - x_\mu)}{s(1 - y^2)} \frac{2C ds dy d\epsilon^\pm}{e^2 m_\eta F_{\eta\gamma\gamma}} \left[ \tilde{\alpha}_R \text{Re } \tilde{F}_{\eta\gamma\gamma^*}(s) + \tilde{\alpha}_I \text{Im } \tilde{F}_{\eta\gamma\gamma^*}(s) \right],$$

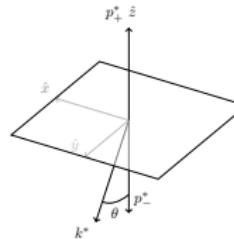
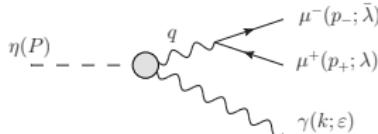
- Richer final state → additional asymmetries

- RETOP:  $10^9 \eta \rightarrow \gamma \mu^+ \mu^-$ ; Noise:  $3 \times 10^{-5}$ ;  $\epsilon_1 \sim 10^{-2}$  and  $c_{\mathcal{O}}^{22st} \sim 1$

$$\eta \rightarrow \mu^+ \mu^- \gamma$$


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- For simplicity SM@LO and interference term



- Along the same steps one finds

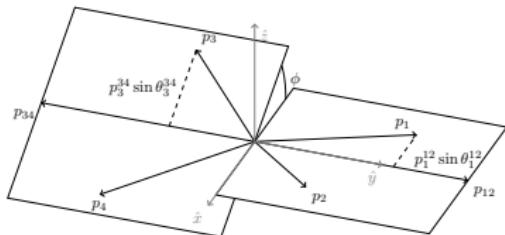
$$\frac{d\Gamma_{CP_H}}{\Gamma_{\gamma\gamma}} = \frac{\alpha}{\pi} \frac{\text{Im } \tilde{F}_{\eta\gamma\gamma^*}(s) \tilde{F}_{\eta\gamma\gamma^*}^{CP1*}(s)}{s} (1 - x_\mu)^3 ds dy d\theta e^\pm \left[ \sqrt{1 - \beta_\mu^2} \sin \theta (b\beta_y - \bar{b}\bar{\beta}_y) - \cos \theta (b\beta_z - \bar{b}\bar{\beta}_z) \right],$$

$$\frac{d\Gamma_{CP_{HL}}}{\Gamma_{\gamma\gamma}} = \frac{\alpha}{\pi} \frac{(1 - x_\mu)}{s(1 - y^2)} \frac{2C ds dy d\theta e^\pm}{e^2 m_\eta F_{\eta\gamma\gamma}} \left[ \tilde{\alpha}_R \text{Re } \tilde{F}_{\eta\gamma\gamma^*}(s) + \tilde{\alpha}_I \text{Im } \tilde{F}_{\eta\gamma\gamma^*}(s) \right],$$

- Richer final state → additional asymmetries
- RETOP:  $10^9 \eta \rightarrow \gamma \mu^+ \mu^-$ ; Noise:  $3 \times 10^{-5}$ ;  $\epsilon_1 \sim 10^{-2}$  and  $c_O^{22st} \sim 1$
- As a bonus: SM P-violating asymmetry reassessed  $10^{-6}$  ( $10^{-2}$  suppression wrt PLB429, 151 (1998) estimate: careful account of  $\mu$  decay!)

$$\eta \rightarrow e^+ e^- \mu^+ \mu^-$$

- CP-violation can be accessed without polarization: lepton plane asymmetry



Expressions lengthy but,  $\propto \sin \phi \cos \phi$

$$A_{\phi/2} = \frac{N(s_\phi c_\phi > 0) - N(s_\phi c_\phi < 0)}{N(\text{all})},$$

- After integration

$$A_{\phi/2}^H = -0.2\epsilon_1 + 0.0003\epsilon_2, \quad A_{\phi/2}^{HL} = -\text{Im}(1.3(c_{\ell equ}^{(1)2211} + c_{\ell edq}^{2211}) - 1.9c_{\ell edq}^{2222}) \times 10^{-5}.$$

- REDTOP:  $10^6 \eta \rightarrow e^+ e^- \mu^+ \mu^-$ ; Noise :  $10^{-4}$ ;  $\epsilon_1 \sim 10^{-3}$  and  $c_{\mathcal{O}}^{22st} \sim 40$

## Summary of sensitivities

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Process	$\epsilon_1 \geq$	$\epsilon_2 \geq$	$\text{Im}\{c_{lequ}^{(1)2211}, c_{ledq}^{2211}\} \geq$	$\text{Im}c_{ledq}^{2222} \geq$
$\eta \rightarrow \mu^+ \mu^-$	$3 \times 10^{-3}$	$8 \times 10^{-3}$	$10^{-2}$	$7 \times 10^{-3}$
$\eta \rightarrow \gamma \mu^+ \mu^-$	0.15	-	14	10
$\eta \rightarrow e^+ e^- \mu^+ \mu^-$	$3 \times 10^{-3}$	2	40	25

- For  $\text{CP}_H$  scenario
  - Stronger bounds for  $\epsilon_1$  ( $\epsilon_2$  only for 2 offshell photons)
  - Stronger bounds from dilepton and double-Dalitz decays
- For  $\text{CP}_{HL}$  scenario
  - Only relevant for dilepton decays (tree vs. loop, helicity suppression)
- Can we set stronger bounds for these parameters? ~~heavy atoms~~ neutron EDM

$$\Gamma^\mu = \gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m_\ell} F_2(q^2) - \frac{\sigma^{\mu\nu} q_\nu}{2m_\ell} \gamma^5 F_E(q^2) + (q^2 \gamma^\mu - q^\mu q^\nu) \gamma^5 F_A(q^2),$$

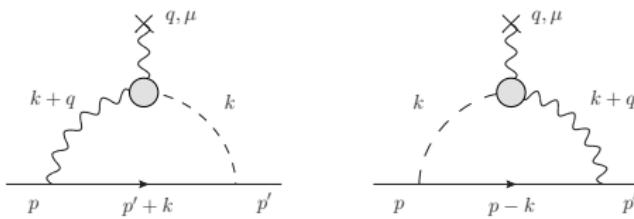
$$e\Gamma^\mu A_\mu^{\text{cl}} \stackrel{\text{NR}}{=} \mathcal{Q}V - \mu \boldsymbol{\sigma} \cdot \boldsymbol{B} - d \boldsymbol{\sigma} \cdot \boldsymbol{E}, \quad \mathcal{Q} = eF_1(0) \quad \mu = \frac{e\hbar}{2m_\ell}(F_1(0) + F_2(0)), \quad d = \frac{e\hbar}{2m_\ell c} F_E(0)$$

## Section 3

nEDM bounds

## nEDM in the $\mathcal{CP}_H$ scenario

- Keep things simple: among other things, this contribution would appear



- Taking  $\eta \bar{N} N$  from  $\chi$ PT and assuming on-shell Dirac and Pauli  $\gamma^* \bar{N} N$  FFs

$$F_E(0) = \epsilon_1 F_{\eta\gamma\gamma} \frac{g_{\eta NN}}{6F_\eta} \frac{\alpha}{\pi} \int_0^\infty dK^2 \frac{K^2}{K^2 + m_{\eta^2}} \tilde{F}_{\eta\gamma^*\gamma^*}^{\mathcal{CP}1}(-K^2, 0)(1 - \beta) \\ \times \left( F_2(-K^2) \frac{3K^2}{16m_N^2} (3 - \beta) - F_1(-K^2) [1 + (1 + \beta)^{-1}] \right)$$

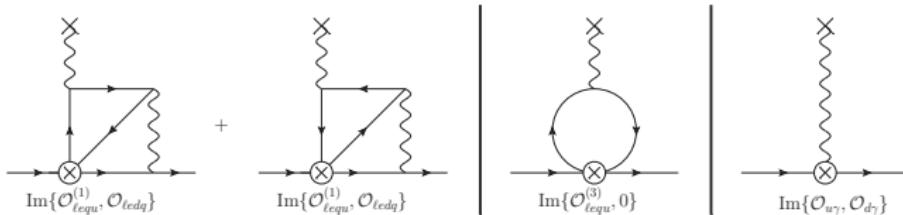
- We obtain  $d_E^n = -6.2 \times 10^{-20} \epsilon_1 \text{ e cm}$  vs  $d_E^n \text{ Exp} \leq 3 \times 10^{-26} \text{ e cm}$
- Thereby  $\epsilon_1 \leq 5 \times 10^{-7}$ , to be compared with found sensitivities  $\epsilon_1 \sim 10^{-3}$

**Cannot measure hadronic-driven CP-violating effects in  $\eta$  muonic decays**

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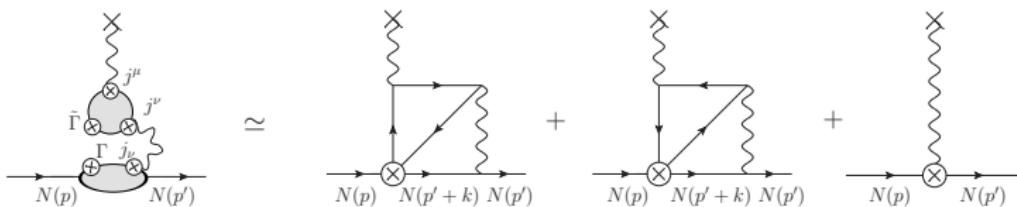
nEDM in the  $\mathcal{CP}_{HL}$  scenario

- More complex: 2-loop effects and account for renormalization



## nEDM in the CP<sub>HL</sub> scenario

- Concerning nucleons: formidable task → approximate



- Low energy via intermediate nucleon state  
 $P$  current:  $\pi, \eta, \eta'$ -dominated and  $\chi$ PT  
 $S$  current:  $\sigma$  terms + resonance saturation

$$d_E^n = \text{Im}(-0.75c_{\ell equ}^{(1)2211} + 0.92c_{\ell edq}^{2211} + 0.08c_{\ell edq}^{2222}) \times 10^{-23}$$

- High energy via the operator product expansion

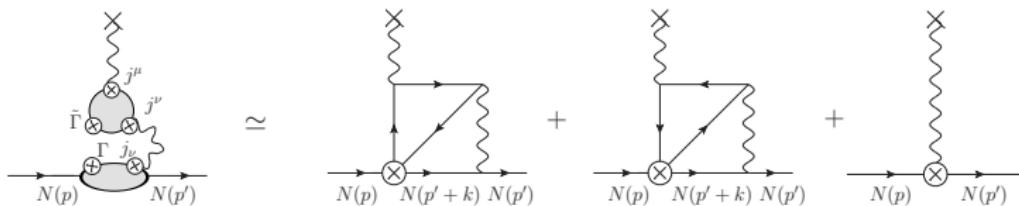
$$\Gamma_{\mu\nu}(\ell) T\{j^\nu(x) S(P)(0)\} \xrightarrow{x \rightarrow 0} \bar{q}\sigma^{\mu\nu}\gamma^5 q$$

Assume 0 at EW, then (RG) + lattice  $\langle n | \bar{q}\sigma^{\mu\nu}\gamma^5 q | n \rangle$

$$d_E^n = \text{Im}(-0.59c_{\ell equ}^{(1)2211} + 0.15c_{\ell edq}^{2211} + 0.001c_{\ell edq}^{2222}) \times 10^{-23}$$

## nEDM in the $\mathcal{CP}_{HL}$ scenario

- Concerning nucleons: formidable task → approximate



- Altogether we find (take it as an order of magnitude)

$$\text{Im } c_{\ell equ}^{(1)2211} < 0.002, \quad \text{Im } c_{\ell edq}^{2211} < 0.003, \quad \text{Im } c_{\ell edq}^{2222} < 0.04. \quad (1)$$

- To be compared with  $\eta \rightarrow \mu^+ \mu^-$

$$\text{Im } c_{\ell equ}^{(1)2211} < 0.01, \quad \text{Im } c_{\ell edq}^{2211} < 0.01, \quad \text{Im } c_{\ell edq}^{2222} < 0.007. \quad (2)$$

**CP-violation possible + Competitive bounds for  $\text{Im } c_{\ell edq}^{2222}$  Wilson Coefficient**

$D_s^+ \rightarrow \mu^+ \nu$ 

- The SMEFT [ $SU(3) \times SU(2) \times U(1)$  symmetry] also has implication for CC

$$\mathcal{O}_{\ell equ}^{(1)} = \frac{c_{\ell equ}^{(1)prst}}{\nu^2} (\bar{\ell}_p^j e_r) \epsilon_{jk} (\bar{q}_s^k u_t) \rightarrow -\frac{\text{Im } c_{\ell equ}^{(1)2222}}{2\nu^2} [(\bar{\nu} i \gamma^5 \mu)(\bar{s} c) + (\bar{\nu} \mu)(\bar{s} i \gamma^5 c)]$$

$$\mathcal{O}_{\ell edq}^{prst} = \frac{c_{\ell edq}^{prst}}{\nu^2} (\bar{\ell}_p^j e_r) (\bar{d}_s q_t^j) \rightarrow \frac{\text{Im } c_{\ell edq}^{2222}}{2\nu^2} [(\bar{\nu} i \gamma^5 \mu)(\bar{s} c) - (\bar{\nu} \mu)(\bar{s} i \gamma^5 c)]$$

- This produces an additional contribution to the SM prediction

$$\text{BR}(D_s^+ \rightarrow \mu^+ \nu) \propto f_{D_s}^2 G_F^2 |V_{cs}|^2 \left| 1 + \frac{m_{D_s}^2 (c_{\ell equ}^{(1)2222} - c_{\ell edq}^{2222})}{2 |V_{cs}| m_\mu (m_c + m_s)} \right|^2 = 5.50(23) \times 10^{-3}$$

- Contribution of the order  $|\text{Im } c_i^{2222}| \leq 0.02$

**CP-violation still possible!**

## Conclusions and Outlook

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- Investigated CP-violation in  $\eta \rightarrow \mu^+ \mu^-$ ,  $\mu^+ \mu^- \gamma$ ,  $e^+ e^- \mu^+ \mu^-$  decays
  - Assumed heavy physics  $\rightarrow$  SMEFT: hadronic/lepton-quark scenarios
  - Estimated sensitivities for REDTOP and amplitudes for MC
  - Stringent bounds from nEDM  $\rightarrow$  only  $\mathcal{O}_{ledq}^{2222}$  plausible
- 
- Lepton origin seems irrelevant due to  $\ell$  EDMs, but might check  $\mathcal{O}_{\ell e}$
  - $\eta \rightarrow \pi^+ \pi^- \mu^+ \mu^-$  not studied, but apriori irrelevant
  - $\eta \rightarrow \mu^+ \mu^- \pi^0$  might be interesting since tree vs loop level
  - A serious calcualtion for the EDM (hadronic model, full RG...)

## Section 4

Backup

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## Some details on the OPE and large logs

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- The nEDM from the loop integral (exact)

$$\begin{aligned}
 \bar{u}_{p'} \Gamma^\mu u_p = e^2 \sum_i \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} & \left[ \frac{1}{i} \int e^{ik \cdot z} \langle N_{p'} | T\{j_\nu(z)(\bar{q}\Gamma_i q)(0)\} | N_p \rangle \right] \\
 & \times \left[ \frac{1}{i} \int e^{-i(q \cdot x + k \cdot y)} \langle 0 | T\{j^\mu(x) j^\nu(y) (\bar{\ell}\tilde{\Gamma}_i \ell)(0)\} | 0 \rangle \right] \\
 \equiv e^2 \sum_i \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} & \Pi_{NNV\Gamma_i}^\rho(-k, k+q) \Pi_{VV\tilde{\Gamma}_i}^{\mu\nu}(k, q) g_{\nu\rho}, \tag{3}
 \end{aligned}$$

- The RG large-logs from cut-off regularization  $\infty \rightarrow \Lambda$

$$\begin{aligned}
 F_E^q(0) = \frac{\alpha}{\pi} \frac{G_F m_\ell m_q}{6\sqrt{2}\pi^2} \mathcal{Q}_q \int_0^\infty \frac{dKK}{m_q^2 \beta_\ell} & \left[ \left( \beta_q - \frac{2}{1+\beta_q} \right) \left[ \frac{1+\beta_\ell^2}{2\beta_\ell} \ln \left( \frac{\beta_\ell+1}{\beta_\ell-1} \right) - 1 \right] \right. \\
 \pm \left( \frac{2\beta_q^2}{1+\beta_q} - 1 \right) \ln & \left. \left( \frac{\beta_\ell+1}{\beta_\ell-1} \right) \right] \text{Im } c_{\ell equ(dq)} \tag{4}
 \end{aligned}$$

$$\rightarrow \frac{\alpha}{\pi} \frac{G_F m_\ell m_q}{6\sqrt{2}\pi^2} \left\{ \text{Im } c_{\ell equ}^{(1)}(\ln^2 \Lambda^2 - \ln \Lambda^2), \text{Im } c_{\ell edq} \ln \Lambda \right\}. \tag{5}$$