QUANTUM GRAVITY PHENOMENOLOGY: THEORETICAL APPROACHES

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QUANTUM GRAVITY PHENOMENOLOGY IN THE MULTI-MESSENGER APPROACH

Working Group 1: Theoretical approaches

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WG1 contribution to the Action

- General aim:
  - Describe scenarios for low-energy (sub-Planckian) effects induced by quantum gravity theories

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  - Improve the mathematical consistency and physical understanding
  - Provide reliable predictions and admit experimental constraints
  - Describe observers, their measurements and measurement comparison in the presence of LIV or DSR scenarios consistently
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- Milestones:
  - Compile a comprehensive review of existing theoretical frameworks for QG phenomenology

    [this is part of one of the Action milestone goals: review on theoretical predictions of possible observations, and of current experimental constraints for each cosmic messenger]

The problem of Quantum Gravity

- Why is there a QG problem

General Relativity is a *classical* theory, describing *spacetime dynamics* and its interaction with classical 'matter'

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Attempts to quantise GR showed that it is fundamentally inconsistent as a perturbative quantum field theory (with or without matter), and attempts to build QFT on a dynamical spacetime have failed

While both GR and QFT use a classical spacetime, they define its points in incompatible ways: in QFT localisation improve with increasing mass of the probe (minimal quantum uncertainties); in GR localisation improves with decreasing mass of the probe (minimal back-reaction on spacetime geometry)

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✦ This might be a problem physicists might be tempted to ignore, since all experimental setups presently available are dominated either by the QM regime or by the GR regime. Indeed arguments to estimate the QG regime show that is it quite extreme.

The problem of Quantum Gravity

- Scale where the QG problem is relevant (a heuristic argument)

The Compton radius limits localisability in QM $r_c = \dfrac{2\pi\hbar}{mc}$

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$$M_P = \sqrt{\frac{\hbar c}{G}} \sim 10^{19}\mathrm{GeV}/c^2$$

For this value of the mass the two scales are of the order of the Planck length

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35}\mathrm{m}$$

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✦ But the QG regime is very relevant also e.g. in the early universe, which is accessible with current observations, and might have signatures in astrophysical observations — that's why we are here!

A (non-exhaustive) list of Quantum Gravity theories

- *Perturbative Quantum Gravity:* quantised gravitational interactions treated at the perturbative level as an Effective Field Theory, no quantum spacetime
- *String Theory*: quantum field theory on a fixed spacetime hoping to recover the (supersymmetric) theory of gravity plus standard model fields in the low energy limit
- *Loop Quantum Gravity:* nonperturbative and background-independent theory of quantum spacetime, without matter
- *Asymptotic Safety:* solve perturbative nonrenormalizability of gravitational interactions in a standard QFT perspective by looking for a nongaussian UV fixed point, no quantum spacetime
- *Noncommutative geometry:* description of quantum properties of spacetime by introducing limitations on their measurability, no gravitational dynamics
- *Causal Sets:* spacetime discretisation based on the definition of a discrete causal structure
- *Causal Dynamical Triangulations:* nonperturbative and background-independent formulation of the path integral describing evolution of a discrete spacetime

From QG theory to phenomenology — and back

- ✦ None of the candidate QG theories is sufficiently developed to produce direct clear predictions (often due to their complicated structure)

- ✦ For the scopes of this Action it is relevant to focus on the aspects of theoretical research in quantum gravity that are closer to phenomenology
(WG1 is called "Theoretical frameworks for QG effects below the Planck energy")

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- <u>At the theory level</u> we can proceed in a two-way fashion

  Construct effective models which are directly testable and try to relate them to features of a given QG theory *(bottom-up approach)*

  Derive semi-classical effects from QG theories, even if with heuristic arguments, and try to link them to phenomenological models *(top-down approach)*

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- Examples of QG-inspired features:

  In string theory there can be an "emergent" spacetime noncommutativity

  In LGQ there are indications that spacetime is somewhat discretised, at least at the level of the area operator

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- ✦ At the phenomenology level we can establish a two-way communication channel between theory and experiments

Current observations cannot rule out specific QG theories, regardless how good the data are, because the link between fundamental theories and QG phenomenology is loose

Still, we can constrain some classes of QG effects, and use the results to inform the theoretical developments

- ✦ We will discuss an explicit example later

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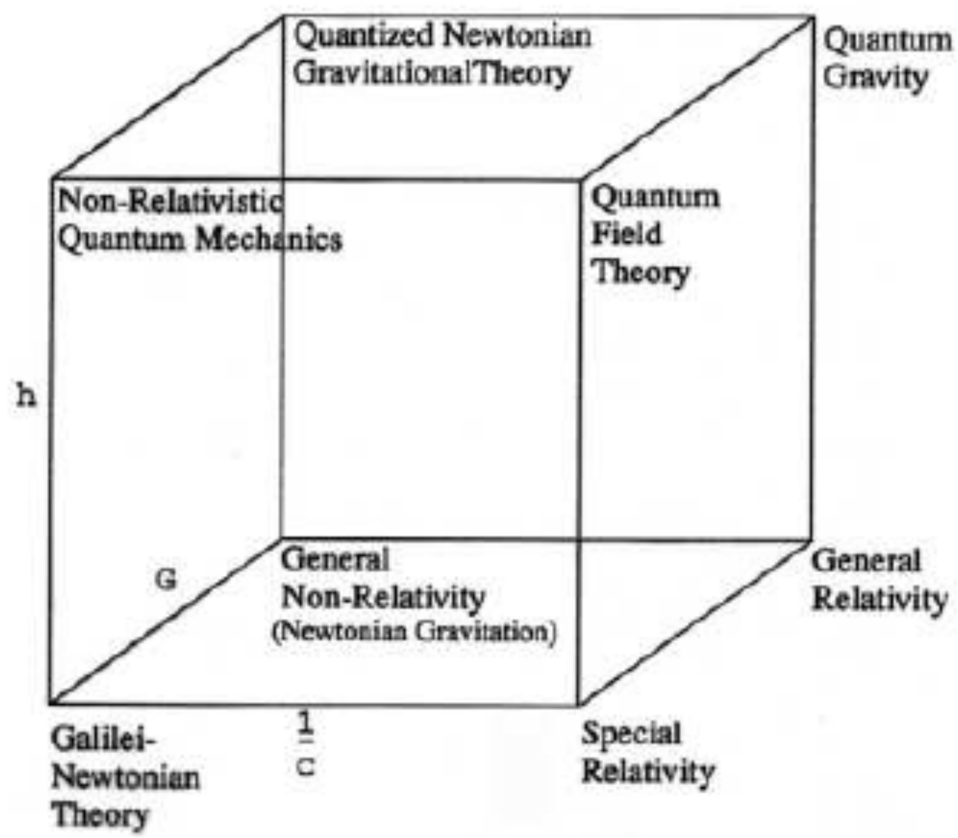

Bronstein cube

[picture taken from J. Stachel, 2001]

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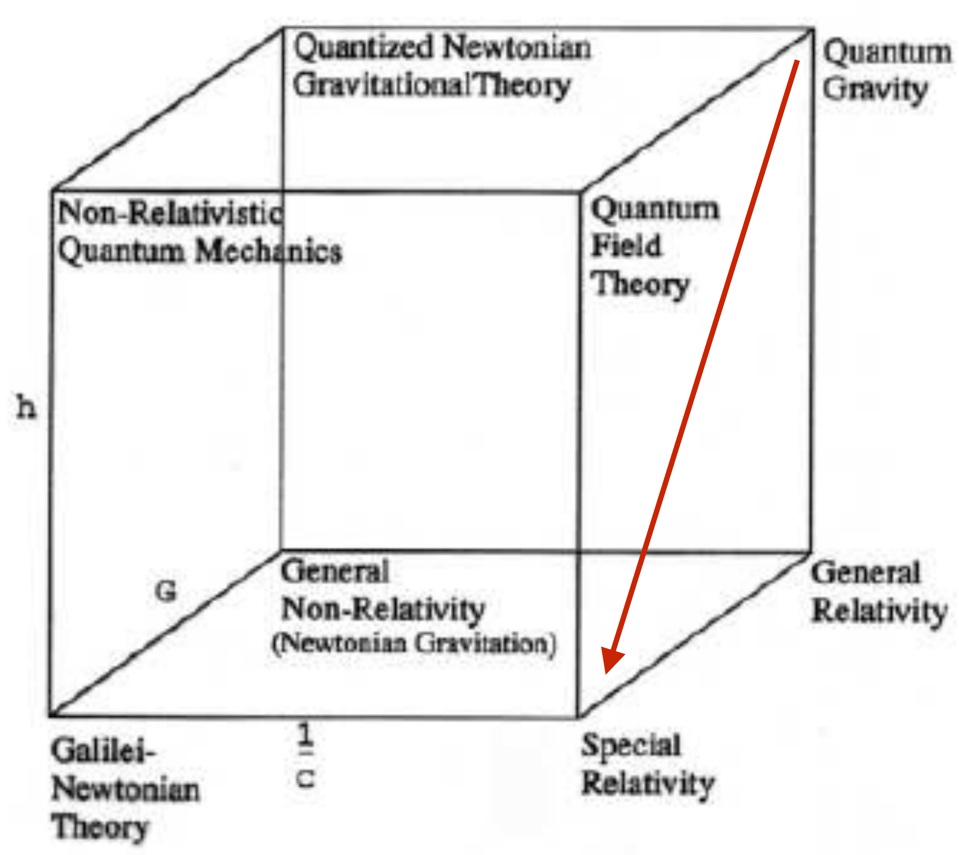

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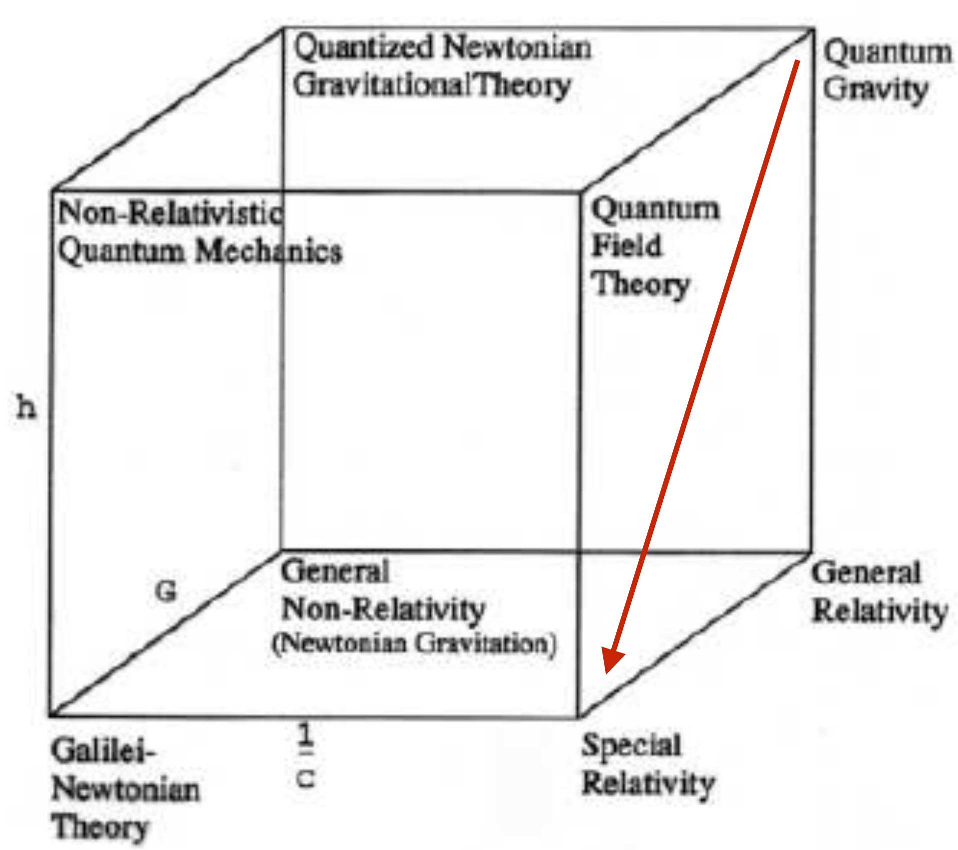

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- Results from black holes might provide access to a different, strong gravity, regime (dynamical gravitational field) where maybe the connection with “fundamental” QG theories can be made stronger [see T. Hinderer talk for WG1]

Most studied* kinds of QG effects

- ✦ Departures from Lorentz symmetries — affecting e.g. dispersion relations of particles and their interactions

- ✦ Fuzziness of spacetime geometry — affecting e.g. the localisability of far-away sources or inducing non-systematic deformations of particles trajectories

- ✦ Departures from CPT symmetry

- ✦ Violations of the equivalence principle

- ✦ Departures from Quantum Mechanics — inducing modifications of the Heisenberg uncertainty principle and of the de Broglie relation

*and promising from the point of view of Planck-scale experimental sensitivity, see WG2 review talk

Theoretical approach to quantum gravity phenomenology

- The effects mentioned in the previous slide have observational consequences that depend on the theoretical framework in which they are embedded
e.g. kinematical assumptions, dynamical assumptions, assumptions on the validity of a Hamiltonian description, etc…

- The role of a theoretical analysis it to systematically characterise possible effects, emphasising how they are related in different theoretical frameworks

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✦ Example1: a useful characterisation is w.r.t. the fate of Lorentz symmetry

Relativistic models which preserve Lorentz invariance
Non-relativistic models which break Lorentz invariance (LIV)
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✦ Example2: another relevant (and possibly related) characterisation concerns the kind of departures from standard physics

Models with systematic departures from standard physics
Models with stochastic (fuzzy) departures from standard physics

Lorentz breaking (LIV) vs. Lorentz deformation (DSR)

✦ In Lorentz breaking theories everything transforms linearly under the appropriate representation of the Lorentz group. On top of this, Lorentz non-invariant fields (e.g. fixed background tensors) are introduced, that identify a *preferred frame of reference* (possibly coincident with that of the CMB) and so manifestly break the symmetry

$$S \propto \int d^4x\sqrt{-g}\left[g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + \left(g^{\alpha\beta} + \tau^{\alpha\beta}\right)\partial_\alpha\psi\partial_\beta\psi\right]$$

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✦ In relativistic theories with Lorentz deformations it is the action of the Lorentz group itself that is modified. The *theory is still relativistic* (i.e. different inertial observers agree on the physics), however the action of the group under which this invariance is realised is different (e.g. the action of the new *Lorentz group linking different inertial observers is deformed* and nonlinear on physical quantities).

This implies the existence of a *new relativistically invariant energy scale besides the speed of light*. This framework leads to apparent Lorentz violating effects, such as an energy-dependent speed of light, which are however compatible with the new relativistic symmetry transformations.

$$m^2 = E^2 - |\vec{p}|^2 - \lambda E|\vec{p}|^2$$

$$\begin{aligned} E &\rightarrow E + \xi p_j \\ p_i &\rightarrow p_i + \xi\left[E\delta_{ij} + \tfrac{\lambda}{2}|\vec{p}|^2\delta_{ij} - \lambda E^2\delta_{ij} - \lambda p_j \textstyle\sum_k p_k\delta_{ik}\right] \end{aligned}$$

Status of Lorentz symmetries in QG theories

✦ No definite statement informed by a rigorous derivation is possible, however our current understanding of the theories allow us to make quite reliable guesses concerning their (semiclassical) Minkowski limit

String theory might entail a breakdown of Lorentz symmetries due to the presence of extra tensor fields

LQG points towards a deformation of Lorentz symmetries, by looking at the structure of the modified hypersurface deformation algebra

Causal sets has been constructed so to retain Lorentz invariance, thanks to an appropriate probability distribution of "spacetime atoms"

Noncommutative geometry predicts either Loretnz breaking or Lorentz deformations, depending on the properties of the commutator between spacetime coordinates

Example: theoretical implications of photon energy-dependent time delays

- Assume that we are able to state that the time of arrival of photons simultaneously emitted from distant sources has a systematic energy dependence (due to Plank-scale physics) which in a flat spacetime approximation can be written as

$$\Delta t = \eta \frac{\Delta E}{E_P} \frac{L}{c}$$

Of course being able to make such a statement is highly nontrivial from an observational point of view (need to account for all possible 'standard' explanations, observational limitations, etc… but we leave these matters to the other WGs)

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✦ This MDR can in principle be associated to either LIV or DSR — the theoretical implications and further observational signals to look for are very different in the two scenarios

Example: MDR in a Lorentz breaking scenario

✦ From a kinematical point of view, in a Lorentz breaking scenario the form of the MDR does not constrain other features, e.g. universality of the deformation parameter, conservation rules in interactions etc…

✦ The most natural assumption is that energy and spatial momenta are conserved as usual: in a process $a + b \rightarrow c + d$ one has

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✦ These combined features have strong implications for threshold reactions, e.g. they allow for photon decay $\gamma \to e^+ e^-$

$$0 = E^2 - |\vec{p}|^2 - \eta \frac{E}{E_P} |\vec{p}|^2 \qquad\qquad m_e^2 = E^2 - |\vec{p}|^2 - \xi \frac{E}{E_P} |\vec{p}|^2$$

$$\Downarrow$$

$$2\left(E_e E_\gamma - |\vec{p}_e||\vec{p}_\gamma| \cos\theta\right) = \eta |\vec{p}_\gamma|^2 E_\gamma + \xi \left(|\vec{p}_e|^2 E_e - |\vec{p}_\gamma - \vec{p}_e|^2 (E_\gamma - E_e)\right)$$

(there is no reason to expect that the deformation parameter in the MDR is universal)

Example: MDR in a Lorentz breaking scenario

✦ One can also embed the Lorentz breaking scenario into a dynamical framework, e.g. an Effective field theory model — more later

✦ However in this case the only way to introduce a linear energy dependence of the velocity is by introducing a MDR with opposite corrections for different photon helicity:

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- In order to explain a measured energy-dependent speed of photons within the EFT framework it is necessary to control for the polarisation of the sources:

  A systematic dependence of the time of arrival on energy would be available only for (circular) polarised sources with the same polarisation

  For unpolarised sources there would be a spread in the time of arrival (advanced and delayed) that increases with energy

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- Moreover one needs to compare to constraints on birefringence

$$A^{\alpha}_{\pm} = e^{-i(\omega_L t - \vec{k}\cdot\vec{z})} \left(\epsilon^{\alpha}_L \pm \epsilon^{\alpha}_R e^{-i(\omega_R - \omega_L)t} \right)$$

if the two circularly polarised components travel at different speed, with a speed that is energy-dependent, a linearly polarised source will have its polarisation direction rotated if observed in a narrow energy range, while if observed in a wide enough energy range will have its polarisation erased

$$\phi(T) = \Delta\omega T = \eta \frac{|\vec{p}|^2}{E_P} T$$

$$\Delta\omega = |\omega_R - \omega_L| = \eta \frac{|\vec{p}|^2}{E_P}$$

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The compatibility condition reads, for a boost B_j along the j direction

$$m^2 = E^2 - |\vec{p}|^2 - \eta \frac{E}{E_P}|\vec{p}|^2 \quad \Longleftrightarrow \quad m^2 = B_j[E]^2 - |B_j[\vec{p}]|^2 - \eta \frac{B_j[E]}{E_P}|B_j[\vec{p}]|^2$$

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This is not satisfied by a standard boost, $B_j[E] = E + \xi p_j, \; B_j[p_i] = p_i + \xi E \delta_{ij}$

$$E^2 - |\vec{p}|^2 - \eta \frac{E}{E_P} |\vec{p}|^2 \to E^2 - |\vec{p}|^2 - \eta \frac{E}{E_P} |\vec{p}|^2 - \xi \eta \frac{p_j}{E_P} \left(2E^2 + |\vec{p}|^2\right)$$

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The (infinitesimal) boost that leaves the dispersion relation invariant has the following nonlinear action on energy and momenta

$$\begin{aligned} B_j[E] &= E + \xi p_j \\ B_j[p_i] &= p_i + \xi \left[E\delta_{ij} + \tfrac{\eta}{2E_P} |\vec{p}|^2 \delta_{ij} - \tfrac{\eta}{E_P} E^2 \delta_{ij} - \tfrac{\eta}{E_P} p_j \textstyle\sum_k p_k \delta_{ik} \right] \end{aligned}$$

Example: MDR in a Doubly Special Relativity scenario

✦ The action of deformed boosts is not compatible with standard conservation of energy and spatial momenta

For the process $a \to b + c$, assuming

$$E^{(a)} = E^{(b)} + E^{(c)} \qquad\qquad p_i^{(a)} = p_i^{(b)} + p_i^{(c)}$$

one finds $B_j[p_i^{(a)}] \neq B_j[p_i^{(b)}] + B_j[p_i^{(c)}]$:

$$\begin{aligned} B_i[p_i^{(a)}] &= p_i^{(a)} + \xi\left[E^{(a)} + \tfrac{\eta}{2E_P}|\vec{p}^{(a)}|^2 - \tfrac{\eta}{E_P}(E^{(a)})^2 - \tfrac{\eta}{E_P}(p_i^{(a)})^2\right] \\ &= p_i^{(b)} + p_i^{(c)} + \xi\left[E^{(b)} + E^{(c)} + \tfrac{\eta}{2E_P}|\vec{p}^{(b)} + \vec{p}^{(c)}|^2 - \tfrac{\eta}{E_P}(E^{(b)} + E^{(c)})^2 - \tfrac{\eta}{E_P}(p_i^{(b)} + p_i^{(c)})^2\right] \end{aligned}$$

$$\begin{aligned} B_i[p_i^{(b)}] + B_i[p_i^{(c)}] &= p_i^{(b)} + \xi\left[E^{(b)} + \tfrac{\eta}{2E_P}|\vec{p}^{(b)}|^2 - \tfrac{\eta}{E_P}(E^{(b)})^2 - \tfrac{\eta}{E_P}(p_i^{(b)})^2\right] \\ &\quad + p_i^{(c)} + \xi\left[E^{(c)} + \tfrac{\eta}{2E_P}|\vec{p}^{(c)}|^2 - \tfrac{\eta}{E_P}(E^{(c)})^2 - \tfrac{\eta}{E_P}(p_i^{(c)})^2\right] \end{aligned}$$

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✦ The action of deformed boosts is not compatible with standard conservation of energy and spatial momenta

For the process $a \rightarrow b + c$, assuming

$$E^{(a)} = E^{(b)} + E^{(c)} \qquad\qquad p_i^{(a)} = p_i^{(b)} + p_i^{(c)}$$

one finds $B_j[p_i^{(a)}] \neq B_j[p_i^{(b)}] + B_j[p_i^{(c)}]$:

$$\begin{aligned} B_i[p_i^{(a)}] &= p_i^{(a)} + \xi \left[E^{(a)} + \tfrac{\eta}{2E_P} |\vec{p}^{(a)}|^2 - \tfrac{\eta}{E_P} (E^{(a)})^2 - \tfrac{\eta}{E_P} (p_i^{(a)})^2 \right] \\ &= p_i^{(b)} + p_i^{(c)} + \xi \left[E^{(b)} + E^{(c)} + \tfrac{\eta}{2E_P} |\vec{p}^{(b)} + \vec{p}^{(c)}|^2 - \tfrac{\eta}{E_P} (E^{(b)} + E^{(c)})^2 - \tfrac{\eta}{E_P} (p_i^{(b)} + p_i^{(c)})^2 \right] \end{aligned}$$

$$\begin{aligned} B_i[p_i^{(b)}] + B_i[p_i^{(c)}] &= p_i^{(b)} + \xi \left[E^{(b)} + \tfrac{\eta}{2E_P} |\vec{p}^{(b)}|^2 - \tfrac{\eta}{E_P} (E^{(b)})^2 - \tfrac{\eta}{E_P} (p_i^{(b)})^2 \right] \\ &\quad + p_i^{(c)} + \xi \left[E^{(c)} + \tfrac{\eta}{2E_P} |\vec{p}^{(c)}|^2 - \tfrac{\eta}{E_P} (E^{(c)})^2 - \tfrac{\eta}{E_P} (p_i^{(c)})^2 \right] \end{aligned}$$

✦ A deformed energy-momentum conservation law that is compatible with the deformed boosts is

$$\begin{aligned} E_a &= E_b + E_c - \eta \vec{p}_b \cdot \vec{p}_c \\ \vec{p}_a &= \vec{p}_b + \vec{p}_c - \eta E_b \vec{p}_c - \eta E_c \vec{p}_b \end{aligned}$$

at least to the first order this is not necessarily unique and there are studies on the allowed compatible forms of MDR, action of deformed boosts and conservation laws

Example: MDR in a Doubly Special Relativity scenario

- The DSR scenario can be distinguished form the LIV one already at this kinematical level

In a DSR setting the interplay between MDR and modified conservation rules weakens the effects on threshold reactions

Moreover the reactions that would be forbidden in special relativity (such as photon decay) are also forbidden in DSR, since this framework does not allow to identify preferred reference frames

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Moreover the reactions that would be forbidden in special relativity (such as photon decay) are also forbidden in DSR, since this framework does not allow to identify preferred reference frames

✦ At a dynamical level we can not make definite statements, since the dynamics in DSR is not sufficiently developed yet

Having deformed conservation rules in interactions does indicate however that the DSR framework lies beyond Effective Field Theory models

LIV: a concrete model

- ✦ The most conservative framework to describe Lorentz breaking in the matter sector is that of Effective Field Theory: Lorentz violations are introduced via extra tensors in the Standard Model action, preserving gauge invariance, covariance of the action and energy-momentum standard conservation (dynamics is fully specified)

- ✦ Adding all possible terms will generate a tower of operators with increasing mass dimension (at tree level these operators are independent, radiative corrections produce in principle lower-order terms)

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✦ Adding all possible terms will generate a tower of operators with increasing mass dimension (at tree level these operators are independent, radiative corrections produce in principle lower-order terms)

✦ Renormalizable operators are part of the *minimal* Standard Model Extension.

Example: the extended QED of photons and electrons

$$\mathcal{L}^{\mathrm{QED}}_{\mathrm{electron}} = \frac{1}{2} i \overline{\psi} \gamma^{\mu} \overleftrightarrow{D}_{\mu} \psi - m_e \overline{\psi} \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$+ \quad \mathcal{L}^{\mathrm{CPT-odd}}_{\mathrm{electron}} = -a_{\mu} \overline{\psi} \gamma^{\mu} \psi - b_{\mu} \overline{\psi} \gamma_5 \gamma^{\mu} \psi \qquad + \quad \mathcal{L}^{\mathrm{CPT-even}}_{\mathrm{electron}} = -\frac{1}{2} H_{\mu\nu} \overline{\psi} \sigma^{\mu\nu} \psi + \frac{1}{2} i c_{\mu\nu} \overline{\psi} \gamma^{\mu} \overleftrightarrow{D}^{\nu} \psi + \frac{1}{2} i d_{\mu\nu} \overline{\psi} \gamma_5 \gamma^{\mu} \overleftrightarrow{D}^{\nu} \psi$$

$$+ \quad \mathcal{L}^{\mathrm{CPT-odd}}_{\mathrm{photon}} = +\frac{1}{2} (k_{AF})^{\kappa} \epsilon_{\kappa\lambda\mu\nu} A^{\lambda} F^{\mu\nu} \qquad + \quad \mathcal{L}^{\mathrm{CPT-even}}_{\mathrm{photon}} = -\frac{1}{4} (k_F)_{\kappa\lambda\mu\nu} F^{\kappa\lambda} F^{\mu\nu}$$

LIV: a concrete model

- The minimal extension of the QED Lagrangian of photons and electrons predicts a modified dispersion relation of photons

$$E = (1+\rho)|\vec{p}| \pm \sqrt{(\sigma^2|\vec{p}|^2 + \tau^2)}$$

The parameters are functions of those appearing in the Lagrangian, and the different signs refer to different polarisations

- Depending on which coefficients are nonzero, photons might or might not have a birefringent behaviour

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✦ Depending on which coefficients are nonzero, photons might or might not have a birefringent behaviour

✦ Because we started with renormalizable operators, the parameters are either dimensionless (ρ and σ) or have dimension of an energy — not Planck-scale suppressed

→ Very strong experimental constraints, even in the laboratory

LIV: a concrete model

✦ Operators that allow to introduce the Planck scale explicitly as a dimensionful deformation parameter) are nonrenormalizable (mass dimension > 4)

Example: the extended QED of photons and electrons with mass dimension 5 operators

$$\mathcal{L}_{(5)}^{QED} = -\frac{\xi}{2E_P} n^\mu F_{\mu\alpha}(n \cdot D)(n_\nu \tilde{F}^{\nu\alpha}) + \frac{1}{2E_P} n^\mu \bar{\psi}\gamma_\mu(\zeta_1 + \zeta_2\gamma_5)(u \cdot D)^2\psi$$

(n is a unitary fixed four-vector, ξ and ζ are dimensionless parameters)

these are the *only* dimension five corrections that are quadratic, gauge invariant, covariant, not reducible to lower dimensional operators nor to total derivatives

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these are the *only* dimension five corrections that are quadratic, gauge invariant, covariant, not reducible to lower dimensional operators nor to total derivatives

- The predicted dispersion relations for photons and electrons are

$$E^2 = |\vec{p}|^2 \pm \frac{\xi}{E_P}|\vec{p}|^3$$

$$E^2 = |\vec{p}|^2 + m^2 + 2\frac{(\zeta_1 \pm \zeta_2)}{E_P}|\vec{p}|^3$$

$\longrightarrow$ photons necessarily have a birefringent behaviour in this framework

In principle nonrenormalisable operators produce lower mass dimension ones via radiative corrections. One needs to find mechanisms to avoid this in order to evade very stringent constraints on renormalisable terms

DSR: a concrete model

✦ A well studied framework for DSR is that of noncommutative geometry, in its realisation in terms of quantum groups: spacetime coordinates are affected by intrinsic limitation on their simultaneous measurability, formalised via a nontrivial commutator, and this requires deformed relativistic symmetries

[see also J. Kowalski-Glikman talk for WG1]

Example: κ-Minkoswki spacetime, $\kappa \sim E_P$

$$[\hat{x}_0, \hat{x}_j] = \frac{\hat{x}_j}{\kappa}$$
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Assuming a standard action of boost transformations on single coordinates, compatibility with spacetime commutators requires a deformed action on products of coordinates:

$$\begin{aligned} B[\hat{x}_0] &\equiv \hat{x}_0 + \xi N \triangleright \hat{x}_0 = \hat{x}_0 + \xi \hat{x}_1 \\ B[\hat{x}_1] &\equiv \hat{x}_1 + \xi N \triangleright \hat{x}_1 = \hat{x}_1 + \xi \hat{x}_0 \end{aligned}$$

$$B[[\hat{x}_0, \hat{x}_1]] = \frac{i}{\kappa} B[\hat{x}_1] \quad \longleftrightarrow \quad \begin{aligned} N \triangleright (\hat{x}_0 \hat{x}_1) &= (N \triangleright \hat{x}_0)\hat{x}_1 + \hat{x}_0 N \triangleright \hat{x}_1 + \tfrac{1}{\kappa} N \triangleright \hat{x}_1 \\ N \triangleright (\hat{x}_1 \hat{x}_0) &= (N \triangleright \hat{x}_1)\hat{x}_0 + \hat{x}_1 N \triangleright \hat{x}_0 \end{aligned}$$

(formulas shown for the 1+1 dimensional case, easily generalisable)

DSR: a concrete model

✦ The nontrivial action of generators on products of coordinates is formalised via the 'coproducts' of symmetry generators

$$\begin{aligned}
\Delta(P_0) &= P_0 \otimes 1 + 1 \otimes P_0 \\
\Delta(P_1) &= P_1 \otimes 1 + e^{-P_0/\kappa} \otimes P_1 \\
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[P_0, P_1] &= 0 \\
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\end{aligned}
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✦ The associated Casimir is nonlinearly deformed

$$
C = 4\kappa^2 \sinh^2\left(\frac{P_0}{2\kappa}\right) - (P_1)^2 e^{P_0/\kappa}
$$

DSR: a concrete model

✦ The translation generators can be represented as an algebra of function over energy-momentum space

$$P_0 = E \qquad P_1 = p_1$$

✦ The structures of the algebra of generators translate into properties of the energy and momenta

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Composition rule from the coproduct

$$\begin{aligned} \Delta(P_0) &= P_0 \otimes 1 + 1 \otimes P_0 \\ \Delta(P_1) &= P_1 \otimes 1 + e^{-P_0/\kappa} \otimes P_1 \end{aligned}$$

$$\begin{aligned} E_p \oplus E_q &= E_p + E_q \\ p_1 \oplus q_1 &= p_1 + e^{-E_p/\kappa} q_1 \\ &\simeq p_1 + (1 - \frac{E_p}{\kappa}) q_1 \end{aligned}$$

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Dispersion relation from the Casimir

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The deformed commutator between boost and spatial translations encodes a nonlinear action of boosts on momenta

$$B[p_1] \equiv p_1 + \xi[N, p_1] = p_1 + \xi\left[\tfrac{\kappa}{2}\left(1 - e^{-2E/\kappa}\right) - \tfrac{1}{2\kappa} p_1^2\right]$$

Conclusions

We do not know how does the correct QG theory look like, and given the status of theoretical research the best we can do at the moment is to get inspiration from the features of candidate QG theories to build a set of interesting observables and point to interesting observational windows

The results of experimental observations can in turn guide the theoretical effort in specific directions, hopefully leading to progress generated by joint experimental/theoretical efforts

For this process to be effective, on the theoretical side we need a careful exploration of the logical relations between different features within any given model we want to test

On the experimental side we need clear statements about what is actually being tested, i.e. what are the assumptions behind the framework that is being used to interpret the data

… during the crises that lead to large-scale changes of paradigm, scientists usually develop many speculative and unarticulated theories that can themselves point the way to discovery. Often, however, the discovery is not quite the one anticipated by the speculative and tentative hypothesis. Only as experiment and tentative theory are together articulated to a match does the discovery emerge and the theory become a paradigm. — T.S. Kuhn, The structure of scientific revolution

REFERENCES (a time-ordered incomplete list)

- G. Amelino-Camelia, *Are we at the dawn of Quantum Gravity phenomenology?* Lect.Notes Phys. 541 (2000), arXiv:gr-qc/9910089

- D. Mattingly, *Modern tests of Lorentz invariance* Living Rev. Relativity, 8, (2005), arXiv:gr-qc/0502097

- S. Hossenfelder, *Experimental search for Quantum Gravity* arXiv:1010.3420

- S. Liberati, D. Mattingly, *Lorentz breaking effective field theory models for matter and gravity: theory and observational constraints* in "Gravity: Where Do We Stand?", Springer, arXiv:1208.1071

- G. Amelino-Camelia, *Quantum-Spacetime Phenomenology* Living Rev. Relativity, 16, (2013)

- S. Carlip et al., *Quantum Gravity: A Brief History of Ideas and Some Prospects* IJMPD, 11 (2015), arXiv:1507.08194

- L. Smolin, *What are we missing in our search for quantum gravity?* arXiv:1705.09208