

*Muonic atoms:  
from atomic to  
to nuclear and particle  
physics*

Aldo Antognini  
ETH Zurich  
for the  
CREMA collaboration

*Muonic atoms:  
from atomic to  
to nuclear and particle  
physics*

$\mu$



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- Muonic hydrogen ( $\mu p$ )
- Muonic deuterium ( $\mu D$ )
- Muonic helium ( $\mu He^+$ )

*Muonic atoms:  
from atomic to  
to nuclear and particle  
physics*

$\mu$

Measure  $\Delta E(2S - 2P)$   
 $\rightarrow r_p$  with  $\delta r_p = 4 \times 10^{-19}$  m

$$\Delta E^{FS} = \frac{2(Z\alpha)^4}{3n^3} m_r^3 r_p^2 \delta_{l0}$$

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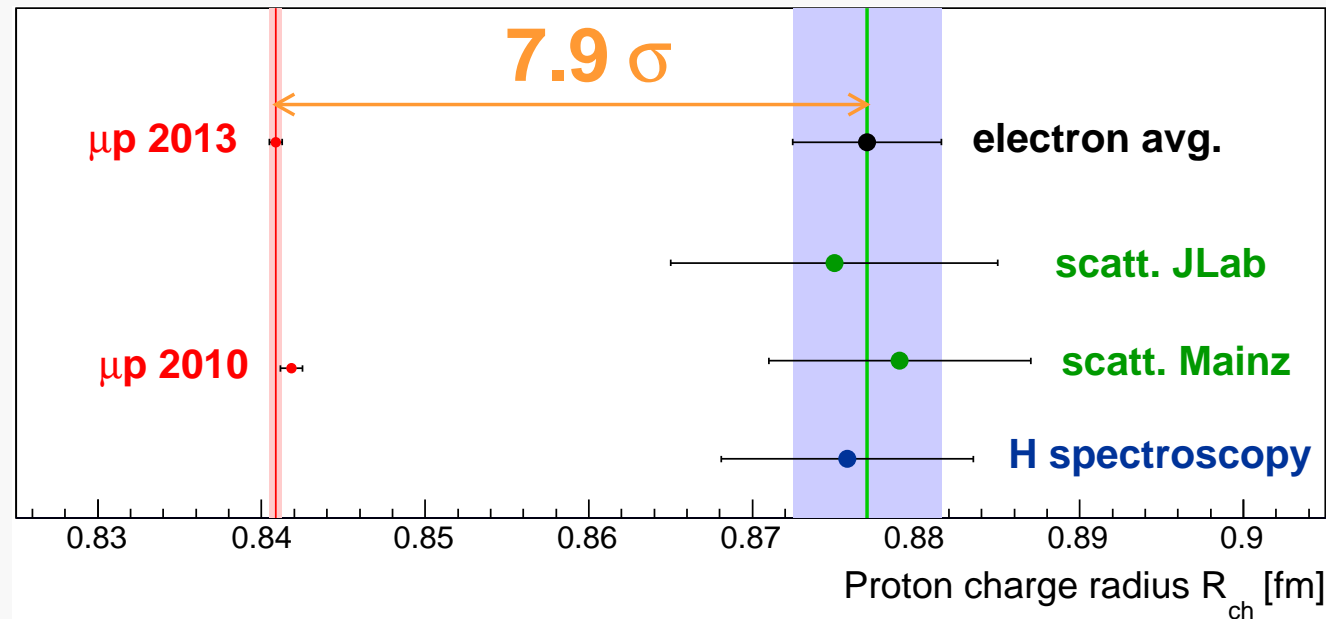
- Muonic hydrogen ( $\mu p$ )
- Muonic deuterium ( $\mu D$ )
- Muonic helium ( $\mu \text{He}^+$ )

# The proton radii puzzle



3 ways to the proton radius

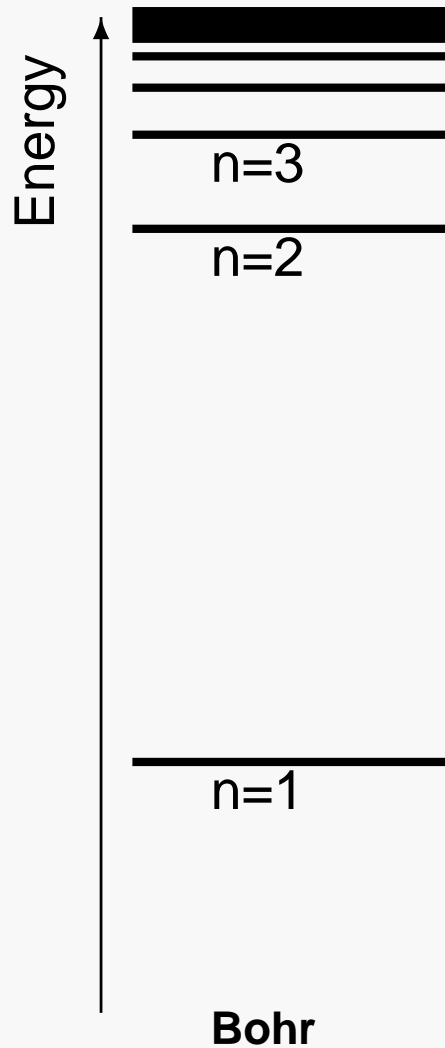
- e-p scattering
- H precision laser spectroscopy
- $\mu p$  laser spectroscopy



Pohl *et al.*, Nature 466, 213 (2010)

Antognini *et al.*, Science 339, 417 (2013)

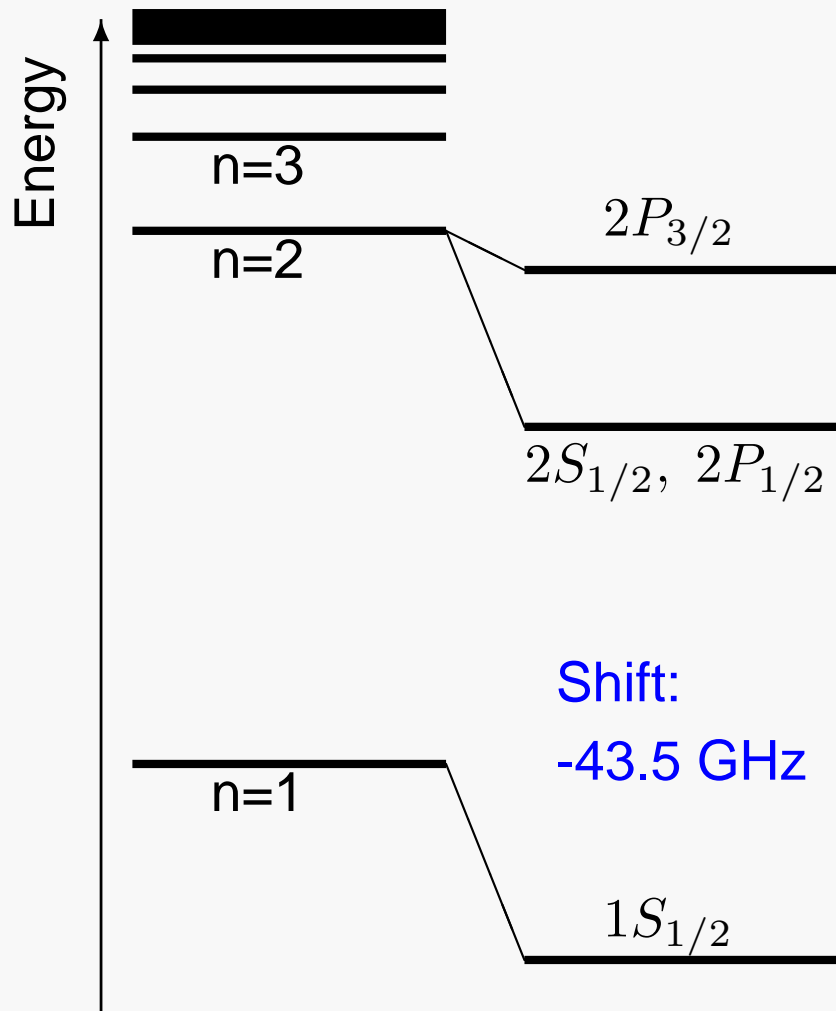
# Hydrogen energy levels and $r_p$



$$E = R_{\infty}/n^2$$

$$V \sim 1/r$$

# Hydrogen energy levels and $r_p$



**Bohr**

$$E = R_{\infty}/n^2$$

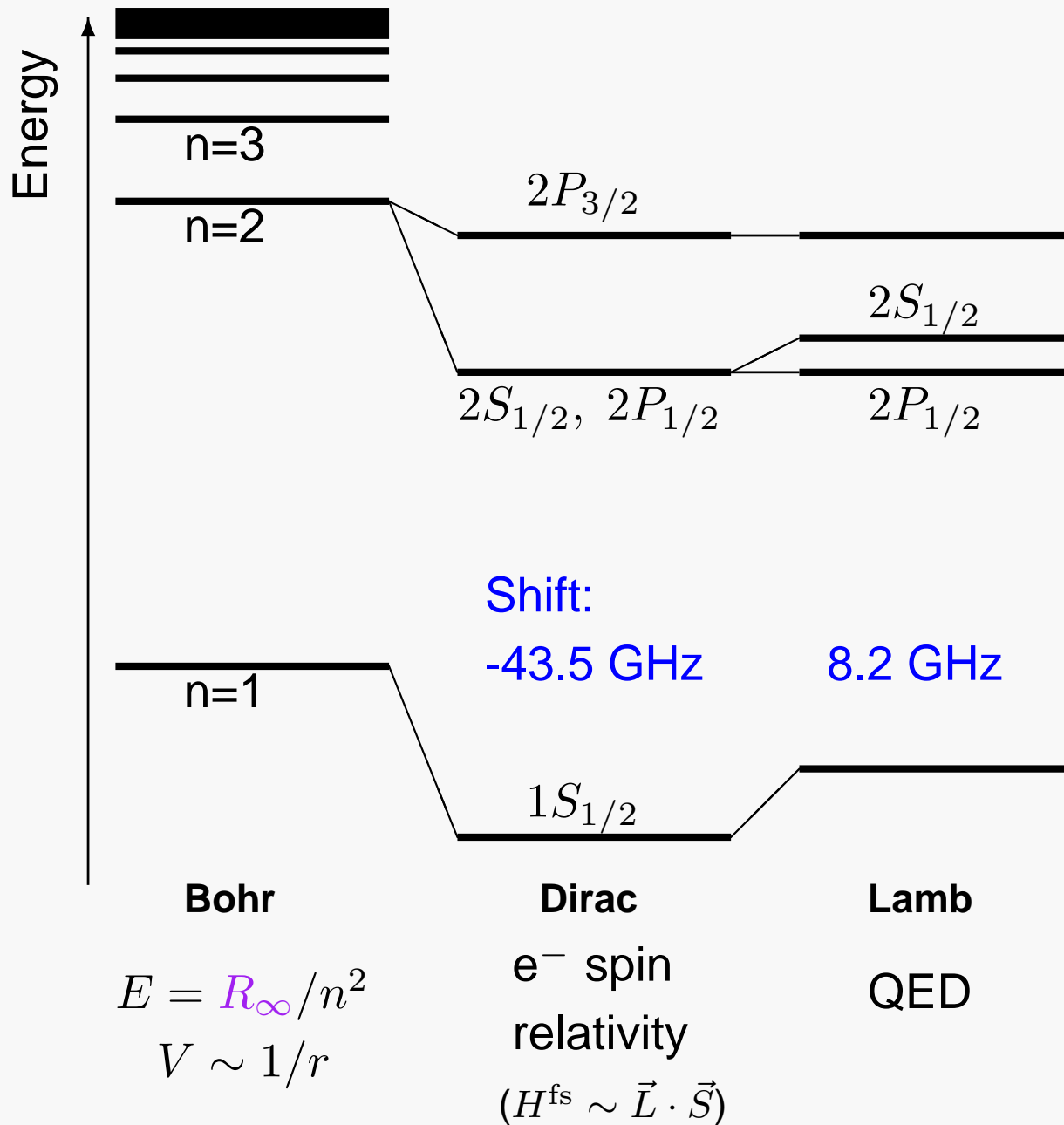
$$V \sim 1/r$$

**Dirac**

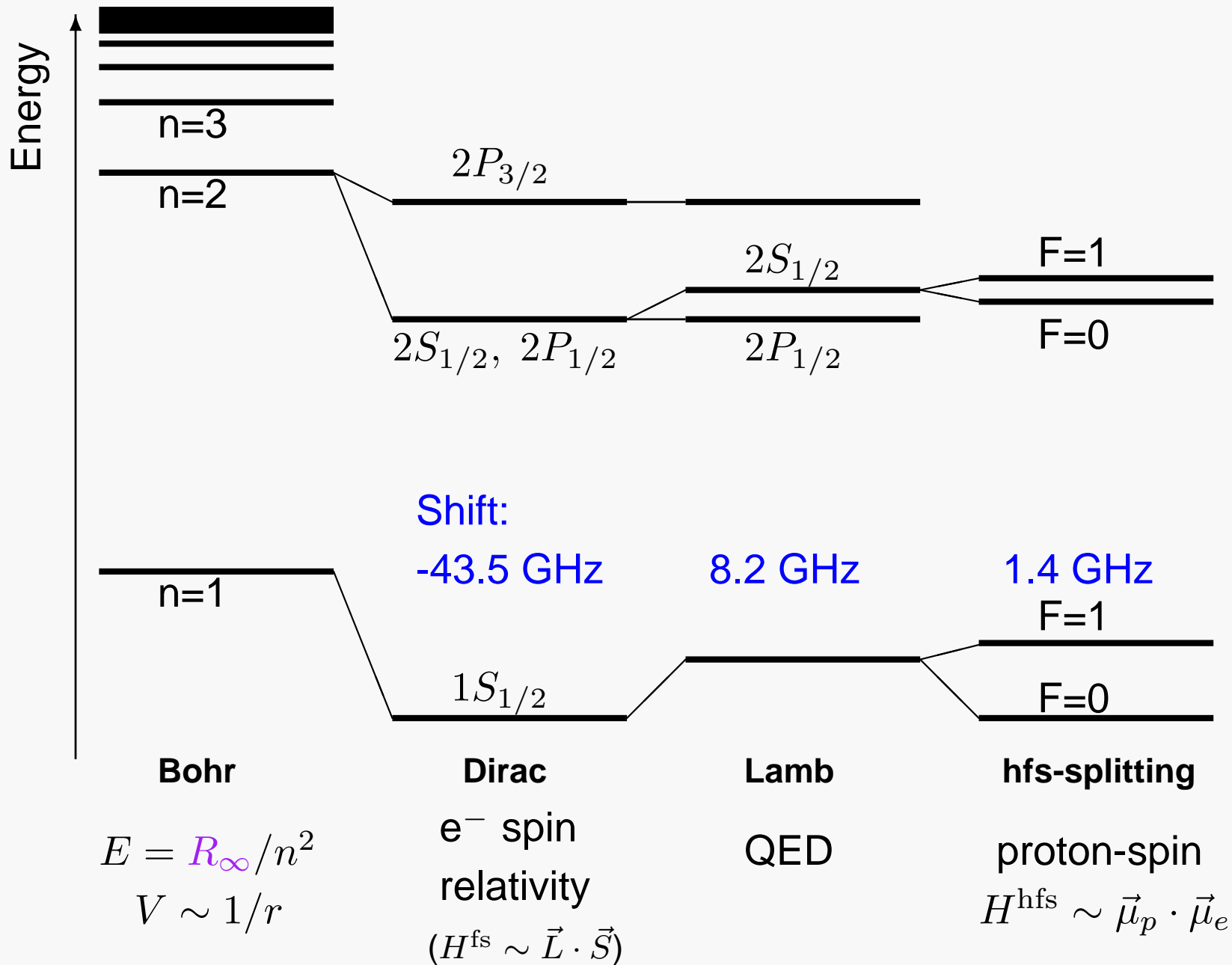
$e^-$  spin  
relativity

$$(H^{\text{fs}} \sim \vec{L} \cdot \vec{S})$$

# Hydrogen energy levels and $r_p$

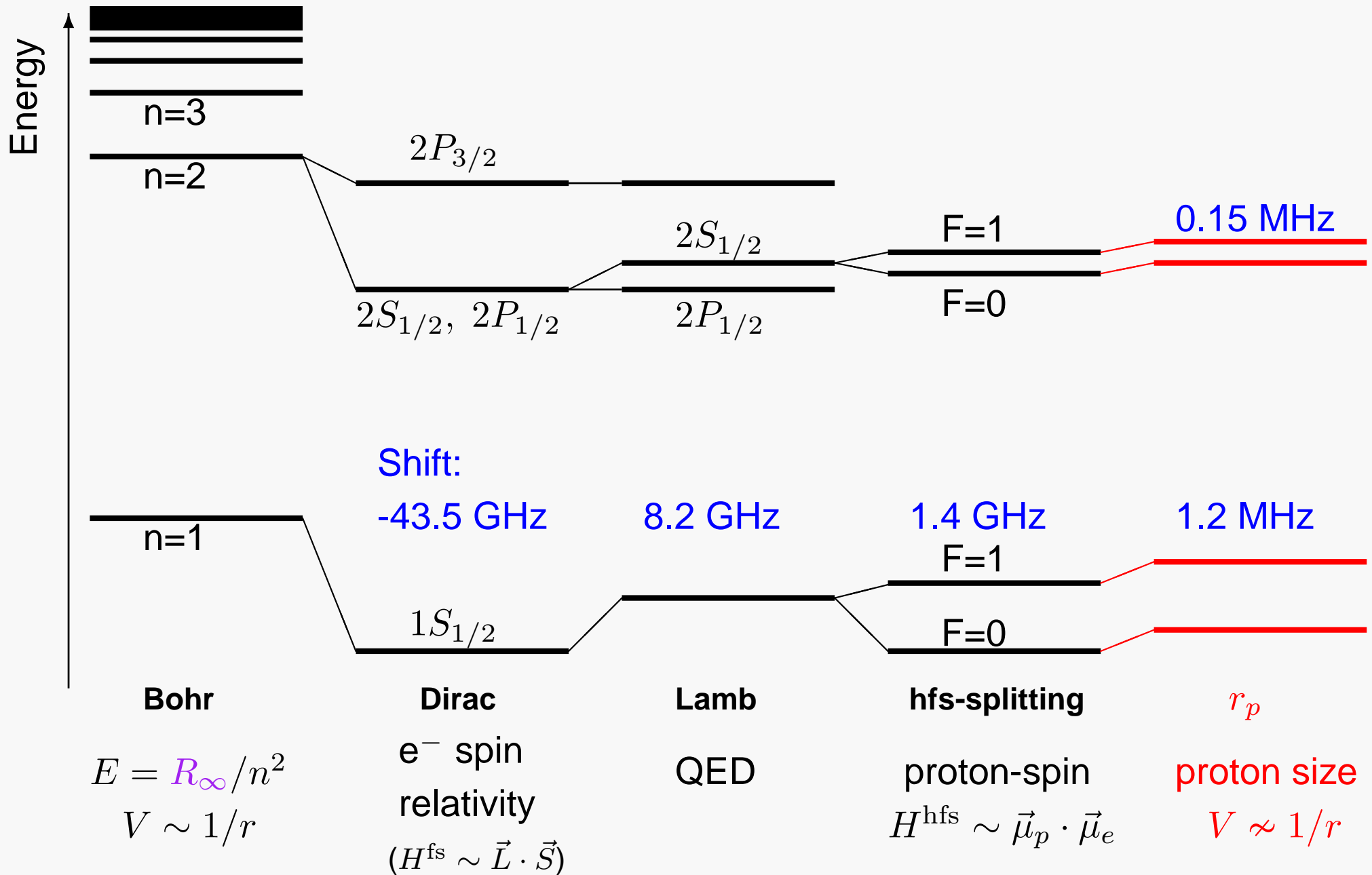


# Hydrogen energy levels and $r_p$

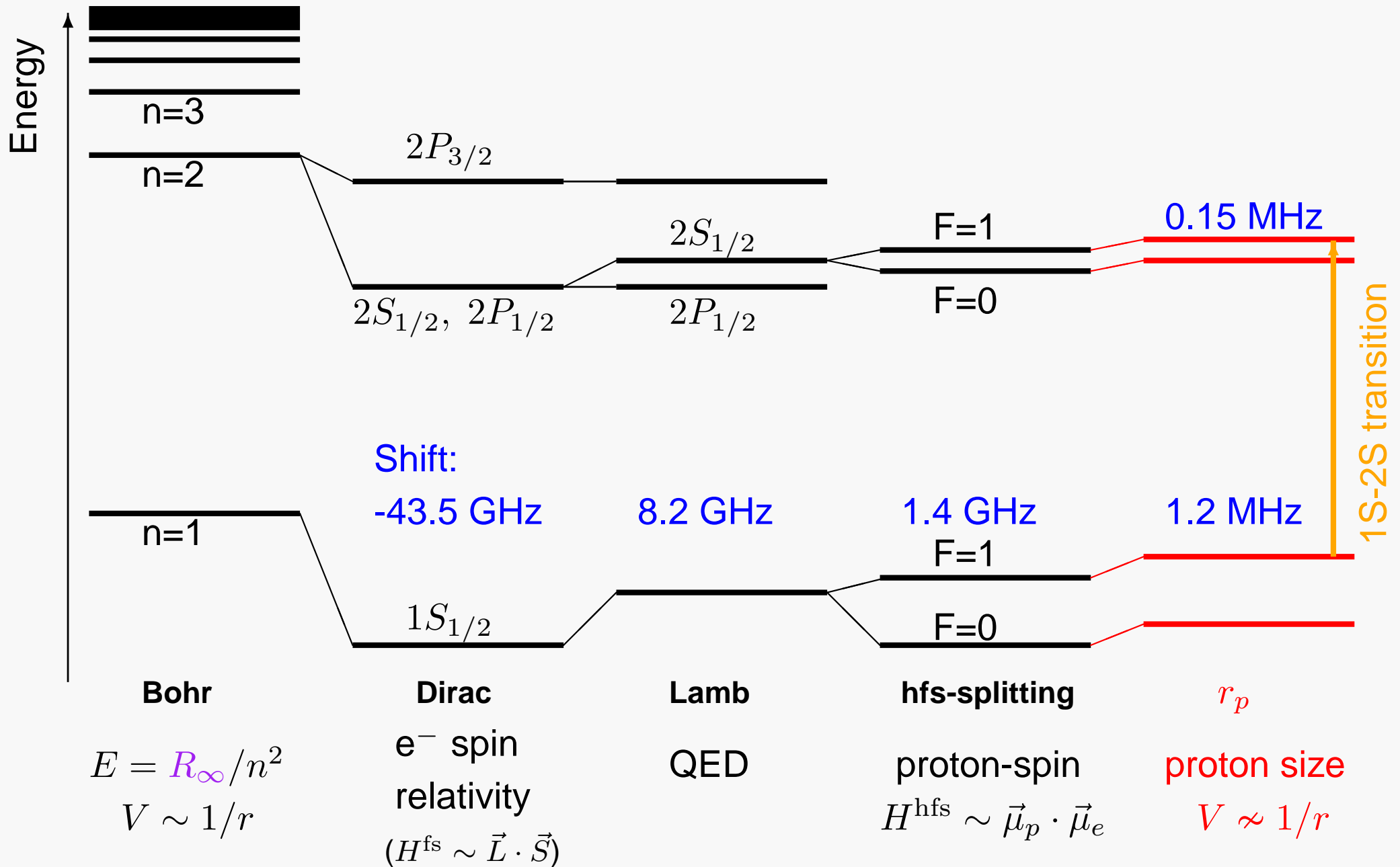




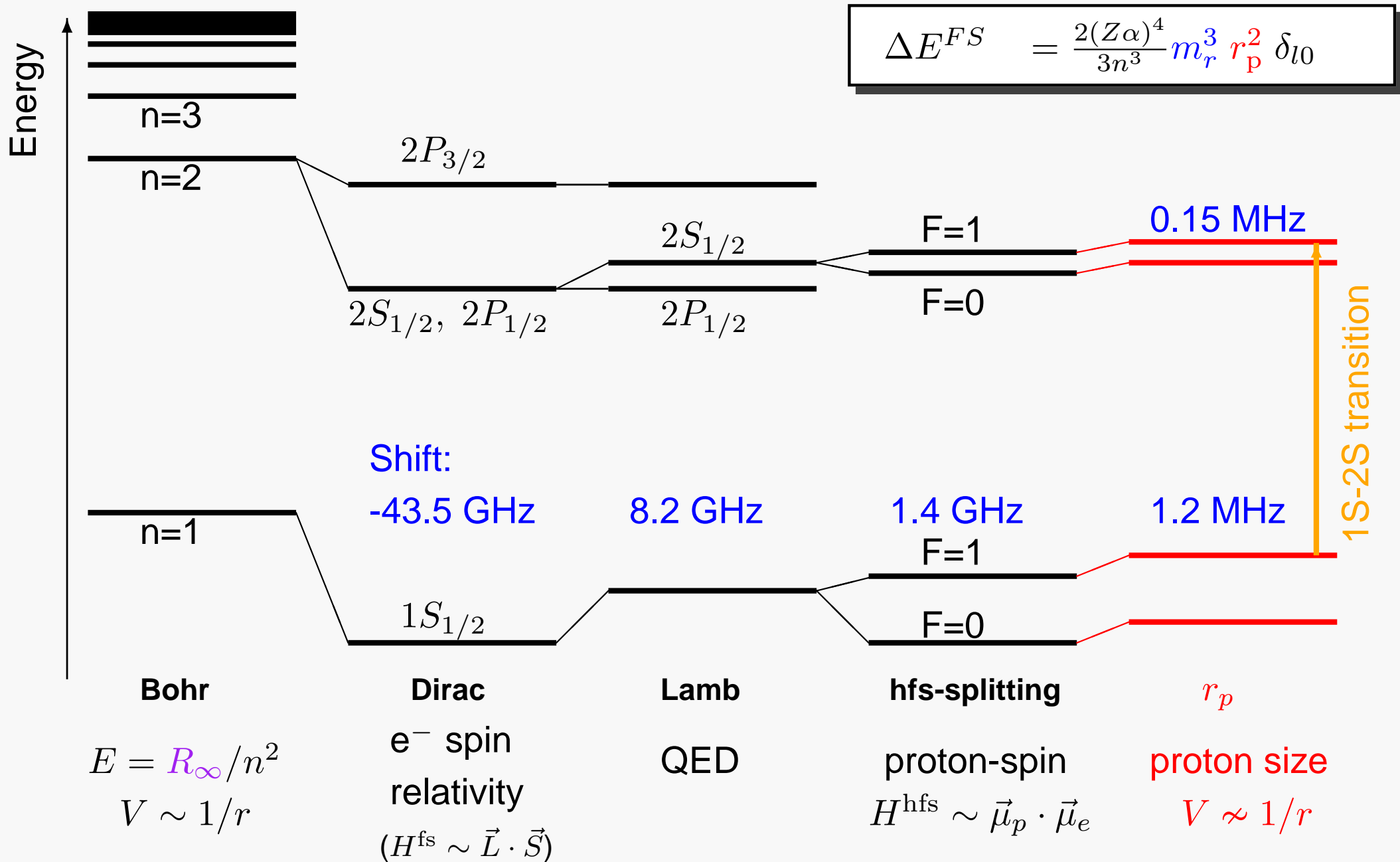
# Hydrogen energy levels and $r_p$



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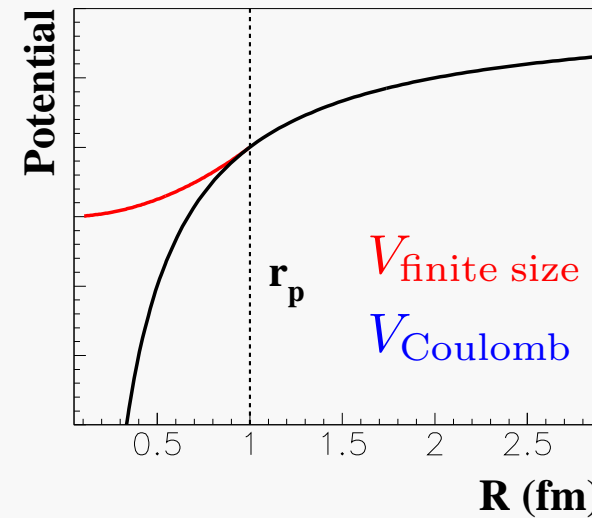


# Atomic energy levels and the proton size

$$\Delta E = \Delta E_{\text{QED}} + \Delta E_{\text{fs}}$$

$$\begin{aligned}\Delta E_{\text{fs}}^{(0)} &= \frac{2\pi(Z\alpha)}{3} \langle r_p^2 \rangle |\Psi_n(0)|^2 \\ &= \frac{2(Z\alpha)^4}{3n^3} m_r^3 \langle r_p^2 \rangle \delta_{l0}\end{aligned}$$

$$m_\mu \approx 200m_e$$



From  $\vec{\nabla} \cdot \vec{E} = 4\pi\rho \rightarrow$  potential  $V$

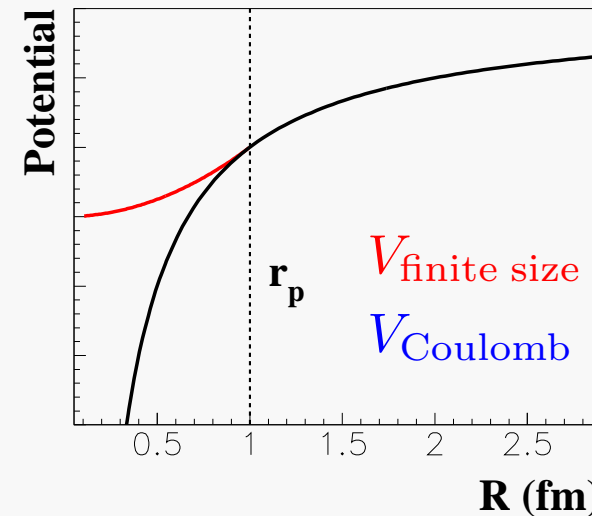
$$\Delta E_{\text{fs}}^{(0)} = \langle \bar{\Psi} | V_{\text{Coulomb}} - V_{\text{fin.size}} | \Psi \rangle$$

# Atomic energy levels and the proton size

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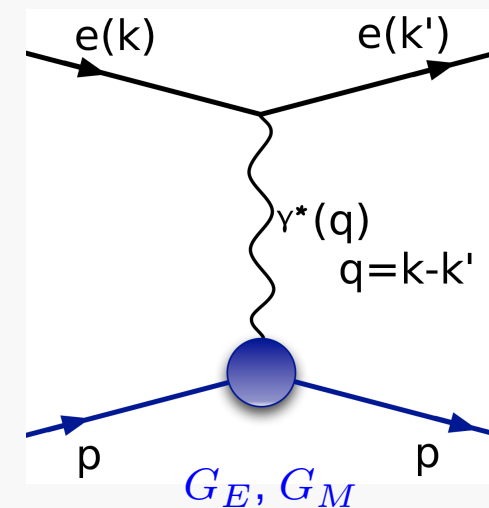
$$G_E(\mathbf{q}^2) = \int d^3r \rho_E(\mathbf{r}) e^{-i\mathbf{q}\cdot\mathbf{r}} \simeq Z(1 - \frac{\mathbf{q}^2}{6} r_p^2 + \dots)$$

$$r_p^2 \equiv \int d^3r \rho_E(\mathbf{r}) r^2$$

$$\Delta V(r) = -\frac{Z\alpha}{r} - V(r)$$

$$\Delta V(\mathbf{q}) = \frac{4\pi Z\alpha}{\mathbf{q}^2} (1 - G_E(\mathbf{q}^2)) \simeq \frac{2\pi(Z\alpha)}{3} r_p^2$$

$$\Delta V(r) = \frac{2\pi(Z\alpha)}{3} r_p^2 \delta(r)$$

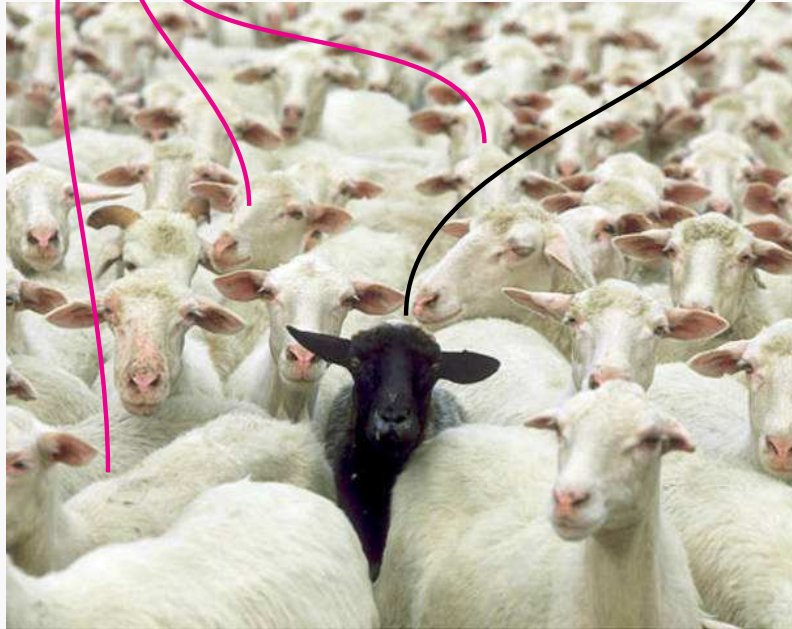


# Proton radius from muonic hydrogen

- Measure  $\Delta E_{2P-2S}^{\text{exp}}$  in  $\mu\text{p}$  with  $u_r = 10^{-5} \leftrightarrow 0.5 \text{ GHz} = \Gamma/20$

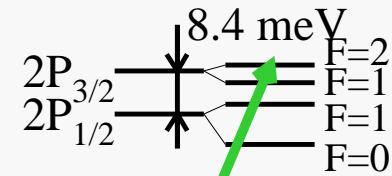
- Compute theoretical prediction

$$\Delta E_{2P-2S}^{\text{th}} = 206.0336(15) - 5.2275(10) r_p^2 + 0.0332(20) \text{ [meV]}$$

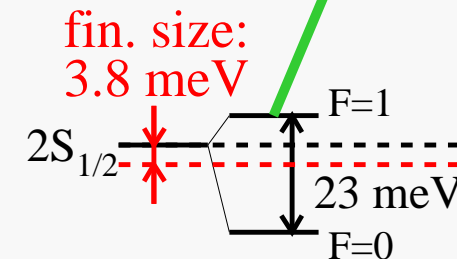


The Lamb shift contributions

Comparing theory with experiment  $\implies r_p$



206 meV  
50 THz  
6  $\mu\text{m}$



# Principle of the $\mu\text{p}$ Lamb shift experiment

- Produce many  $\mu^-$

PSI accelerator

- Stop  $\mu^-$  in 1 mbar  $\text{H}_2$  gas

→  $\mu\text{p}$  formation

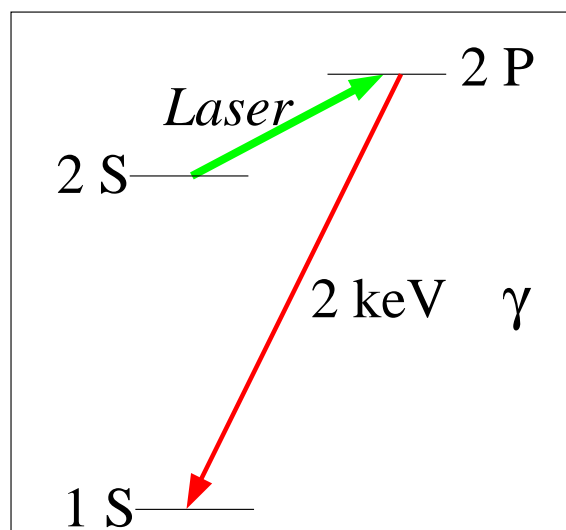
(1% in the 2S-state with  $1\ \mu\text{s}$  lifetime)

Dedicated low-energy  $\mu^-$  beam line

- Fire laser at  $\lambda = 6\ \mu\text{m}$

→ to induce  $\mu\text{p}(2\text{S}) \rightarrow \mu\text{p}(2\text{P})$  transition

Dedicated laser system with “strange” requirements



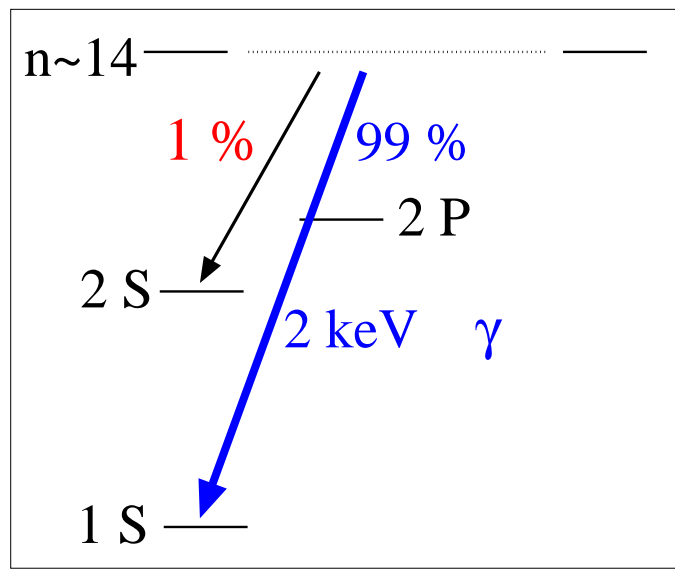
2 keV x-ray detectors

- If laser resonant

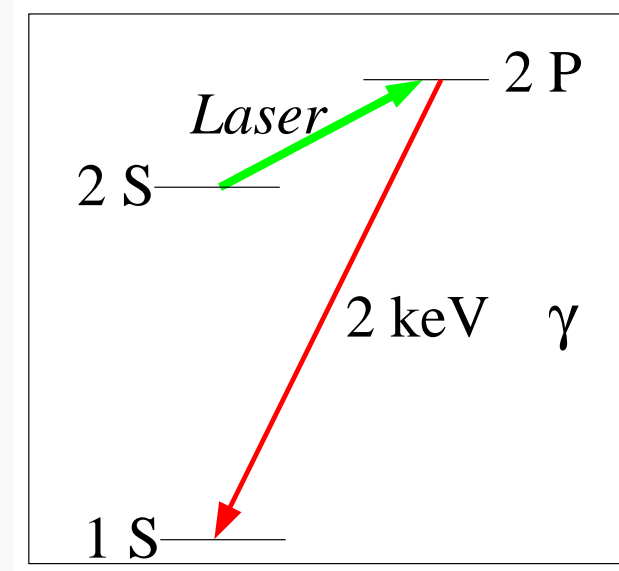
→ observe 2 keV x-rays

# Principle of the $\mu\text{p}$ Lamb shift experiment

“prompt” ( $t = 0$ )



“delayed” ( $t \approx 1\mu\text{s}$ )



$\mu^-$  stop in  $\text{H}_2$  gas

$\rightarrow \mu\text{p}^*$  formation ( $n \sim 14$ )

99% cascade to  $\mu\text{p}(1\text{S})$

emitting prompt  $K_\alpha, K_\beta \dots$

1% long-lived  $\mu\text{p}(2\text{S})$

$\tau_{2\text{S}} \approx 1\mu\text{s}$  at 1 mbar  $\text{H}_2$  pressure

fire laser at  $\lambda = 6\mu\text{m}$ ,  $\Delta E = 0.2 \text{ eV}$

$\Rightarrow$  induce  $\mu\text{p}(2\text{S}) \rightarrow \mu\text{p}(2\text{P})$  transition

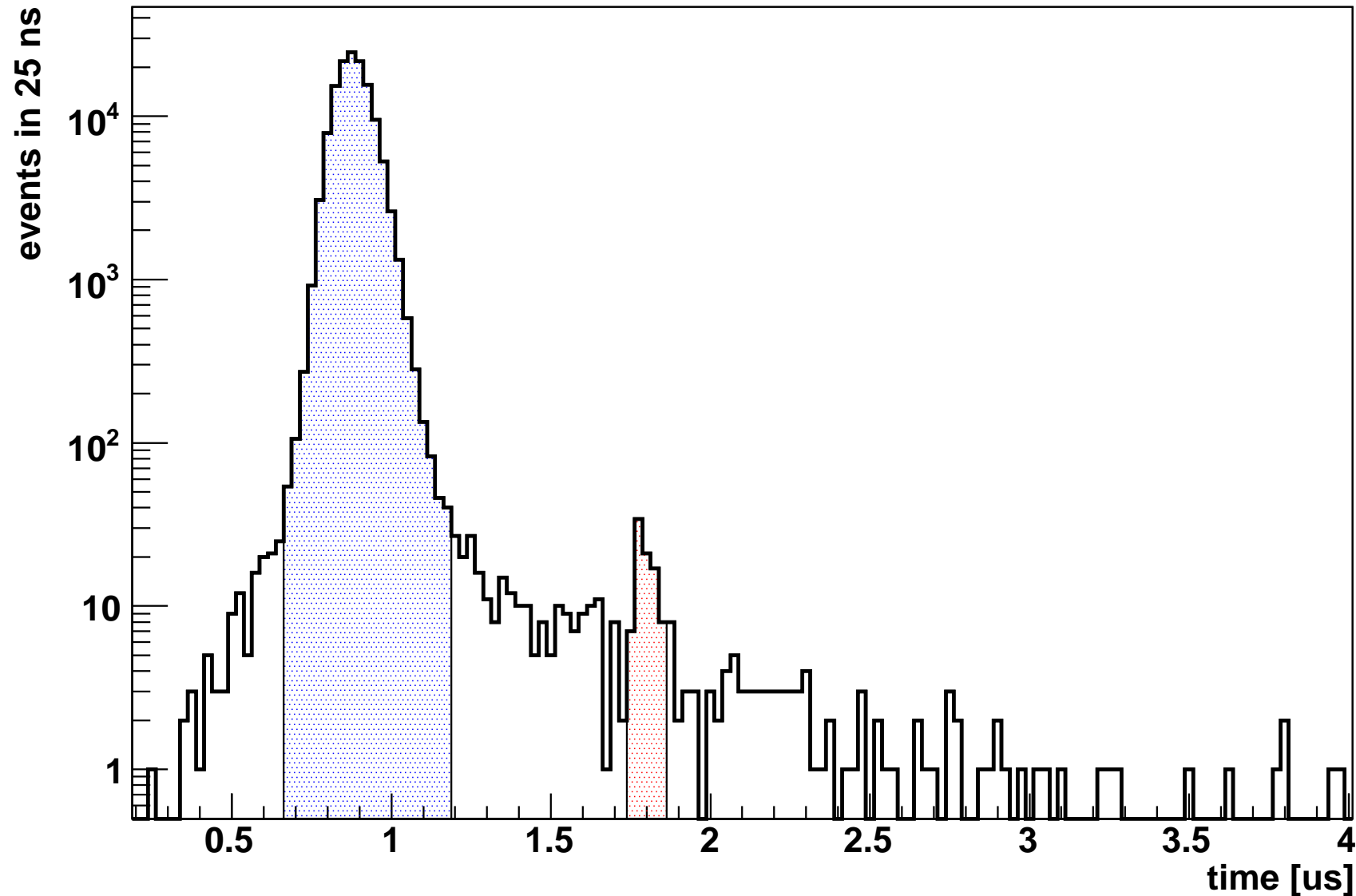
$\Rightarrow$  observe delayed  $K_\alpha$  x-rays

$\Rightarrow$  normalize  $\frac{\text{delayed } K_\alpha}{\text{prompt } K_\alpha}$



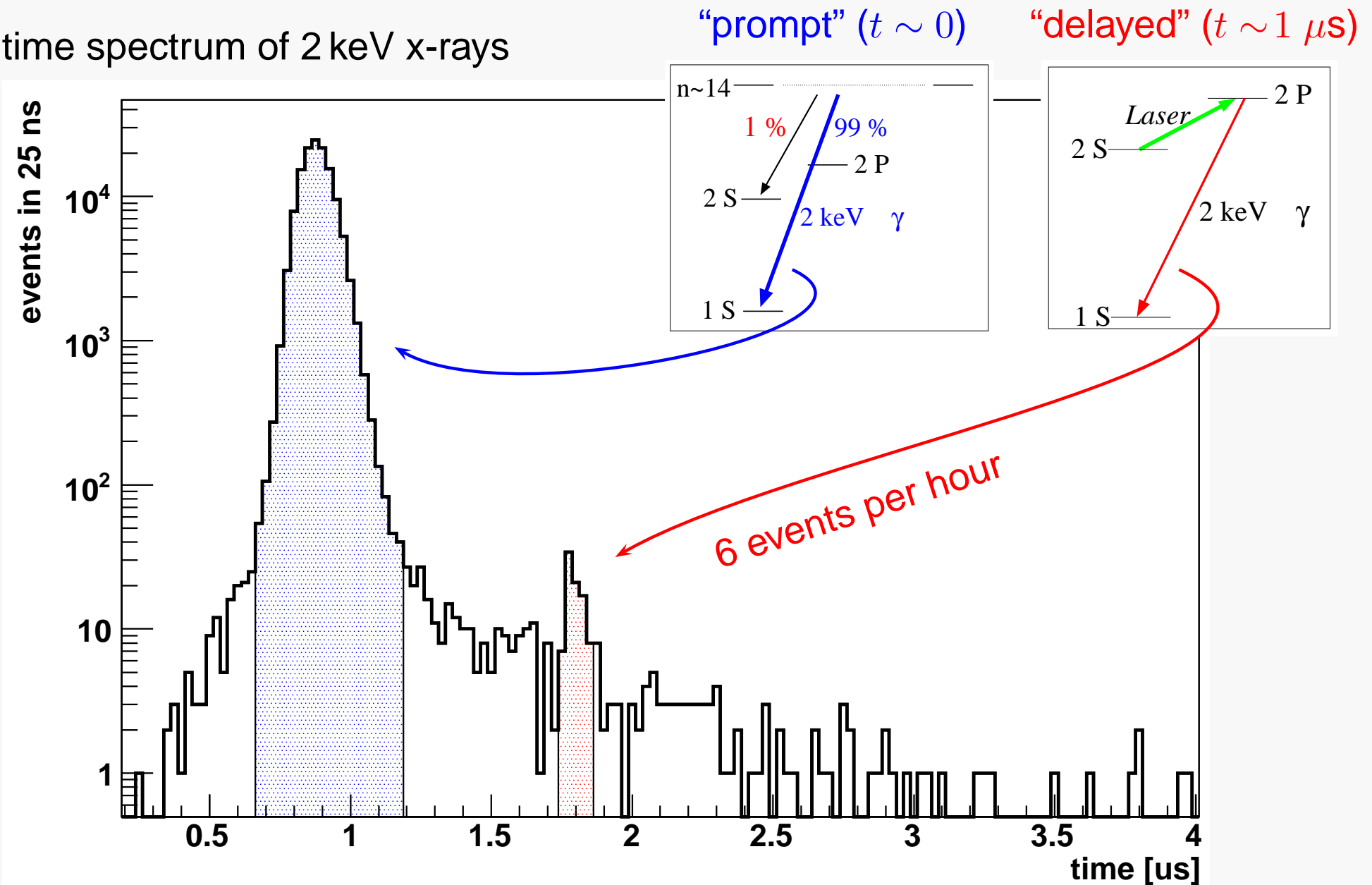
# Principle of the $\mu p$ Lamb shift experiment

time spectrum of 2 keV x-rays



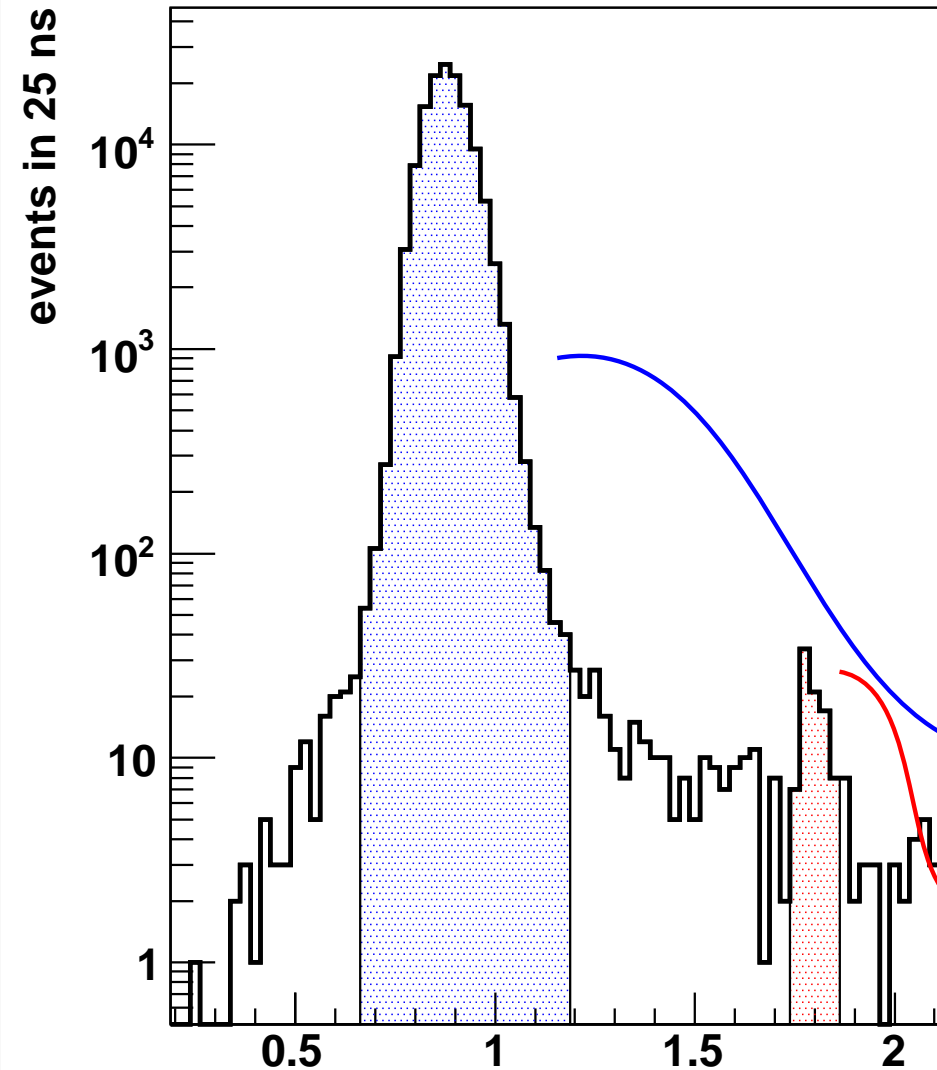
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time spectrum of 2 keV x-rays



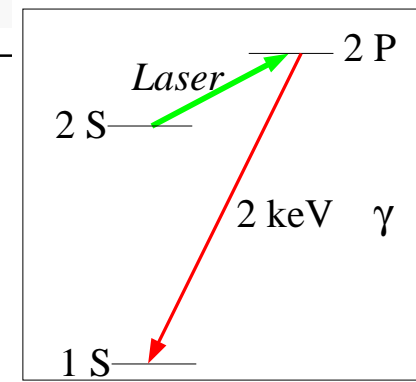
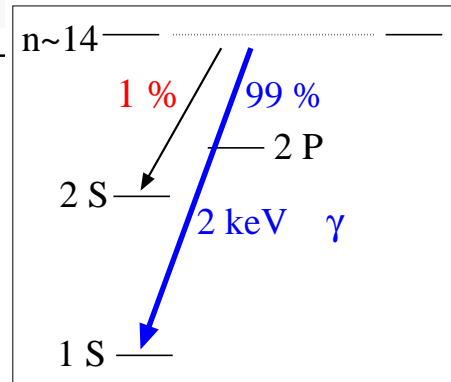
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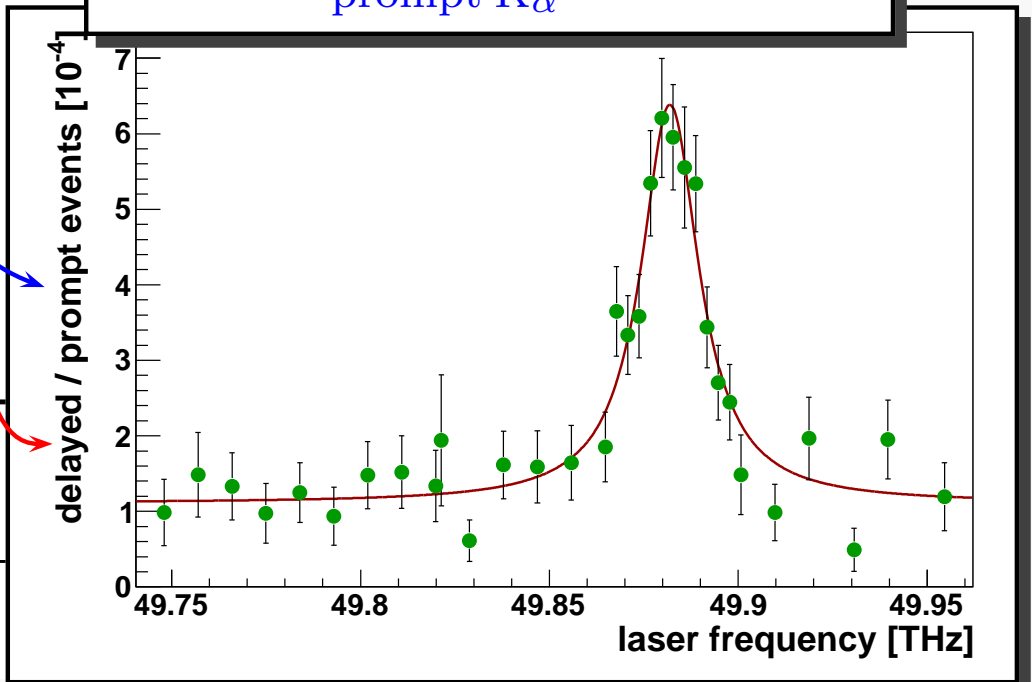


“prompt” ( $t \sim 0$ )

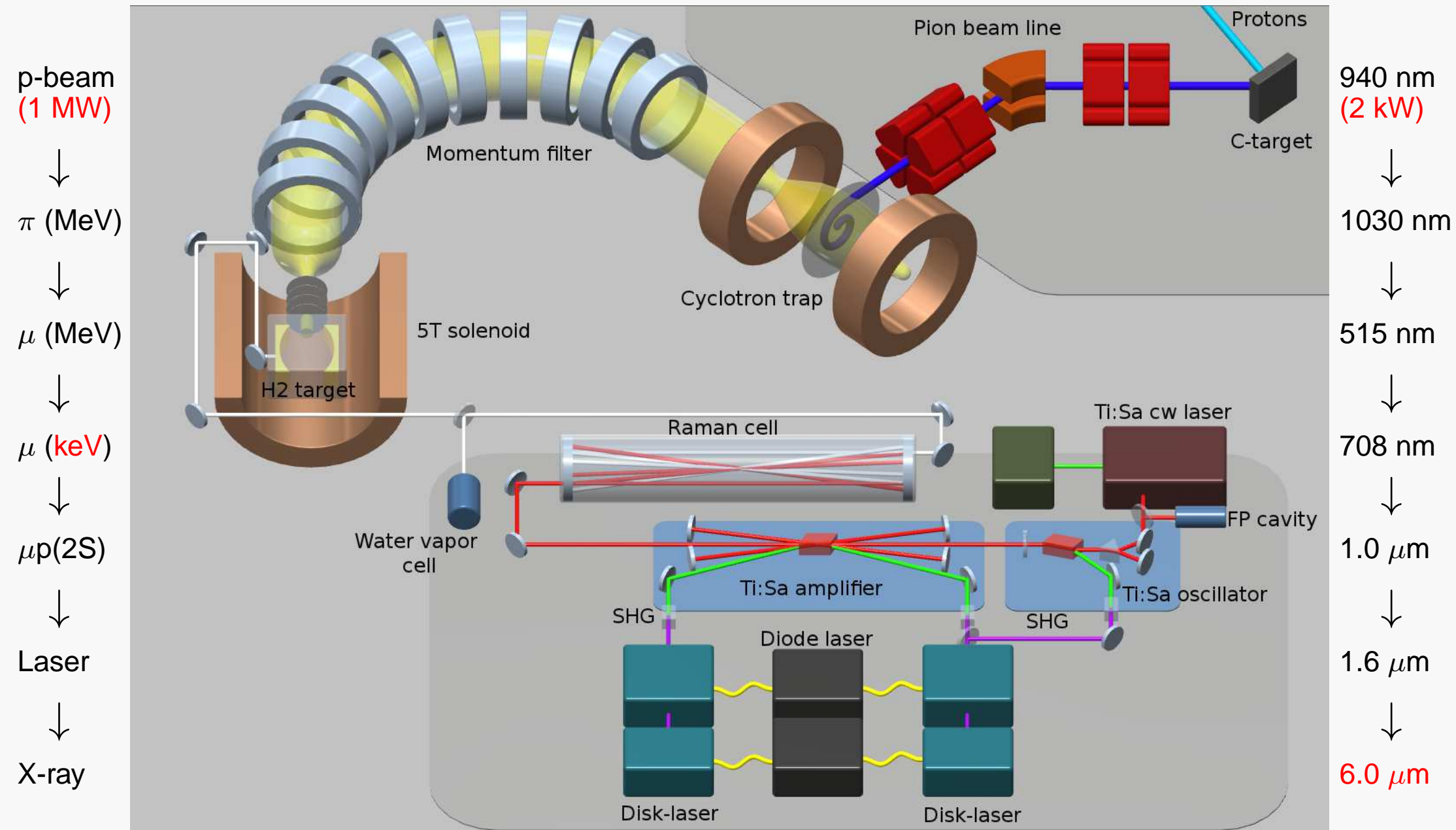
“delayed” ( $t \sim 1 \mu s$ )



normalize  $\frac{\text{delayed } K_{\alpha}}{\text{prompt } K_{\alpha}} \Rightarrow \text{Resonance}$



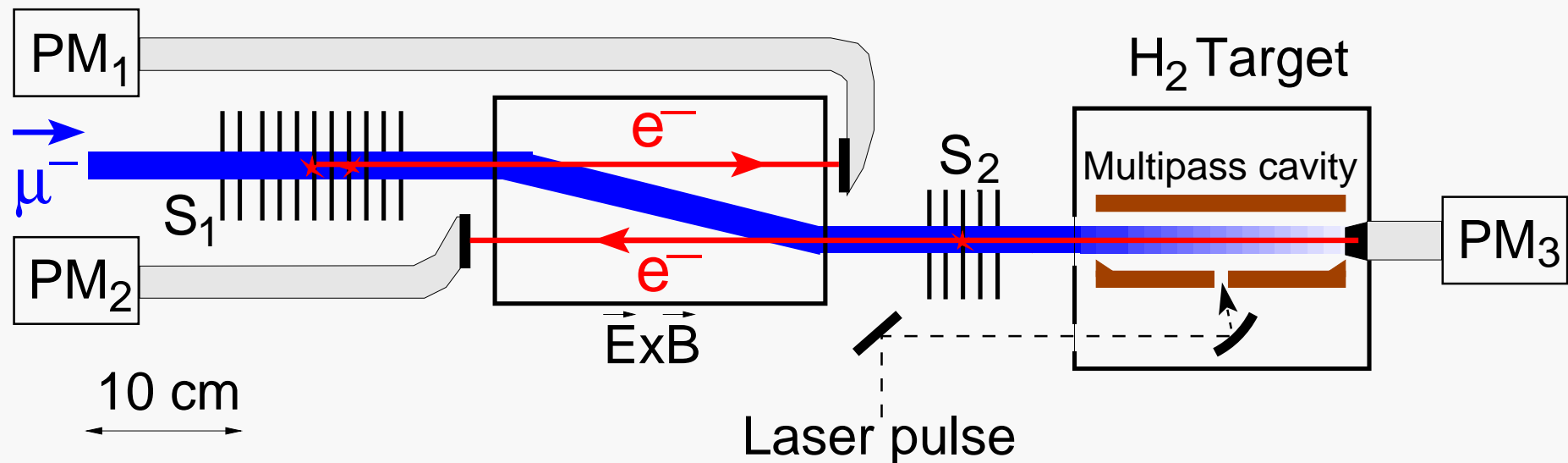
# The $\mu p$ Lamb shift setup



# Inside the 5 Tesla solenoid

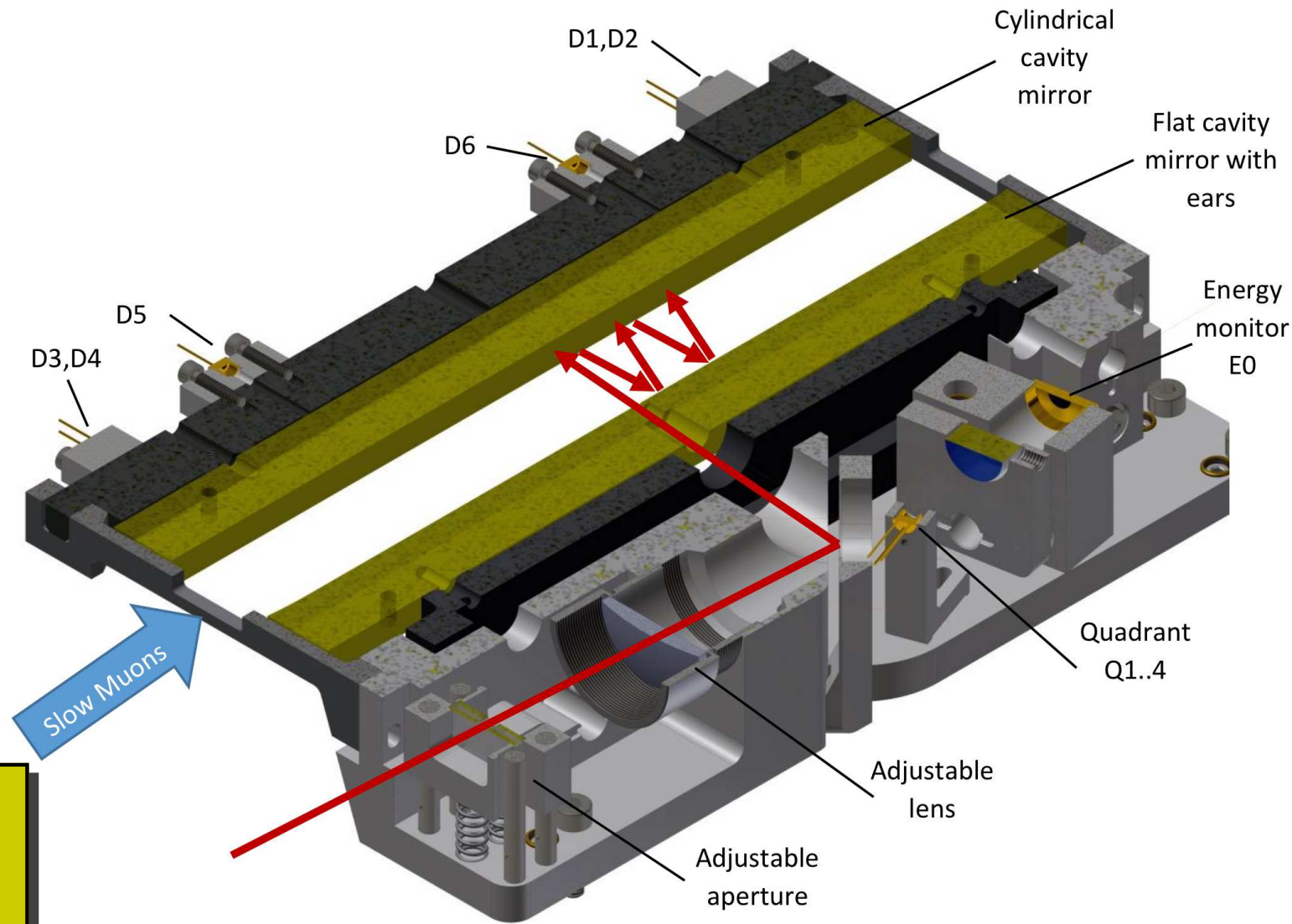
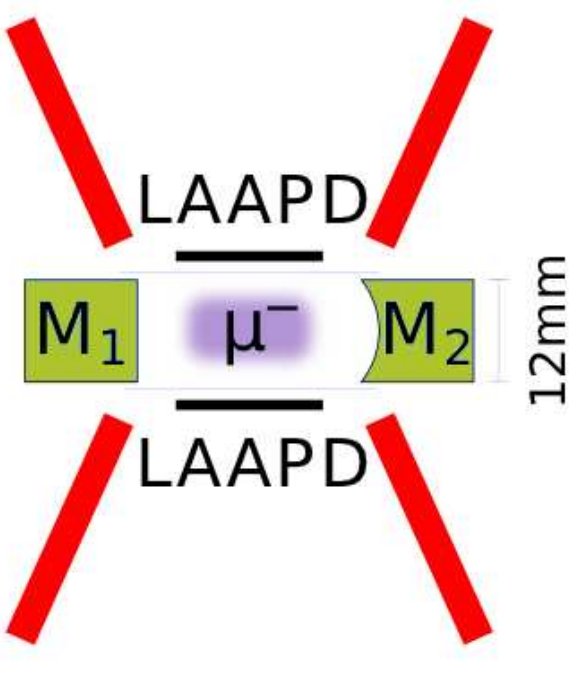
5 keV  $\mu^-$  with a rate of 500  $s^{-1}$

- Stacks of C foils are used as non-destructive muon detector
- Laser is triggered by the electrons signals from the C stacks (coincidence with TOF)



Isn't trivial to stop muons in 1 mbar  $H_2$

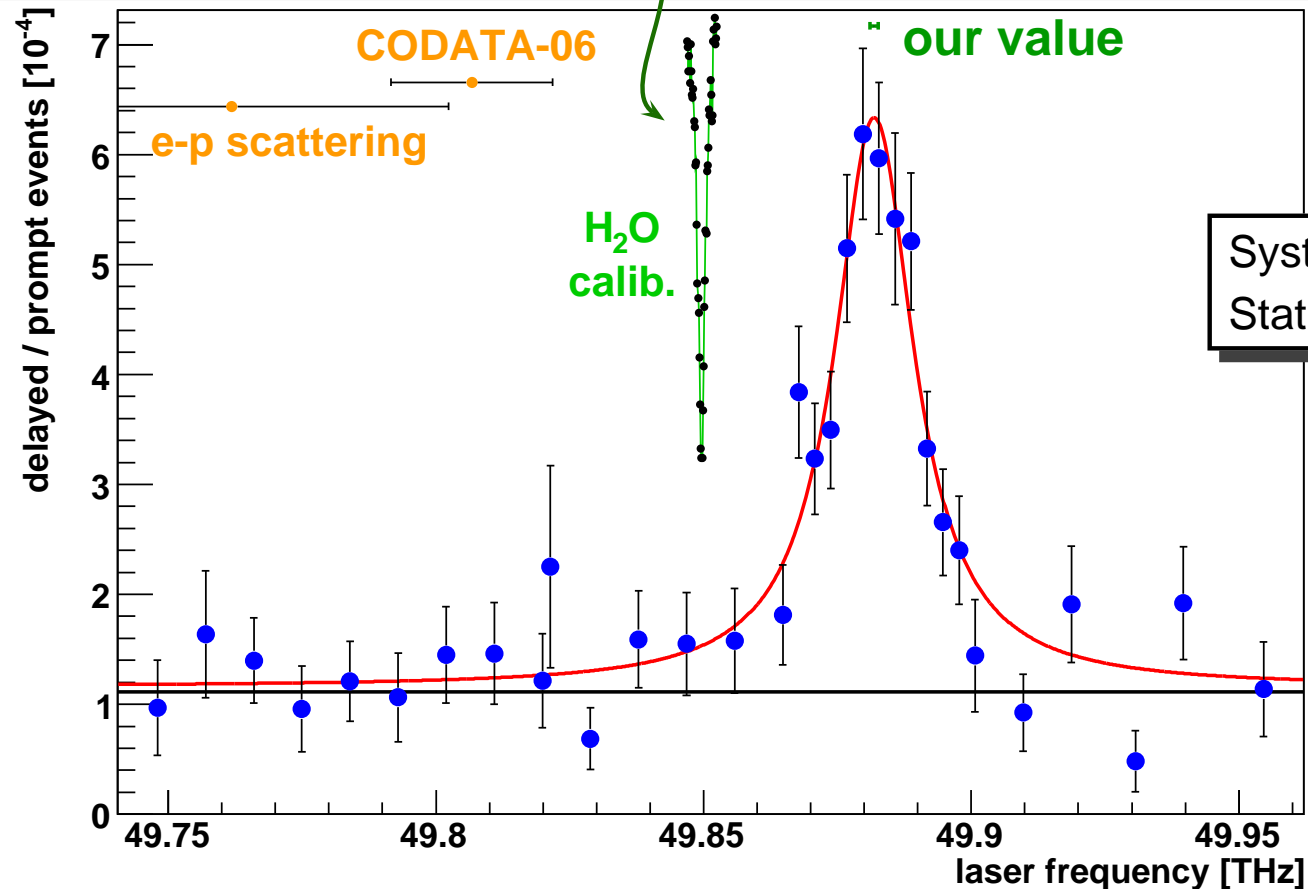
# Inside the 5 Tesla solenoid



- $\mu_p$  formation
- laser drives 2S-2P transition
- 2P-1S x-ray deexcitation
- detect  $e^-$  from  $\mu^-$  decay

# The resonance: discrepancy, sys., stat. (2010)

Water line scan:  
Laser frequency known with 300 MHz uncertainty

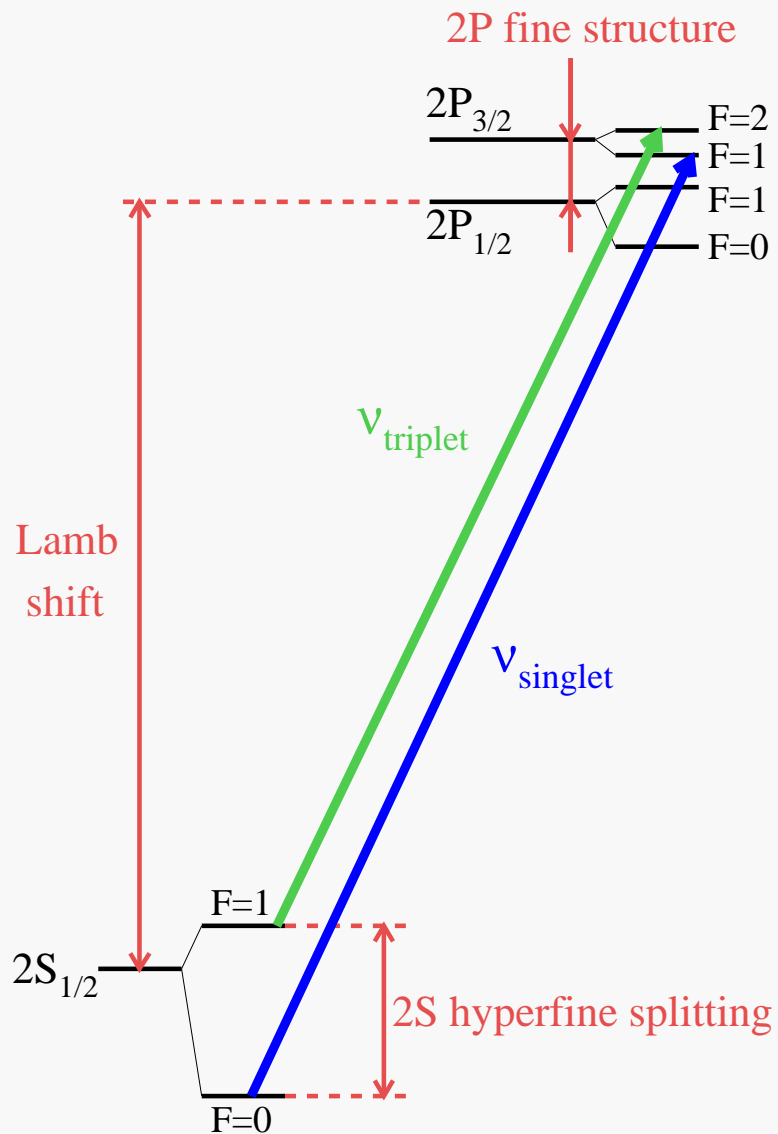


Discrepancy:

$$5.0 \sigma \leftrightarrow \sim 75 \text{ GHz} \leftrightarrow \delta\nu/\nu = 1.5 \times 10^{-3}$$

Pohl *et al.*, Nature 466, 213 (2010)

# We have measured two transitions in $\mu\text{p}$





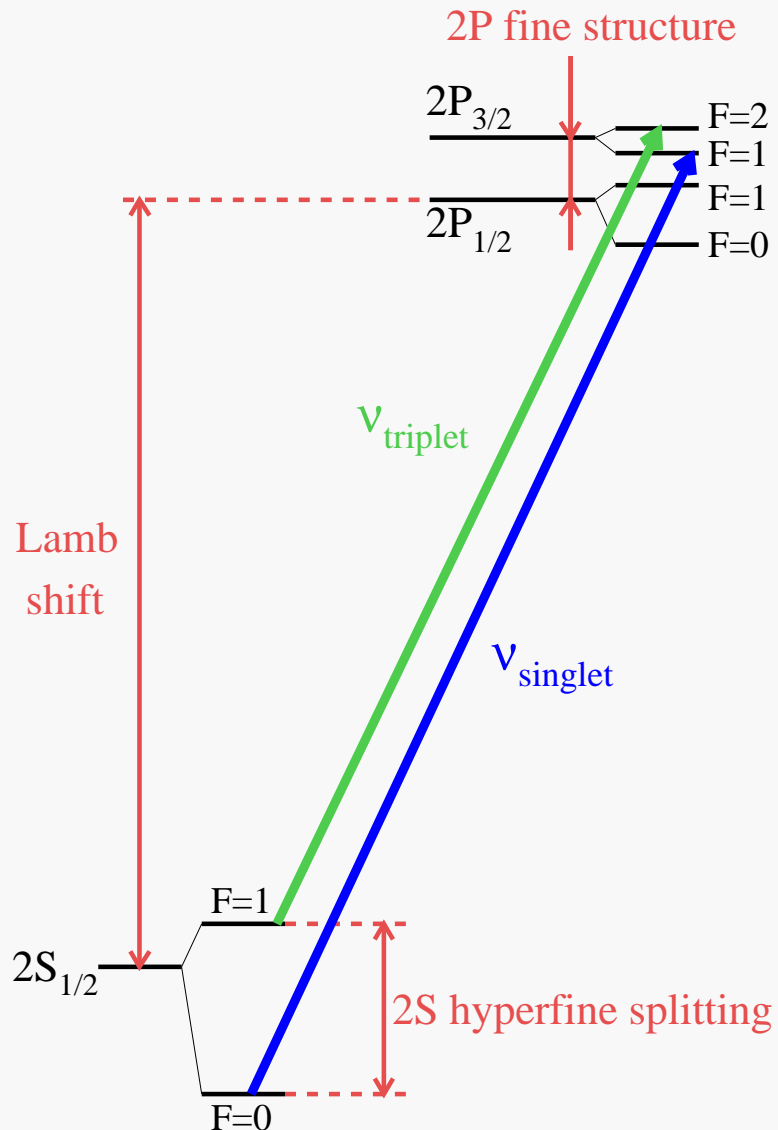
# We have measured two transitions in $\mu\text{p}$

- Considering the two measurements separately

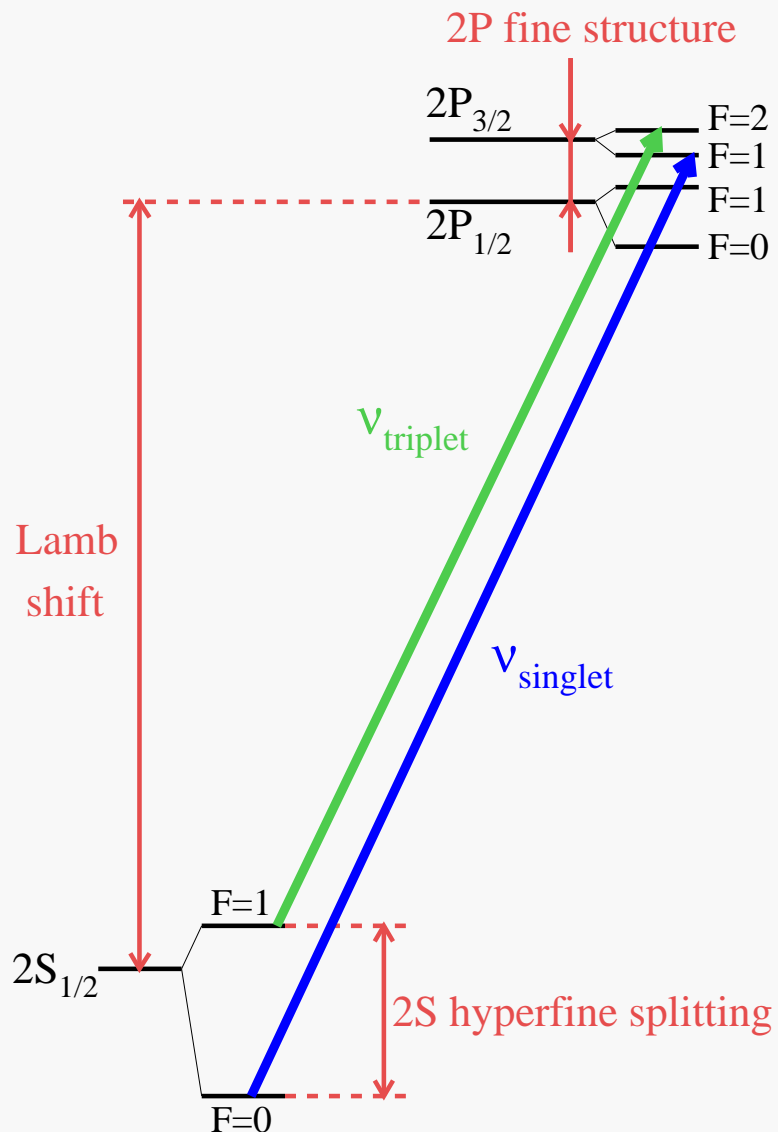
Two independent determinations of  $r_p$

$$(\nu_t \rightarrow r_p, \nu_s \rightarrow r_p)$$

Consistent results !!!



# We have measured two transitions in $\mu\text{p}$



- Considering the two measurements separately

Two independent determinations of  $r_p$

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Consistent results !!!

- Combining the two measurements

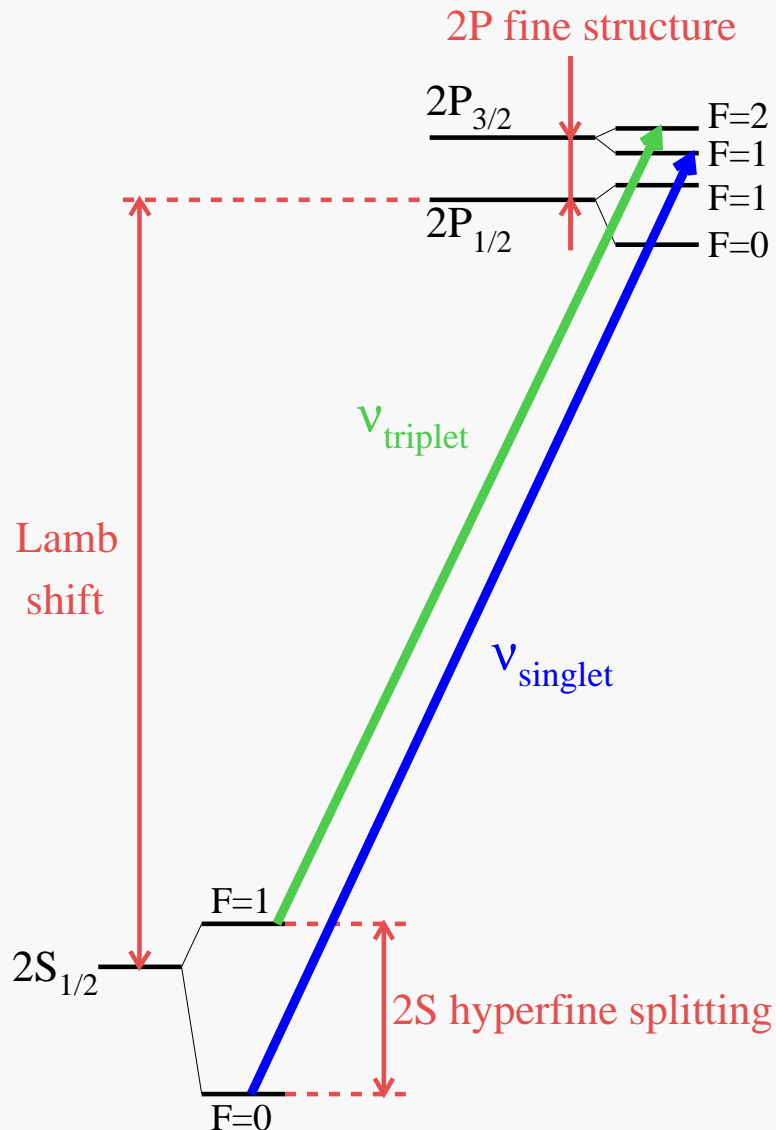
Two measurements  $\rightarrow$  determine two parameters

$$\nu_t, \nu_s \rightarrow \Delta E_L, \Delta E_{\text{HFS}} \rightarrow r_p, r_Z$$

$$r_Z = \int d^3r_1 d^3r_2 \rho_E(r_1) \rho_M(r_2) |r_1 - r_2|$$

$$\begin{aligned} \frac{3}{4} \nu_t + \frac{1}{4} \nu_s &= \Delta E_L(r_p) + 8.8123 \text{ meV} \\ \nu_s - \nu_t &= \Delta E_{\text{HFS}}(r_Z) - 3.2480 \text{ meV} \end{aligned}$$

# We have measured two transitions in $\mu\text{p}$



- Considering the two measurements separately

Two independent determinations of  $r_p$

$$(\nu_t \rightarrow r_p, \nu_s \rightarrow r_p)$$

Consistent results !!!

Using the 2S-HFS prediction

- Combining the two measurements

Two measurements  $\rightarrow$  determine two parameters

$$\nu_t, \nu_s \rightarrow \Delta E_L, \Delta E_{\text{HFS}} \rightarrow r_p, r_Z$$

$$r_Z = \int d^3r_1 d^3r_2 \rho_E(r_1) \rho_M(r_2) |r_1 - r_2|$$

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New  $r_p$  does NOT depend on 2S-HFS prediction

# Results on $\mu p$ : $r_p$

$$\nu(2S_{1/2}^{F=1} \rightarrow 2P_{3/2}^{F=2}) = 49881.88(76) \text{ GHz}$$

Pohl *et al.*, Nature 466, 213 (2010)

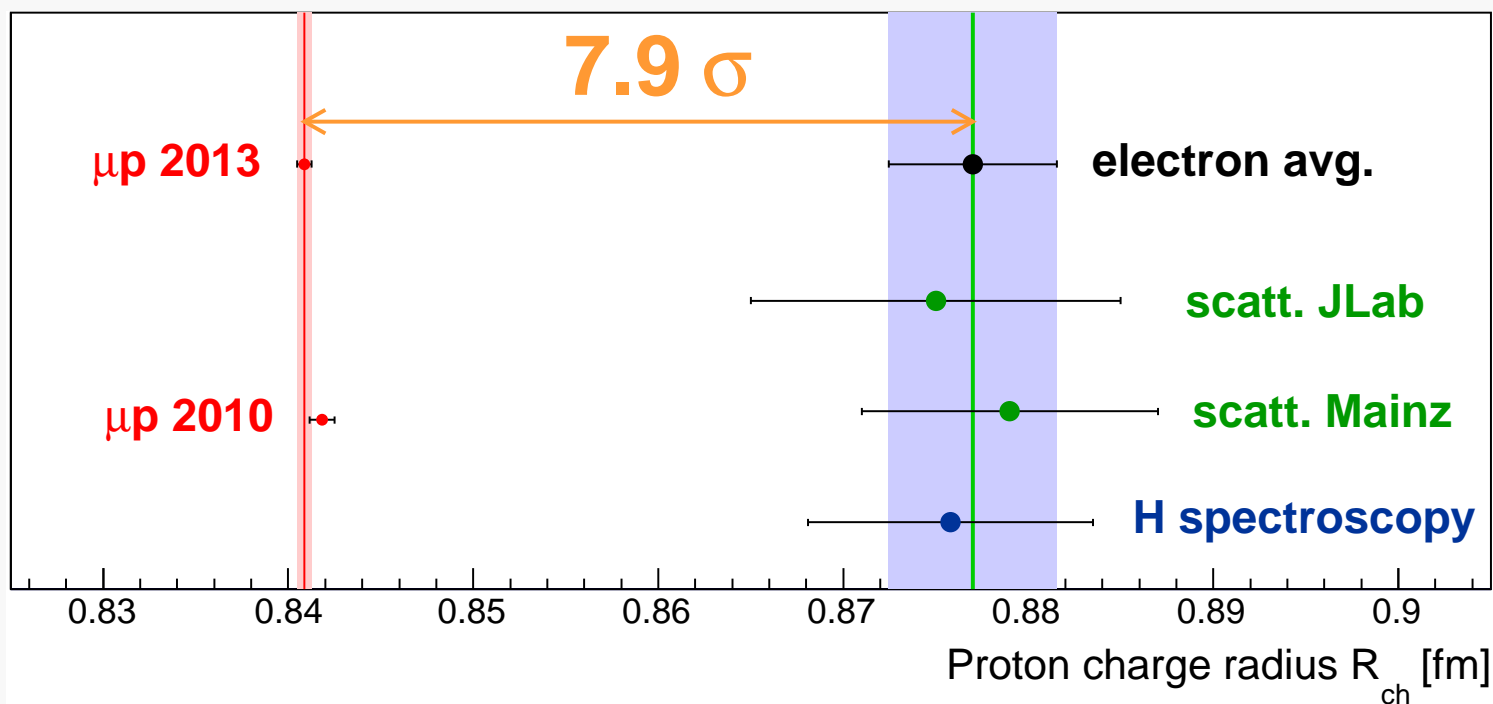
$$49881.35(65) \text{ GHz}$$

$$\nu(2S_{1/2}^{F=0} \rightarrow 2P_{3/2}^{F=1}) = 54611.16(1.05) \text{ GHz}$$

Antognini *et al.*, Science 339, 417 (2013)

⇒ Proton charge radius:  $r_p = 0.84087(26)_{\text{exp}}(29)_{\text{th}} = 0.84087(39) \text{ fm}$

using  $\mu p$  theory summary: Antognini *et al.*, Ann. Phys. 331, 127 (2013) [arXiv:1208.2637]



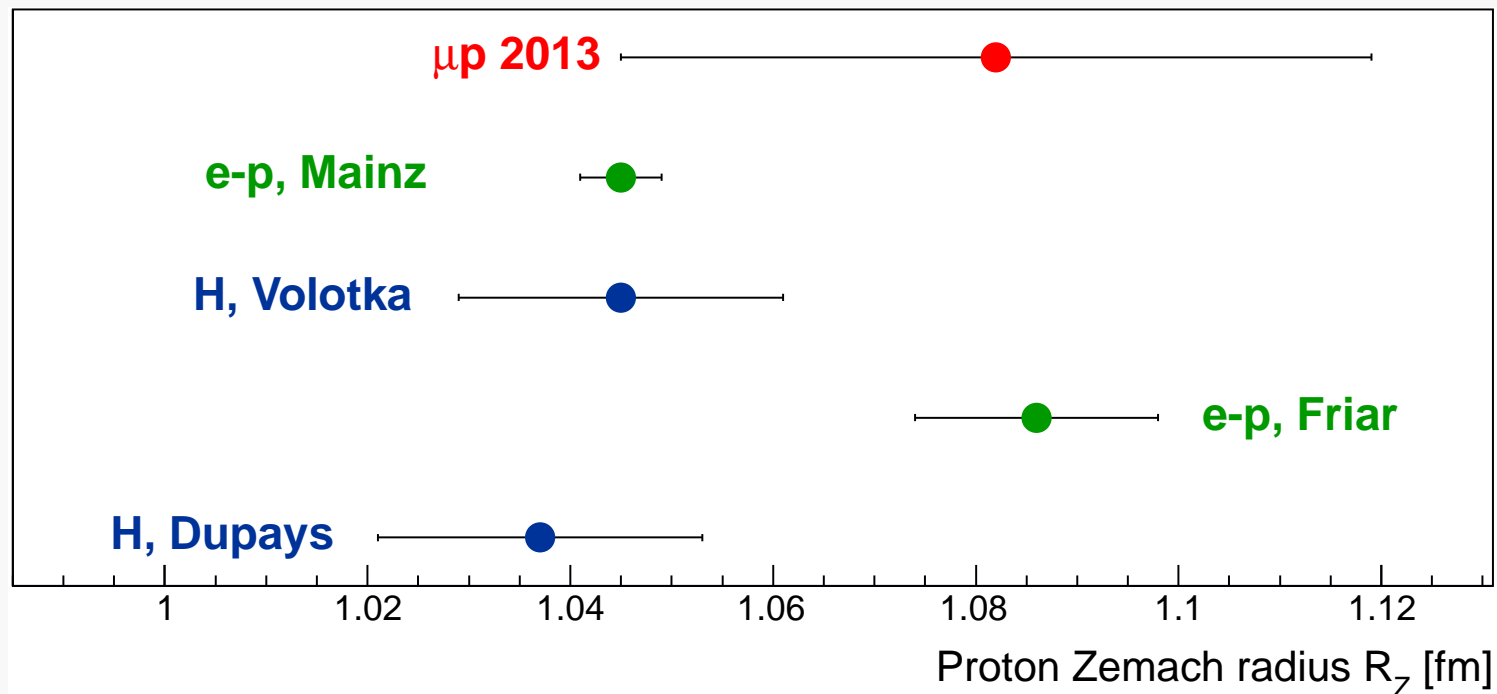
# The 2S-HFS in $\mu p$ and Zemach radius $r_Z$

Difference of the two transitions  $\rightarrow$  2S-HFS in  $\mu p$ :  $\Delta E_{\text{HFS}} = 22.8089(51) \text{ meV}$

$\Rightarrow$  Proton Zemach radius:  $r_Z = 1.082(31)_{\text{exp}}(20)_{\text{th}} = 1.082(37) \text{ fm}$

$$r_Z = \int d^3r_1 d^3r_2 \rho_E(r_1) \rho_M(r_2) |r_1 - r_2|$$

Contains information of the magnetic distributions of the proton



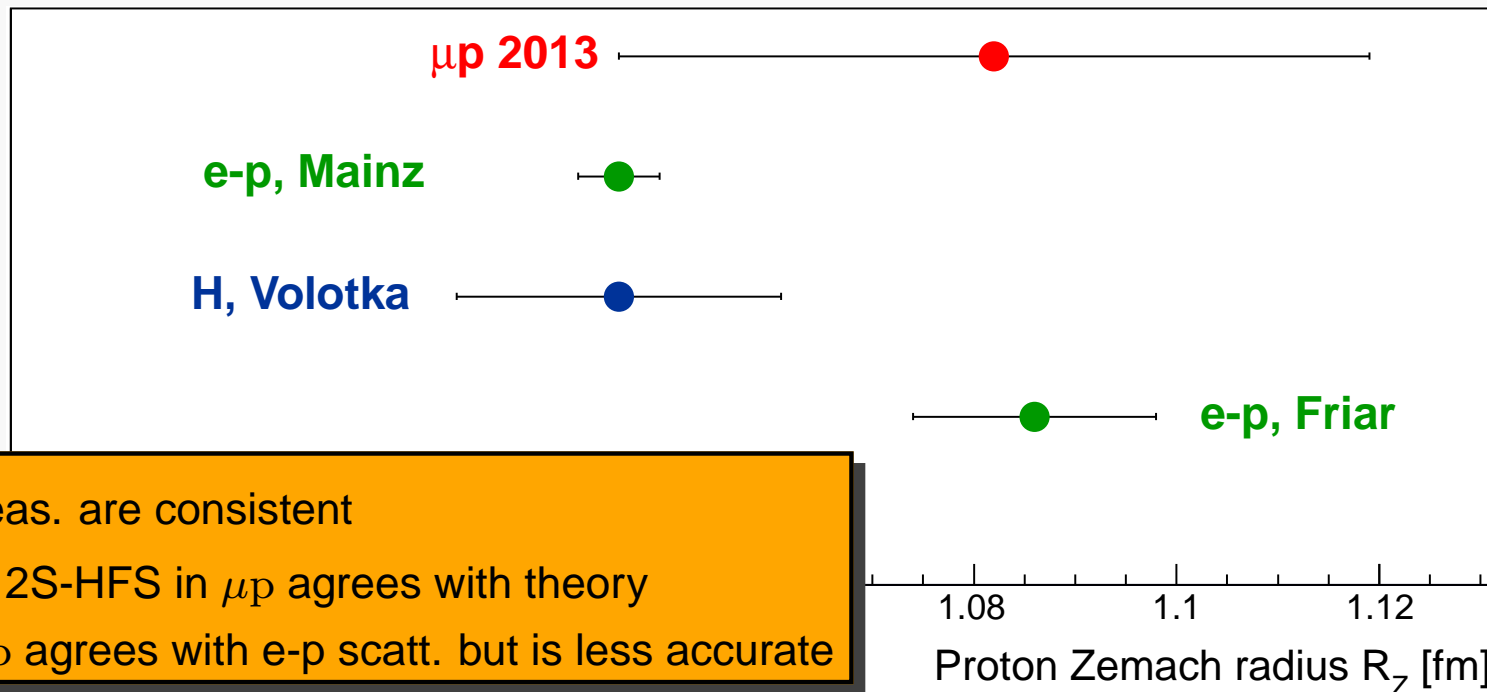
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Contains information of the magnetic distributions of the proton



- the two  $\mu p$  meas. are consistent
- the measured 2S-HFS in  $\mu p$  agrees with theory
- the  $r_Z$  from  $\mu p$  agrees with e-p scatt. but is less accurate

# Proton radius puzzle: What may be wrong?



Bound-state QED?

Proton structure?

Measurements?

Definition p-radius?

“New physics”?

More than 250 publications

# Politically correct discussion



Everybody is right!..?



# Proton radius puzzle: What may be wrong?

(2)  $\mu\text{p}$  theory wrong? but

- mainly pure QED (vac.pol., etc.)
- 'huge' relative discrepancy
- hadronic terms small
- pol. term = 0.015(4) meV

(1)  $\mu\text{p}$  exp. wrong? but

- good statistics ( $\sigma = 0.76 \text{ GHz} \ll$  discrepancy)
- linewidth  $\sim 19 \text{ GHz} \ll$  discrepancy
- several methods for frequency calibration
- another  $\mu\text{p}(2\text{S}-2\text{P})$  measured!

$$\Delta E_{\mu\text{p}}^{\text{th.}}(r_p^{\text{CODATA}}) - \Delta E_{\mu\text{p}}^{\text{exp.}} = \begin{cases} 75 \text{ GHz} \\ 0.31 \text{ meV} \\ 0.15 \% \end{cases}$$

(3) e-p scattering wrong? but

- new Mainz and JLab results ...

(4) H spectroscopy wrong? but

- 2S-8S, 2S-8D, 2S-12S, etc. all consistent ...

(5) H theory wrong? but

- uncertainties at least  $25\times$  smaller than discrepancy ...



# $r_p$ puzzle (1): Is the $\mu p$ experiment wrong ?

## • Systematics?

- laser frequency calibration 300 MHz
- Zeeman effect ( $B = 5$  Tesla) 30 MHz
- AC-Stark, DC-Stark shift  $< 1$  MHz
- Doppler shift  $< 1$  MHz
- pressure shift (1 mbar) 1 MHz

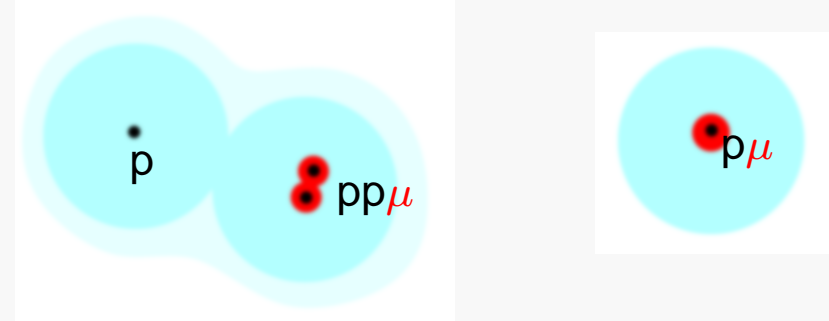
Systematics shift  $\sim 1/m$

Finite size shift  $\sim m^3$

## • Frequency mistake by 75 GHz ?

- **Huge** difference for laser spectroscopy accuracies
- Two ways to calibrate the frequency (consistent)

## • Spectroscopy of $pp\mu$ molecules or $p\mu e$ ions?



Do not exist or too short lived (in 2S state)

Karr and Hilico, PRL 109, 103401 (2012)

Pohl *et al.*, PRL 97, 193402 (2006)

Discrepancy = 75 GHz  $\approx 4\Gamma$

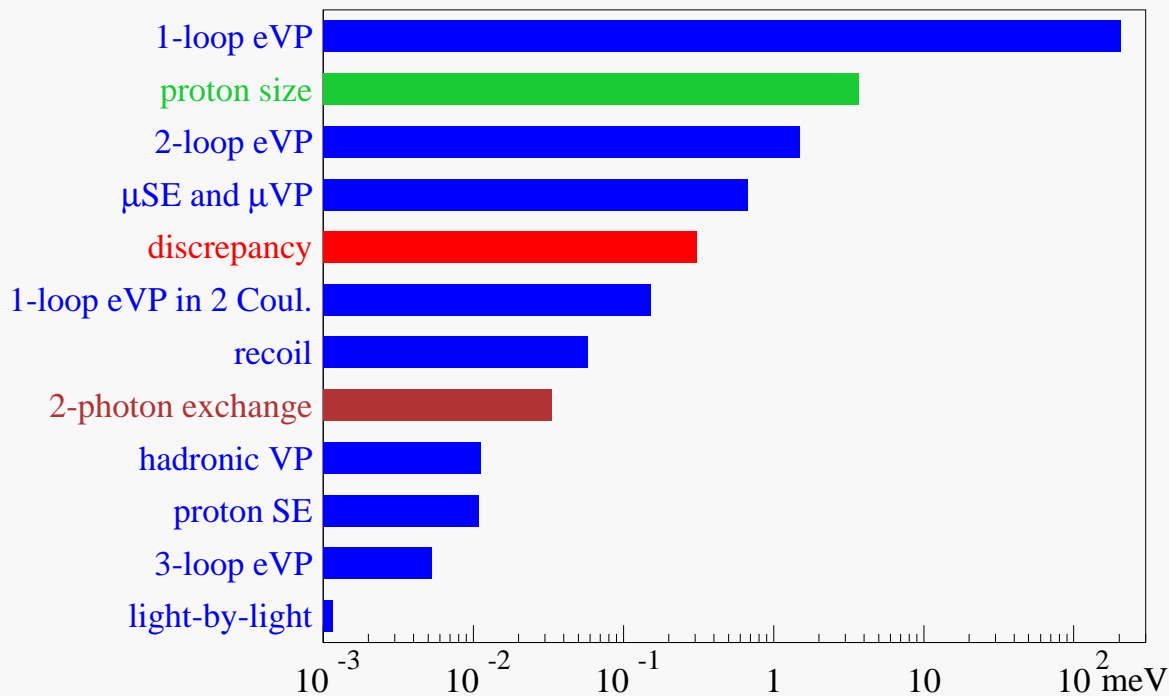
Two consistent  $\mu p$  transition measurements

$\mu p$  experiment is probably not wrong by 100  $\sigma$

# $r_p$ puzzle (2): Is the $\mu p$ theory wrong?

Discrepancy = 0.31 meV  
Theory uncertainty = 0.0025 meV  
 $\Rightarrow 120\delta(\text{theory})$  deviation?

$$\Delta E^{\text{th}} = 206.0668(25) - 5.2275(10) r_p^2 \text{ [meV]}$$



Pachucki, PRA 60, 3593 (1999)

Borie, arXiv: 1103.1772-v6

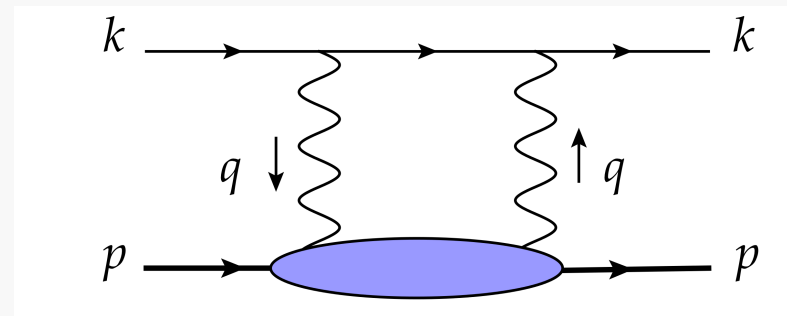
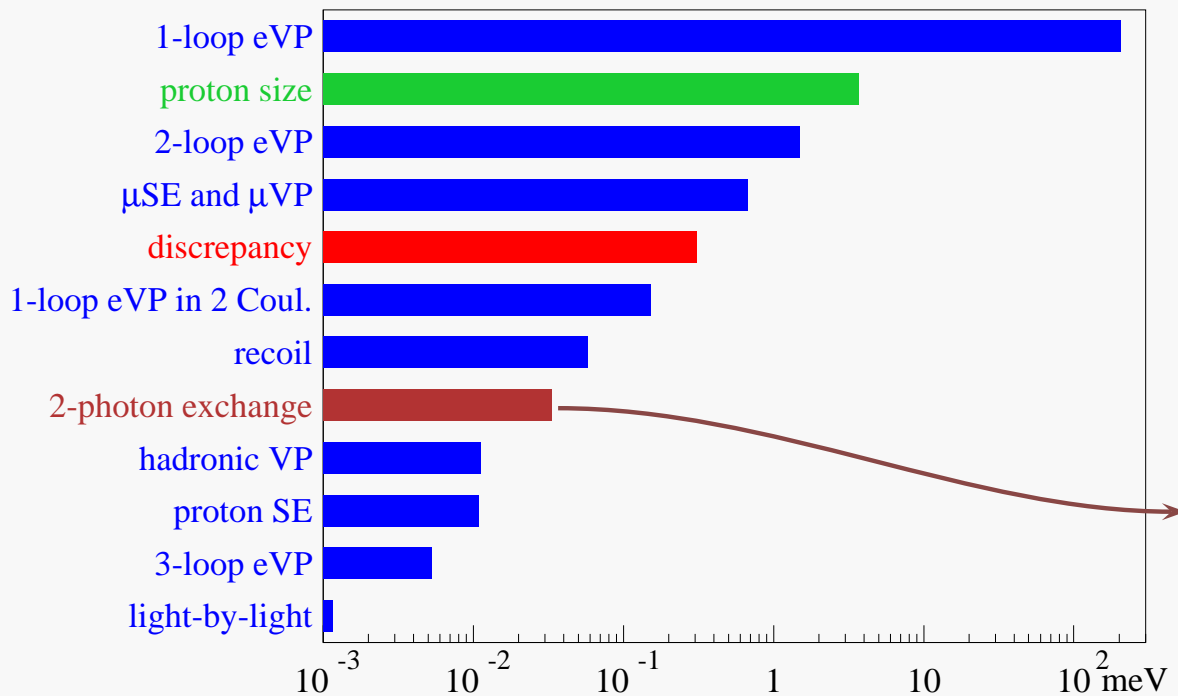
Jentschura, Ann. Phys. 326, 500 (2011)

Karshenboim *et al.*, PRA 85, 032509 (2012)

# $r_p$ puzzle (2): Is the $\mu p$ theory wrong?

Discrepancy = 0.31 meV  
 Theory uncertainty = 0.0025 meV  
 $\Rightarrow 120\delta(\text{theory})$  deviation?

$$\Delta E^{\text{th}} = 206.0668(25) - 5.2275(10) r_p^2 \text{ [meV]}$$



Pachucki, PRA 60, 3593 (1999)  
 Borie, arXiv: 1103.1772-v6  
 Jentschura, Ann. Phys. 326, 500 (2011)  
 Karshenboim *et al.*, PRA 85, 032509 (2012)

Carlson *et al.*, PRA 84, 020102 (2011)  
 McGovern and Birse, EPJA 48 120 (2012)  
 Peset and Pineda, arXiv:1406.4524  
 Alarcon *et al.*, arXiv:1312.1219

# $r_p$ puzzle (2): Is $\mu_p(2S-2P)$ theory wrong?

Higher order finite size effects

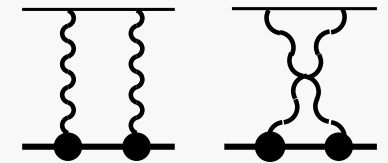
Potential corr.

Wave function corr.

$$\Psi(r) \approx \Psi(0) \left( 1 - m_r \alpha \int d^3r' \rho(\vec{r}') |\vec{r} - \vec{r}'| + \dots \right)$$



$$E_{FS} = -\frac{2\pi\alpha}{3} |\Psi(0)|^2 \left[ r_p^2 - \frac{\alpha}{2} m_r \langle r_p^3 \rangle_{(2)} + \dots \right]$$



Discrepancy = 0.31 meV

3.7 meV

0.02 meV

Third Zemach moment:

$$\langle r_p^3 \rangle_{(2)} = \int d^3r \int d^3r' \rho(\vec{r}) \rho(\vec{r}') |\vec{r} - \vec{r}'|^3$$

This term is important for  $\mu_p$

# $r_p$ puzzle (2): Is $\mu_p(2S-2P)$ theory wrong?

Can we find a p-shape to solve the discrepancy?

In principle yes  $\Leftrightarrow \langle r_p^3 \rangle_{(2)} = 37(7) \text{ fm}^3$

[PL B 693, 555 (2010)]

Third Zemach moment:

$$\langle r_p^3 \rangle_{(2)} = \int d^3r \int d^3r' \rho_E(\vec{r}) \rho_E(\vec{r}') |\vec{r} - \vec{r}'|^3$$

## Ever-changing proton



### BEFORE JULY 2010

Experiments with hydrogen suggest proton radius is 0.877 femtometres and halo is 1.394 fm



### JULY 2010

Exotic-hydrogen experiments suggest radius is 4% smaller. Halo unchanged



### AUGUST 2010

New calculations bring back former proton radius, but with a halo that is  $\sim 4\frac{1}{2}$  times as large

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In principle yes  $\Leftrightarrow \langle r_p^3 \rangle_{(2)} = 37(7) \text{ fm}^3$

[PLB 693, 555 (2010)]



$\Leftrightarrow$  But community not very happy

Measurable

$$\langle r_p^3 \rangle_{(2)} = \frac{48}{\pi} \int \frac{dq}{q^4} [G_E^2(q^2) - 1 + \frac{1}{3} q^2 \langle r_p^2 \rangle]$$

$$\langle r_p^3 \rangle_{(2)} = 2.71(13) \text{ fm}^3 \quad [\text{PRA } 72 \text{ 040502 (2005)}]$$

$$\langle r_p^3 \rangle_{(2)} \leq 4.5 \text{ fm}^3 \quad [\text{PRC } 83, \text{ 012201 (2011)}]$$

$$\langle r_p^3 \rangle_{(2)} = 2.85(8) \text{ fm}^3 \quad [\text{PLB } 696, \text{ 343 (2011)}]$$

$$\langle r_p^3 \rangle_{(\chi\text{PT})} \sim \langle r_p^3 \rangle_{(\text{experiments})} \quad [\text{hep-ph/0412142}]$$

## Ever-changing proton



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# $r_p$ puzzle (2): Is $\mu_p(2S-2P)$ theory wrong?

## Two ways to the $2\gamma$ exchange

Chiral EFT

Phenomenological:  
dispersion relations  
+ data



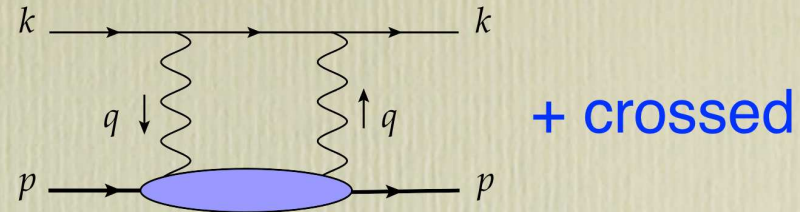
Both agree but...



# $r_p$ puzzle (2): Is $\mu_p(2S-2P)$ theory wrong?

## Two photon exchange contribution to Lamb shift

Kinematics: 2 loop variables  
 $q^2$  and  $\nu=(pq)/M$



$$\mathcal{M} = e^4 \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^4} \bar{u}(k) \left[ \gamma^\nu \frac{1}{\not{k} - \not{q} - m_l + i\epsilon} \gamma^\mu + \gamma^\mu \frac{1}{\not{k} + \not{q} - m_l + i\epsilon} \gamma^\nu \right] u(k) T_{\mu\nu}$$

### Forward virtual Compton amplitude

$$\begin{aligned} T^{\mu\nu} &= \frac{i}{8\pi M} \int d^4 x e^{iqx} \langle p | T j^\mu(x) j^\nu(0) | p \rangle \\ &= \left( -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) T_1(\nu, Q^2) + \frac{1}{M^2} \left( p - \frac{pq}{q^2} q \right)^\mu \left( p - \frac{pq}{q^2} q \right)^\nu T_2(\nu, Q^2) \end{aligned}$$

### Lamb shift (nS-nP)

$$\Delta E = -\frac{\alpha^2}{2\pi m_l M_d} \phi_n^2(0) \int d^4 q \frac{(q^2 + 2\nu^2) T_1(\nu, q^2) - (q^2 - \nu^2) T_2(\nu, q^2)}{q^4 [(q^2/2m_l)^2 - \nu^2]}$$

[Slide stolen from Gorshteyn]

# $r_p$ puzzle (2): Is $\mu_p(2S-2P)$ theory wrong?

## Two photon exchange contribution to Lamb shift

$T_1, T_2$  - the imaginary parts known (Optical theorem)

$$\text{Im}T_1(\nu, Q^2) = \frac{1}{4M} F_1(\nu, Q^2) \quad \text{Inelastic structure functions = data}$$
$$\text{Im}T_2(\nu, Q^2) = \frac{1}{4\nu} F_2(\nu, Q^2) \quad (\text{real and virtual photoabsorption, FF})$$

Real parts - from forward dispersion relation

$$F_1(\nu \rightarrow \infty, q^2) \sim \nu^{1+\epsilon} \quad \text{- subtraction needed}$$

$$F_2(\nu \rightarrow \infty, q^2) \sim \nu^\epsilon \quad \text{- no subtraction}$$

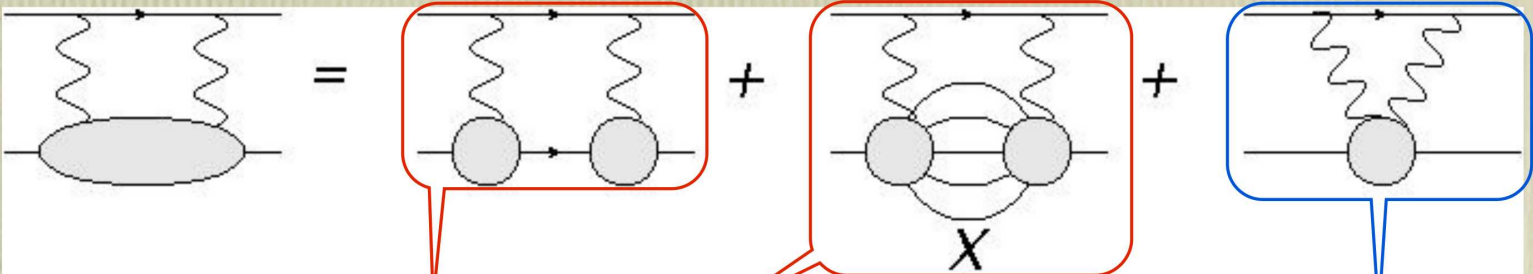
$$\text{Re}T_1(\nu, Q^2) = \bar{T}_1(0, Q^2) + T_1^{pole}(\nu, Q^2) + \frac{\nu^2}{2\pi M} \int_{\nu_0}^{\infty} \frac{d\nu'}{\nu(\nu'^2 - \nu^2)} F_1(\nu', Q^2)$$

$$\text{Re}T_2(\nu, Q^2) = T_2^{pole}(\nu, Q^2) + \frac{1}{2\pi} \int_{\nu_0}^{\infty} \frac{d\nu'}{\nu'^2 - \nu^2} F_2(\nu', Q^2)$$

[Slide stolen from Gorshteyn]

# $r_p$ puzzle (2): Is $\mu_p(2S-2P)$ theory wrong?

## TPE from Dispersion Relations



The diagram shows the decomposition of a transition polarizability (TPE) into three parts: a dispersive part, a subtraction constant, and a model part. The first part is labeled 'Dispersion Relation + Data' and the second 'Subtraction Constant'. The third part is labeled 'Model + data'.

$$\Delta E = \int_0^\infty dQ^2 \int_{\nu_0}^\infty d\nu [\text{DATA}]$$

Model + data

[Slide stolen from Gorshteyn]

The controversy:  
How well do we know the subtraction term?

# $r_p$ puzzle (2): Is $\mu_p(2S-2P)$ theory wrong?

- The subtraction term  $W_1(0, Q^2)$  is NOT determined by the imaginary part (data)

$W_1(0, Q^2)$	known at small $Q^2$	via <b>NRQED</b> + Wilson coeff. from data
	<b>NOT</b> known at intermediate $Q^2$	( $\gamma p \rightarrow l^+ l^- p'$ planned at HIGS, Duke)
	known at large $Q^2$	from OPE expansion

Uncertainty of this term underestimated? [PRL107,160402 (2011), Miller PLB 2012]

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- Could the two-photon exchange explain the discrepancy?

<b>Unknown:</b>	Could be MUCH larger as previously assumed	Hill <i>and</i> Paz, PRL 107, 160402 (2011), Miller
<b>Under control:</b>	Direct calc. of whole contribution in LO $\chi$ PT	Nevado <i>and</i> Pineda, PRC 77, 035202 (2008)
<b>Under control:</b>	$\chi$ PT expansion to bridge low- $Q^2$ to high- $Q^2$	McGovern <i>and</i> Birse, EPJA 48 120 (2012)
<b>Under control:</b>	Sum rule + Regge +...photoabsorbtion data	Gorchtein <i>et al</i> , PRA 84, 052501 (2013)
<b>Under control:</b>	Barion $\chi$ PT + $\Delta(1232)$ contribution	Alarcón <i>et al</i> , arXiv 1312.1219
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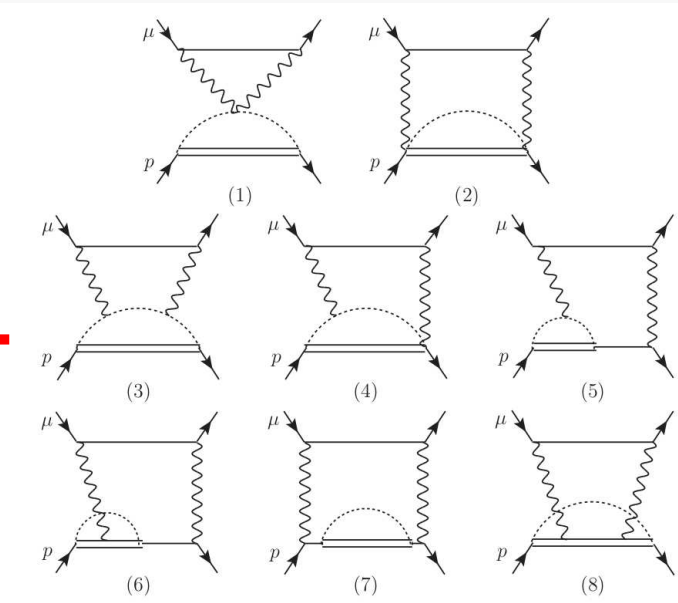
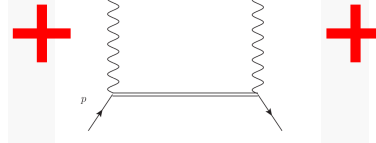
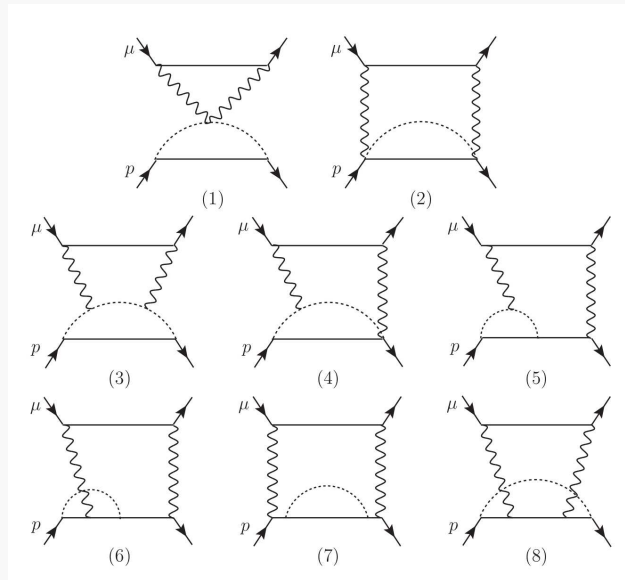
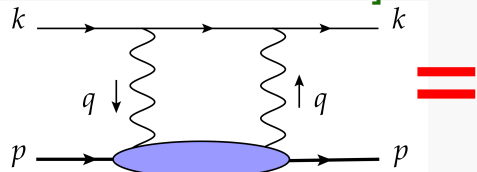
$$\Delta E_{\text{sub}} = -0.0042(10) \text{ meV} \longleftrightarrow \text{Discrepancy}=0.3 \text{ meV}$$

# $r_p$ puzzle (2): Is $\mu p(2S-2P)$ theory wrong ?

$B\chi PT$  vs  $HB\chi PT$ :

Main part of pol. contribution comes from the low  $Q^2$  regime  $\rightarrow$  Chiral EFT

[Peset and Pineda,  
arXiv:1406.4524]



Two approaches have been developed:

$B\chi PT$  [Pascalutsa, Lensky, Alarcon] and  $HB\chi PT$  [Pineda, Nevado, Peset]

There are some not yet understood disagreement between the two approaches.

However when summing up all contributions to the TPE

$$\Delta E_{TPE} = 33(2) \mu eV \text{ (Dispersive approach)}$$

$$\Delta E_{TPE} = 34(12) \mu eV \text{ (HB}\chi PT)$$

# $r_p$ puzzle (2): Is $\mu_p(2S-2P)$ theory wrong ?

## Polarizability contribution

( $\mu\text{eV}$ )	DR + Model	[33]	[34]	[35]	[36]	$B\chi\text{PT}$ [22] ( $\pi$ )	HBET [6] ( $\pi$ )	[12] ( $\pi\&\Delta$ )
$\Delta E_{\text{pol}}$		12(2)	11.5	7.4(2.4)	15.3(5.6)	8.2( $^{+1.2}_{-2.5}$ )	18.5(9.3)	26.2(10.0)

- [33] Pachucki, PRL A 60, 3593, (1999)
- [34] Martynenko, hep-ph/0509236
- [35] Carlson and Vanderhaeghen, PRA 84, 020102 (2011)
- [36] Gorchtein et al., PRA 87, 052501
- [22] Alarcon et al., EPJC 74, 2854 (2014)
- [6] Nevado and Pineda, PRC 77, 035202 (2008)
- [12] Peset and Pineda, arXiv:1403.3408

Very interesting  
physics

## Proton charge moments

	$\langle r^3 \rangle$	$\langle r^4 \rangle$	$\langle r^5 \rangle$	$\langle r^6 \rangle$	$\langle r^7 \rangle$	$\langle r^3 \rangle_{(2)}$
$\pi$	0.4980	0.6877	1.619	5.203	20.92	0.9960
$\pi\&\Delta$	0.4071	0.6228	1.522	4.978	20.22	0.8142
[25]	0.7706	1.083	1.775	3.325	7.006	2.023
[26]	0.9838	1.621	3.209	7.440	19.69	2.526
[27]	1.16(4)	2.59(19)(04)	8.0(1.2)(1.0)	29.8(7.6)(12.6)	— — —	2.85(8)

- [25] Janssens et al., PR 142, 922 (1966)
- [26] Kelly, PRC 70, 068202 (2004)
- [27] Distler et al, PLB 696, 343 (2011)

[Peset and Pineda,  
arXiv:1406.4524]



# $r_p$ puzzle (2): Is the $\mu p$ theory wrong?

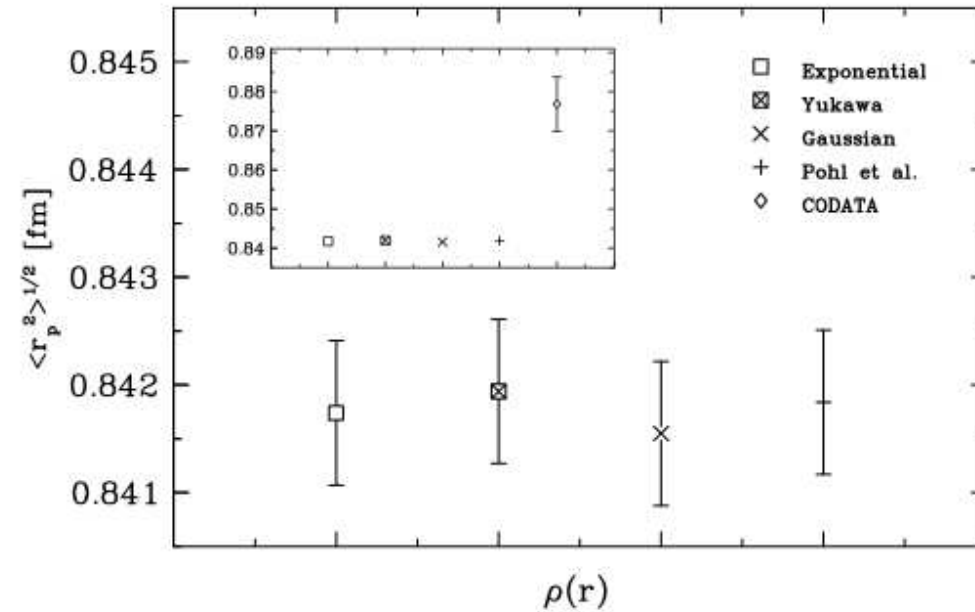
Can we find a p-shape to solve the discrepancy?

NO, but the question is interesting. DeRujula

How does the radius extracted from  $\mu p$  depends on the assumed proton shape? [Miller]

Finite size contributions

$$\Delta E_{\text{finite size}} = \sum_n a_n \langle r_p^n \rangle$$



bound-state QED expansion	→	$a_n$ decreases rapidly	Friar, Indelicato
e-p scattering data	→	$\langle r_p^n \rangle$ sufficiently small for $n < 6$	Distler, Miller
$\chi$ PT	→	$\langle r_p^n \rangle(\text{HB}\chi\text{PT}) < \langle r_p^n \rangle(\text{scatt})$	Peset, Pineda

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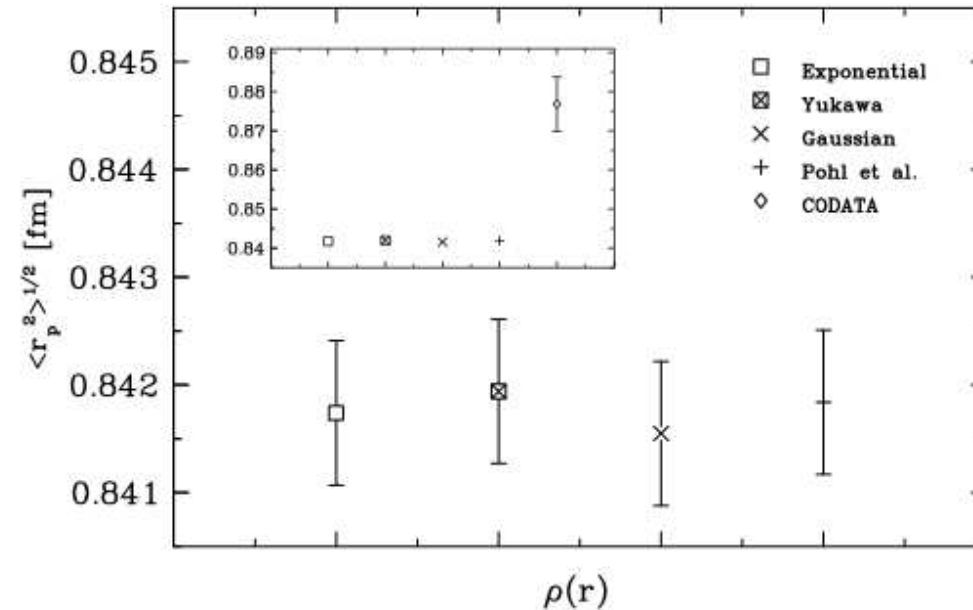
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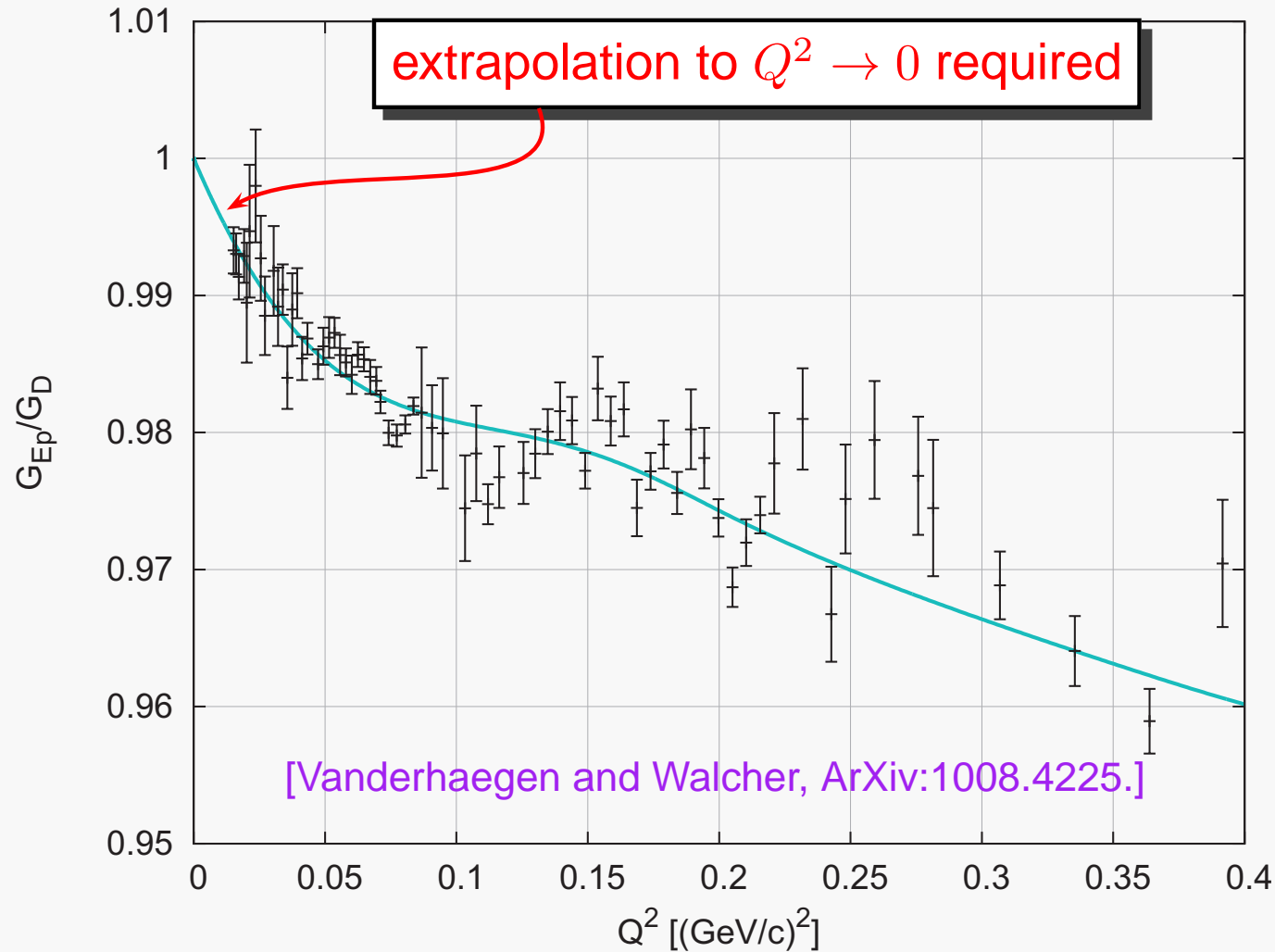
Chiral EFT are important for 3 reasons:

- provide a value of the TPE (polarizability + elastic) contribution
- give values of the various charge moments of the proton
- provide a model of the proton shape that could be used to analyze scattering data

# $r_p$ puzzle (5): Is e-p scattering wrong ?

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Ros.}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{1}{(1 + \tau)} \left( \varepsilon G_E^2(Q^2) + \tau G_M^2(Q^2) \right)$$

$$\langle r_p^2 \rangle = -6\hbar^2 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0}$$



— Spline fit      —+— Rosenbluth Separation

# $r_p$ puzzle (5): Is e-p scattering wrong ?

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$$\langle r_p^2 \rangle = -6\hbar^2 \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0}$$

Needs a fit  
Model dependence?

Sick, PLB 576, 62 (2003)

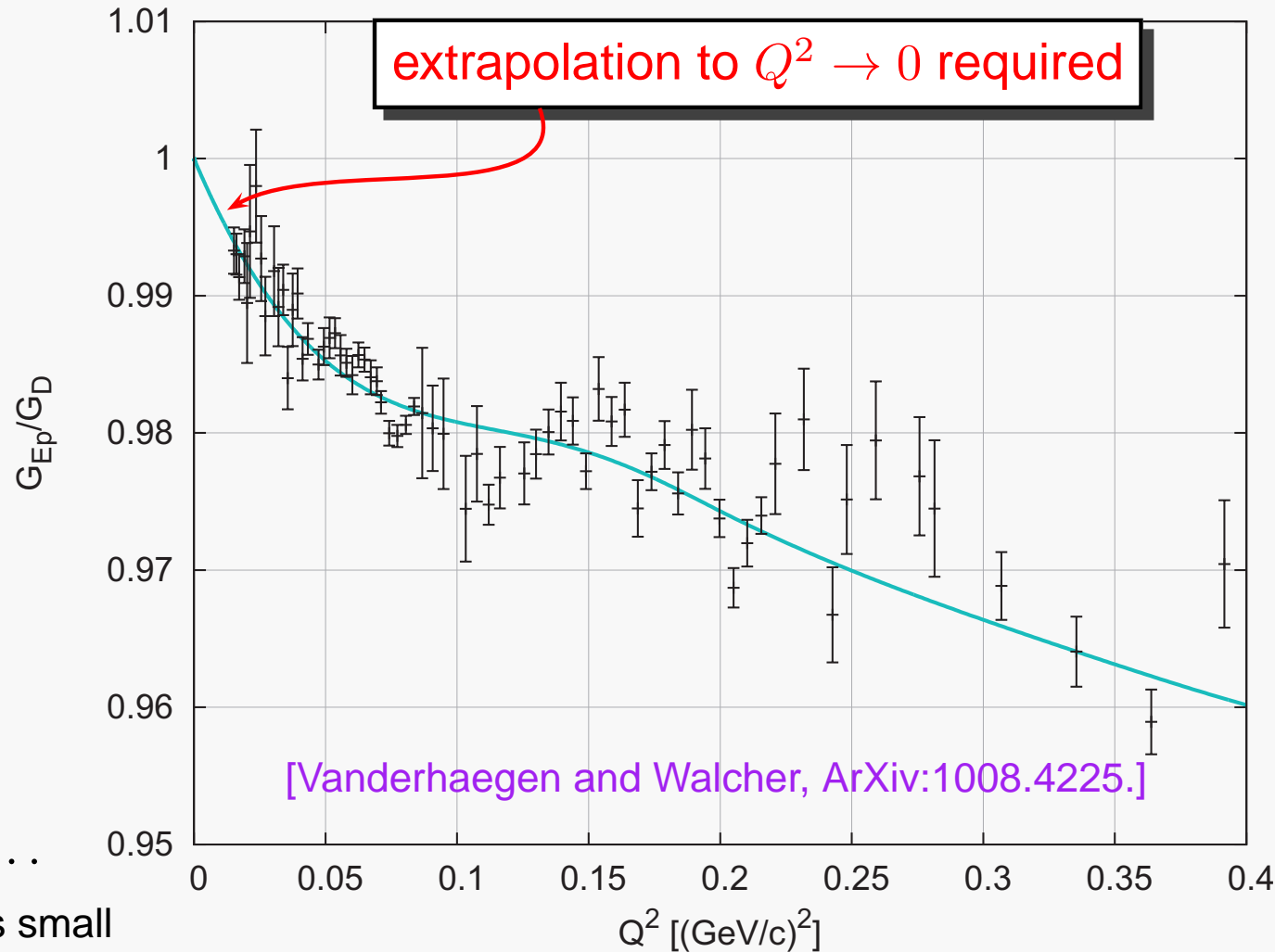
Hills and Paz, PRD 82, 113005 (2010)

Bernauer et al, PRL 105, 242001 (2010)

Lorenz and Meissner, arXiv:1406.2962

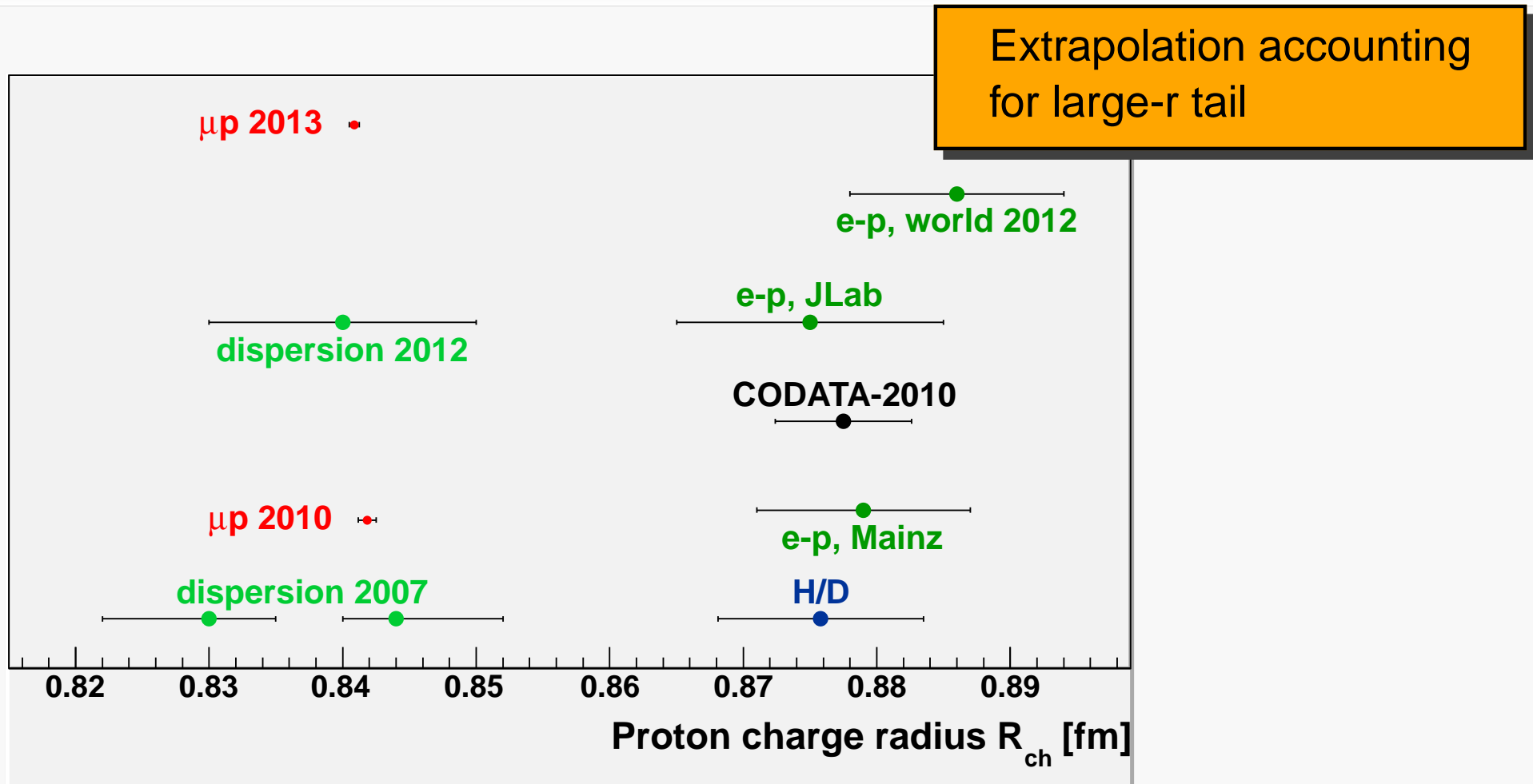
$$G_E(Q^2) = 1 + \frac{Q^2}{6} \langle r_p^2 \rangle + \frac{Q^4}{120} \langle r_p^4 \rangle + \dots$$

- Very low  $Q^2$  yields slope but sensitivity is small
- Larger  $Q^2$  more sensitive but larger higher-order terms



— Spline fit      —+— Rosenbluth Separation

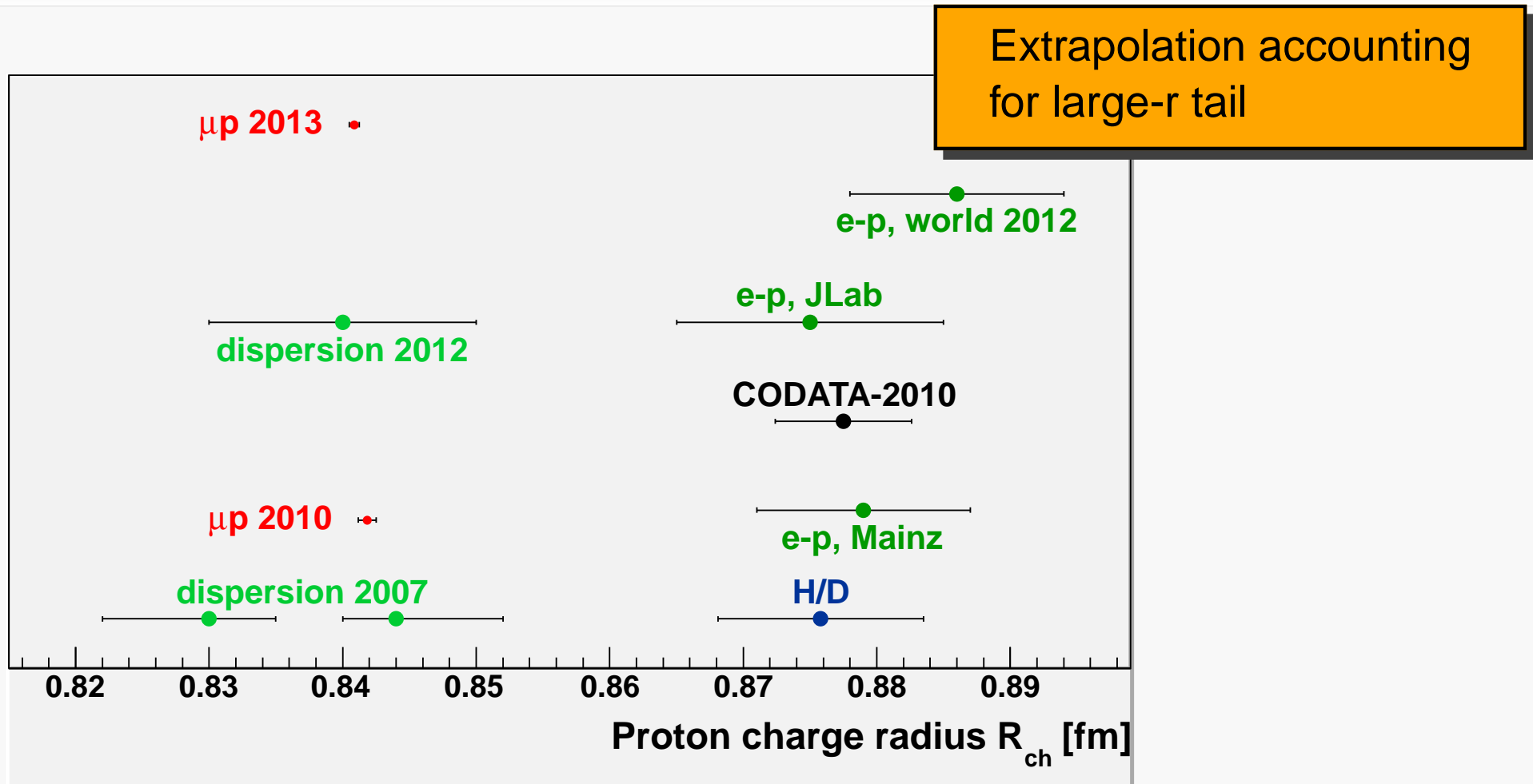
# Proton charge radii



Analysis of e-p, e-n scattering data using VMD and dispersion relations gives radii in agreement with  $\mu p$  (arXiv:1406.2962)

Extrapolation of scattering data?

# Proton charge radii



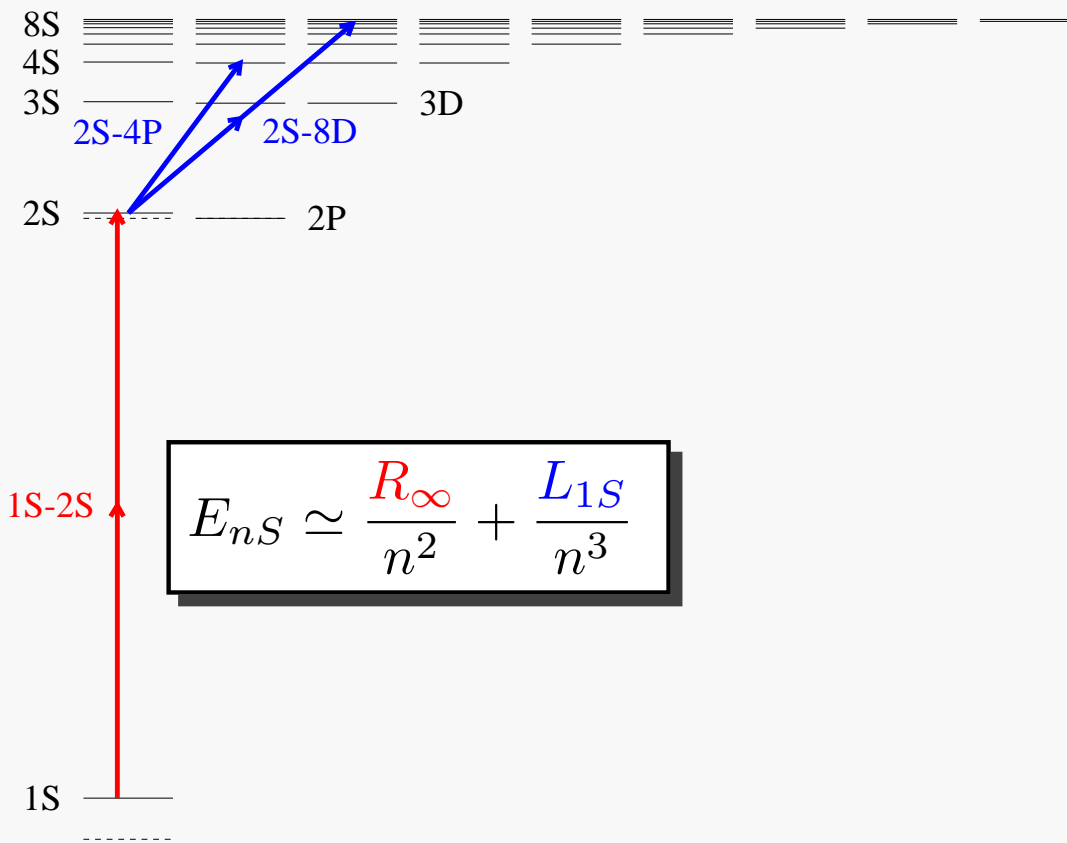
Analysis of e-p, e-n scattering data using VMD and dispersion relations gives radii in agreement with  $\mu p$  (arXiv:1406.2962)

Extrapolation of scattering data?

What about H?

# $r_p$ puzzle (3): Is H-spectroscopy wrong ?

Two measurements  $\rightarrow$  two unknown:  $R_\infty$  and  $L_{1S}^{\text{exp}}$



$$L_{1S}^{\text{th}}(r_p) = 8171.636(4) + 1.5645 r_p^2 \text{ MHz}$$

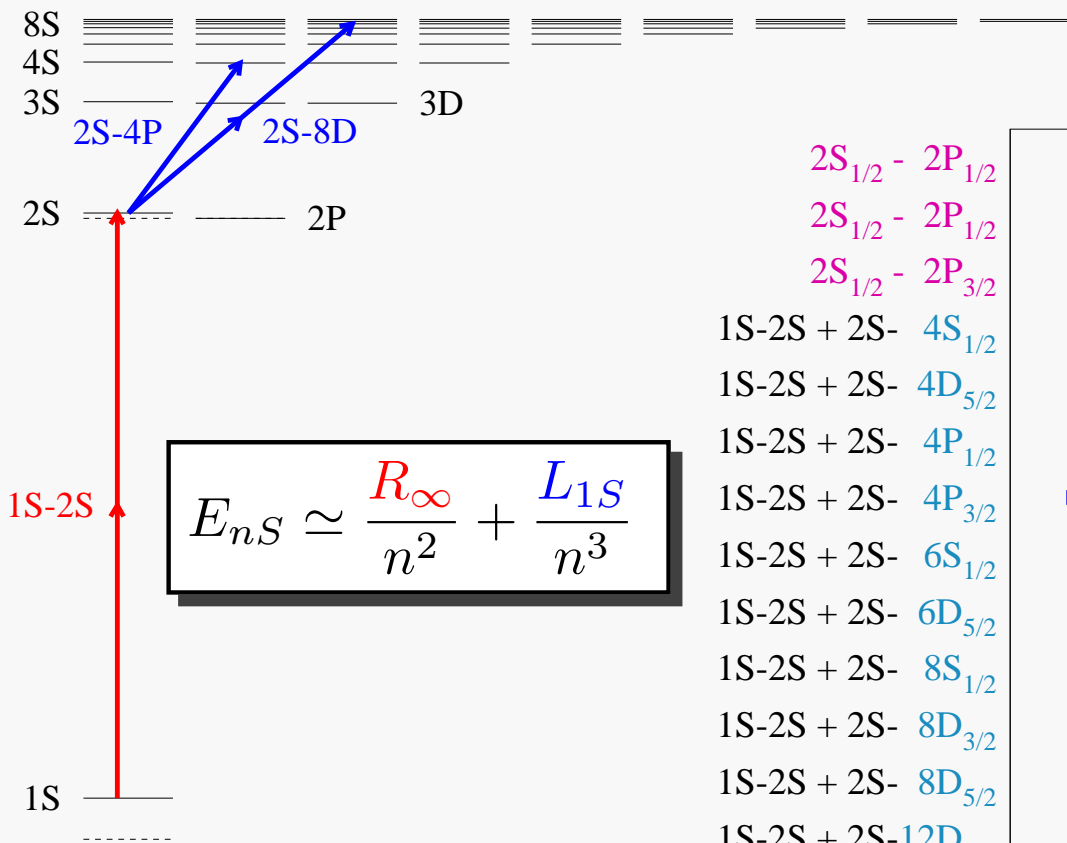
$$E_{nS} \simeq \frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}$$

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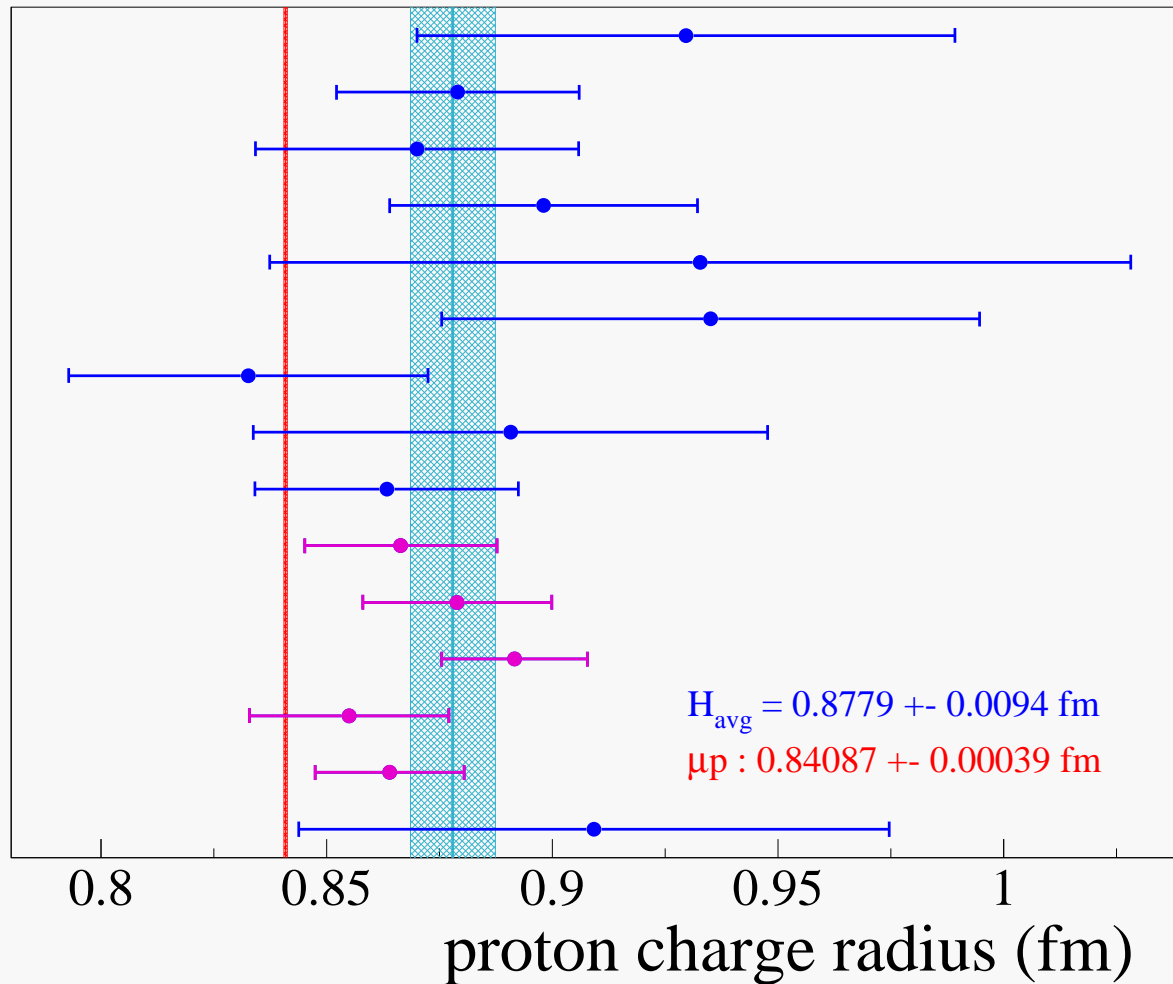


$$L_{1S}^{\text{th}}(r_p) = 8171.636(4) + 1.5645 r_p^2 \text{ MHz}$$



$$E_{nS} \simeq \frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}$$

- $2S_{1/2} - 2P_{1/2}$
- $2S_{1/2} - 2P_{1/2}$
- $2S_{1/2} - 2P_{3/2}$
- $1S-2S + 2S-4S_{1/2}$
- $1S-2S + 2S-4D_{5/2}$
- $1S-2S + 2S-4P_{1/2}$
- $1S-2S + 2S-4P_{3/2}$
- $1S-2S + 2S-6S_{1/2}$
- $1S-2S + 2S-6D_{5/2}$
- $1S-2S + 2S-8S_{1/2}$
- $1S-2S + 2S-8D_{3/2}$
- $1S-2S + 2S-8D_{5/2}$
- $1S-2S + 2S-12D_{3/2}$
- $1S-2S + 2S-12D_{5/2}$
- $1S-2S + 1S-3S_{1/2}$



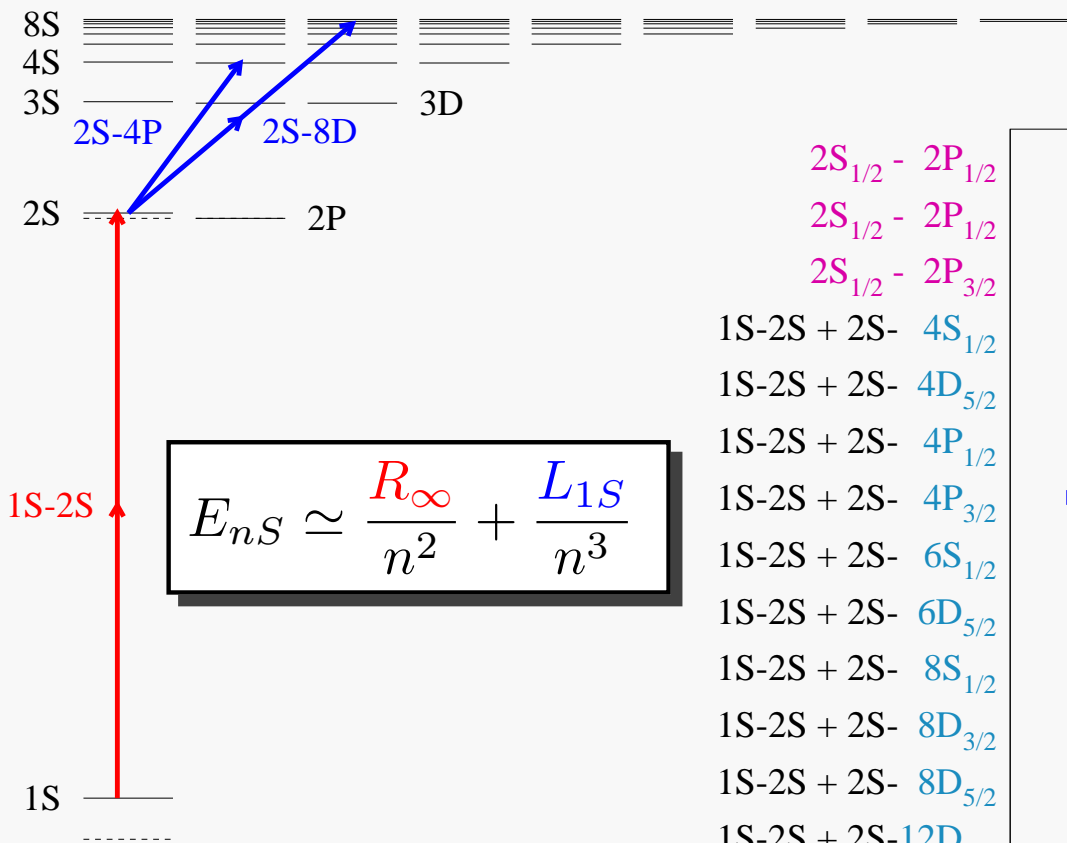


# $r_p$ puzzle (3): Is H-spectroscopy wrong?

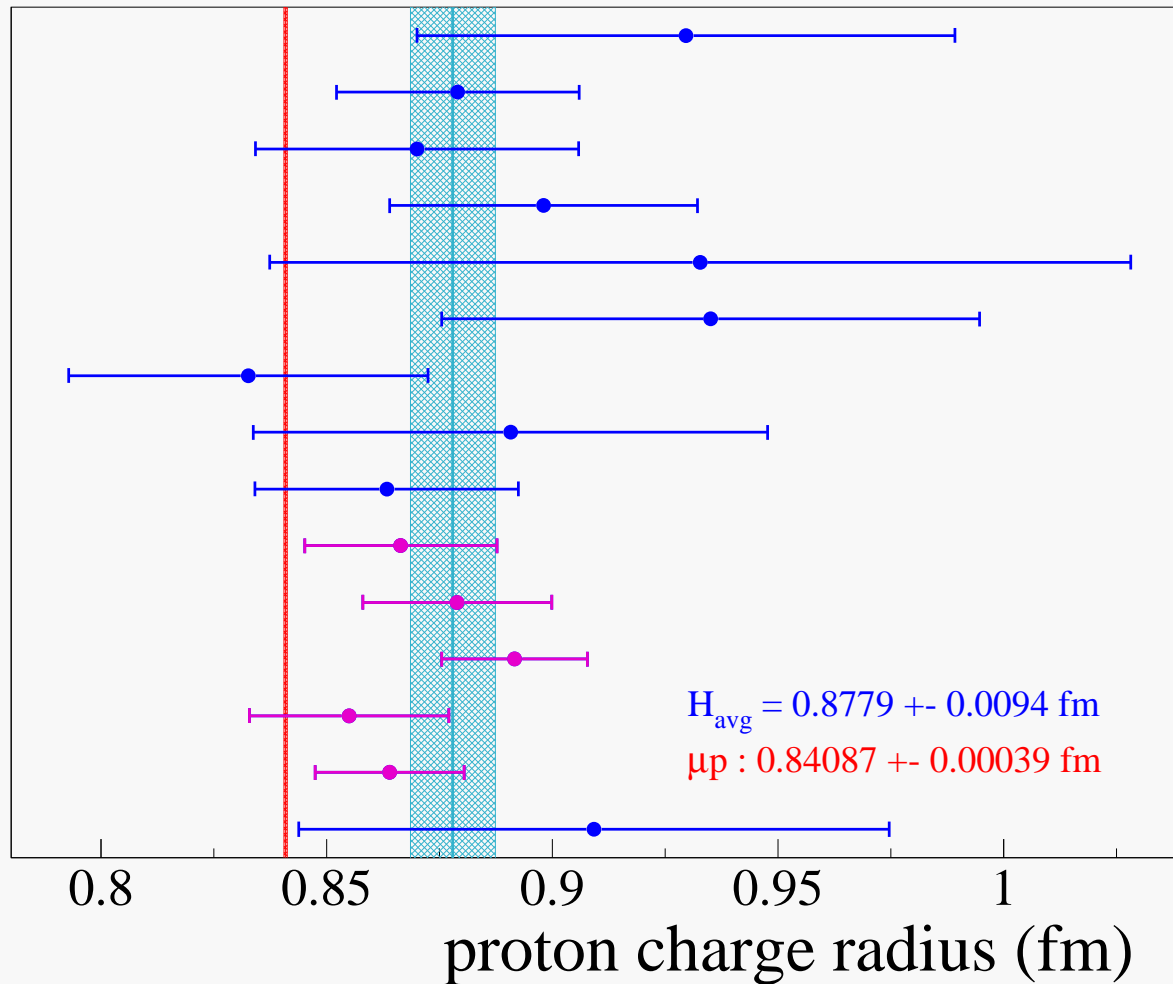
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- $2S_{1/2} - 2P_{1/2}$
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- $1S-2S + 2S-4P_{1/2}$
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- $1S-2S + 1S-3S_{1/2}$



Discrepancy  $< 3\sigma$   
for individual H meas.

# $r_p$ puzzle (6): New physics?

- Several models have been discussed and discarded because of low energy constraints  $(g - 2)_{\mu/e}$ ,  $\mu e$ , H,  $\mu$ Si spectroscopy,  $J/\Psi$ ,  $\pi$ ,  $K$ ,  $\eta$  decay widths, n-scattering ...

Models exist which escape the many constraints but at “high price”:

- Tuning (e.g. vector vs axial-vector) and target coupling
- No UV completion and no full SM gauge invariance

$$m_x \sim \text{MeV}$$
$$\text{coupling} \sim 10^{-4}$$

[arXiv:1401.6154 / PRL 107, 011803 (2011) / PRD 86, 035013 (2012) / PRD 83, 101702 (2011)]

Window for new physics is very small.

BUT more “natural” extensions could come into play IFF  $r_p^{\text{H}} < r_p^{\mu\text{P}} < r_p^{\text{scatt}}$

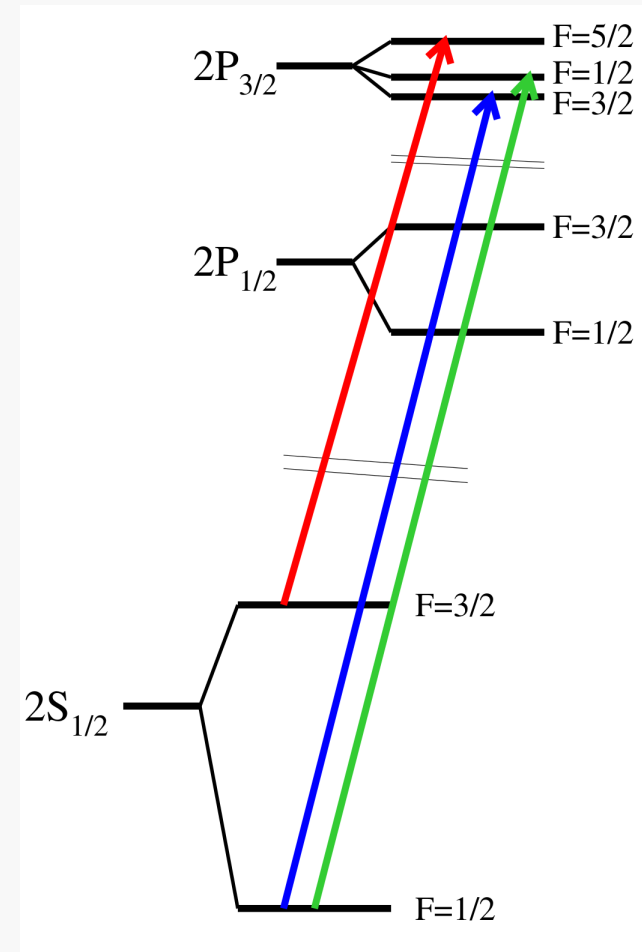
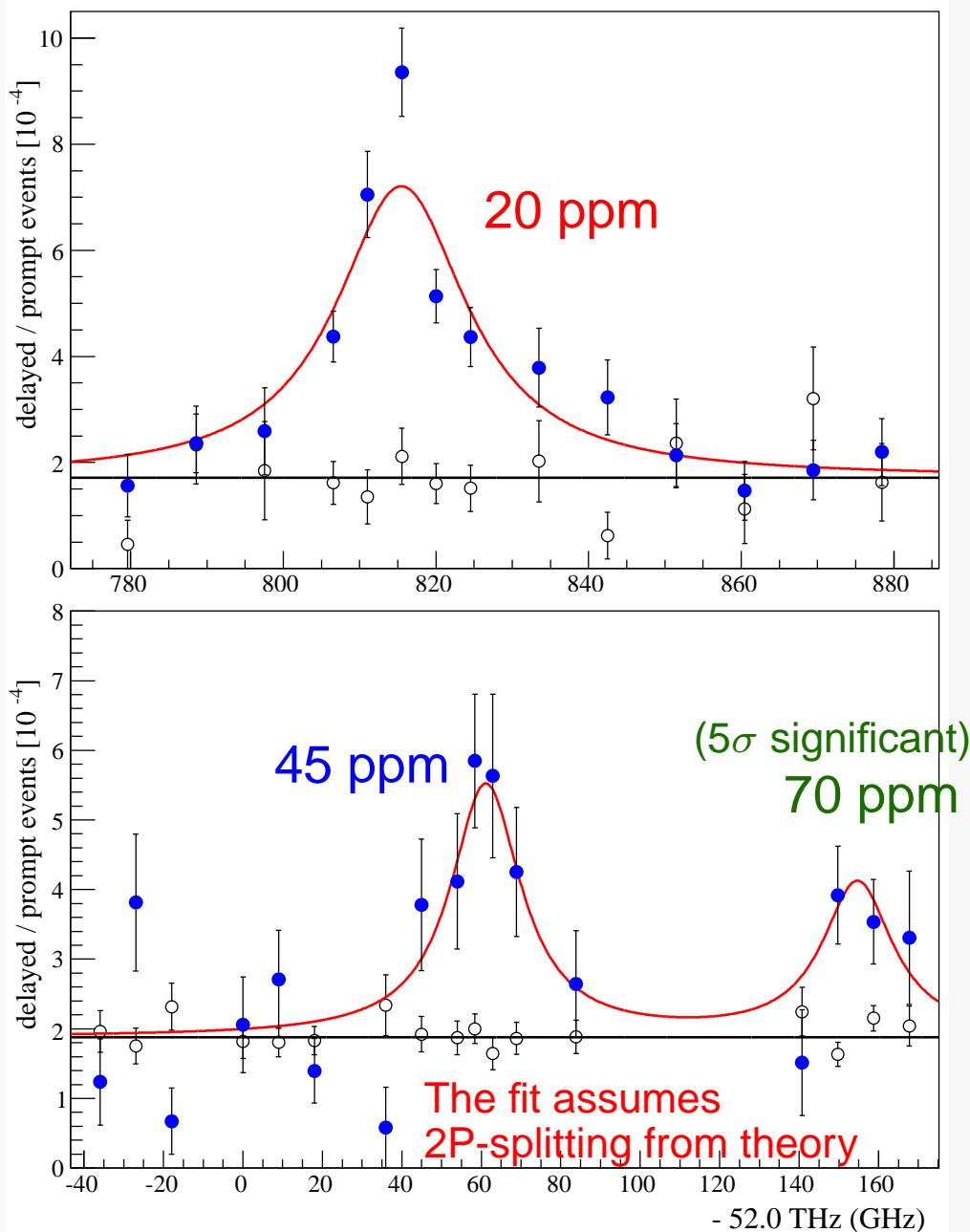
→ e.g. dark photons

[Pospelov]

- Maybe the “new physics” or new effects have to be searched elsewhere: strange proton structure, non-perturbative QED inside proton, quantum gravity etc.

Muonic deuterium and muonic helium  
will soon provide stringent additional information

# Measurements in muonic deuterium $\mu d$



In the last week of 2009 beam time we measured 2.5 transitions in  $\mu d$

# Deuteron radius from $\mu\text{d}$ and $\mu\text{p}$ (**preliminary**)

Directly from  $\mu\text{d}$  spectroscopy

$$\Delta E^{\text{th}} = 230.495(30) - 6.109(1)r_{\text{d}}^2 \text{ meV}$$

$$\Delta E^{\text{exp}} = 202.8759(34) \text{ meV}$$

# Deuteron radius from $\mu\text{d}$ and $\mu\text{p}$ (**preliminary**)

$$\left. \begin{array}{l} \text{H-D shift: } r_{\text{d}}^2 - r_{\text{p}}^2 = 3.820\,07(65) \text{ fm}^2 \\ \mu\text{p: } r_{\text{p}} = 0.84087(39) \text{ fm} \end{array} \right\} \Rightarrow r_{\text{d}} = 2.12771(22) \text{ fm}$$

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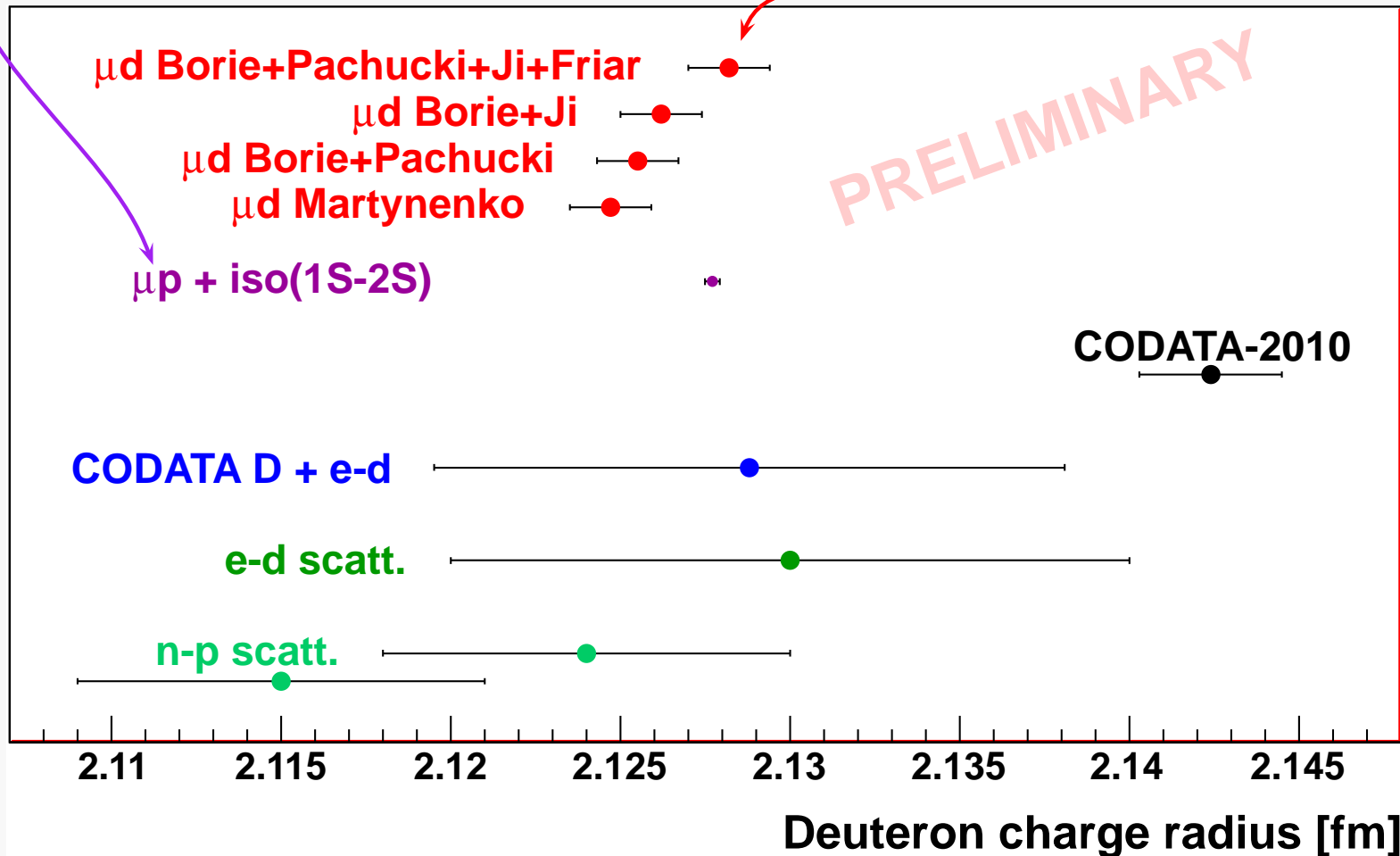
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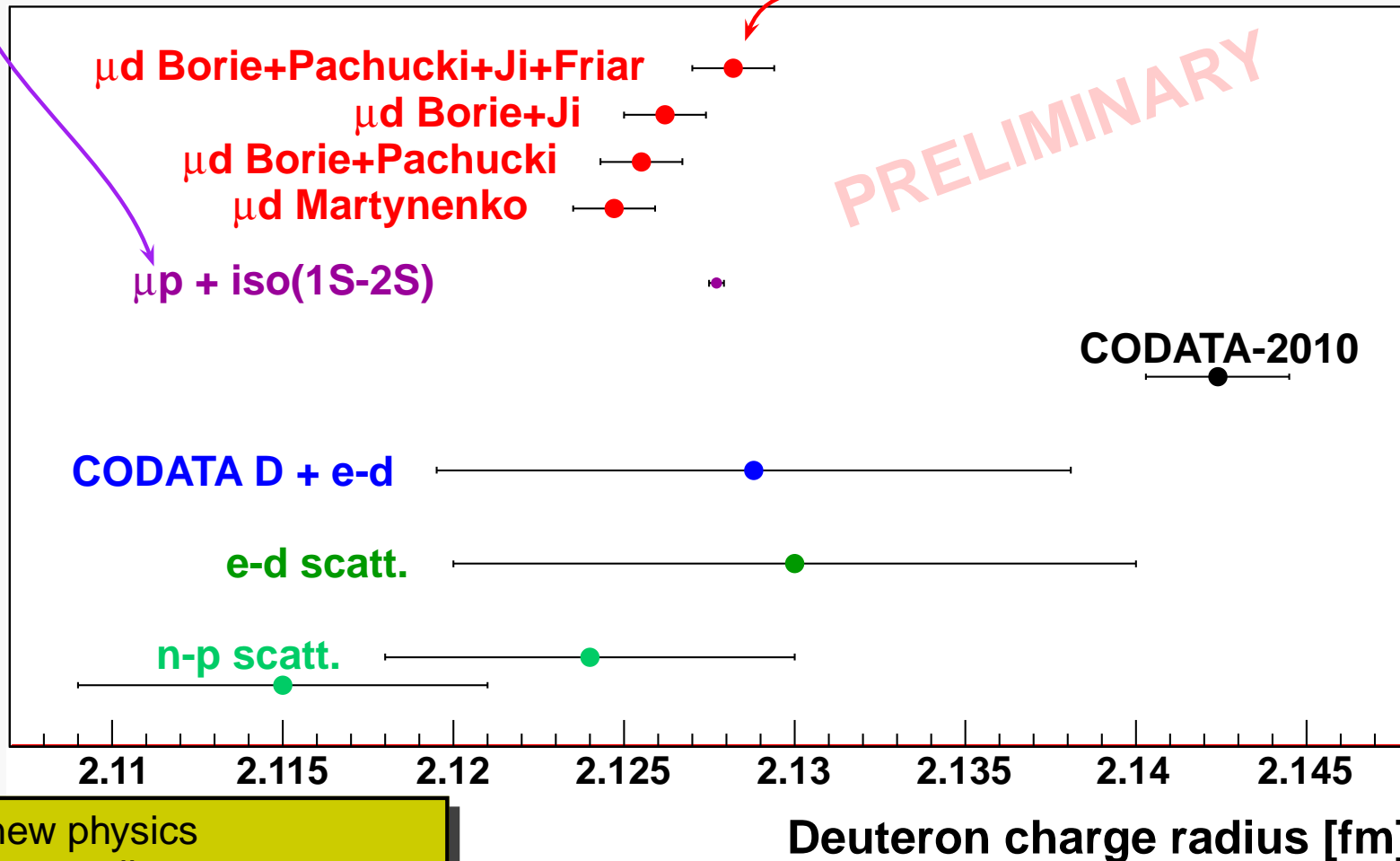
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Consistency  
of muonic results!

IFF new physics  
→ not coupling to neutrons

# $\mu\text{He}^+$ Lamb shift

Measure  $\Delta E(2S-2P)$  in  $\mu^3\text{He}^+$  and  $\mu^4\text{He}^+$  with 50 ppm



$r_{^3\text{He}}$  and  $r_{^4\text{He}}$  with  $u_r = 3 \times 10^{-4} \iff 0.0005 \text{ fm}$

if polarisability contribution known with  $u_r = 5\%$

Antognini et al., Can. J. Phys. 89, 47 (2011)



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Proton radius puzzle  
- new muonic force?

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Benchmark for few-nucleon theories  
- absolute radii of  $^3\text{He}$ ,  $^4\text{He}$   
and  $^6\text{He}$ ,  $^8\text{He}$  via isotopic shifts

R. van Rooij et al. Science 333, 196 (2011)  
Cancio Pastor et al., arXiv:1201.1362  
Müller, Wang, Shiner...

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Cancio Pastor et al., arXiv:1201.1362  
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Enhanced bound-state QED test when combined with  $\text{He}^+(1S-2S)$

- Finite size  $\sim Z^4 R^2$

[MPQ and Amsterdam]

- Bohr structure  $\sim Z^2 R_\infty$

- Challenging QED contributions  $\sim (Z\alpha)^{5\dots 6}$

# Why testing bound-state QED?

## • Free QED

$$a_e = C_1 \left(\frac{\alpha}{\pi}\right) + C_2 \left(\frac{\alpha}{\pi}\right)^2 + C_3 \left(\frac{\alpha}{\pi}\right)^3 + C_4 \left(\frac{\alpha}{\pi}\right)^4 + C_5 \left(\frac{\alpha}{\pi}\right)^5 + \Delta(\text{had.}, \dots)$$

## • Bound-state QED

- Binding effects ( $Z\alpha$ )                      bad convergence, all-order approach/expansion
- Radiative corrections ( $\alpha$  and  $Z\alpha$ )
- Recoil corrections ( $m/M$  and  $Z\alpha$ )                      relativity  $\Leftrightarrow$  two-body system
- Radiative–recoil corrections ( $\alpha$ ,  $m/M$  and  $Z\alpha$ )
- Nuclear structure corrections

→ Cannot develop the calculation in a systematic way  
 → Corrections are mixed up:  $\alpha^x \cdot (Z\alpha)^y \cdot (m/M)^z$   
 → Difficulty in finding out the desired order of corrections

## New development: NRQED

QED	$g - 2$ free particle particle mass only perturbative around free particle	Lamb shift bound-state particle three scales, hierarchy non-perturbative
QCD	deep inelastic scattering pQCD	hadron lattice, Chiral perturbation

[after Nio]

# Few-nucleon theories and He-radius

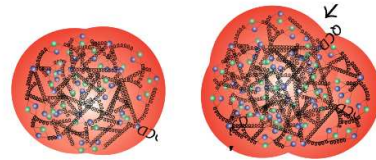
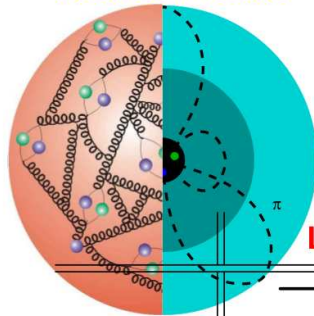
## (a) Few-Nucleon Interactions in $\chi$ EFT

Weinberg, Ordóñez/Ray/van Kolck, Friar/Coon, Kaiser/Brockmann/Weise, Epelbaum/Glöckle/Meißner, Entem/Machleidt, Kaiser, Higa/Robilotta, Epelbaum, ...

typ. momentum  
breakdown scale  $\ll 1$

**Long-Range:** correct symmetries and IR degrees of freedom: **Chiral Dynamics**

**Short-Range:** symmetries constrain contact-ints to simplify UV: **Minimal parameter-set**



**Hierarchy: 2NF-effects  $\gg$  3NF-effects  $\gg$  4NF-effects**

[from Griesshammer]

	LO	NLO	N <sup>2</sup> LO	N <sup>3</sup> LO
2N ints	 2 parameter	 +7 parameter	 +0 parameter	 +15 = 24 param.
3N ints	—	—	 2 parameter	 parameter-free, in progress

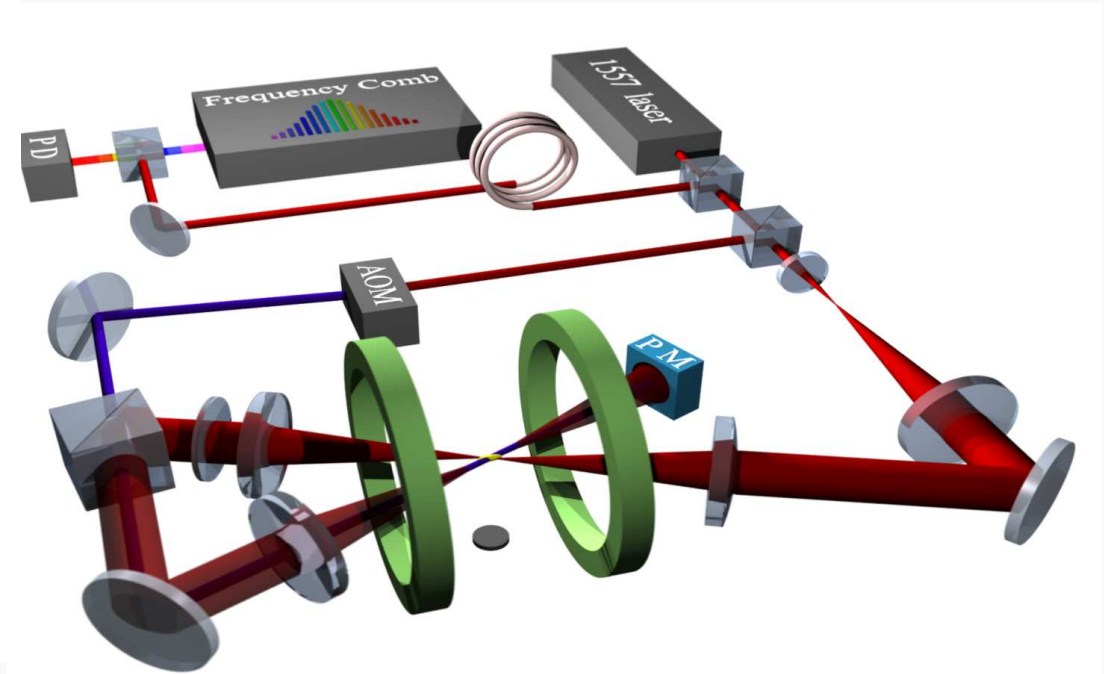
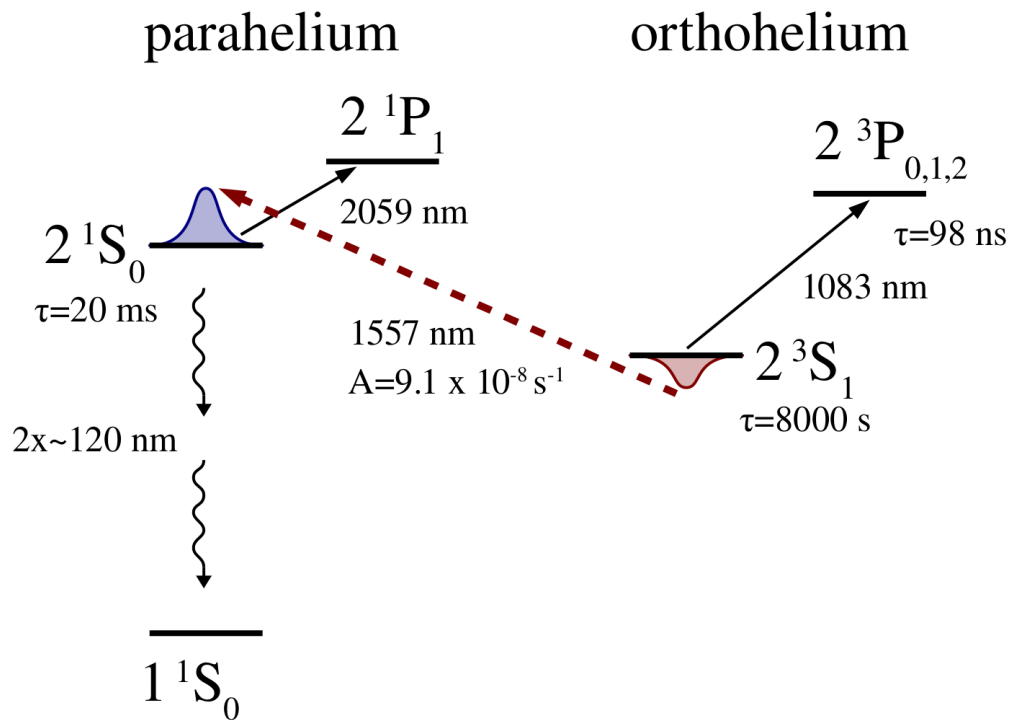
From  $r_{\text{He}} \rightarrow c_D$  or  $c_E$ .

Radii are “clean” benchmarks to test few-nucleons theories or to fix LEC

[Navratil et al., PRL99, 042501 (2007)]

[Gazit, Marcucci, Forssen, Kievsky, Stadler, Krebs...]

# Helium spectroscopy in Amsterdam



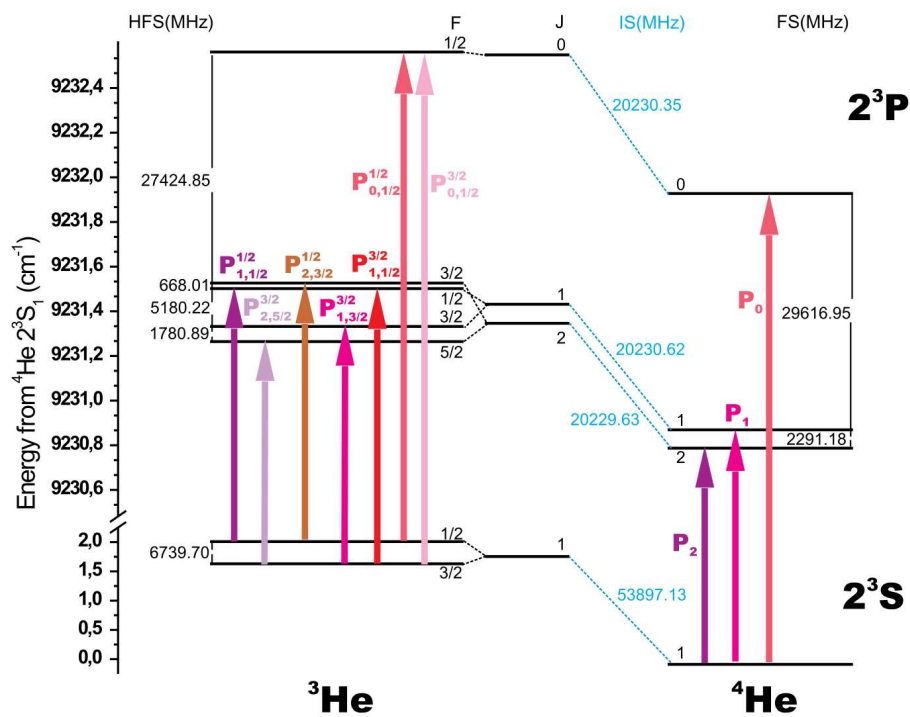
- Trap  $\mu\text{K}$  cold  $^4\text{He}^*$  and  $^3\text{He}^*$ .
- Measure the double forbidden 1557 nm line (M1 transition between two metastable states). (200'000 times narrower than  $2^3P$  states)
- Precision of  $\nu_r = 8 \times 10^{-12}$  (1.5 kHz).

From isotope shift

$$R_{^3\text{He}}^2 - R_{^4\text{He}}^2 = 1.028(11) \text{ fm}^2$$

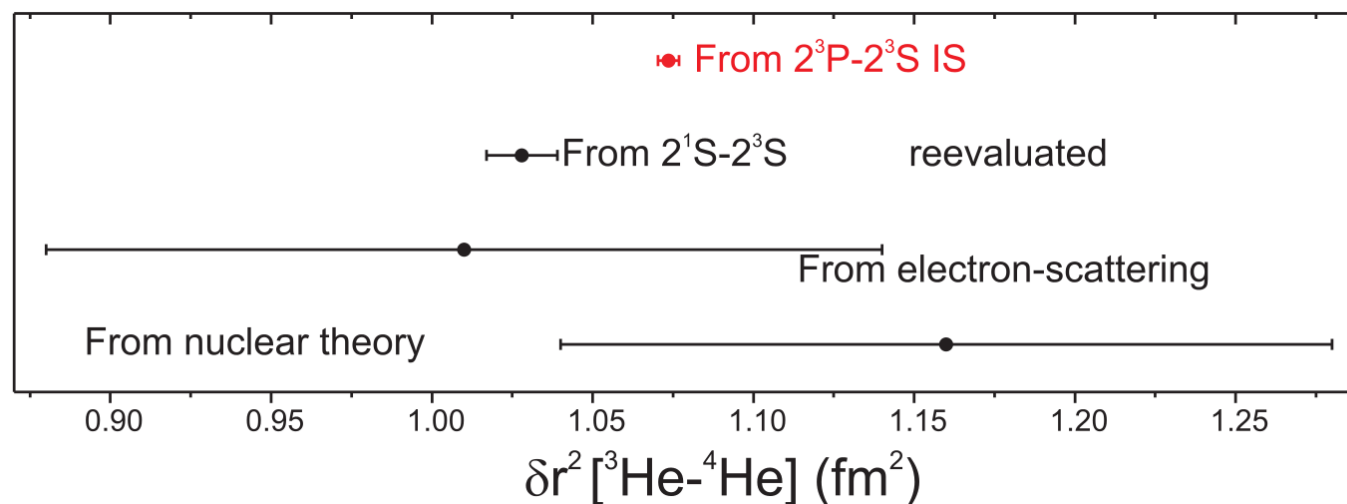
[R. van Rooij et al., Science 333, 196 (2011)]

# 2S-2P metrology of $^3\text{He}$ and $^4\text{He}$ in Florence



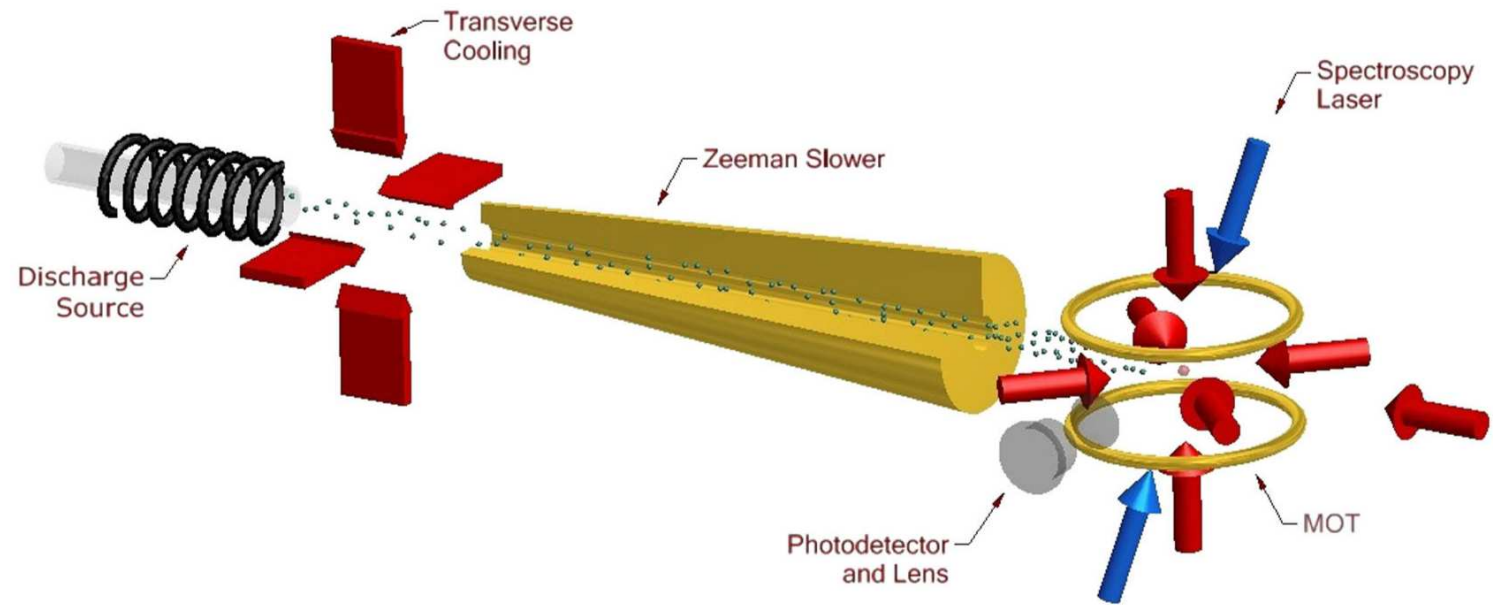
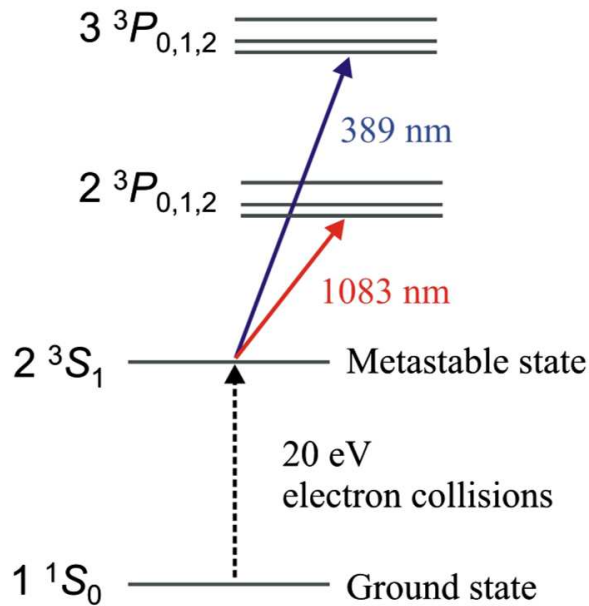
- Measure transitions between the 2S-2P hyperfine manifolds in metastable  $^3,4\text{He}$  beams with  $u_r \approx 5 \times 10^{-12}$  (2.5 kHz) using saturation spectroscopy at 1083 nm
- From isotope shift theory
 
$$R_{^3\text{He}}^2 - R_{^4\text{He}}^2 = 1.074(3) \text{ fm}^2$$
- Test of three-body bound-state QED

[Cancio Pastor et al., PRL 108, 143001 (2012)]



**4 $\sigma$  discrepancy**

# ${}^6\text{He}$ and ${}^8\text{He}$ spectroscopy at GANIL



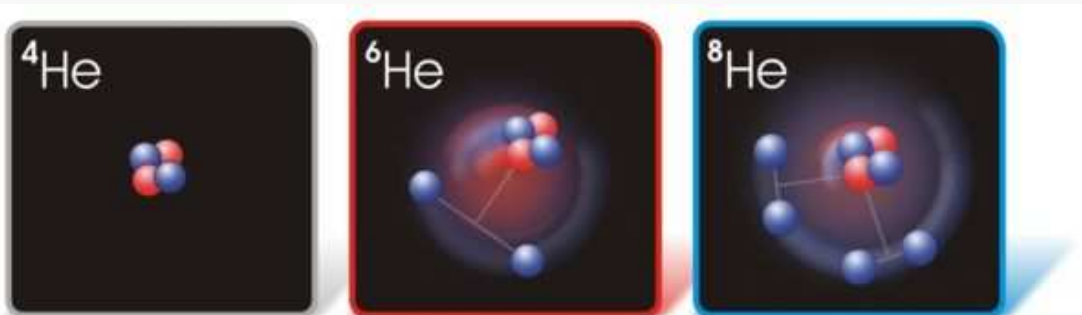
- Finite size shift: 1 MHz
- Mass shift: 50 GHz

- Measure the 389 nm transitions with 10...70 kHz precision.
- From isotope shift theory and knowledge of  ${}^4\text{He}$  charge radius

$$R_{{}^6\text{He}} = 2.059(8) \text{ fm}$$

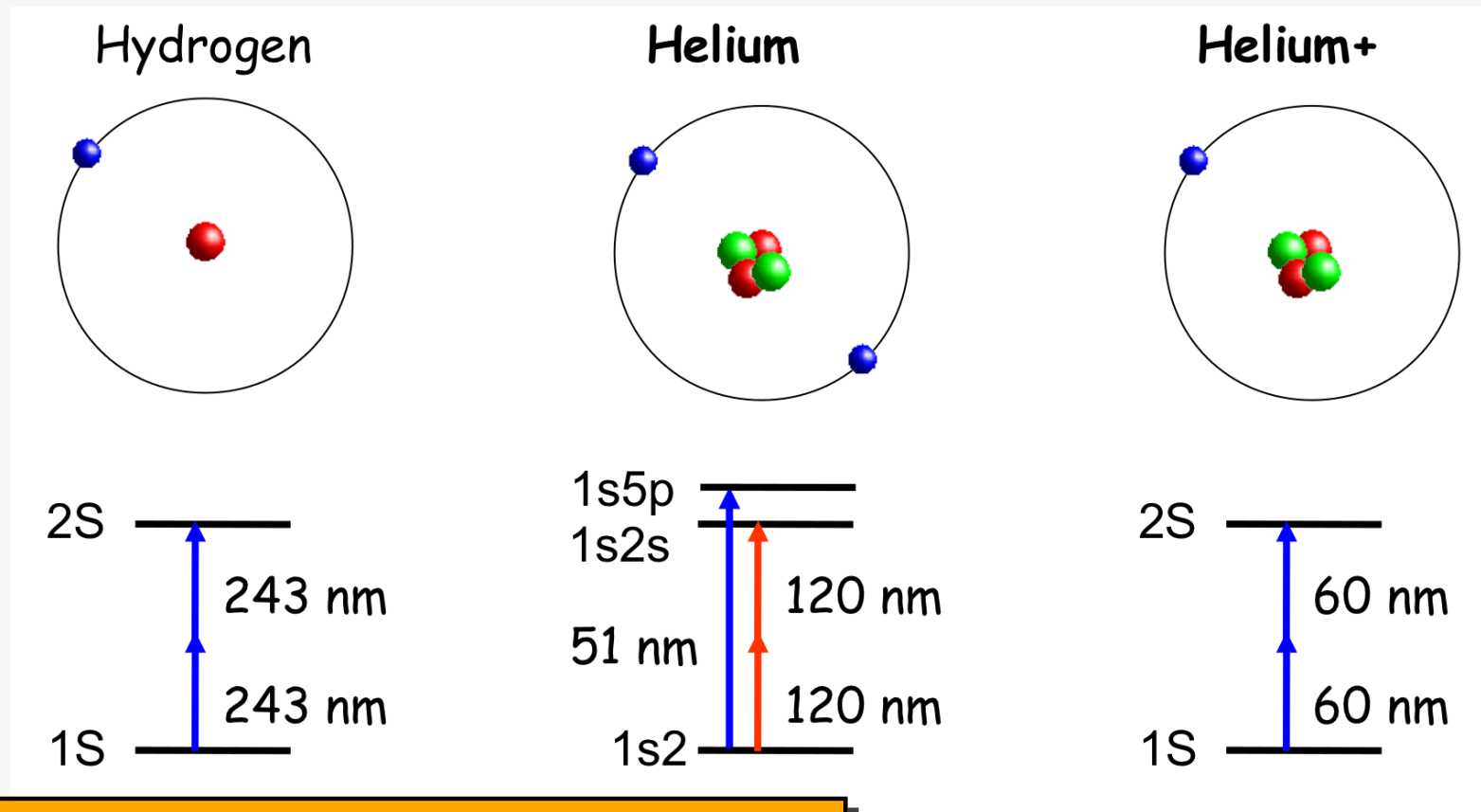
$$R_{{}^8\text{He}} = 1.958(16) \text{ fm}$$

[Lu, Müller, Drake et al., RMP 85 1383 (2013)]





# He<sup>+</sup>(1S-2S) and He(1S2-1S5P)



Both experiments are performing XUV comb spectroscopy:

- two-photon, on a trapped He<sup>+</sup> ion (MPQ)
- one-photon Ramsey technique, on a He jet (Amsterdam)

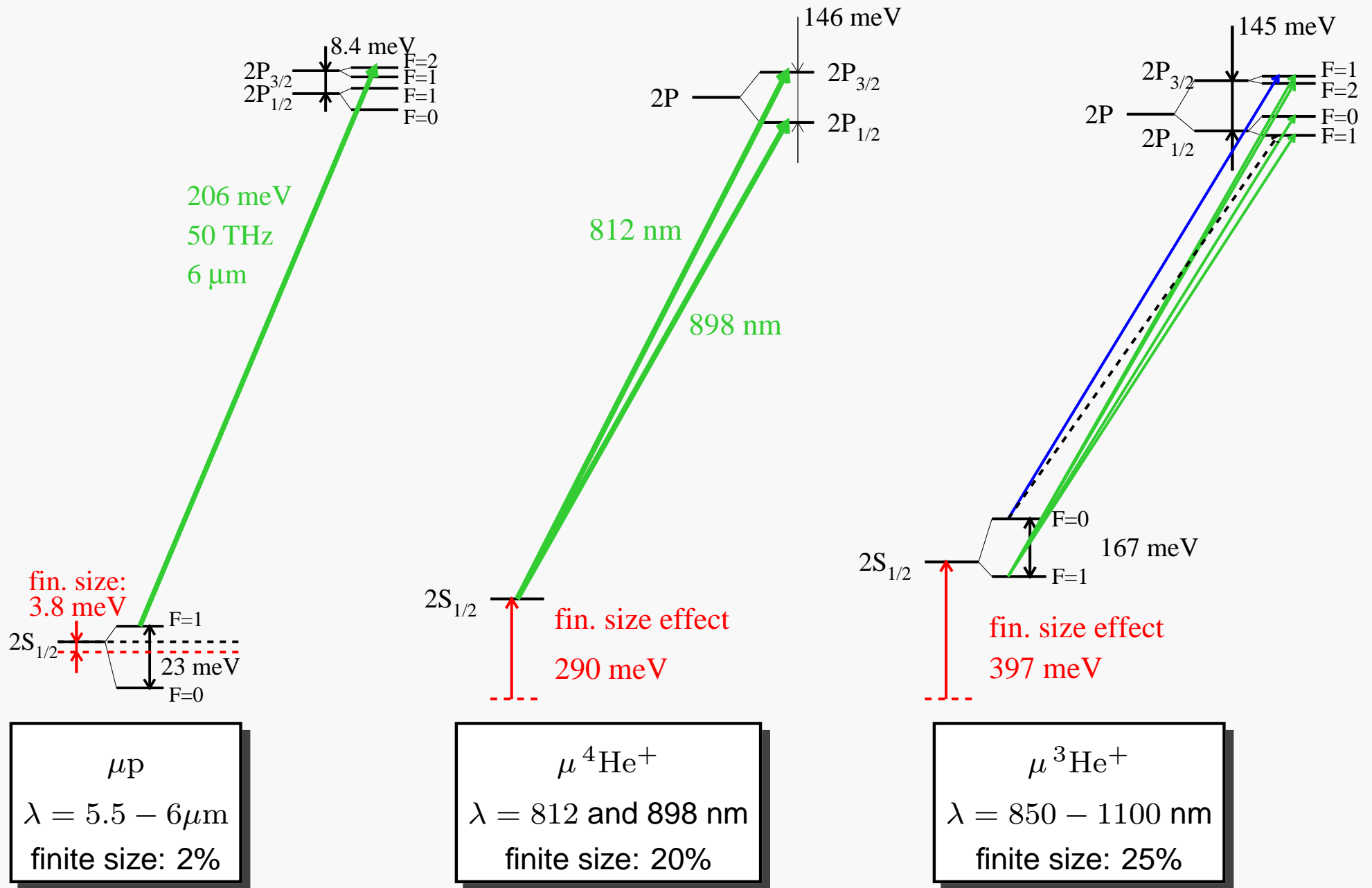
From these measurements

→  $R_{4\text{He}}$  or one/two-electrons bound-state QED test

[Hermann et al., PRA 79, 052505 (2009)]

[Kandula et al., PRA 84, 062512 (2011)]

# Muonic helium transitions

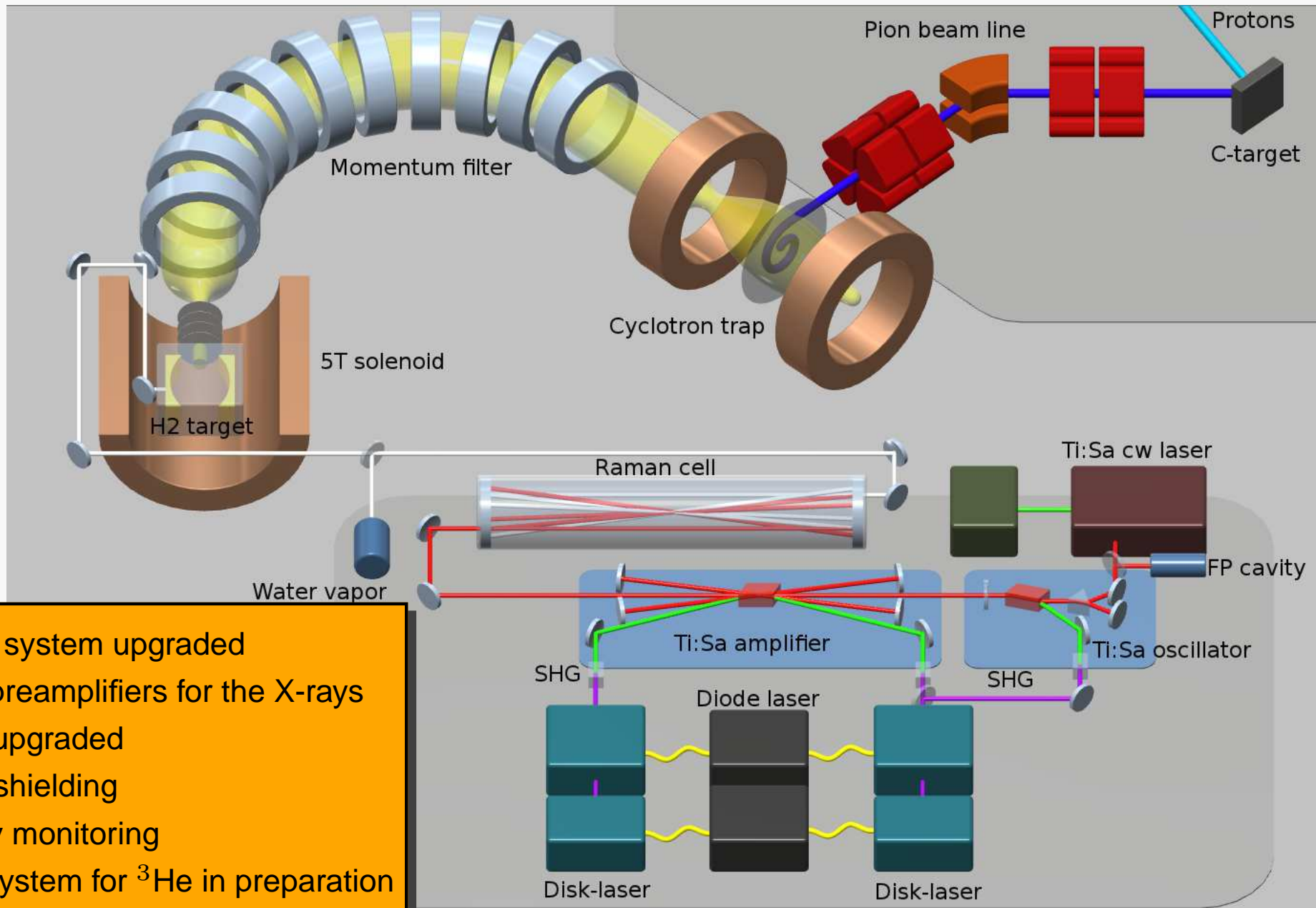


$\mu p$   
 $\lambda = 5.5 - 6 \mu\text{m}$   
 finite size: 2%

$\mu^4\text{He}^+$   
 $\lambda = 812 \text{ and } 898 \text{ nm}$   
 finite size: 20%

$\mu^3\text{He}^+$   
 $\lambda = 850 - 1100 \text{ nm}$   
 finite size: 25%

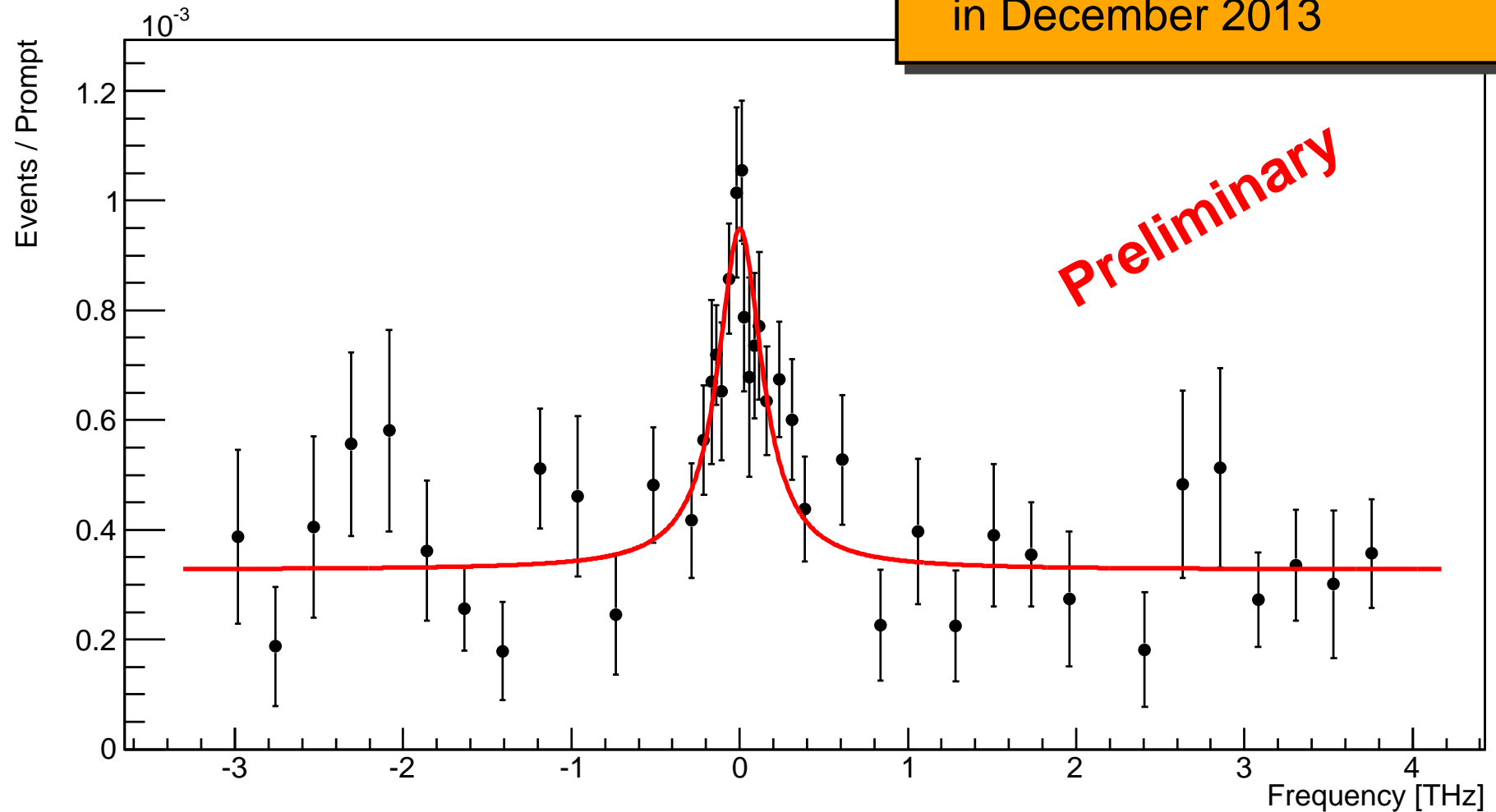
# The setup for $\mu\text{He}^+$ is similar to $\mu\text{p}$



- Laser system upgraded
- New preamplifiers for the X-rays
- DAQ upgraded
- Light shielding
- Cavity monitoring
- Gas system for  $^3\text{He}$  in preparation

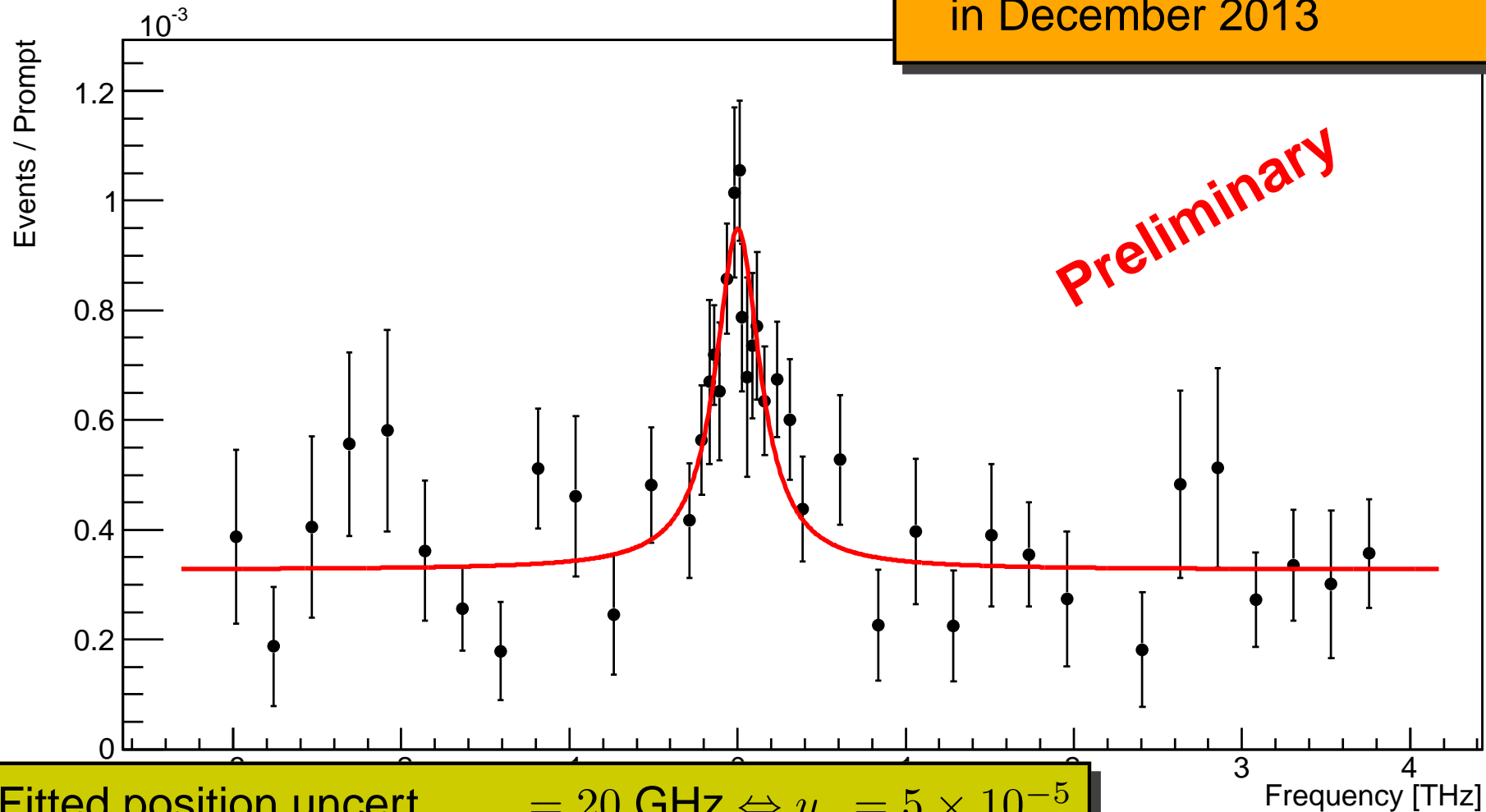
# The $2S_{1/2} - 2P_{3/2}$ resonance in $\mu^4\text{He}^+$

Two weeks of measurements  
in December 2013



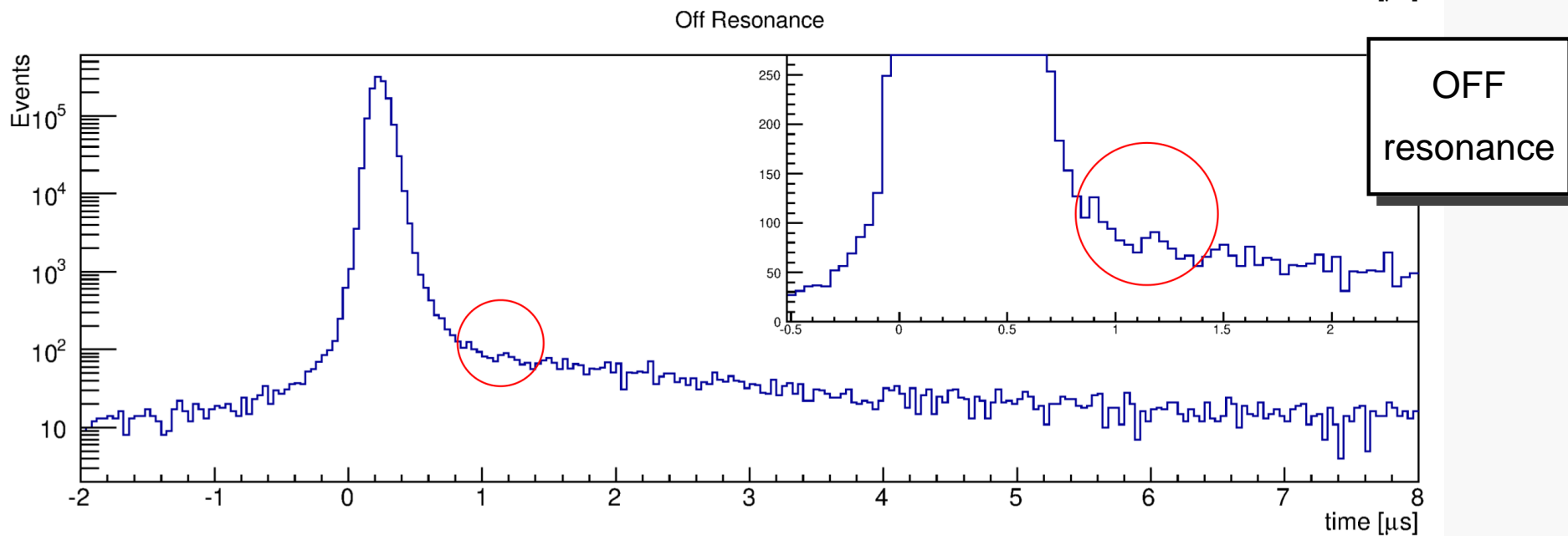
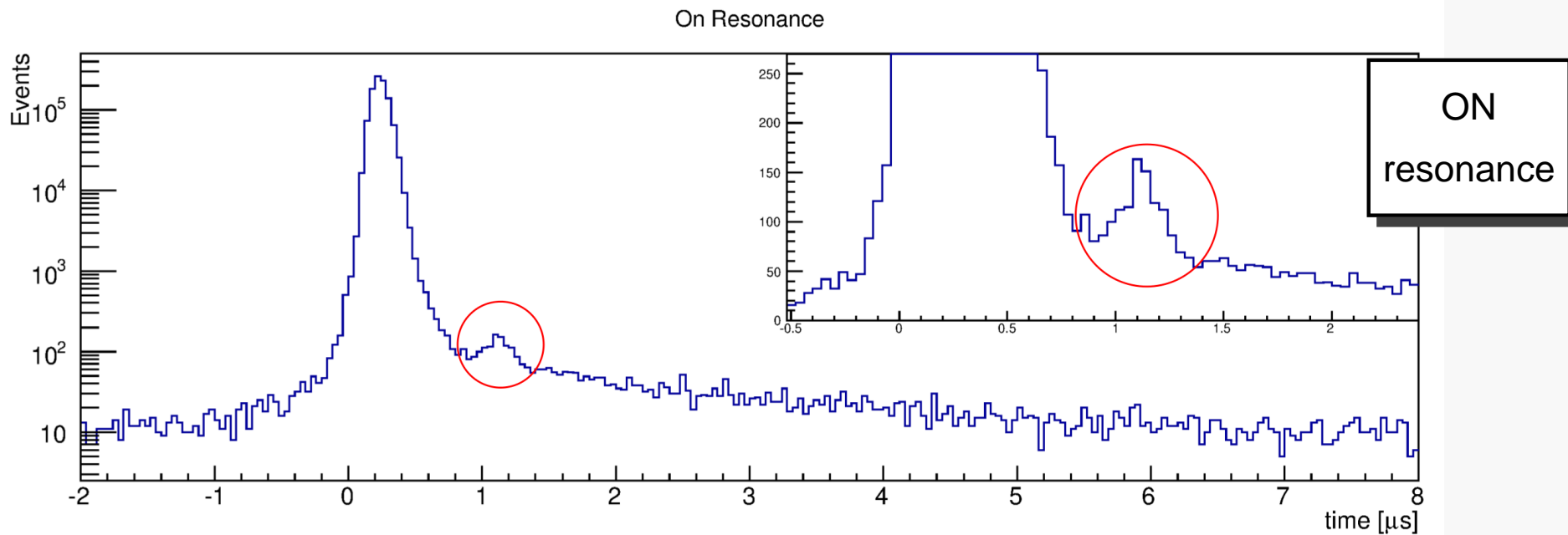
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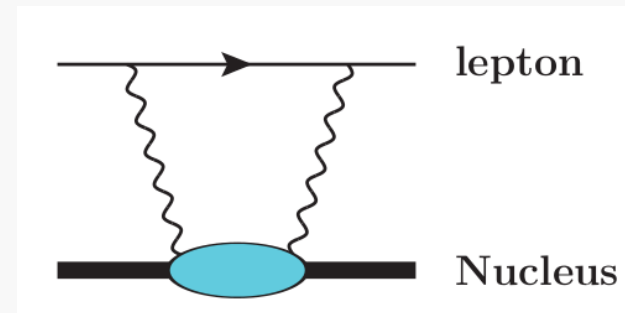
Fitted position uncert. = 20 GHz  $\Leftrightarrow u_r = 5 \times 10^{-5}$   
Laser frequency uncert. < 100 MHz  
Systematics < 10 MHz

# The $K_\alpha$ time spectra



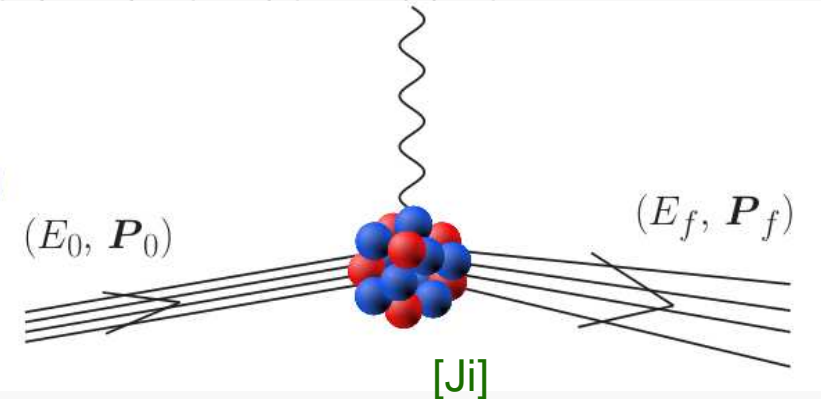
# Nuclear polarization contribution in $\mu\text{He}^+$

$$\Delta E_{\text{LS}}^{\text{th}} = \Delta E_{\text{QED}} - \frac{m_r^3}{12} (Z\alpha)^4 \langle r^2 \rangle + \frac{m_r^3}{12} (Z\alpha)^4 \langle r^3 \rangle_{(2)} + \delta_{\text{pol}}$$



- From nuclear response function  $S_0(\omega) \rightarrow$  nuclear polarization contribution

$$S_0(\omega) = \sum_f |\langle \psi_f | \hat{O} | \psi_0 \rangle|^2 \delta(E_f - E_0 - \omega)$$



- Two ways to get the response function:

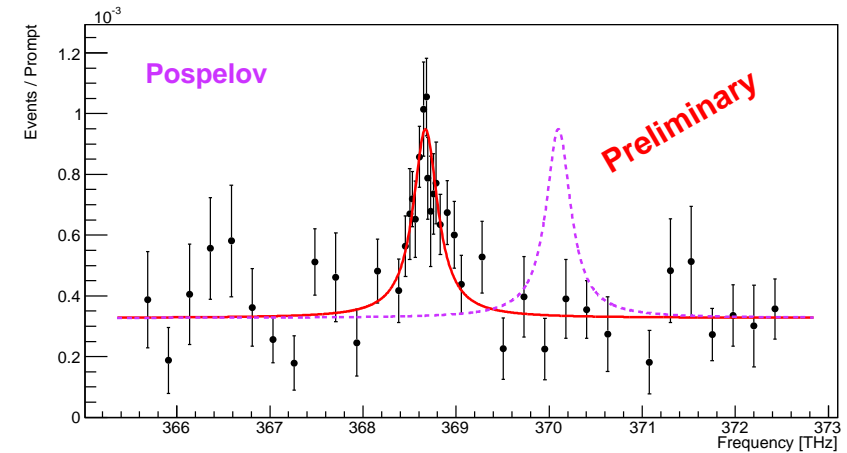
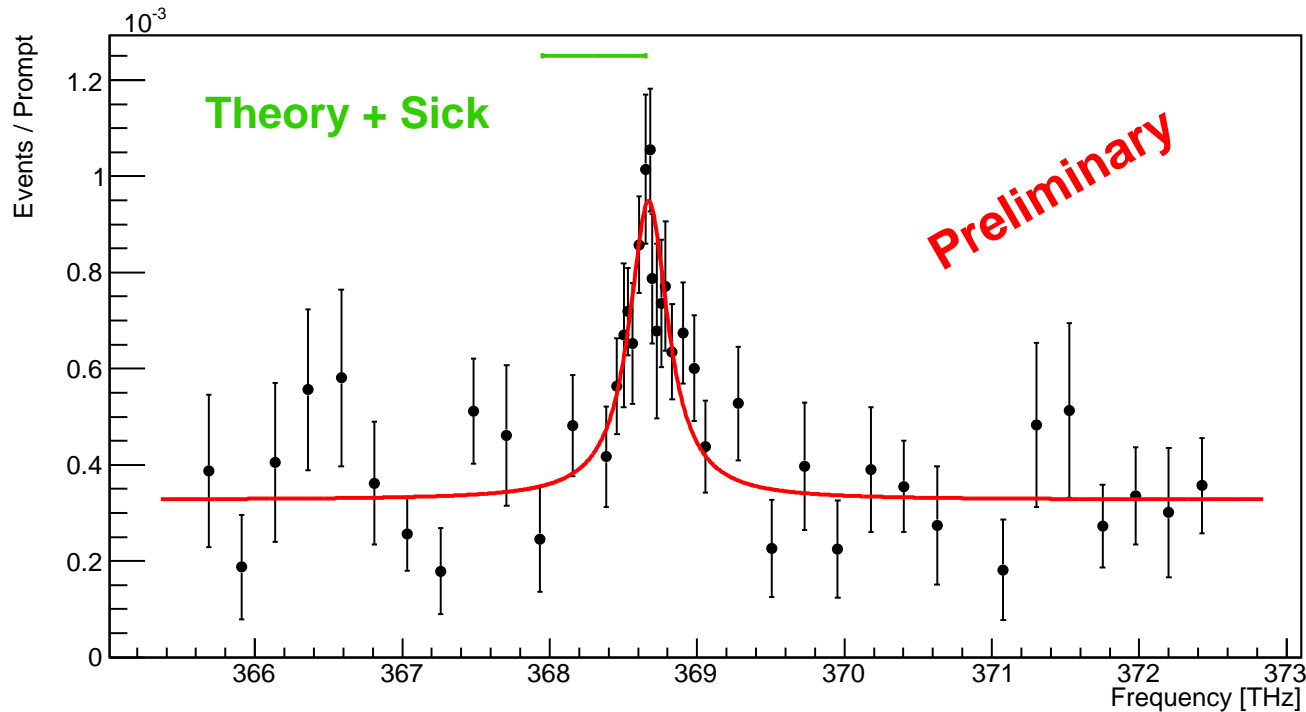
- From photo-absorption [Bernabeu & Jarlskog, Rinker, Friar]

$$\delta_{\text{pol}} = 3.1 \text{ meV} \pm 20\%$$

- From state-of-the-art potentials (chiral EFT, AV18/UIX) [Ji, Nevo Dinur, Bacca...]

$$\delta_{\text{pol}} = 2.47 \text{ meV} \pm 6\%$$

# Secret results!



- The transition has been found at the expected position i.e., within the uncert. given by  $r_{\text{He}}$  from  $e\text{-He}$  scattering.
- New physics model of Pospelov excluded
- Zavattini value from old  $\mu\text{He}^+$  experiment excluded

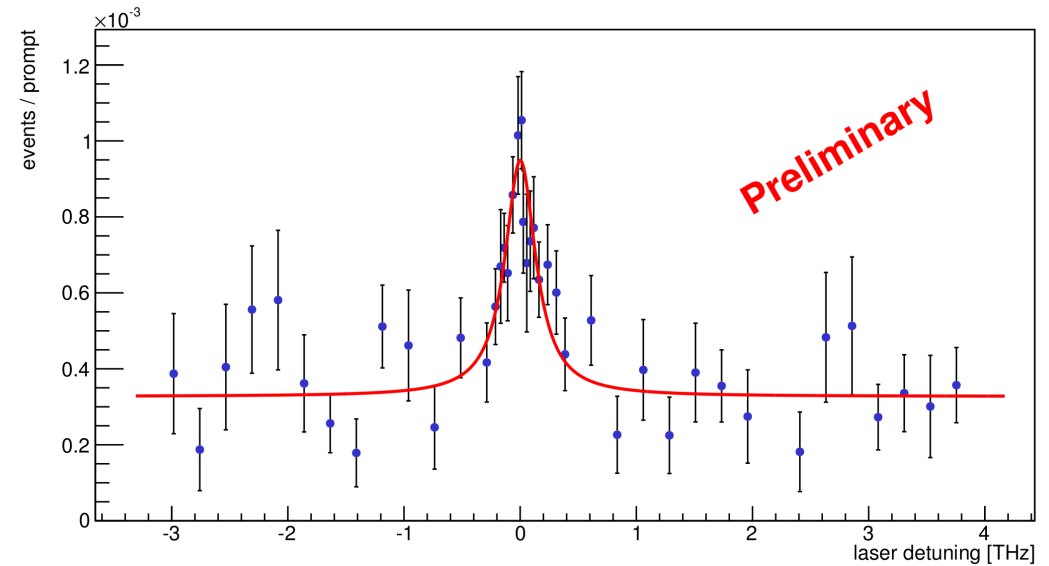
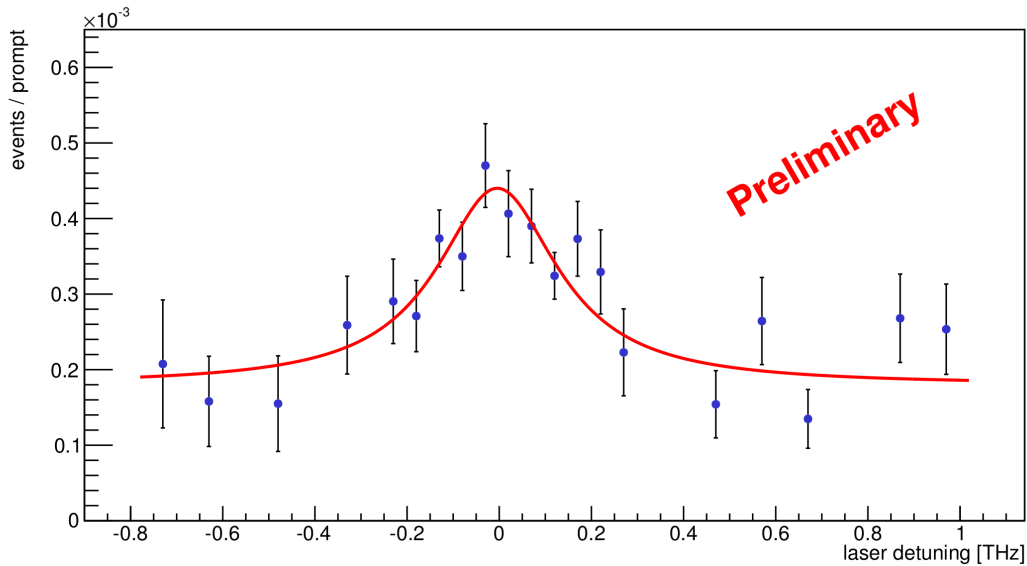
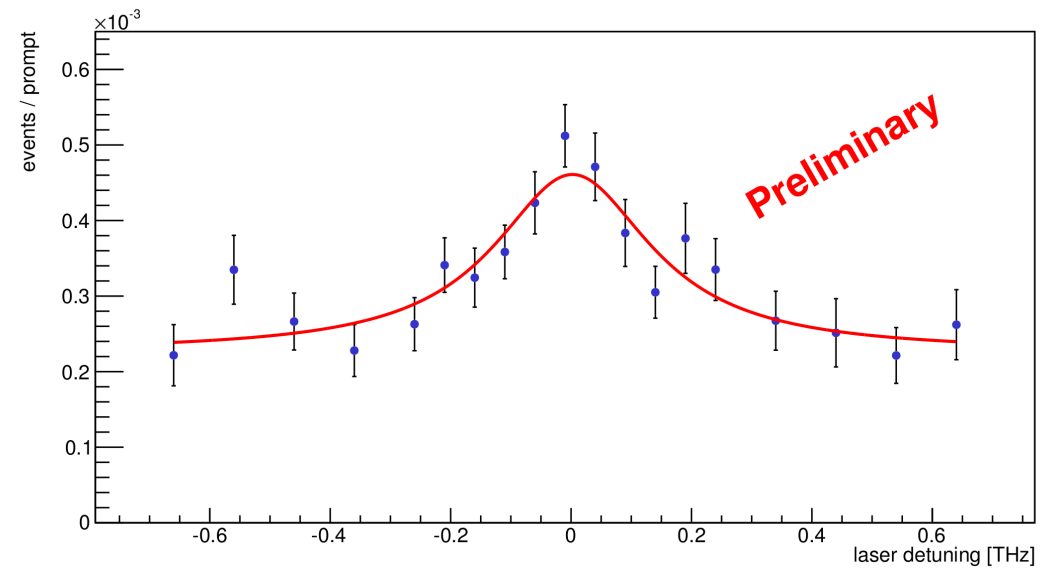
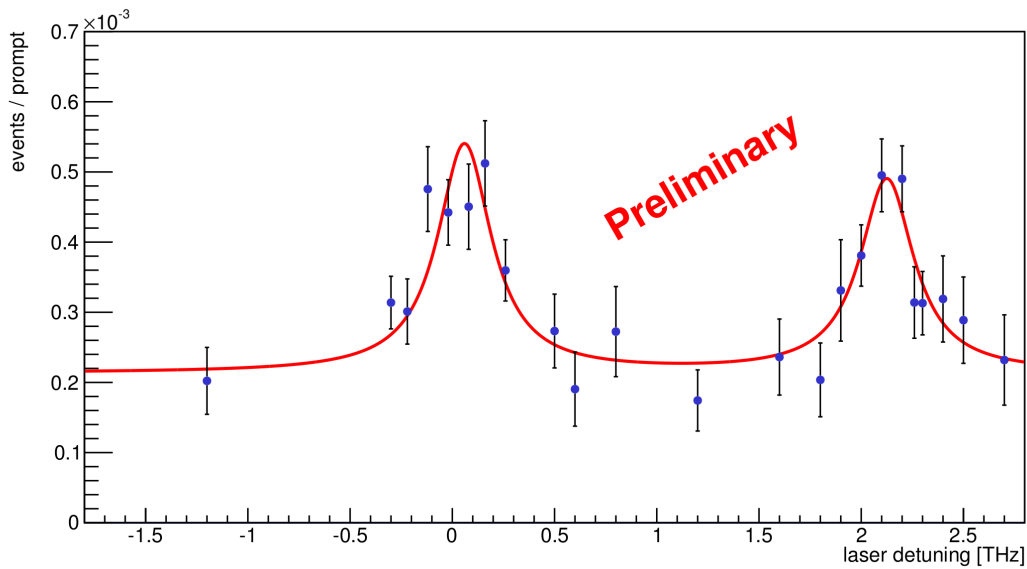
Need to summarize all 2S-2P contributions

$^4\text{He}$  nuclear charge radius

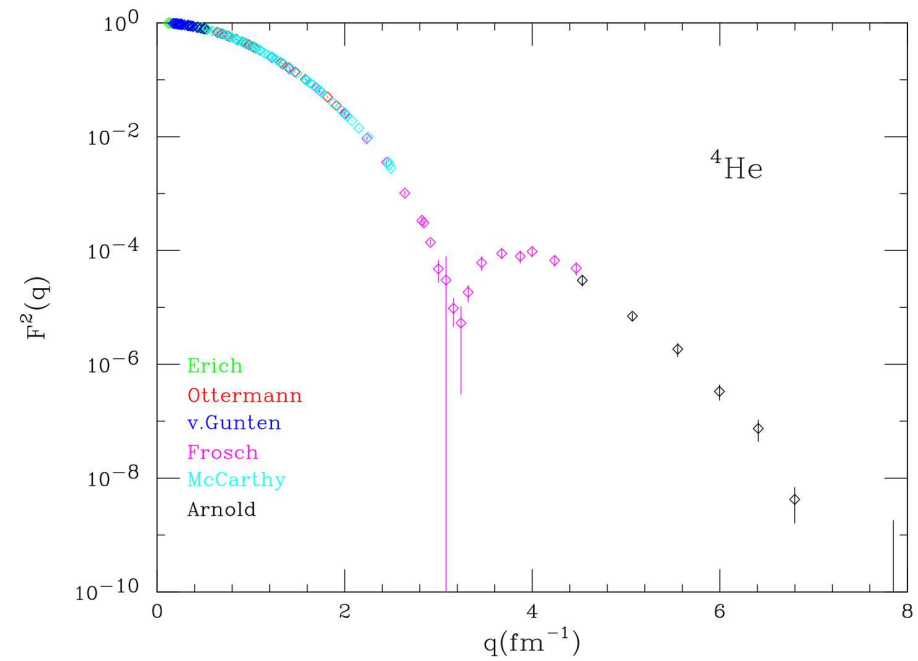
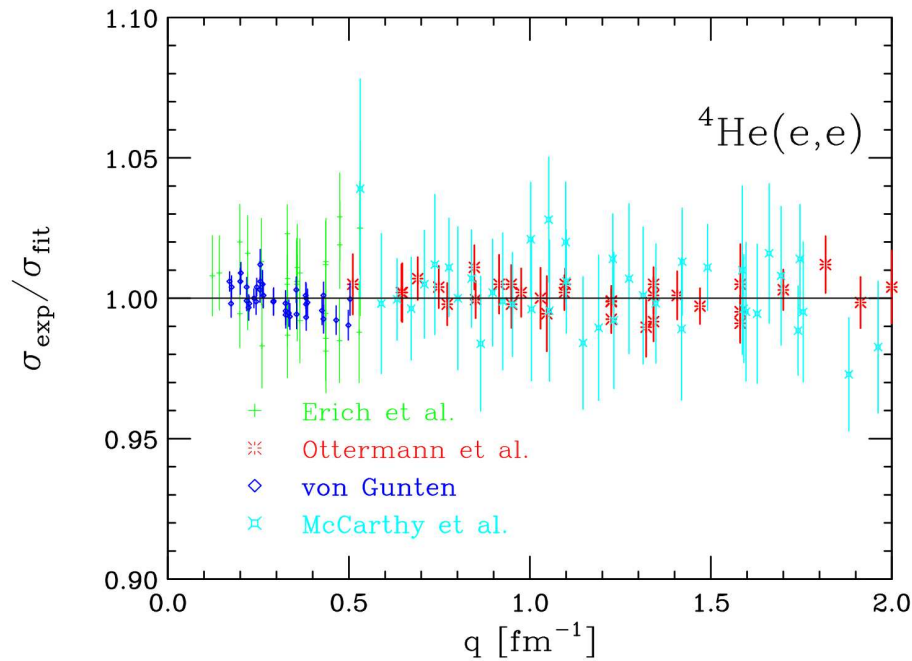
1.681(4) fm	$u_r = 2 \times 10^{-3}$	[Sick]
1.677(1) fm	(VERY preliminary)	$[\mu\text{He}^+]$



# Measured $\mu^4\text{He}^+$ and $\mu^3\text{He}^+$ resonances



# He radius from e-scattering



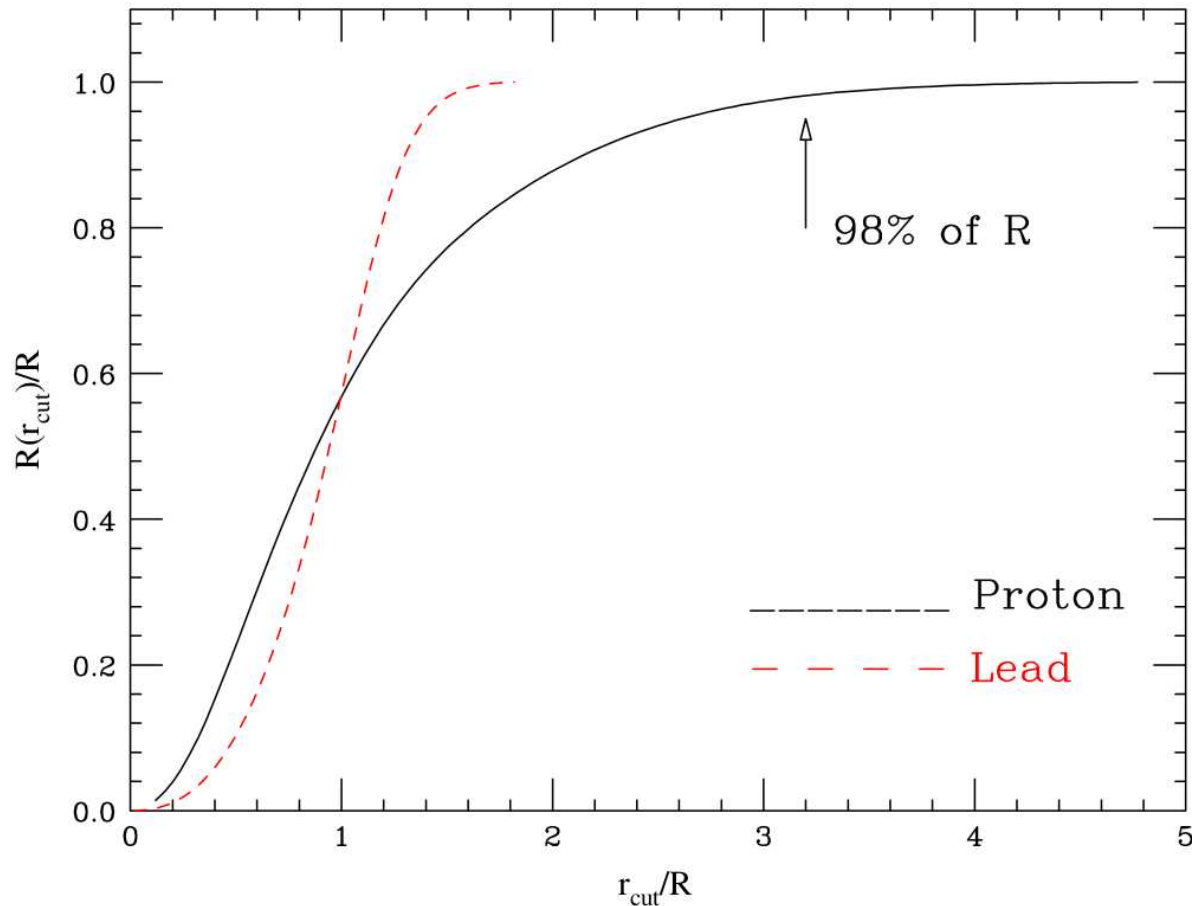
- world data of e-scattering.
- constraints density at large  $r$ :
  - shape: from p-wavefunction  $\sim$  Whittaker.
  - absolute density: from p-He scattering + FDR.
- point density from potential + GFMC (small  $r$ ) + FDR (large  $r$ ).
- fold point density with charge density distribution of p and n.
- include Coulomb distortions.

Fit with SOG  
 $\rightarrow R = 1.681(4) \text{ fm}$   
 (best known radius from e-scattering)

[Sick, PRC 77, 941392(R) (2008)]

# Difficulties due to large- $r$ tail (from I. Sick)

I. Sick / Progress in Particle and Nuclear Physics 67 (2012) 473–478



The extrapolation from finite  $q$  to  $q = 0$  is much more difficult for  $p$  than for nuclei with  $A > 2$

Extrapolation of  $G(q)$  is not fully reliable. Needs to consider  $\rho(r)$  at large  $r$ . Most  $e - p$  scattering fits have not been checked for large- $r$  behavior

Need a physical model to constrain the large- $r$  behavior

Slow convergence of the  $p$  rms radius vs upper cutoff  $r_{\text{cut}}$  calculated over the integral of the charge density  $\rho(r)$

# Conclusions

From two transitions in muonic hydrogen:

- Proton charge radius:  $r_E = 0.84087(39)$  fm
- Proton Zemach radius:  $r_Z = 1.802(37)$  fm

deducing also

- Deuteron charge radius:  $r_d = 2.12771(22)$  fm
- $R_\infty = 3.289\,841\,960\,249\,5(10)^{\text{radius}}(25)^{\text{QED}} \times 10^{15}$  Hz/c

Proton radius puzzle persist:

- Experimental problem(s)?
- New physics?
- Weird QCD or bound-state QED?
- Proton structure?

From 2.5 transitions in muonic deuterium:

→ deuteron radius

The deuteron and proton radii extracted from  $\mu_p$  and  $\mu_d$  are consistent with the 1S-2S isotope shift in H

From transitions in  $\mu^4\text{He}^+$  with  $u_r = 5 \times 10^{-5}$ .

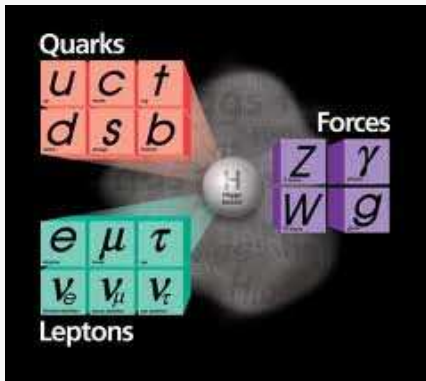
→  $^4\text{He}$  charge radius with  $u_r = 3 \times 10^{-4}$

→ agreement with the e-scattering value ( $u_r = 2 \times 10^{-3}$ )

→ important information for the proton puzzle (spin-, isospin-dependence etc.)

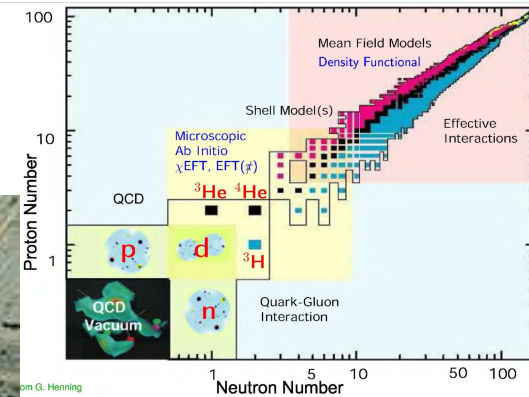
→ interesting information for few-nucleons theory, to disentangle potentials....

# Motivation, summary, outlook



Test of H energy levels  
Bound-state QED

$$\begin{aligned} \text{Mu} &= \mu^+ e^- \\ \text{Ps} &= e^+ e^- \end{aligned}$$



New physics?

Scattering  
 $e + p \rightarrow e + p$   
 $e + d \rightarrow e + d$   
 $\mu + p \rightarrow \mu + p$   
 $\gamma + p \rightarrow \gamma + p$   
 ...



Low-energy QCD  
 EFT,  $\chi$ pt, lattice  
 strong bound-state  
 p-structure  
 few-nucleon th.

H-spectroscopy

$\mu p$  and  $\mu d$

Proton charge radius  
 Proton Zemach radius  
 Deuteron charge radius

$\mu \text{He}^+$

$R_\infty = 3.2898419602495(10)(25)10^{15} \text{ Hz/c}$   
 combining  $\mu p$  with H spectroscopy





F. Biraben, S. Galtier, P. Indelicato, L. Julien, Labor. Kastler Brossel, Paris  
F. Nez, C. Szabó

M. Diepold, B. Franke, J. Götzfried, T.W. Hänsch, MPQ, Garching, Germany  
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## Scattering

- E08-007 @ JLAB, e-p at very low  $Q^2$
- A1-1/12 @ Mainz, e-d at very low  $Q^2$
- MUSE @ PSI,  $\mu$ -p/e-p
- E05-015 and CLASS @ JLAB, test  $2\gamma$
- OLYMPUS@ DESY and VEPP3, test  $2\gamma$
- Structure functions
- Compton scattering

## Theory and theoretical theory

- Bound-state QED
- Few-nucleon theories
- New physics, including weird QCD and QED
- Hadronic effects and proton structure (EFT,  $\chi$ PT, lattice?...)
- Analysis of scattering data

## Atomic physics

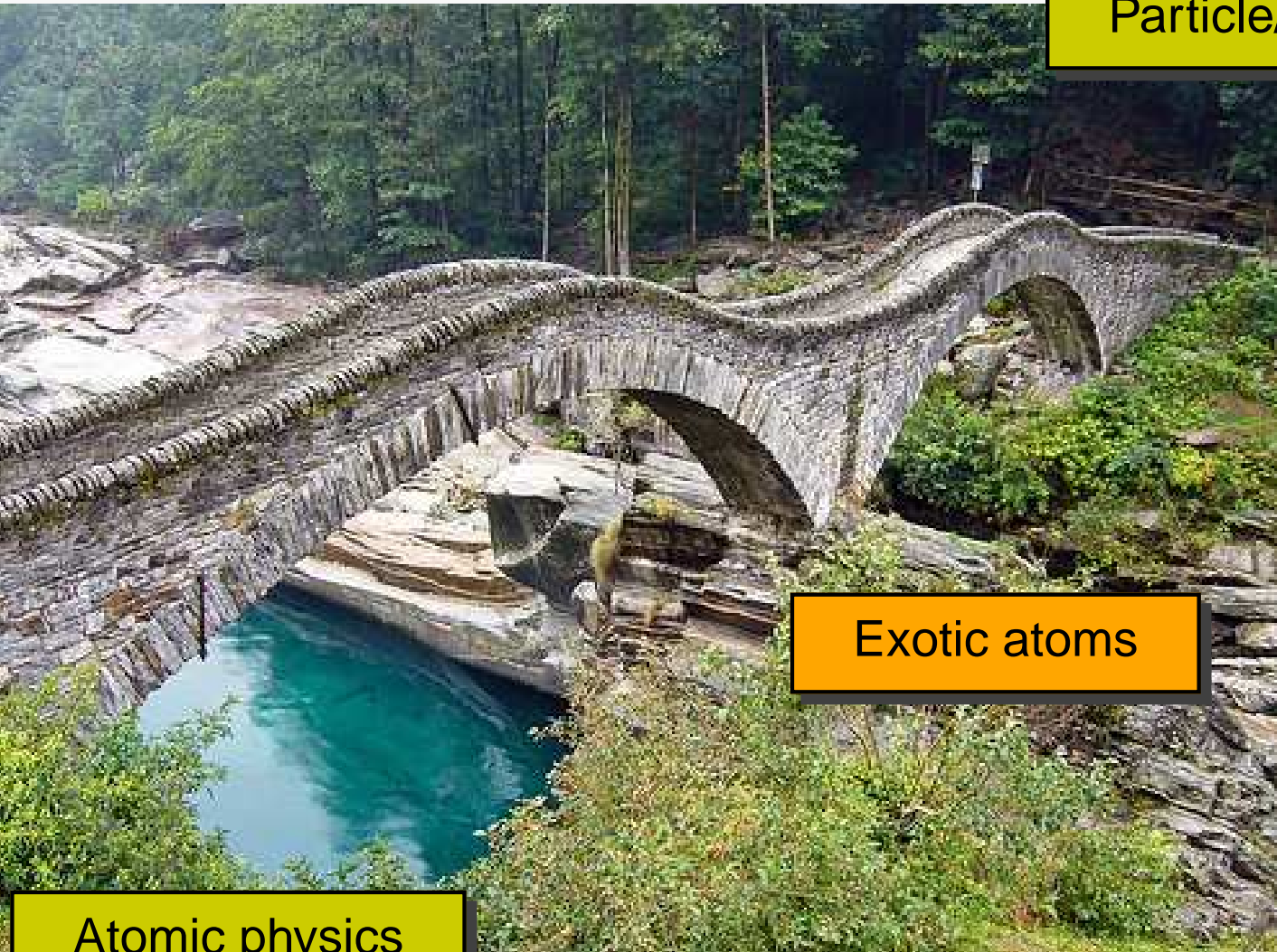
- Tan @ NIST:  $\text{Ne}^{9+}$
- Hänsch @ MPQ:  $2S - 4P$
- Nez @ LKB:  $1S - 3S$
- Hessels @ York:  $2S - 2P$
- Udem @ MPQ:  $\text{He}^+$
- Eikema @ Amsterdam:  $\text{He}^+$
- Cancio @ Florence: He
- Müller @ Ganil: halo He nuclei
- Ubachs @ Laserlab:  $\text{H}_2$
- Hilico @ LKB:  $\text{H}_2^+$

## Exotic atoms spectroscopy

- CREMA,  $\mu\text{He}^+$
- ETHZ-PSI-MPQ, Muonium and positronium

# Exotic atoms

Particle/Nuclear physics

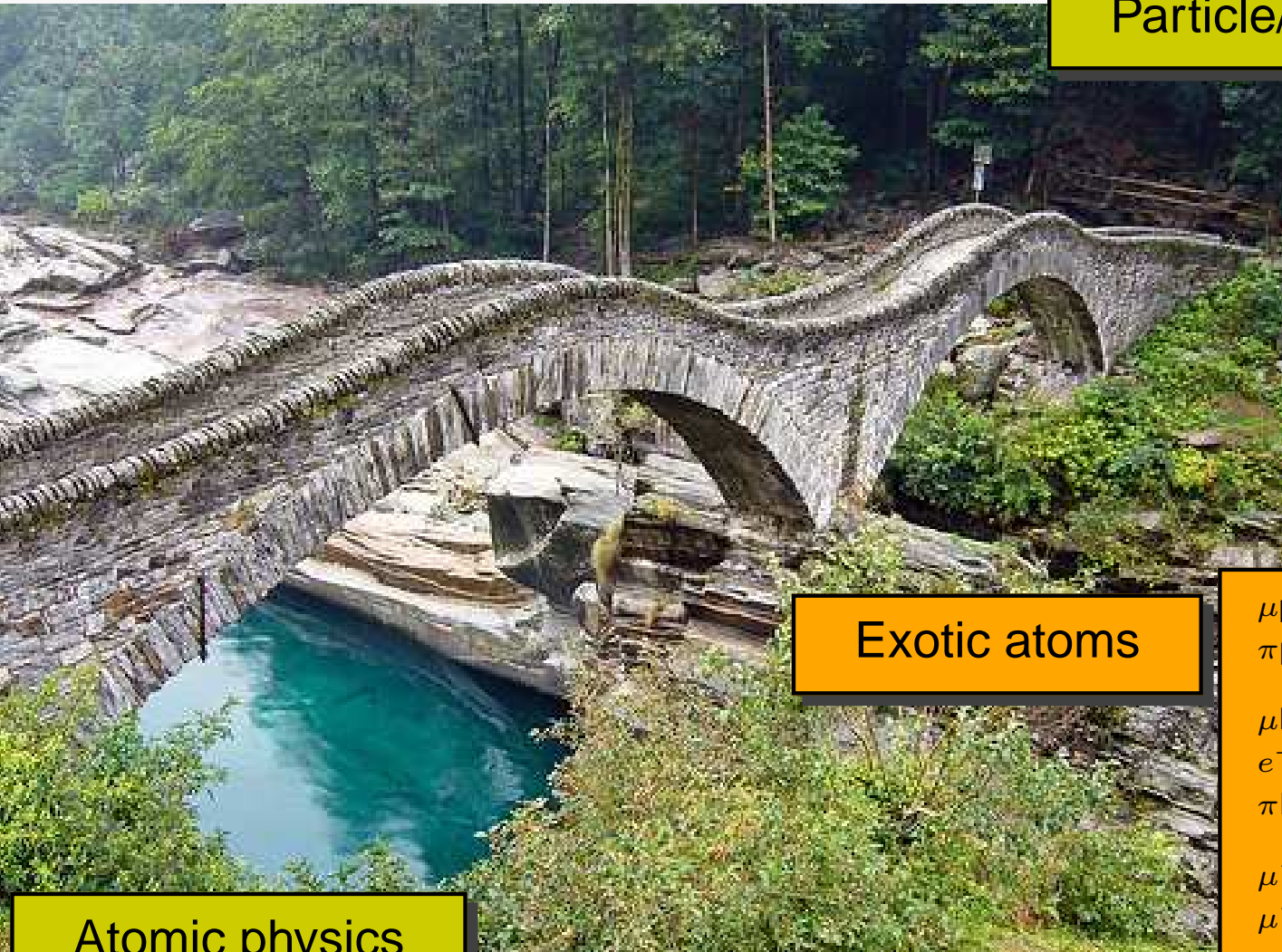


Exotic atoms

Atomic physics



# Exotic atoms



## Particle/Nuclear physics

- Bound-state QED
- Low-energy QCD
- EFT theories,  $\chi$ PT...
- Lattice QCD
- Ab-initio few nucleon th.
- Fundamental constants
- Symmetry test
- New physics

## Exotic atoms

## Atomic physics

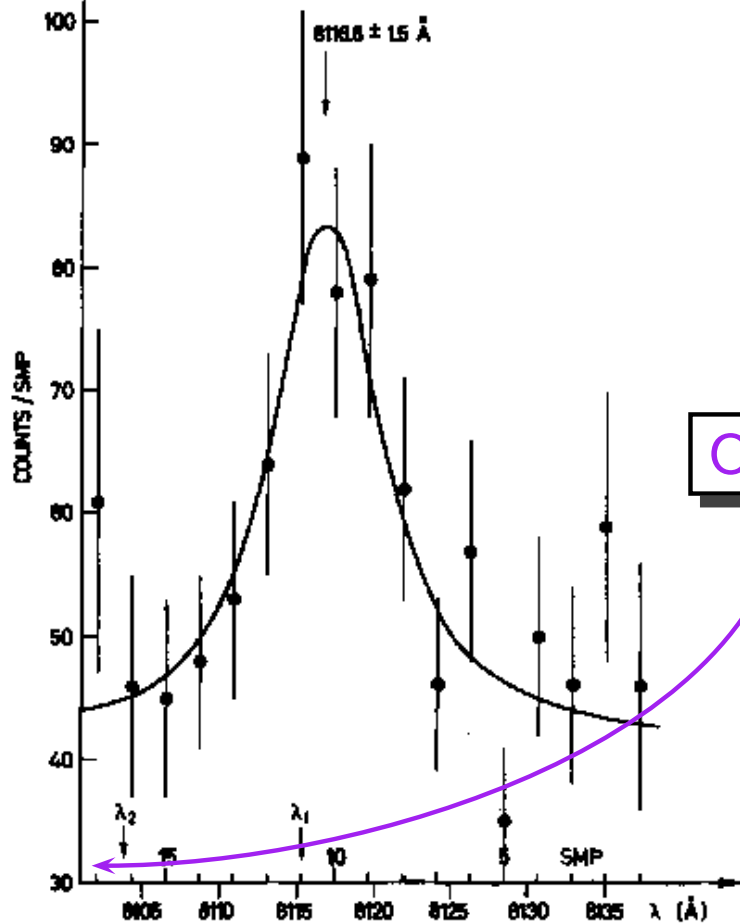
$\mu p$ (2S-2P)	(2010)	$[r_p]$
$\pi p$ and $\pi D$	(2011)	[scatt. length]
$\mu He^+$ (2S-2P)	(2014)	$[r_{He}]$
$e^+ e^-$ (1S-2S)	(ongoing)	
$\pi He$	(ongoing)	[pion mass]
$\mu^+ e^-$ (1S-2S)	(in preparation at PSI)	
$\mu^+ e^-$ (HFS)	(ongoing at JPARK)	
$\mu p, \mu^3 He$ (HFS)	(PSI?, JPARK)	[Zemach rad.]
$\mu Li$	(PSI?)	
$\mu Ra$	(PSI?)	[for Ra EDM]
$\bar{H}, \bar{p} He$	(CERN, ongoing)	

Back up slides

# Zavattini “resonance”

Zavattini radius seems apparently correct  
but it results from a wrong experiment  
combined with an incomplete theory!

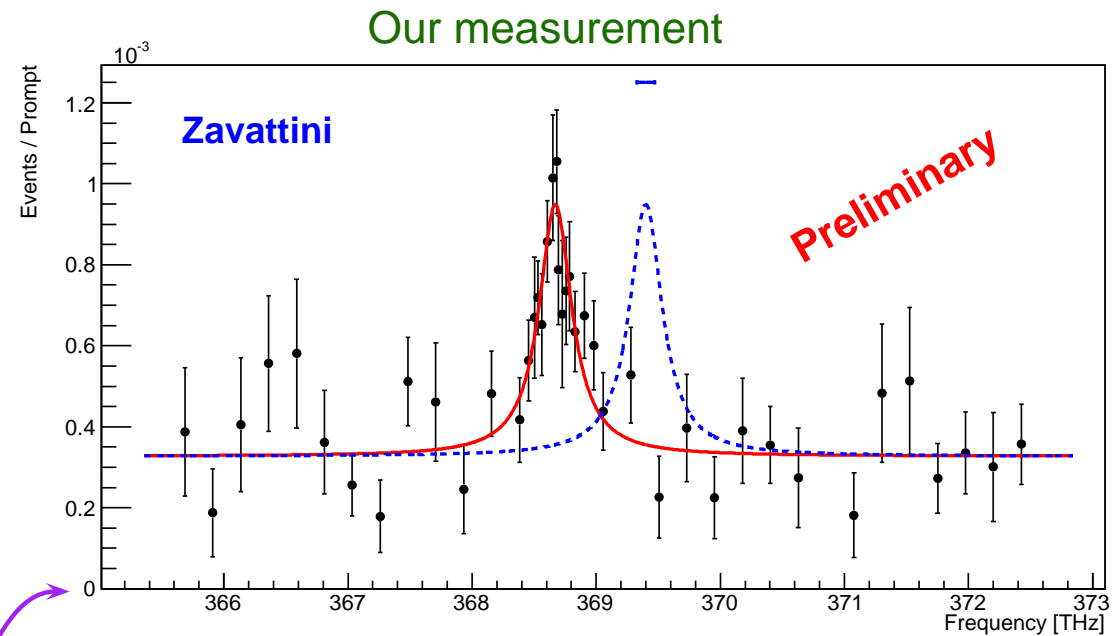
[Carboni et al., Nucl. Phys. A 278, 381 (1977)]



OFFSET

Zavattini experiment was performed at **50 bar** pressure:  
 $\Rightarrow$  2S-population is collisionally quenched.  
 $\Rightarrow$  No population left for a laser experiment.  
 (for comparison: we are measuring  $\mu\text{He}^+$  at **2 mbar**)

[Hauser et al., PRA 46, 2363 (1992)]



# Precision test of $B_{50}$ , $B_{60}$ ... contributions

	H [kHz]	He <sup>+</sup> [kHz]	ratio
$\Delta E_{2S-1S}$	$2.466 \times 10^{12}$	$9.869 \times 10^{12}$	$Z^2$ [= $\frac{3}{4}Z^2 R_\infty + \delta(L_{1S} - L_{2S})$ ]
$\delta(L_{1S} - L_{2S})^{\text{exp}}$ (from $\delta R_\infty$ )	<b>16</b> (2.2 ppm)	<b>65</b> (0.7 ppm)	$Z^2$ [= $\delta(\Delta E_{2S-1S} - \frac{3}{4}Z^2 R_\infty)$ ]
$(L_{1S} - L_{2S})^{\text{th}}$	7 127 887(44)	93 856 127(348)	$Z^{3.7}$ [Jentschura, 2006]
$\delta(L_{1S} - L_{2S})^{\text{th}}$	(6.3 ppm)	(3.7 ppm)	
$B_{60}$ and $B_{7i}$ terms	-8(3)	-543(185)	$Z^{6\dots}$
nuclear size (p, $^4\text{He}$ )	1102(44)	62 079(295)	$Z^4 r^2$

after  $\mu\text{p}$  ↓ ↓  $\mu\text{He}$  experiments

uncert. of nucl. size

(2)

(40)

$\mu\text{He}^+$ -pol. 5%

(16)

$\mu\text{He}^+$ -pol. 2%

check  $B_{60}$  and  $B_{7i}$  with

25%

7%

$\mu\text{He}^+$ -pol. 5%

3%

$\mu\text{He}^+$ -pol. 2%

# Precision test of $B_{50}$ , $B_{60}$ ... contributions

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$\Delta E_{2S-1S}$	$2.466 \times 10^{12}$	$9.869 \times 10^{12}$	$Z^2$ [= $\frac{3}{4}Z^2 R_\infty + \delta(L_{1S} - L_{2S})$ ]
$\delta(L_{1S} - L_{2S})^{\text{exp}}$ (from $\delta R_\infty$ )	16 (2.2 ppm)	<del>10 65</del> (0.7 ppm) (0.1 ppm)	$Z^2$ [= $\delta(\Delta E_{2S-1S} - \frac{3}{4}Z^2 R_\infty)$ ]  ( $H_{1S-2S} + \mu p$ )
$(L_{1S} - L_{2S})^{\text{th}}$	7 127 887(44)	93 856 127(348)	$Z^{3.7}$ [Jentschura, 2006]
$\delta(L_{1S} - L_{2S})^{\text{th}}$	(6.3 ppm)	(3.7 ppm)	
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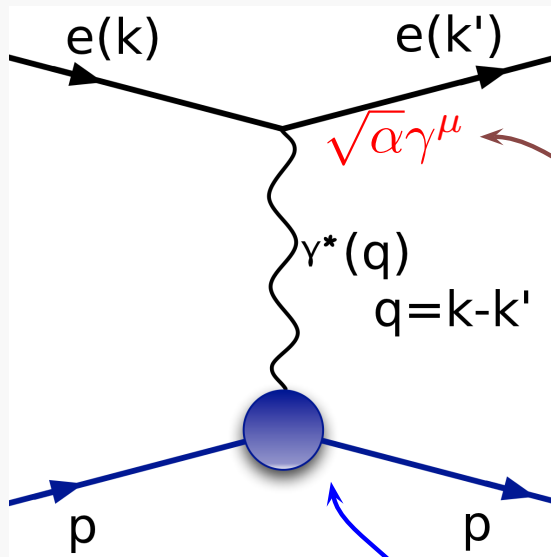
7%

$\mu\text{He}^+$ -pol. 5%

3%

$\mu\text{He}^+$ -pol. 2%

# Leptonic probes to determine the p structure

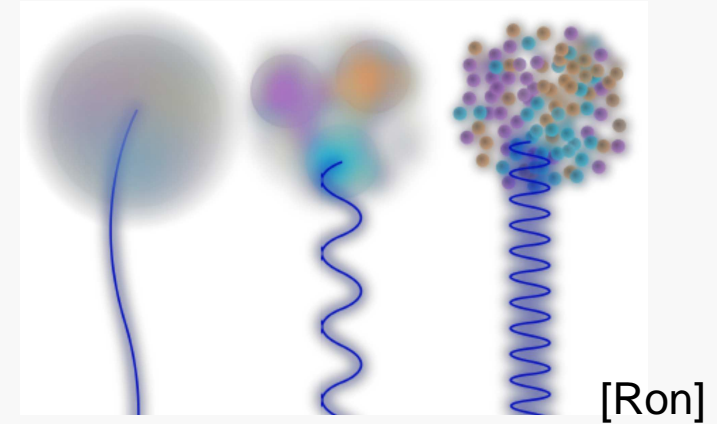


Electron vertex  
well known from QED  
and  $(g - 2)_e$

$$\sqrt{\alpha} \left[ \gamma^\mu F_1(q^2) + i\sigma^{\mu\nu} \frac{q_\nu}{2m} F_2(q^2) \right]$$

$$Q^2 [(\text{GeV}/c)^2] \sim \begin{cases} < 0.1 & \text{(Static Properties)} \\ 0.1 - 10 & \text{(Distributions, structure)} \\ \geq 20 & \text{(Perturbative QCD)} \end{cases}$$

$$Q^2 [(\text{GeV}/c)^2] \sim \begin{cases} (4 \cdot 10^{-6})^2 & (H) \\ (8 \cdot 10^{-4})^2 & (\mu p) \\ (> 6 \cdot 10^{-2})^2 & (e-p \text{ scatt.}) \end{cases}$$



Resolving power:  $\lambda = \hbar / \sqrt{-q^2}$

