

$$[P_\mu, P_\nu] = 0,$$

$$\Delta P_0 = P_0 \otimes \mathbb{1} + \mathbb{1} \otimes P_0, \quad \Delta P_j = P_j \otimes \mathbb{1} + e^{-P_0/\kappa} \otimes P_j,$$

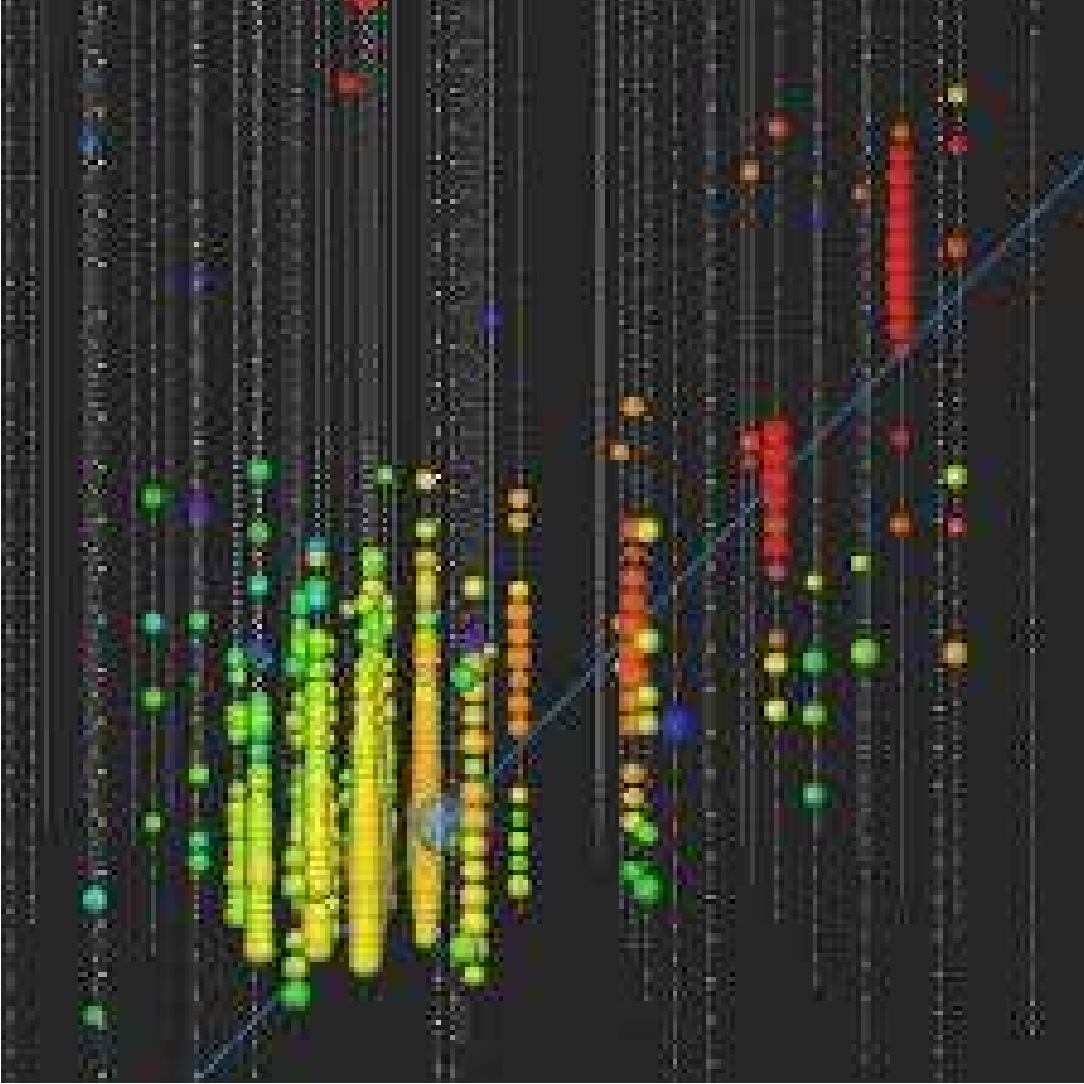
$$\varepsilon(P_\mu) = 0, \quad S(P_0) = -P_0, \quad S(P_j) = -e^{P_0/\kappa} P_j,$$

$$[N_j, P_k] = i \delta_{jk} \left(\frac{\kappa}{2} (1 - e^{-2P_0/\kappa}) + \frac{1}{2\kappa} |\vec{P}|^2 \right) - \frac{i}{\kappa} P_j P_k,$$

$$[N_j, P_0] = i P_j, \quad [R_j, P_k] = i \epsilon_{jkl} P_l$$

$$\Delta N_k = N_k \otimes \mathbb{1} + e^{-P_0/\kappa} \otimes N_k + \frac{i}{\kappa} \epsilon_{klm} P_l \otimes R_m, \quad \Delta R_j = R_j \otimes \mathbb{1} + \mathbb{1} \otimes R_j,$$

$$\varepsilon(N_j) = 0, \quad \varepsilon(R_k) = 0, \quad S(N_j) = -e^{P_0/\kappa} N_j + \frac{i}{\kappa} \epsilon_{jkl} e^{\lambda P_0} P_k R_l, \quad S(R_k) = -R_k$$



Planck length as the minimum allowed value for wavelengths:

- suggested by several indirect arguments combining quantum mechanics and GR
- found in some detailed analyses of formalisms in use in the study of the QG problem

But the minimum wavelength is the Planck length for which observer?

**GAC, ModPhysLettA (1994)
PhysLettB (1996)**

one or another form of “granularity” of spacetime, inducing departures from standard relativistic symmetries

- models with spacetime noncommutativity certainly have it
- group-field-theory models certainly have it
- expected in loopQG when curvature/cosmological constant is turned on (and expected also by some authors in the limit of vanishing curvature)
- expected in noncritical string theory
- critical string theory is formulated in a classical Minkowski spacetime by axiom (but unclear relativistic implications of stringy spacetime fuzziness, generalizedUP....)

fate of relativistic symmetry at the Planck scale needs to be investigated...
might well be “broken”...

but from 2000 onwards together with broken relativistic symmetries
there starts to be a literature on the possibility
of “Planck-scale deformations of relativistic symmetry”

[jargon: “DSR”, for “doubly-special”, or “deformed-special”, relativity]

GAC, grqc0012051, IntJournModPhysD11,35

hep-th/0012238, PhysLettB510,255

KowalskiGlikman, hep-th/0102098, PhysLettA286,391

Magueijo+Smolin, hep-th/0112090, PhysRevLett88,190403

gr-qc/0207085, PhysRevD67,044017

GAC, gr-qc/0207049, Nature418,34

change the laws of transformation between observers so that the new properties
are observer-independent

- * a law of minimum wavelength can be turned into a DSR law
- * could be used also for properties other than minimum wavelength,
such as deformed on-shellness, deformed uncertainty relations...

The notion of DSR-relativistic theories is best discussed in analogy with the transition
from Galileian Relativity to Special Relativity

analogy with Galilean-SR transition

introduction to DSR case is easier starting from reconsidering the Galilean-SR transition (the SR-DSR transition would be closely analogous)

Galilean Relativity

on-shell/dispersion relation $E = \frac{p^2}{2m} \quad (+m)$

linear composition of momenta $p_\mu^{(1)} \oplus p_\mu^{(2)} = p_\mu^{(1)} + p_\mu^{(2)}$

linear composition of velocities $\vec{V} \oplus \vec{V}_0 = \vec{V} + \vec{V}_0$

Special Relativity

special-relativistic law of composition of momenta is still linear $p_{\mu}^{(1)} \oplus p_{\mu}^{(2)} = p_{\mu}^{(1)} + p_{\mu}^{(2)}$

but the on-shell/dispersion relation takes the new form $E = \sqrt{p^2 + m^2}$

of course (since c is invariant of the new theory) the special-relativistic boosts act nonlinearly on velocities (whereas Galilean boosts acted linearly on velocities)

and the special-relativistic law of composition of velocities is nonlinear, noncommutative and nonassociative

$$\mathbf{w} = \mathbf{v} \oplus \mathbf{u} \quad \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \frac{1}{1 + \frac{v_1 u_1 + v_2 u_2 + v_3 u_3}{c^2}} \left\{ \left[1 + \frac{1}{c^2} \frac{\gamma_{\mathbf{v}}}{1 + \gamma_{\mathbf{v}}} (v_1 u_1 + v_2 u_2 + v_3 u_3) \right] \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} + \frac{1}{\gamma_{\mathbf{v}}} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \right\}$$

much undervalued in most textbooks,
which only give composition of parallel velocities:

$$\frac{v + u}{1 + (vu/c^2)}$$

from Special Relativity to DSR

If there was an observer-independent scale E_p (inverse of length scale ℓ) then, for example, one could have a modified on-shell relation as relativistic law

$$m^2 = \Lambda(E, p; E_p) = E^2 - p^2 - \frac{E}{E_p} p^2 + O\left(\frac{E^4}{E_p^2}\right)$$

For suitable choice of $\Lambda(E, p; E_p)$ one can easily have a **maximum allowed value of momentum**, **i.e. minimum wavelength**

($p_{\max} = E_p$ for $\ell = -1/E_p$ in the formula here shown)

$$\cosh(\ell m) = \cosh(\ell p_0) - \frac{\ell^2}{2} e^{-\ell p_0} p_1^2$$

it turns out that such laws could still be relativistic, part of a relativistic theory where not only c (“speed of massless particles in the infrared limit”) but also E_p would be a nontrivial relativistic invariant

action of boosts on momenta must of course be deformed so that

$$[N_k, \Lambda(E, p; E_p)] = 0$$

then it turns out to be necessary to correspondingly deform the law composition of momenta

$$p_\mu^{(1)} \oplus p_\mu^{(2)} \neq p_\mu^{(1)} + p_\mu^{(2)}$$

in 3D quantum gravity

see, e.g., **Freidel+Livine**,
PhysRevLett96,221301(2006)

consider a matter field ϕ coupled to gravity,

$$Z = \int Dg \int D\phi e^{iS[\phi,g]+iS_{GR}[g]}, \quad (1)$$

where g is the space-time metric, $S_{GR}[g]$ the Einstein gravity action and $S[\phi, g]$ the action defining the dynamics of ϕ in the metric g .

integrate out
the quantum gravity fluctuations and derive an *effective action* for ϕ taking into account the quantum gravity correction:

$$Z = \int D\phi e^{iS_{eff}[\phi]}.$$

the effective action obtained through this constructive procedure gives matter fields in a noncommutative spacetime (similar to, but not exactly given by, kappa-Minkowski) and with curved momentum space, as signalled in particular by the deformed on-shellness

(anti-deSitter momentum space) $\cos(E) - e^{\ell E} \frac{\sin(E)}{E} P^2 = \cos(m)$

dual redshift on Planck-scale-curved momentum spaces (but with flat spacetime) produces time-of-arrival effects which at leading order are of the form ($n \in \{1,2\}$)

$$\Delta T = \left(\frac{E}{E_P} \right)^n T$$

and could be described in terms of an energy-dependent “physical velocity” of ultrarelativistic particles

$$v = c + s_{\pm} \left(\frac{E}{E_P} \right)^n c$$

these are very small effects but (at least for the case $n=1$) they could cumulate to an observably large ΔT if the distances travelled T are cosmological and the energies E are reasonably high (GeV and higher)!!!

GRBs are ideally suited for testing this:

cosmological distances (established in 1997)

photons (and neutrinos) emitted nearly simultaneously

with rather high energies (GeV.....TeV...100 TeV...)

large variety of phenomenological models

- * quantum-gravity scale could be bigger or smaller than E_{planck}
- * can be brokenSR or deformedSR
 - notice that no quantum-spacetime picture has been shown rigorously to lead to brokenSR
 - notice that threshold anomalies (e.g. anomalous transparency... $\gamma\gamma\rightarrow e^+e^-$) are only possible with brokenSR (protected by a theorem in any deformedSR scenario, GAC, PhysRevD85,084034)
 - for time-of-flight analyses techniques borrowed from propagation of light in media might not apply to deformedSR
- *the redshift dependence may be different from the Jacob-Piran ansatz
- *the effects can be spin/helicity/polarization dependent
- *the effects can be particle-type dependent (different for photons and neutrinos)
- *the effects should be fuzzy but theory work at present only provides essentially the deformation of the lightcone, without being able to establish the fuzziness of the deformed lightcone

problem:

solid theory is for (curved momentum space and) flat spacetime

phenomenological opportunities are for propagation over cosmological distances, whose analysis requires curved spacetime

study of theories with both curved momentum space and curved spacetime still in its infancy

GAC+Rosati, PhysRevD86,124035(2012)

KowalskiGlikman+Rosati, ModPhysLettA28,135101(2013)

Heckman+Verlinde, arXiv:1401.1810(2014)

Jacob and Piran [JCAP0801,031(2008)] used a compelling heuristic argument for producing a formula of energy-dependent time delay applicable to FRW spacetimes, which has been the only candidate so far tested

$$\Delta T = -s_{\pm} \frac{E}{M_{QG}} \frac{c}{H_0} \int_0^z d\zeta \frac{(1+\zeta)}{\sqrt{\Omega_{\Lambda} + (1+\zeta)^3 \Omega_m}}$$

where as usual H_0 is the Hubble parameter, Ω_{Λ} is the cosmological constant and Ω_m is the matter fraction.

Jacob-Piran formula is surely not the most general possibility. It is important for phenomenology to understand this issue, but it requires handling the interplay between curvature of spacetime and curvature of momentum space in subtle ways

GAC+Rosati, PhysRevD

Jacob-Piran formula in dS spacetime (it actually assumes modification only affects boosts, with translations unaffected...not what we see in explicit quantum-spacetime models...GAC+Marcianò+Matassa+Rosati,PhysRevD86,124035)

$$\Delta t = \ell|p| \frac{z + \frac{z^2}{2}}{H}$$

example of logically-consistent alternative

$$\Delta t = \ell|p| \frac{\ln(1+z)}{H}$$

and combinations are also logically consistent

$$\Delta t = \ell|p| \left(\alpha \frac{\ln(1+z)}{H} + \beta \frac{z + \frac{z^2}{2}}{H} \right)$$

this is for deSitter expansion...we reported observations relevant for FRW expansion in Rosati+GAC+Marcianò+Matassa, arxiv:1507.02056, Phys.Rev. D92 (2015) 124042

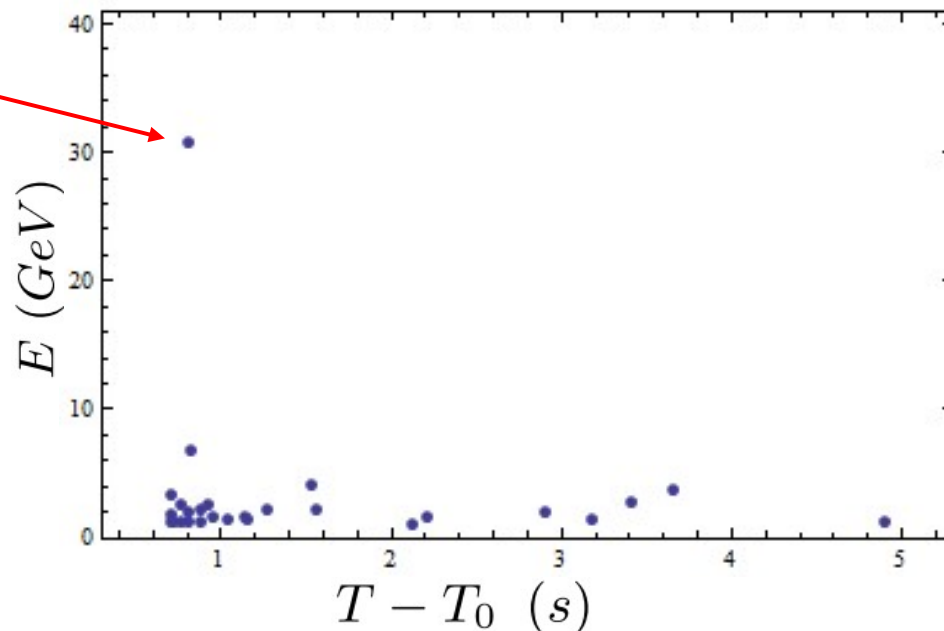
testing Jacob-Piran formula:

focus on n=1 case (sensitivity to the n=2 case still far beyond our reach presently
but potentially within reach of future neutrino astrophysics)

first came GRB080916C data providing a limit of $M_{QG} > 10^{-1} M_{\text{planck}}$ for
hard spectral lags and $M_{QG} > 10^{-2} M_{\text{planck}}$ for soft spectral lags

analogous studies of blazars lead to comparable limits

then came GRB090510 (magnificent short burst) allowing to establish a
limit at M_{planck} level on both signs of dispersion (soft and hard spectral lags)



a test with accuracy of
about one part in 10^{20} !!!

this Planck-scale sensitivity is illustrative of how we have learned that there are ways for achieving in some cases sensitivity to Planck-scale-suppressed effects, something that was thought to be impossible up to the mid 1990s

Quantum-Gravity Phenomenology exists!!!

a collection of other plausible quantum-gravity effects and of some associated data analyses where Planck-scale sensitivity was achieved (or is within reach) can be found in my “living review”

GAC, LivingRev.Relativity16,5(2013)

<http://www.livingreviews.org/lrr-2013-5>

still makes sense to test in-vacuo dispersion statistically...
our “quantum-gravity phenomenological models” will turn out
to be (at best!!) like the Bohr-Somerfeld quantization...

in order to best setup the statistical analysis it is convenient to notice that we are testing
**a linear relationship between Δt
and the product of energy and the redshift-dependent function $D(z)$**

$$\Delta t = \eta \frac{E}{M_P} D(z) \quad \text{with} \quad D(z) = \int_0^z d\zeta \frac{(1 + \zeta)}{H_0 \sqrt{\Omega_\Lambda + (1 + \zeta)^3 \Omega_m}}$$

we can absorb the redshift dependence into an “accordingly rescaled energy”,
which we call E^*

$$E^* \equiv E \frac{D(z)}{D(1)}$$

This then affords us the luxury of analysing data in terms of a linear relationship
between Δt and E^*

$$\Delta t = \eta D(1) \frac{E^*}{M_P}$$

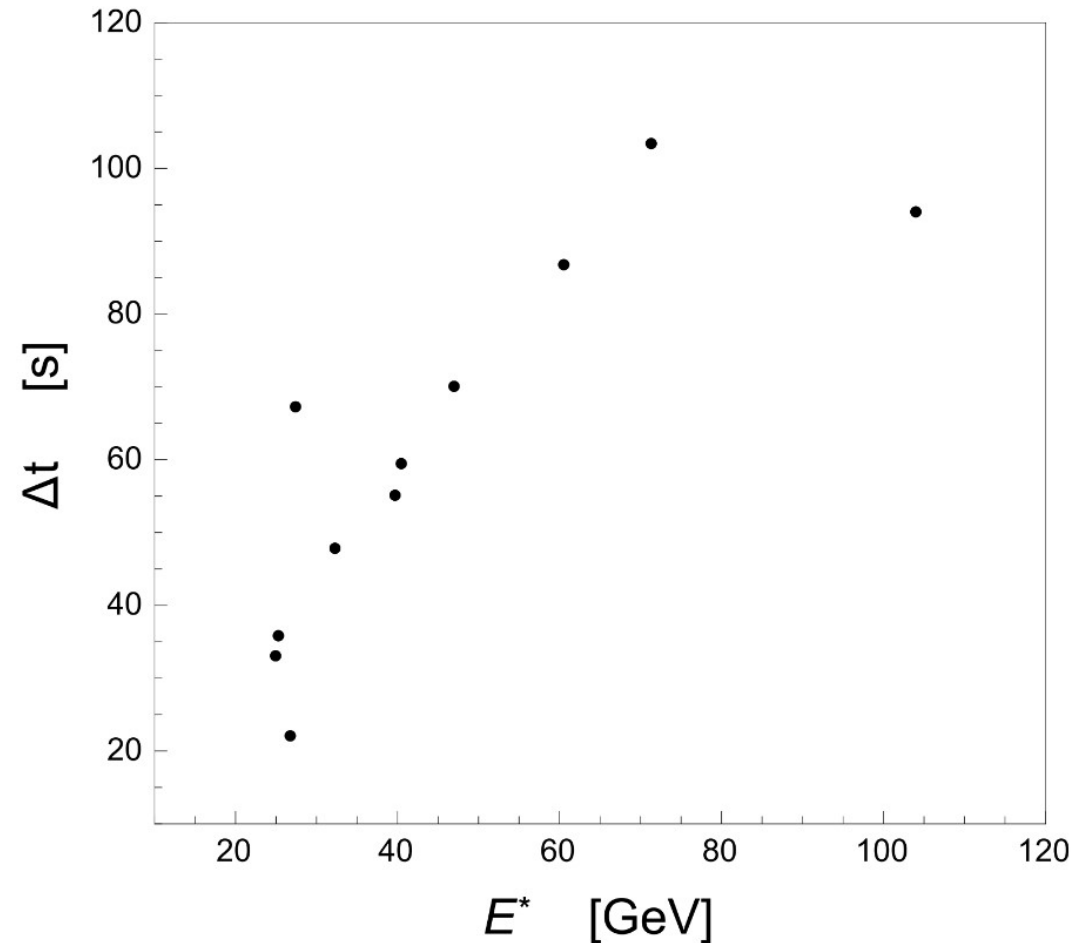
H.Xu+B.Q.Ma, PhysLettB760(2016)602

GAC+G.D'Amico+G.Rosati+N.Loret, arXiv:1612.02765, NatureAstronomy1,0139

criteria:

- focus on photons whose energy at emission was greater than 40 GeV
- take as Δt the time-of-observation difference between such high-energy photons and the first peak of the (mostly low-energy) signal

[note that this makes sense only for photons which were emitted in (near) coincidence with the first peak...not all those with >40GeV will ...and surely only a rather small percentage of all photons...]

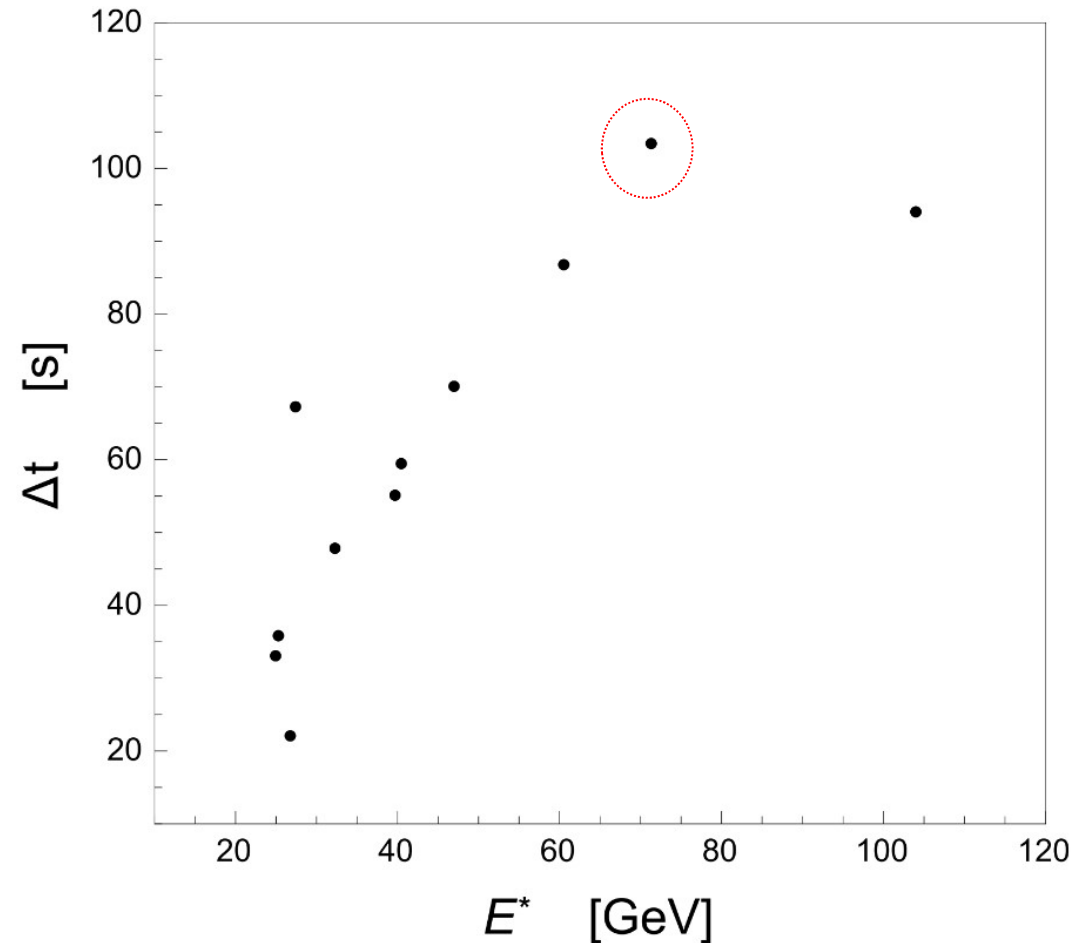


in order to get a sense of how striking this data situation is one can ask how often such high correlation between Δt and E^* would occur if the pairing of values of Δt and E^* was just random: overall having such high correlation would happen in less than 0.1% of cases, and correlation as high as seen for the best 8 out of 11 in 0.0013% of cases

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NEW

GAC+D'Amico+Fiore+Puccetti+Ronco, arXiv:1707.02413

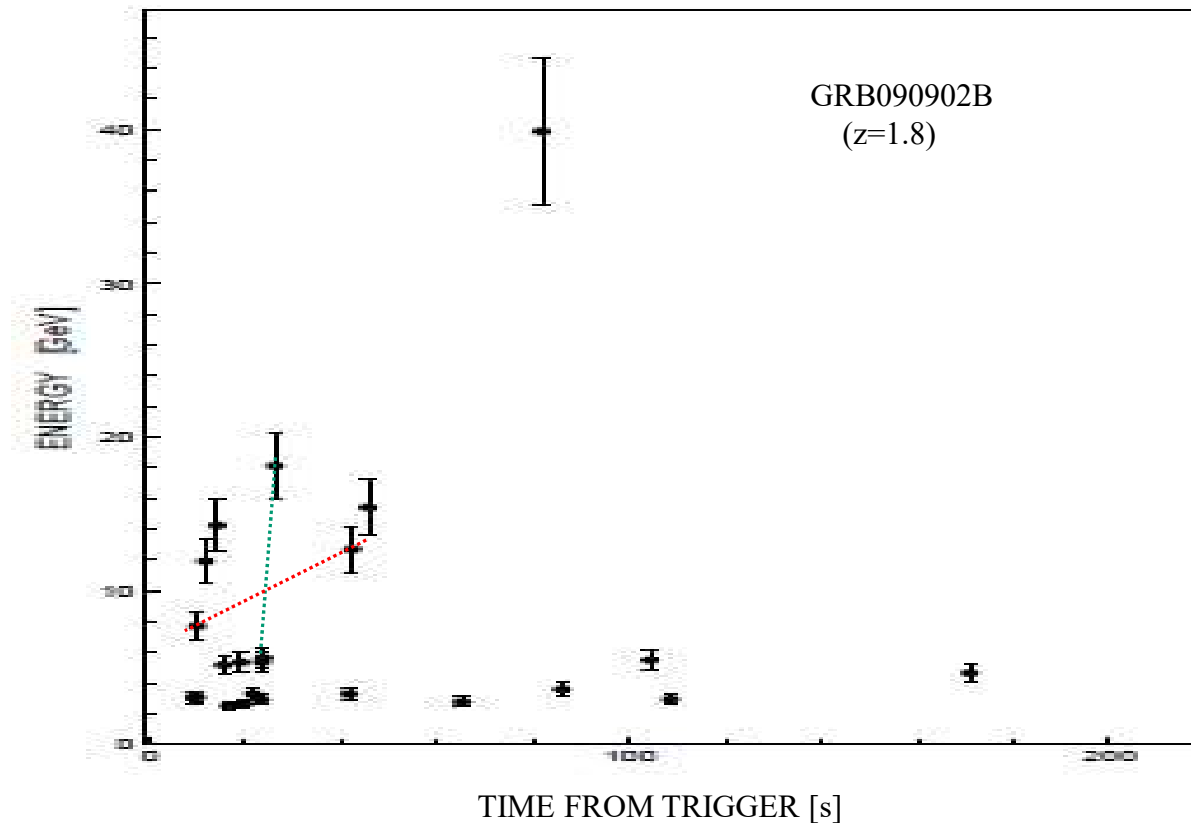
there is no reason to dwell much on statistical significance since more data will be available in a rather near future...

actually we already have more data to analyse, the GRB photons with energy at emission lower than 40 GeV, but for those it would be absurd to assume emission in near coincidence with the first peak of the GRB

previous graph gives η_γ of 30 ± 6

and note that each pair of photons in a GRB nominally determines a value of η_γ , though the large majority of them will be “spurious” for our analysis (photons emitted in different phases of the GRB)

we can still see if the frequency of occurrence of η_γ of about 30 is particularly high



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GAC+D'Amico+Fiore+Puccetti+Ronco, arXiv:1707.02413

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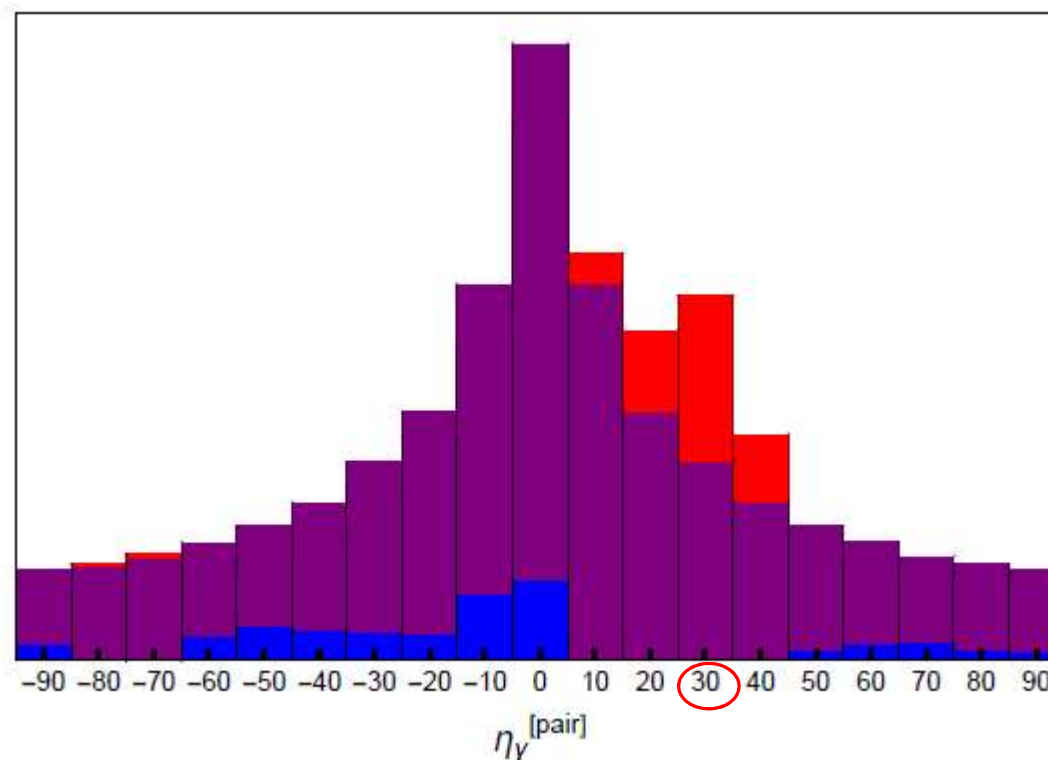
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for bins where the observed population is higher than expected we color the bar in purple up to the level expected, showing then the excess in red;
for bins where the observed population is lower than expected the bar height gives the expected population, while the blue portion of the bar quantifies the amount by which the observed population is lower than expected



threshold anomaly for $\gamma\gamma \rightarrow e^+e^-$

assume the relevant photon and electrons are all governed by on-shellness

$$E^2 \approx m^2 + p^2 + \eta \frac{2}{n+1} \left(\frac{E}{M_P} \right)^n p^2$$

and make the additional assumption that energy-momentum is trivially conserved

$$E_1 + \epsilon = E_2 + E_3$$

$$p_1 - q = p_2 + p_3$$

then, also assuming $n=1$, one finds that the threshold-energy requirement E_1 for a hard photon to produce an electron-positron pair in interaction with a soft photon of energy ϵ is

$$E_1 - \eta \frac{E_1^3}{\epsilon M_P} \approx \frac{m_e^2}{\epsilon}$$

I formulate a bold proposal concerning GRB190114C: our cost action requests access to MAGIC's data and a paper is written by all cost-action members about the possible relevance of this observation for QG phenomenology

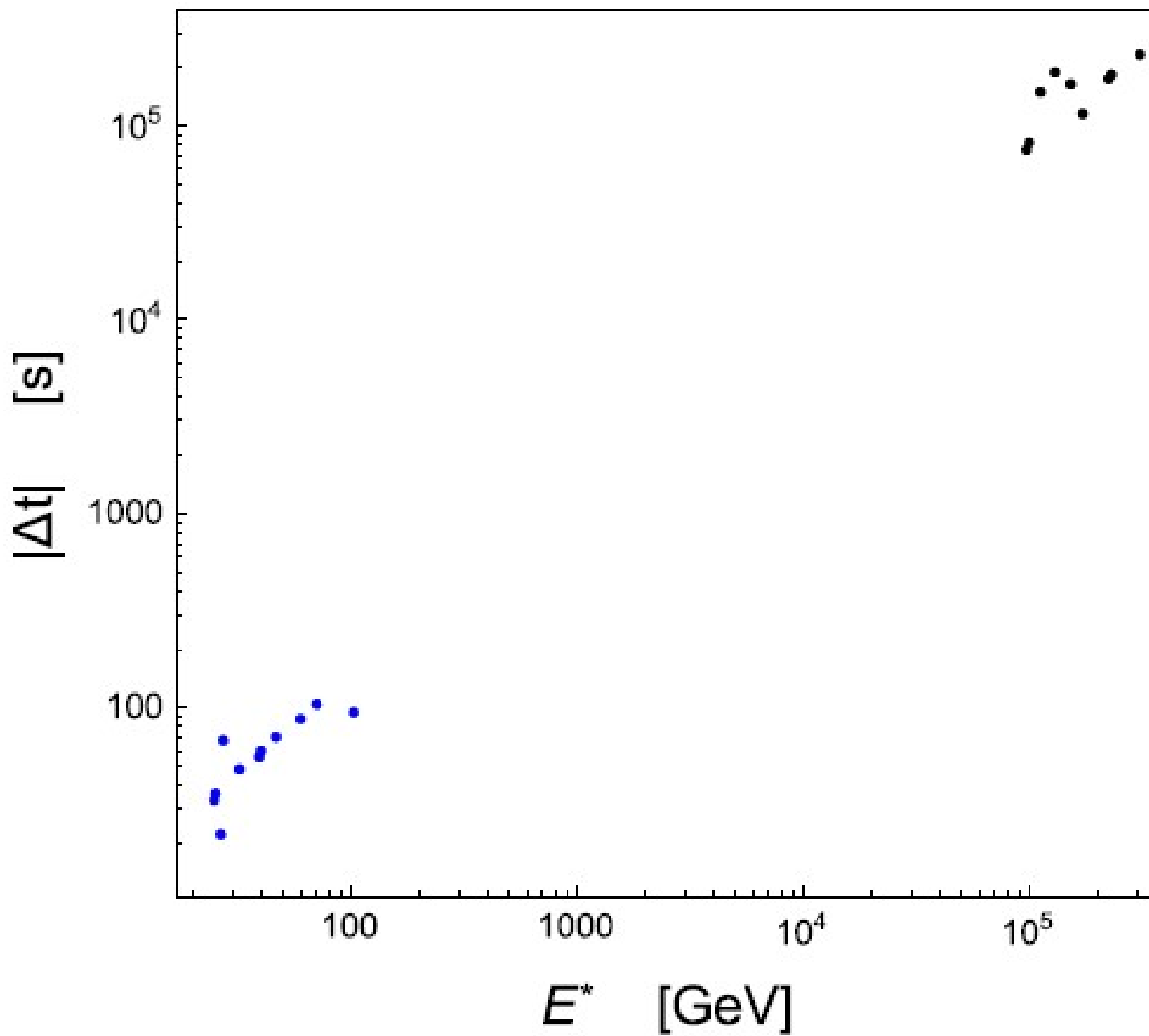
GAC+Barcaroli+D'Amico+Loret+Rosati, arXiv1605.00496, PhysicsLettersB761,318
GAC+D'Amico+Rosati +Loret, arXiv:1612.02765, NatureAstronomy1,0139
[these use latest data release by IceCube....also see previous exploratory analysis
on 2008-2010 IceCube data **GAC+Guetta+Piran**, Astrophys.J.806,269]

IceCube still found no GRB neutrinos (expected at least a dozen at this point)

If effect is of seconds for GeV photons it can be very large for 300TeV neutrinos...the time window adopted by IceCube would never catch such GRB neutrinos...

IceCube has reported so far 21 shower neutrinos with energy between 60 and 500 TeV

**we found that 9 of them could be “GRB-neutrino candidates” (direction compatible with the GRB direction and time of observation within 3 days of the GRB)
so let's see if they provided some support for the linear dependence between Δt and E^***



CLOSING REMARKS

the “preliminary statistical evidence” is strong enough to encourage us to think about alternative phenomenological models, giving a better description of the data situation...would have to be a case such that my simple-minded in-vacuo-dispersion formula is like the Bohr-Somerfeld description of atoms:

- what about the 31GeV event from GRB090510? Should we ascribe it to a remarkable conspiracy?
 - is the effect intrinsically statistical/non-systematic?
 - Does the effect depend on polarization?
 - Does the effect depend on direction? Do we need to look beyond the Jacob-Piran formula?
(most of the data that give more strength to the statistical evidence are from very distant GRBs)
- 4 out of our 9 neutrinos are “early neutrinos”...are they background?
 - Or would the effect for neutrinos have both signs?
 - If so why would the effect have only one sign for photons?

**working on quantum gravity
one cannot avoid getting the
feeling that Nature might have
hidden very well some of its
most fascinating secrets**

**still we have no other
option but to keep looking**

**and maybe we are wrong and
the secrets are not so well
hidden**

$$v \approx c + s_{\pm} \left(\frac{E}{E_P} \right)^n c + \dots$$



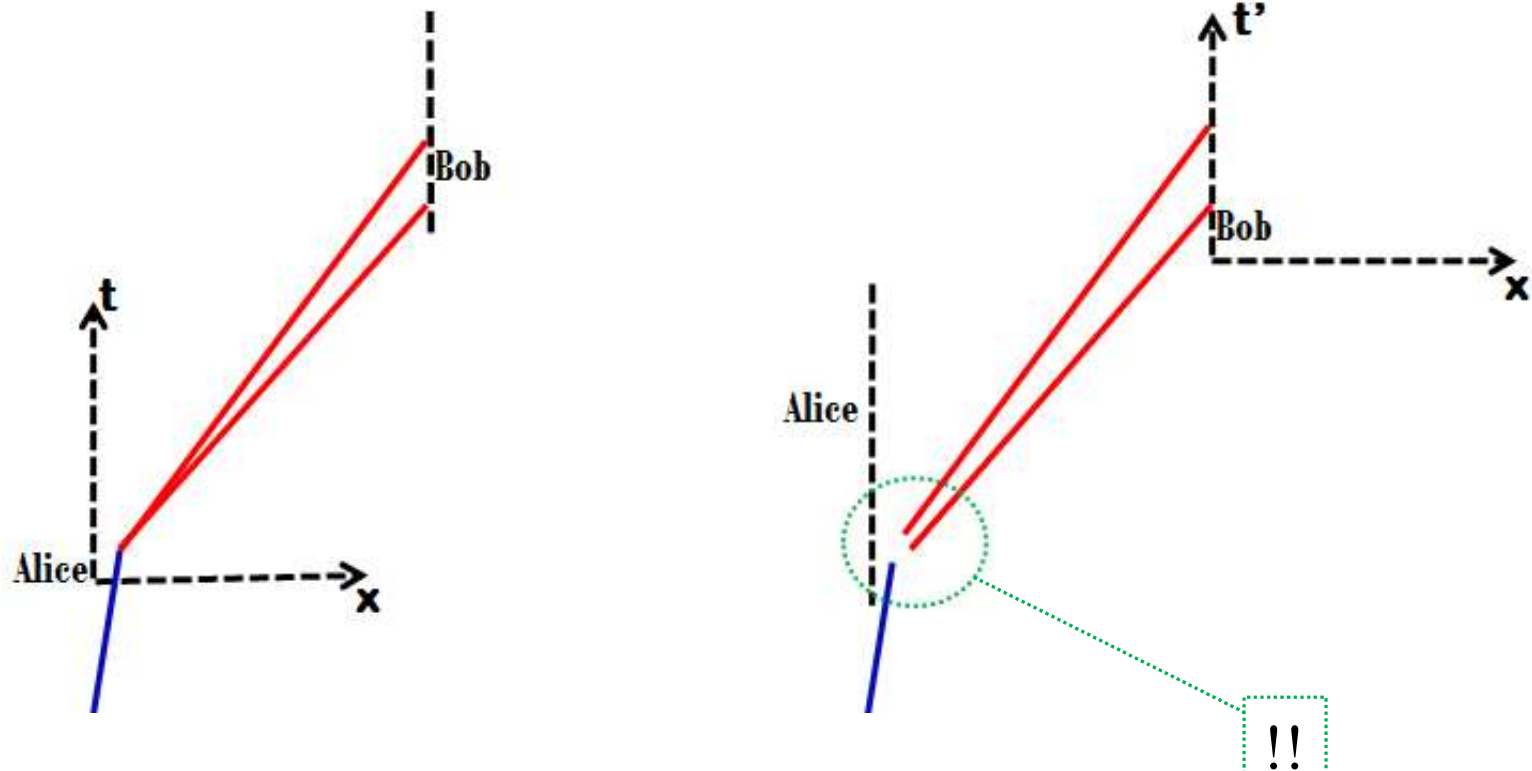
relative locality

proper introduction to the notion of relative locality would require a full talk...
famously from viewpoint of a “Galileian physicist” the fact that special relativity introduces an invariant velocity scale produces artifacts about the simultaneity (time coincidence) of events, that is “relative simultaneity”...
from viewpoint of “Einsteinian physicist” a relativistic theory which introduces also an invariant energy scale produces artifacts about the spacetime coincidence (locality) of events

GAC+Matassa+Mercati+Rosati, *PhysicalReviewLetters*106,07301

GAC+Freidel+Kowalski+Smolin, *PhysicalReviewD*84,087702

illustrative example:



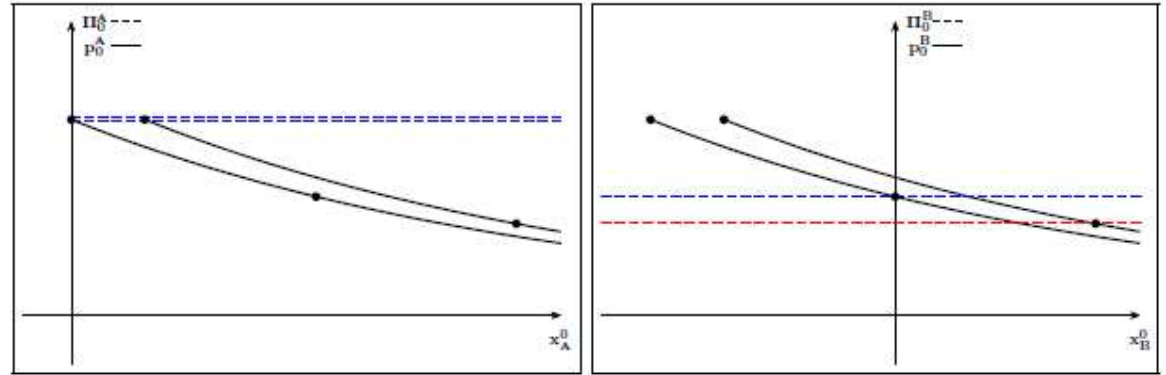


Figure 4. We here visualize our preferred illustrative example of a relative momentum-space locality analysis, which concerns the evolution in time of Π_0 and p_0 on the worldlines of massless particles. Alice's descriptions of some worldlines are in the left panel, while Bob's description of the same worldlines is in the right panel. Both with p_0, p_1 coordinates (solid lines) and with Π_0, Π_1 coordinates (dashed lines), Alice has that the emitted particles have the same energy and Bob has that the energies at detection are different (same energy difference within both coordinatizations). The peculiarities introduced by the coordinatization Π_0, Π_1 , affected by relative momentum-space locality, only play a role in the inferences the observers make about distant events: adopting Π_0, Π_1 coordinates, Alice would describe the distant detections at Bob as having the same energy and Bob would describe the distant emissions at Alice as having different energy. Equations in support of these figures will be derived in section 4.

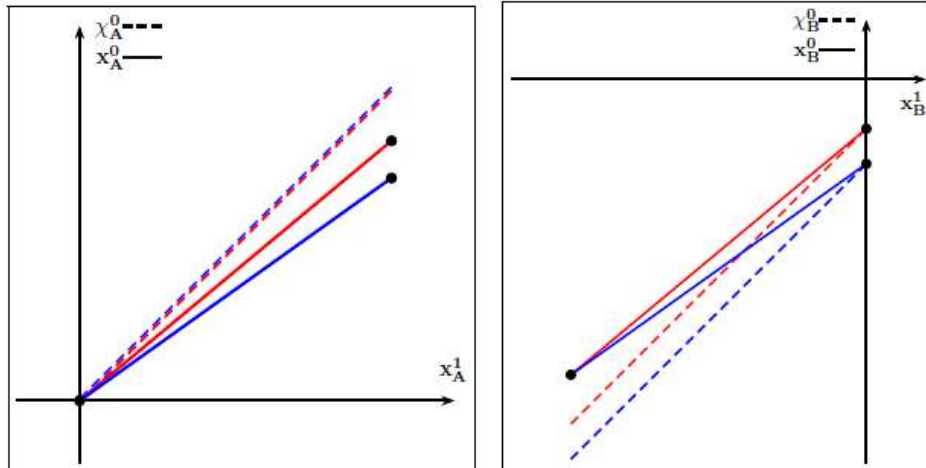


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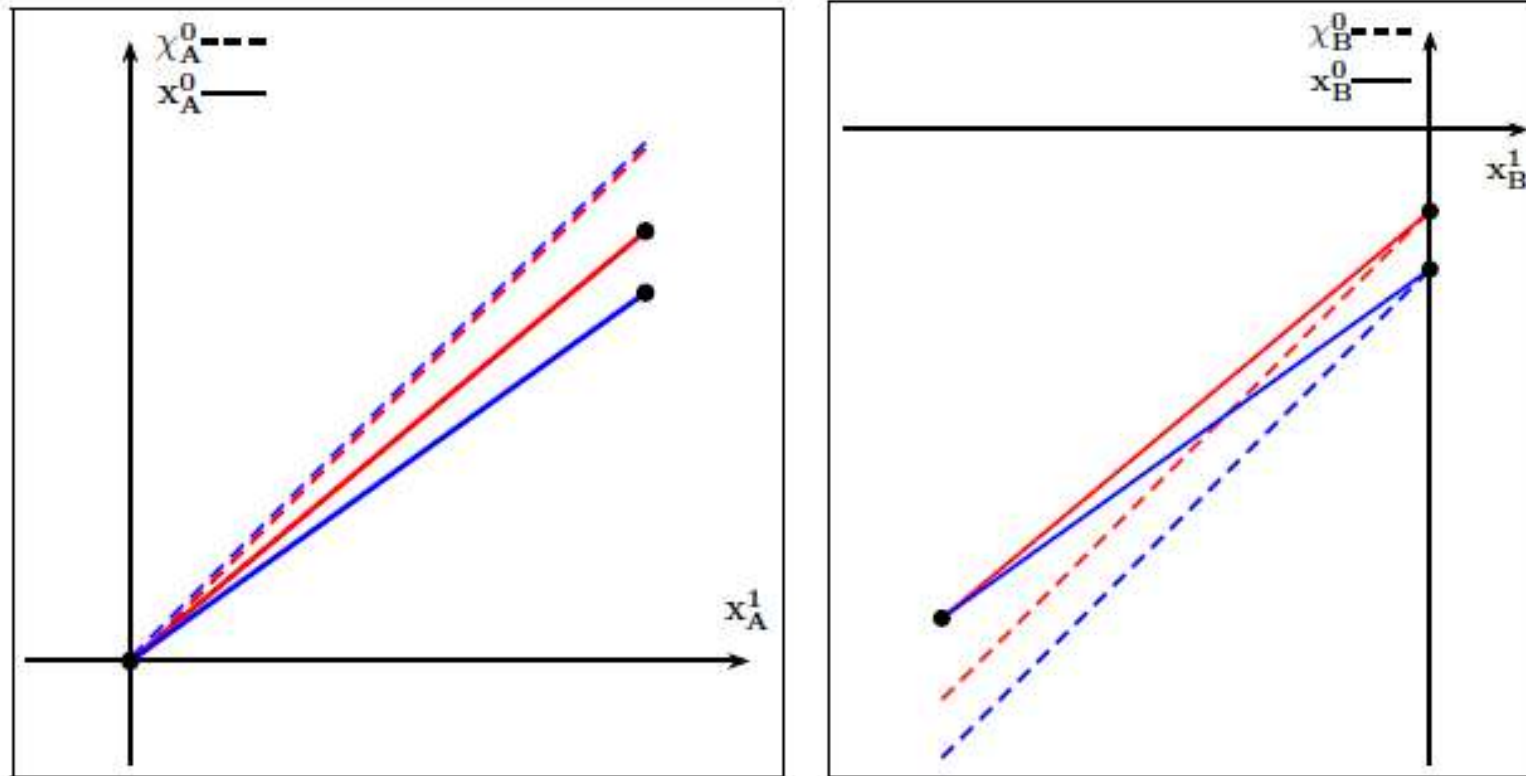


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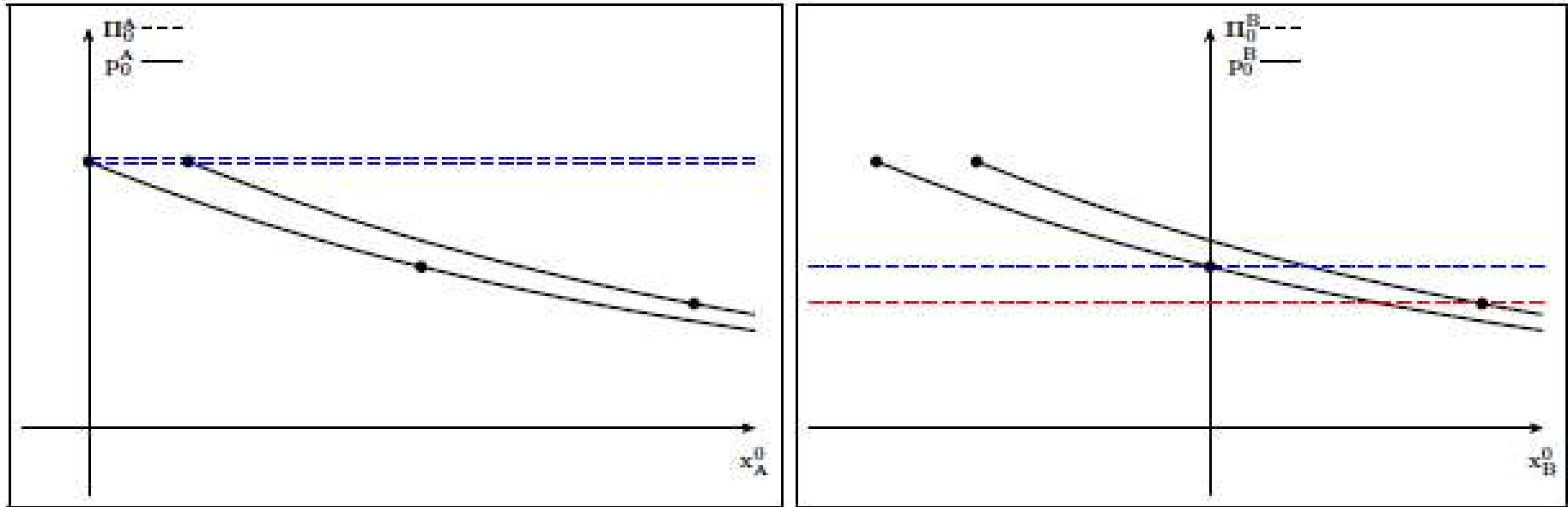


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