On classification of effective field theories

Karol Kampf

Charles University, Prague

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Outline:

- Introduction of Amplitude methods: gluon scattering
- Effective Field Theories
- From NLSM to Periodic Table of Scalar Theories
- Spin-1
- Further avenues
- Summary

Introduction: Amplitudes

Objective of amplitude community:

Study a priori known objects from different perspective

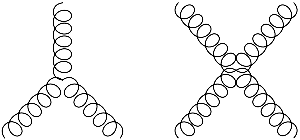
Example in mind: gluon amplitudes

- 1986: Parke and Taylor calculated 6-point gluon-scattering
- simplification: tree-level, no-fermions
- final result: extremely simple
- other way of calculation?

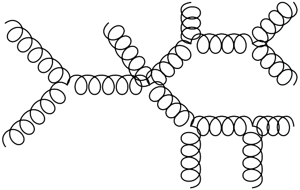
Example: gluon amplitudes

standard method of calculating n-gluon scattering processes:

- dominated by pure-gluon interactions in QCD
- elementary 3pt and 4pt vertices



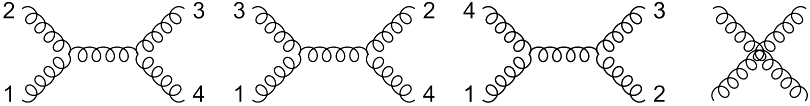
- construct all possible Feynman diagrams, e.g. 9pt:



- complicated already for the tree level diagrams even for small number of external legs

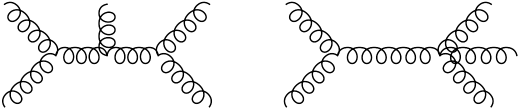
History: gluon amplitude, tree-level

- 3pt: 1 diagram, on-shell = 0
- 4pt: 4 diagrams can be calculated by hand:



intermediate steps complicated but differential cross section nice

- 5pt: calculated in '80, calculation blows up on several pages



structure of the numerators, schematically:

$$\text{double-propagator: } (p_i \cdot p_j)(p_k \cdot \epsilon)(\epsilon \cdot \epsilon)(\epsilon \cdot \epsilon),$$

$$\text{single-propagator: } (p_k \cdot \epsilon)(\epsilon \cdot \epsilon)(\epsilon \cdot \epsilon),$$

- 6pt: impossible by standard methods, but...

History: gluon amplitude, tree-level, 6pt

SSC approved in 1983 (to be cancelled 10 years later) motivated the following work

THE CROSS SECTION FOR FOUR-GLUON PRODUCTION BY GLUON–GLUON FUSION

Stephen J. PARKE and T.R. TAYLOR

Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510 USA

Received 13 September 1985

The cross section for two-gluon to four-gluon scattering is given in a form suitable for fast numerical calculations.

Theoretical predictions for four-jet production at hadron colliders allow detailed tests of QCD. Moreover, at SSC energies, four jets become a serious background to many interesting processes which probe new physics, e.g. pair production of electroweak bosons [1]. Hence a detailed knowledge of four-jet event characteristics is crucial for good background rejection. Although some individual contributions to four-jet production have already been analysed (see e.g. ref. [2]), the two-gluon to four-gluon scattering, which is the dominant contribution for a wide range of subprocess energies, has remained beyond the scope of previous computational techniques. Here we outline our calculation of the cross section for this process, in the tree approximation of perturbative QCD. The final cross section is presented in a form suitable for fast numerical calculations.

Our calculation makes use of techniques developed in ref. [3], based on the application of extended supersymmetry. We adopt the convention that all particles

History: gluon amplitude, tree-level, 6pt

Parke and Taylor finished the article with:

> Details of the calculation, together with a full exposition of our techniques, will be given in a forthcoming article. Furthermore, we hope to obtain a simple analytic form for the answer, making our result not only an experimentalist's, but also a theorist's delight.

History: gluon amplitude, tree-level, 6pt

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Indeed it was given a year later [Parke, Taylor '86] for the MHV:

$$A_n(- - + \ldots +) = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \ldots \langle n1 \rangle}$$

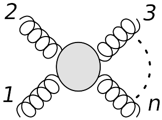
One line formula!

The so-called spinor-helicity formalism was introduced (reasonable variables for massless particles) cf. [Mangano,Parke,Xu '87]

$$\langle ij \rangle = \sqrt{|2p_i \cdot p_j|} \mathrm{e}^{i\phi_{ij}}$$

Is there some better way to calculate?

Example: gluon amplitudes



Important simplification at tree level:

- colour ordering → stripped amplitude

$$M^{a_1 \dots a_n}(p_1, \dots p_n) = \sum_{\sigma/Z_n} \mathrm{Tr}(t^{a_{\sigma(1)}} \dots t^{a_{\sigma(n)}}) M_\sigma(p_1, \dots, p_n)$$

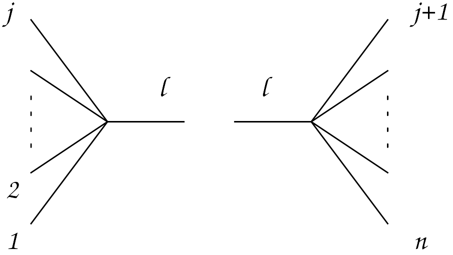
- $M_\sigma(p_{\sigma(1)}, \dots, p_{\sigma(n)}) = M(p_1, \dots, p_n) \equiv M(1, 2, \dots n)$
- propagators ⇒ the only poles of M_σ
- thanks to ordering the only possible poles are:

$$P_{ij}^2 = (p_i + p_{i+1} + \dots + p_{j-1} + p_j)^2$$

Pole structure

Weinberg's theorem (one particle unitarity): on the factorization channel

$$\lim_{P_{1j}^2 \to 0} M(1,2,\ldots n) = \sum_{h_l} M_L(1,2\ldots j,l) \times \frac{1}{P_{1j}^2} \times M_R(l,j+1,\ldots n)$$



BCFW relations, preliminaries

[Britto, Cachazo, Feng, Witten '05]

Reconstruct the amplitude from its poles (in complex plane)

- shift in two external momenta

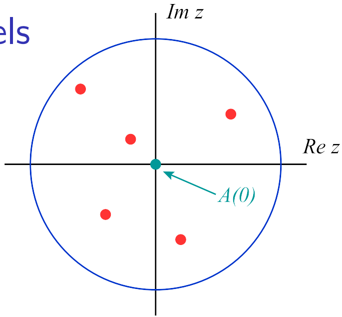
$$p_i \to p_i + zq, \qquad p_j \to p_j - zq$$

- keep p_i and p_j on-shell, i.e.

$$q^2 = q \cdot p_i = q \cdot p_j = 0$$

- amplitude becomes a meromorphic function $A(z)$
- only simple poles coming from propagators $P_{ab}(z)$
- original function is $A(0)$

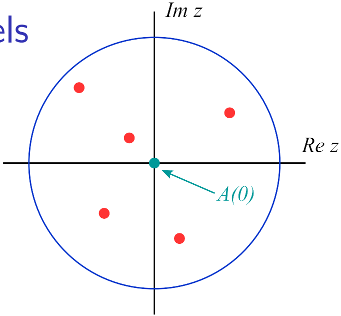
BCFW relations: factorization channels



Cauchy's theorem

$$\frac{1}{2\pi \mathrm{i}} \int \frac{dz}{z} A(z) = A(0) + \sum_k \frac{\mathrm{Res}\,(A, z_k)}{z_k}$$

BCFW relations: factorization channels



Cauchy's theorem

$$0 = \frac{1}{2\pi \mathrm{i}} \int \frac{dz}{z} A(z) = A(0) + \sum_k \frac{\mathrm{Res}\,(A, z_k)}{z_k}$$

If $A(z)$ vanishes for $z \to \infty$

$$A = A(0) = -\sum_k \frac{\mathrm{Res}\,(A, z_k)}{z_k}$$

BCFW relations

$\boxed{P_{ab}^2(z) = 0}$ if one and only one i (or j) in $(a, a+1, \ldots, b)$.

Suppose $i \in (a, \ldots, b) \not\ni j$

$$\boxed{P_{ab}^2(z) =} (p_a + \ldots + p_{i-1} + p_i + zq + p_{i+1} + \ldots + p_b)^2 =$$
$$= P_{ab}^2 + 2q \cdot P_{ab} z = \boxed{0}$$

solution

$$z_{ab} = -\frac{P_{ab}^2}{2(q \cdot P_{ab})} \qquad \Rightarrow \qquad P_{ab}^2(z) = -\frac{P_{ab}^2}{z_{ab}}(z - z_{ab})$$

Thus

$$\mathrm{Res}(A, z_{ab}) = \sum_s A_L^{-s}(z_{ab}) \times \frac{-z_{ab}}{P_{ab}^2} \times A_R^s(z_{ab})$$

and for allowed helicities it factorizes into two subamplitudes

BCFW relations

Using Cauchy's formula, we have finally as a result

$$A = \sum_{k,s} A_L^{-s_k}(z_k) \frac{1}{P_k^2} A_R^{s_k}(z_k)$$

- based on two-line shift (convenient choice: adjacent i,j)
- recursive formula (down to 3-pt amplitudes)
- number of terms small = number of factorization channels
- all ingredients are on shell

BCFW Example: gluon amplitudes

od diagrams for n-body gluon scatterings at tree level

n	3	4	5	6	7	8
# diagrams (inc.crossing)	1	4	25	220	2485	34300
# diagrams (col.ordered)	1	3	10	38	154	654
# BCFW terms	–	1	2	3	6	20

[C.Cheung: TASI Lectures '17]

BCFW recursion relations: problems

We have assumed that

$$A(z) \to 0, \qquad \text{for} \qquad z \to \infty$$

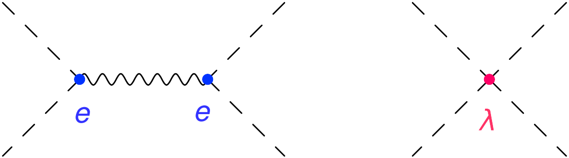
More generally we have to include a boundary term in Cauchy's theorem.

This is intuitively clear: we can formally use the derived BCFW recursion relations to obtain any higher n amplitude starting with the leading interaction. But this does not have to be the correct answer.

BCFW recursion relations: problems

example: scalar-QED

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |D\phi|^2 - \frac{1}{4}\lambda|\phi|^4$$



Due to the power-counting the boundary term is proportional to

$$B \sim 2e^2 - \lambda$$

In order to eliminate the boundary term we have to set $\lambda = 2e^2$, then the original BCFW works.

I.e. we needed some further information (e.g. supersymmetry) to determine the λ piece.

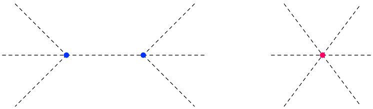
Effective field theories

Effective field theories: general picture

Now we have infinitely many unfixed "λ" terms. Schematically

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \lambda_4(\partial^{m_4}\phi)^4 + \lambda_6(\partial^{m_6}\phi)^6 + \dots$$

Example: 6pt scattering, Feynman diagrams



Corresponding amplitude:

$$\mathcal{M}_6 = \sum_{I=poles} \lambda_4^2 \frac{\dots}{P_I} + \lambda_6(\dots)$$

λ_6 part: not fixed by the pole behaviour.

Task: to find a condition in order to link these two terms

Effective field theories: introduction

Usual steps:

Symmetry $\rightarrow$ Lagrangian $\rightarrow$ Amplitudes $\rightarrow$ physical quantities
(cross-section, masses, decay constants, ...)

In our work – opposite direction:

Amplitudes $\rightarrow$ physical quantities ($\rightarrow$ Lagrangian $\rightarrow$ Symmetry)

Our aim: classification of interesting EFTs

works done in collaborations with Clifford Cheung, Jiri Novotny, Chia-Hsien Shen, Jaroslav Trnka and Congkao Wen

Effective field theories: scalar theories

As simple as possible: a spin-0 massless degree of freedom with a three-point interaction.

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General formula for three-particle amplitude

$$A(1^{h_1}2^{h_2}3^{h_3}) = \begin{cases} \langle 12\rangle^{h_3-h_1-h_2}\langle 23\rangle^{h_1-h_2-h_3}\langle 31\rangle^{h_2-h_3-h_1}, & \Sigma h_i \leq 0 \\ [12]^{h_1+h_2-h_3}[23]^{h_2+h_3-h_1}[31]^{h_3+h_1-h_2}, & \Sigma h_i \geq 0 \end{cases}$$

n.b. again the spinor-helicity notation, e.g. $p_i \cdot p_j \sim \langle ij\rangle[ij]$

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For scalars ($h_i = 0$) this is a constant - corresponding to $\mathcal{L}_{int} = \lambda\phi^3$.

All derivatives can be removed by equations of motions (boxes)

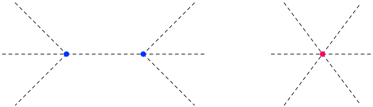
$$\mathcal{L}_{int} = (\partial_\alpha \ldots \partial_\omega \phi)(\partial^\alpha \ldots \partial^\omega \phi)\phi \quad \rightarrow \quad \mathcal{L}_{int} = (\Box\phi)(\ldots)$$

Effective field theories: scalar theories

We start with (m counts number of derivatives)

$$\mathcal{L}_{int} = \lambda_4 \partial^m \phi^4$$

n.b. we want to connect this four-point vertex with the 6-point contact terms



This rules out again the no-derivative terms, as the powercounting dictates:

$$\partial^m \times \frac{1}{\partial^2} \times \partial^m \quad \rightarrow \quad \partial^{2m-2}\phi^6$$

and we have to start at least with $m = 2$, i.e. two derivatives

Simplest example: two derivatives, single scalar

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi + \lambda_4\partial^2\phi^4 + \lambda_6\partial^2\phi^6 + \dots$$

How to connect λ_4 and λ_6?
Well Lagrangian, an infinite series, looks complicated, but it is not the case. It represents a free theory:

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi\,\underbrace{(1 + \lambda_4\phi^2 + \dots)}_{F(\phi)}$$

$F(\phi)$ can be removed by a field redefinition

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Summary: the non-trivial simplest cases:

- more than two derivatives
- more flavours ($\phi \to \phi_1, \phi_2, \dots$)

More flavours

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi^i\partial^\mu\phi^i + \lambda_{ijkl}\partial_\mu\phi^i\partial^\mu\phi^j\phi^k\phi^l + \lambda_{i_1\dots l_6}\partial_\mu\phi^{i_1}\partial^\mu\phi^{i_2}\phi^{i_3}\dots\phi^{i_6} + \dots$$

- Can be used for systematic studies of two species, three species, etc.
- Very complicated generally
- Assume some simplification, organize using a group structure

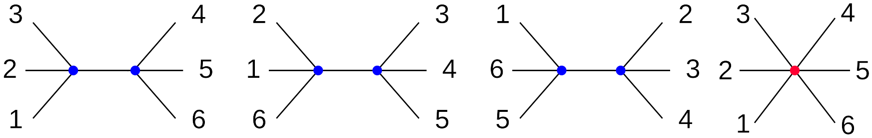
$$\phi = \phi^a T^a$$

- motivated by the 'gluon case': flavour ordering [KK,Novotny,Trnka'13]

$$A^{a_1\dots a_n} = \sum_{perm} \mathrm{Tr}(T^{a_1}\dots T^{a_n})A(p_1,\dots p_n)$$

More flavours: stripped amplitude

first non-trivial case 6pt scattering:



power-counting is ok:

$$\lambda_4^2\, p^2 \frac{1}{p^2} p^2 + \lambda_6\, p^2$$

in order to combine the pole and contact term we need to consider some limit. The most natural candidate: we will demand soft limit, i.e.

$$A \to 0, \qquad \text{for} \quad p \to 0$$

$$\Rightarrow \quad \lambda_4^2 \sim \lambda_6$$

Standard direction(s)

Assuming the shift symmetry

$$\phi^a \to \phi^a + \epsilon^a$$

$\Rightarrow$ Noether current

$$A_\mu^a = \frac{\delta \mathcal{L}}{\delta \partial^\mu \phi^a}$$

$\Rightarrow$ Ward identity $\Rightarrow$ LSZ

$$\langle 0|A_\mu^a(x)|\phi^b(p)\rangle = iF\delta^{ab}p_\mu \mathrm{e}^{-ipx}$$

$\Rightarrow$ Adler zero

$$\lim_{p\to 0}\langle f|i+\phi^a(p)\rangle = 0$$

$\Rightarrow$ CCWZ: non-linear sigma model

$$\mathcal{L} = \frac{F^2}{2}\mathsf{Tr}(\partial_\mu U^\dagger \partial^\mu U), \qquad U = \mathrm{e}^{\frac{i}{F}\phi^a T^a}$$

[Weinber'66], [Ian Low '14-'15]

Natural classification: σ and ρ

Soft limit of one external leg of the tree-level amplitude

$$A(tp_1, p_2, \ldots, p_n) = \mathcal{O}(t^{\sigma}), \qquad \text{as} \quad tp_1 \to 0$$

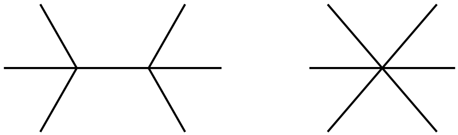
Interaction term

$$\mathcal{L} = \partial^m \phi^n$$

Then another natural parameter is (counts the homogeneity)

$$\rho = \frac{m-2}{n-2} \qquad \text{"averaging number of derivatives"}$$

e.g. $\mathcal{L} = \partial^m \phi^4 + \partial^{\widetilde{m}} \phi^6$



so these two diagrams can mix: $p^{2m-2} \sim p^{\widetilde{m}}$

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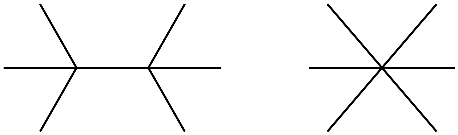

so these two diagrams can mix: $p^{2m-2} \sim p^{\widetilde{m}}$
$2m - 2 - 2 = \tilde{m} - 2 \Rightarrow \frac{2m-4}{4} = \frac{\tilde{m}-2}{4} \Rightarrow$

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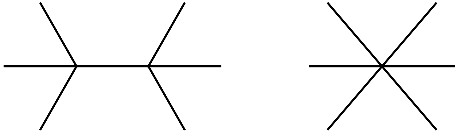

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$2m - 2 - 2 = \tilde{m} - 2 \Rightarrow \frac{2m-4}{4} = \frac{\tilde{m}-2}{4} \Rightarrow \rho = \widetilde{\rho}$

rho is same if they talk to each other

Non-trivial cases

$$\text{for: } \mathcal{L} = \partial^m \phi^n : \qquad m < \sigma n$$

or

$$\sigma > \frac{(n-2)\rho + 2}{n}$$

i.e.

ρ	σ at least
0	1
1	2
2	2
3	3

i.e. non-trivial regime for $\rho \leq \sigma$

First case: $\rho = 0$ (i.e. two derivatives)

Schematically for single scalar case

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \sum_i \lambda_4^i(\partial^2\phi^4) + \sum_i \lambda_6^i(\partial^2\phi^6) + \dots$$

similarly for multi-flavour (ϕ_i: $\phi_1, \phi_2, \dots$).
non-trivial case

$$\sigma = 1$$

Outcome:

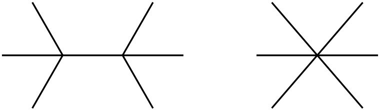
- single scalar: free theory
- multiple scalars (with flavour-ordering): non-linear sigma model

n.b. it represents a generalization of [Susskind, Frye '70], [Ellis, Renner '70]

Second case: $\rho = 1$, $\sigma = 2$ (double soft limit)

1. focus on the lowest combination and fix the form:

$$\mathcal{L}_{int} = c_2(\partial\phi \cdot \partial\phi)^2 + c_3(\partial\phi \cdot \partial\phi)^3 \qquad \text{condition: } c_3 = 4c_2^4$$



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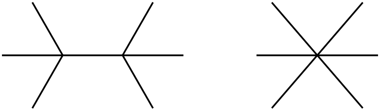

2. find the symmetry

$$\phi \to \phi - b_\rho x^\rho + b_\rho \partial^\rho \phi\, \phi \quad \text{(again up to 6pt so far)}$$

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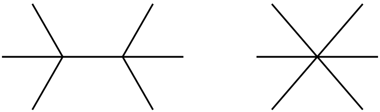

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3. ansatz of the form

$$c_n(\partial\phi \cdot \partial\phi)^n + c_{n+1}(\partial\phi \cdot \partial\phi)^n \partial\phi \cdot \partial\phi$$

4. in order to cancel: $2(n+1)c_{n+1} = (2n-1)c_n$
   i.e. $c_1 = \frac{1}{2} \Rightarrow c_2 = \frac{1}{8}, c_3 = \frac{1}{16}, c_4 = \frac{5}{128}, \ldots$

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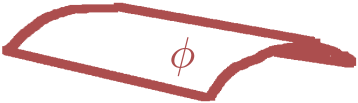
solution:

$$\mathcal{L} = -\sqrt{1 - (\partial\phi \cdot \partial\phi)}$$

This theory known as a scalar part of the Dirac-Born-Infeld [1934] – DBI action
Scalar field can be seen as a fluctuation of a 4-dim brane in five-dim Minkowski space



Remark: soft limit and symmetry are “equivalent”

Third case: $\rho = 2$, $\sigma = 2$ (double soft limit)

Similarly to previous case we will arrive to a unique solution: the Galileon Lagrangian

$$\mathcal{L} = \sum_{n=1}^{d+1} d_n \phi \mathcal{L}^{\mathrm{der}}_{n-1}$$

$$\mathcal{L}^{\mathrm{der}}_n = \varepsilon^{\mu_1 \ldots \mu_d} \varepsilon^{\nu_1 \ldots \nu_d} \prod_{i=1}^{n} \partial_{\mu_i} \partial_{\nu_i} \phi \prod_{j=n+1}^{d} \eta_{\mu_j \nu_j} = -(d-n)! \det \left\{ \partial^{\nu_i} \partial_{\nu_j} \phi \right\}.$$

It possesses the Galilean shift symmetry

$$\phi \to \phi + a + b_\mu x^\mu$$

(leads to EoM of second-order in field derivatives)

Surprise: $\rho = 2$, $\sigma = 3$ (enhanced soft limit)

- general galileon: three parameters (in 4D)
- only two relevant (due to dualities [de Rham, Keltner, Tolley '14] [KK, Novotny '14])

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- symmetry explanation: hidden symmetry [K. Hinterbichler and A. Joyce 1501.07600]

$$\phi \to \phi + s_{\mu\nu} x^\mu x^\nu - 12\lambda_4 s^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

New recursion for effective theories

[Cheung, KK, Novotny, Shen, Trnka 2015]

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Can we instead use the soft limit directly? yes!
The standard BCFW not applicable, we propose a special shift:

$$p_i \to p_i(1 - za_i) \qquad \text{on all external legs}$$

This leads to a modified Cauchy formula

$$\oint \frac{dz}{z} \frac{A(z)}{\Pi_i (1 - a_i z)^\sigma} = 0$$

note there are no poles at $z = 1/a_i$ (by construction).
Now we can continue in analogy with BCFW

Vector EFTs

[Cheung, KK, Novotny, Shen, Trnka, Wen '18]

Spin-1 sector

- we have followed same strategy as for spin-0
- first focused on single field, massless, A_μ
- again start at 4-pt
- gauge invariance $\Rightarrow$ in Lagrangian we have $F_{\mu\nu}$, i.e.

$$\mathcal{L} = f(\partial_\mu A^\nu)$$

- $\Rightarrow$ soft-limit trivial

Spin-1 sector

- general gauge invariant Lagrangian

$$\mathcal{L} = -\frac{1}{4}\langle FF\rangle + g_4^{(1)}\langle FFFF\rangle + g_4^{(2)}\langle FF\rangle^2 + g_6^{(1)}\langle FF\rangle^3 + g_6^{(2)}\langle FFFF\rangle\langle FF\rangle + g_6^{(3)}\langle FFFFFF\rangle + \ldots,$$

$\langle \ldots \rangle$: traces over Lorentz indices

- trivial soft limit
- $\Rightarrow$ we have tried $O(t^2) \rightarrow$ no solution

Spin-1 sector

- important difference from the spin-0 case: two polarizations
- spinor helicity variables

$$p^\mu = \sigma^\mu_{a\dot{a}} \lambda^a \tilde{\lambda}^{\dot{a}}$$

- little group scaling

$$\lambda \to t\lambda, \ \tilde{\lambda} \to \frac{1}{t}\tilde{\lambda}$$

- momenta invariant under the little group scaling

$$p^\mu \to p^\mu$$

- but not the polarization vector (and consequently neither amplitude)

$$\epsilon^+_{a\dot{a}} \to \frac{1}{t^2}\, \epsilon^+_{a\dot{a}}, \qquad \epsilon^-_{a\dot{a}} \to t^2\, \epsilon^-_{a\dot{a}}$$

Spin-1 sector

- focusing on $D = 4$, defining

$$f = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu}, \qquad g = -\frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

- Cayley-Hamilton relation⇒ these are the only two building blocks:

$$\mathcal{L} = f + a_1 f^2 + a_2 g^2 + b_1 f^3 + b_2 f g^2 + \ldots$$

- three 4-pt possibility ($+/-$: positive/negative helicity)

$$- - - -, \qquad - - - +, \qquad - - + +$$

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$$----, \qquad ---+, \qquad --++$$

trivially zero

- thus only two possible 4-pt amplitudes

$$A_{----} = (a_1 - a_2)(\langle 12 \rangle^2 \langle 34 \rangle^2 + perm)$$
$$A_{--++} = (a_1 + a_2)(\langle 12 \rangle^2 [34]^2)$$

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$$A_{--++} = (a_1 + a_2)(\langle 12\rangle^2[34]^2)$$

- simplificaton: focus only on the spinor-conserving models

Spin-1 sector

- we can study quite generic amplitudes and its properties
- single soft limit doesn't work, but we can study more combinations, depending on helicities
- two basic possibilities

$$p^\mu = \sigma^\mu_{a\dot{a}} \lambda^a \tilde{\lambda}^{\dot{a}} \to 0 \quad \begin{cases} \lambda \to 0 \\ \tilde{\lambda} \to 0 \end{cases}$$

- so-called holomorphic or anti-holomorphic soft-limit
- we can study combinations of multiple soft limits

Spin-1 sector

- Result, what finally worked: multichiral soft limit

$$A(1^-2^-\dots(n/2)^-(n/2+1)^+\dots n^+) = O(\epsilon) \quad \text{for } \tilde{\lambda}_i \to 0$$

- i.e. anti-holomorphic soft-limit of all negative helicity photons
- (or vice versa)
- can be easily generalized to non-concerving helicity models

Spin-1 sector

- the only reasonable soft limit leads to the following theory

$$\mathcal{L}_{\mathrm{BI}} = 1 - \sqrt{(-1)^{D-1}\det(\eta_{\mu\nu} + F_{\mu\nu})},$$

- very well-known (from 1934!): the Born-Infeld model
- $U(1)$ gauge field on the brane
- so far we don't have a symmetry explanation of this behaviour
- proved using SUSY breaking $\mathcal{N}$=2 to $\mathcal{N}$=1
- the soft limit property is strong enough for new on-shell recursion relation

Spin-1 sector

- alternatively we used dim reduction to fix the amplitude
- we have also studied the Galileon-like theories
- (no-go theorem on vector Galileon [Deffayet et al'14])
- the starting amplitude is easy

$$A_{--++} = \langle 12 \rangle^2 [34]^2 s_{12}$$

- however, there is no signal of an exceptional theory

Further avenues

- so far avoided the fermionic degrees of freedom (see e.g. Elvang et al.'18)
- multiple flavours – especially without flavour ordering
- only two-flavour case fully classified
- preliminary study of the mixed scalar-vector case (Galileon-BI): more promising than the pure Galileon-like BI
- spin-2: similar to Galileon-like studies – no exceptional candidate
- non-abelian Born-Infeld
- non-zero masses (technically possible)
- more generally: breaking the shift symmetries
- loop corrections – focused on the exceptional theories
- connection with CHY [Cachazo, He, Yuan] formalism

Flavour of Phenomenology...

Our works so far for hep-th.
Principal connections with phenomenology mainly ih these directions:

- Chiral Perturbation Theory: multiple pion scattering
- "Beyond Standard Gravity"
- High-energy SM precision tests
    - modification of the Standard EFT expansion

$$\mathcal{L}^{\text{eff}} = \mathcal{L}^{SM} + \sum c_i^6 \frac{O^6}{\Lambda^2} + \sum c_i^8 \frac{O^8}{\Lambda^4} + \dots$$

    - Higgs non-linear dynamics
    - cf. works of F.Riva and collaborators
    - for spin-1: see e.g. recent Ellis, Ge studies on the BISM extensions focusing on $gg \to \gamma\gamma$ scattering at LHC

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- motivated by the amplitude methods employed for renormalizable theories
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Thank you!