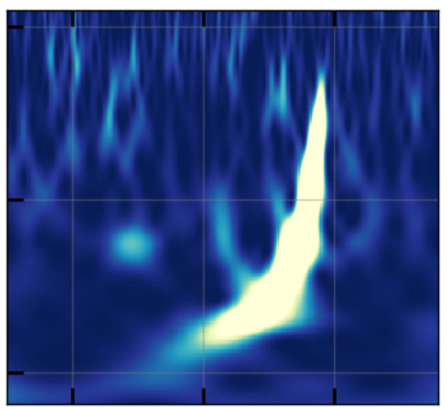
Data Analysis Techniques for Testing General Relativity with GWs



Michael Pürrer
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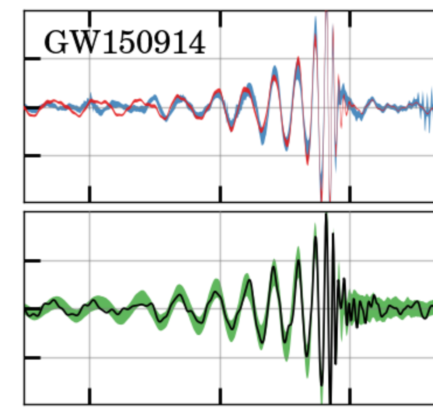
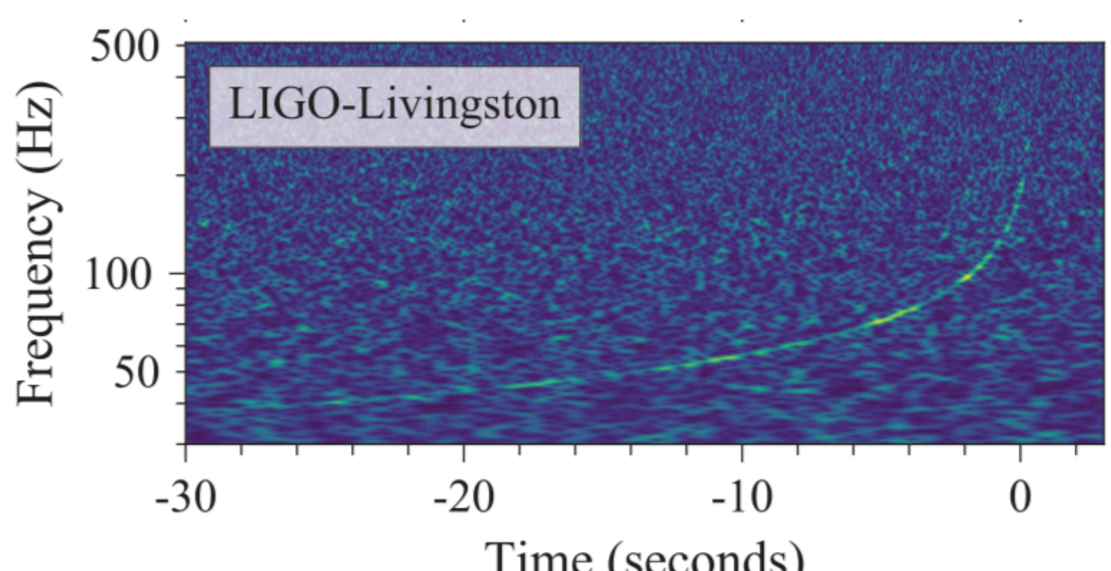


COST CA18108 Kickoff Meeting
Barcelona, October 2, 2019



MAX-PLANCK-GESELLSCHAFT

Motivation

- GW astronomy:
  - We want to get the **“fingerprints” of GW sources** from the **shapes of their waveforms**
  - Want to learn about **binary parameters** and **astrophysics and fundamental physics** of these systems
  - **LIGO / Virgo** have set the stage with about a dozen observations in O1 & O2, more coming as we speak in O3.

- Testing General Relativity with GWs:
  - GWs emitted from merging compact binaries allow us to **test GR in the strong field regime**
  - So far, **no evidence for deviations from GR**

- First, let’s look at **basic GW data analysis:**
  - How can we **infer binary parameters** given a GW signal?
  - **Ingredients:** matched filtering, model of (GR) waveform and noise

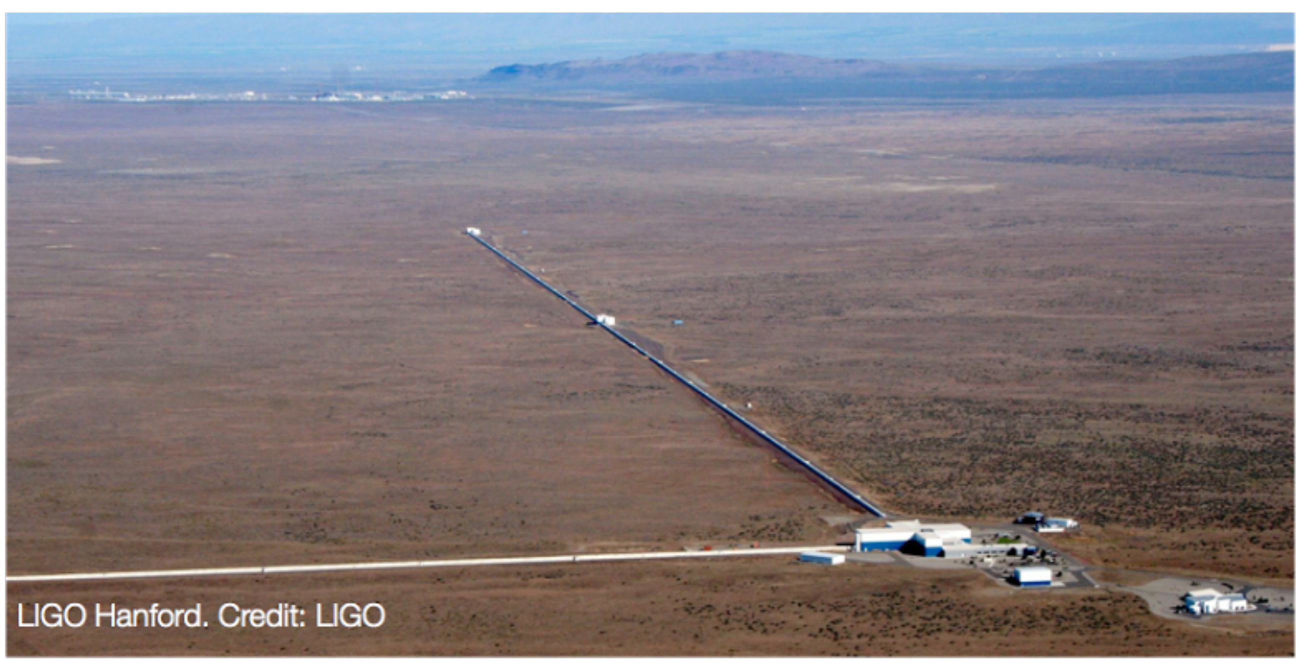
Introduction to GW data analysis for compact binary mergers

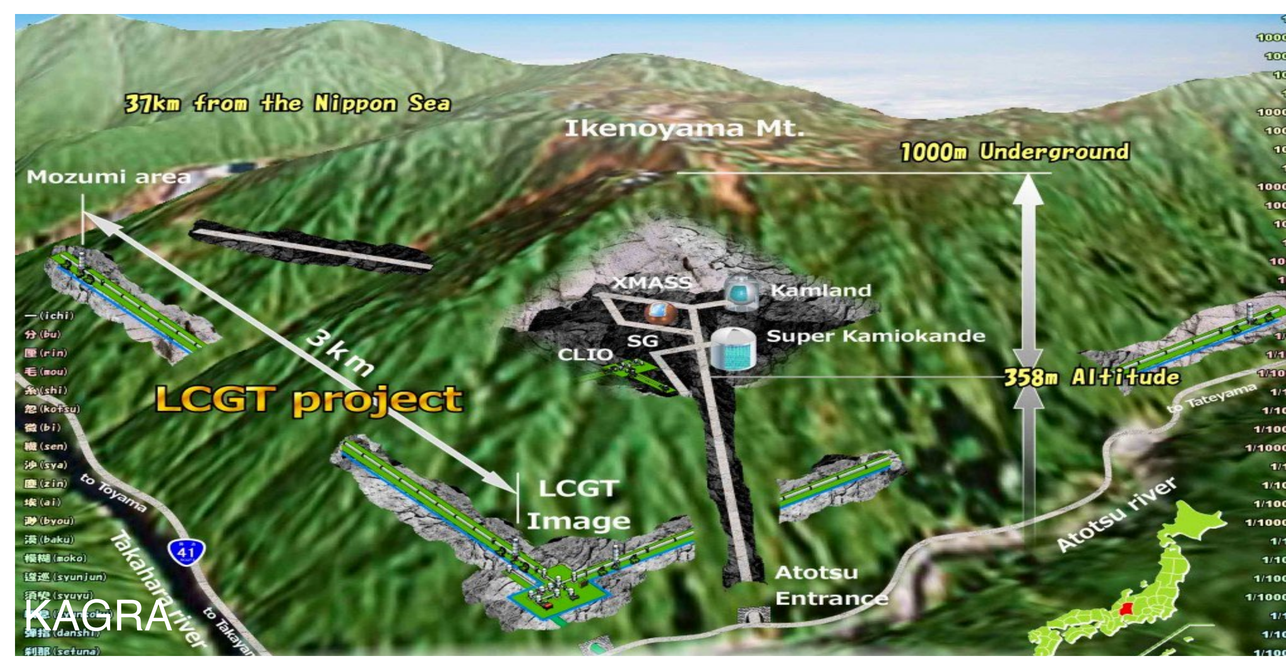
See also: "Guide to LIGO data analysis", arXiv:1908.11170

Current GW detector network


LIGO Livingston. Credit: LIGO


Virgo observatory. Credit: Virgo

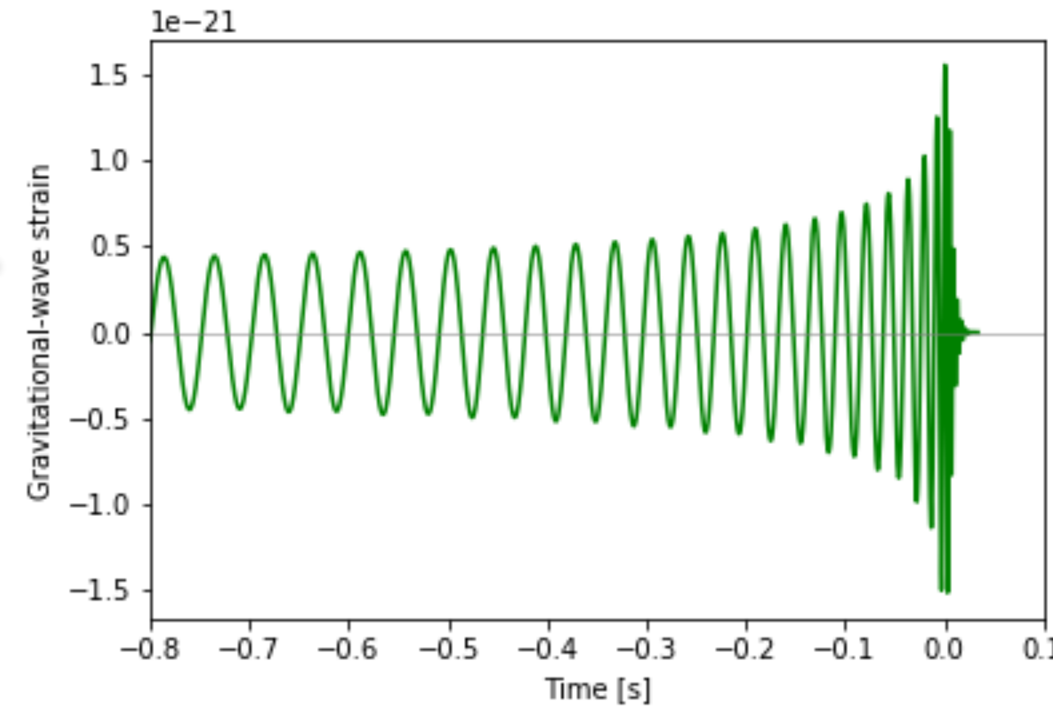

LIGO Hanford. Credit: LIGO



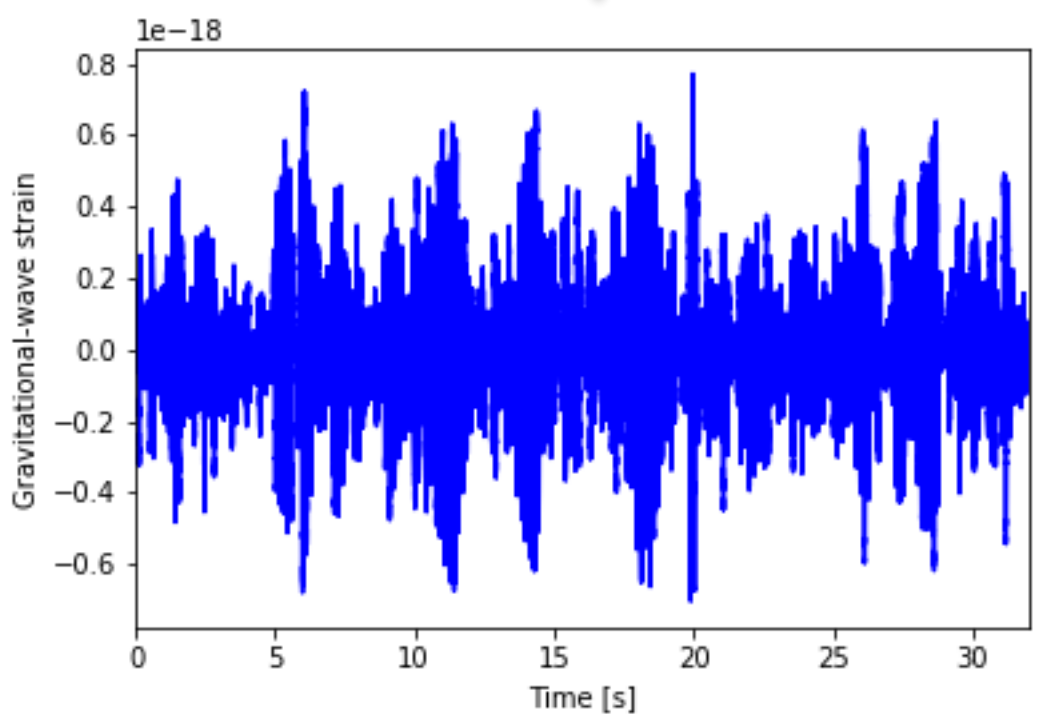
Matched filtering

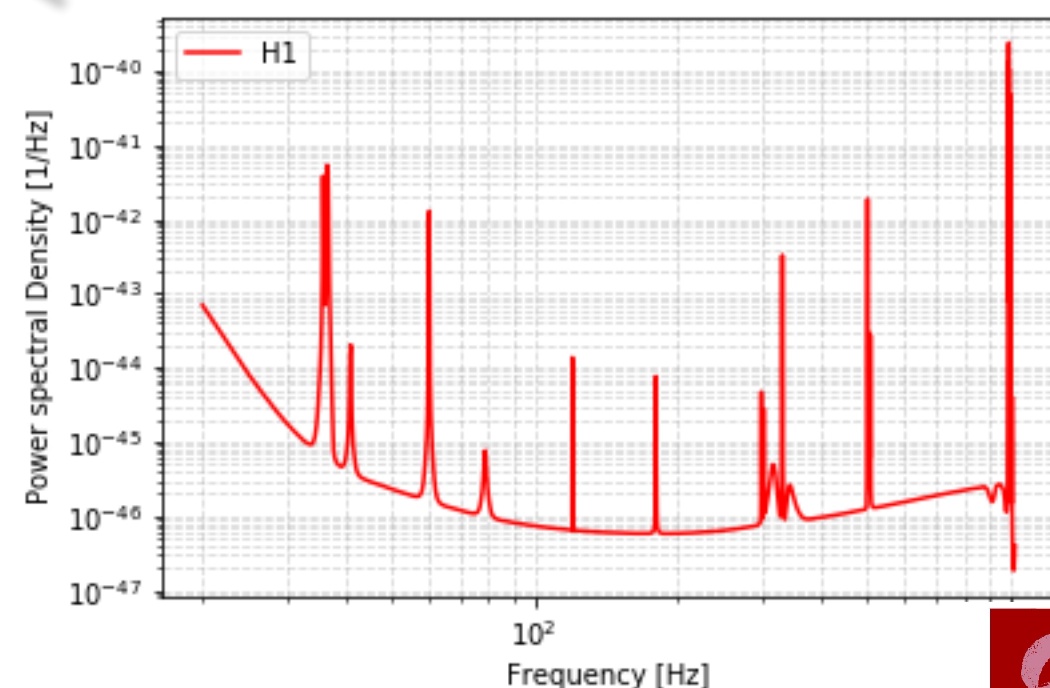
Inner product:

$$\langle d \,|\, h \rangle = 4\,\mathrm{Re} \int_0^\infty \frac{\tilde{d}(f)\tilde{h}^*(f)}{S_n(f)} \mathrm{d}f$$



SNR: $\rho = \|h\|$



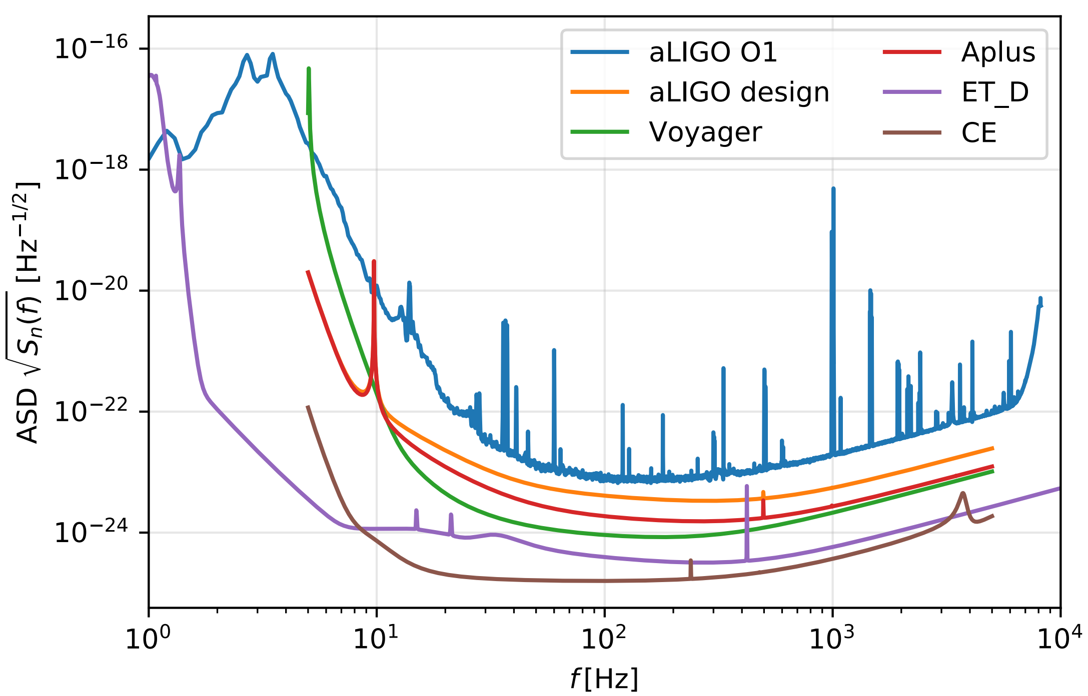


GW data analysis

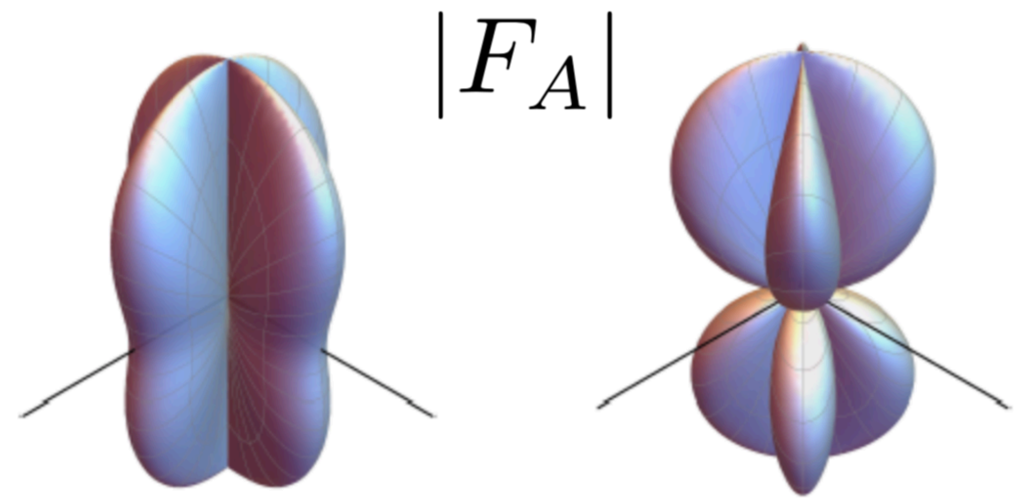
- Detection:
  - Build a **bank of template waveforms** that covers parameter space
  - Want to **keep fraction of missed signals small**
  - **Cross-correlate templates against detector data**
- Parameter estimation:
  - Given a likely detection follow it up with **Bayesian methods** to find the **probability distribution of the binary parameters given the signal in the detectors and the PSD of the detector noise**
  - Need high accuracy waveforms to make sure that we don't infer wrong / biased parameters
- Both need **fast & accurate waveform models**
  - Require $O(10^7)$ - $O(10^8)$ waveform evaluations!

Interferometric GW detectors

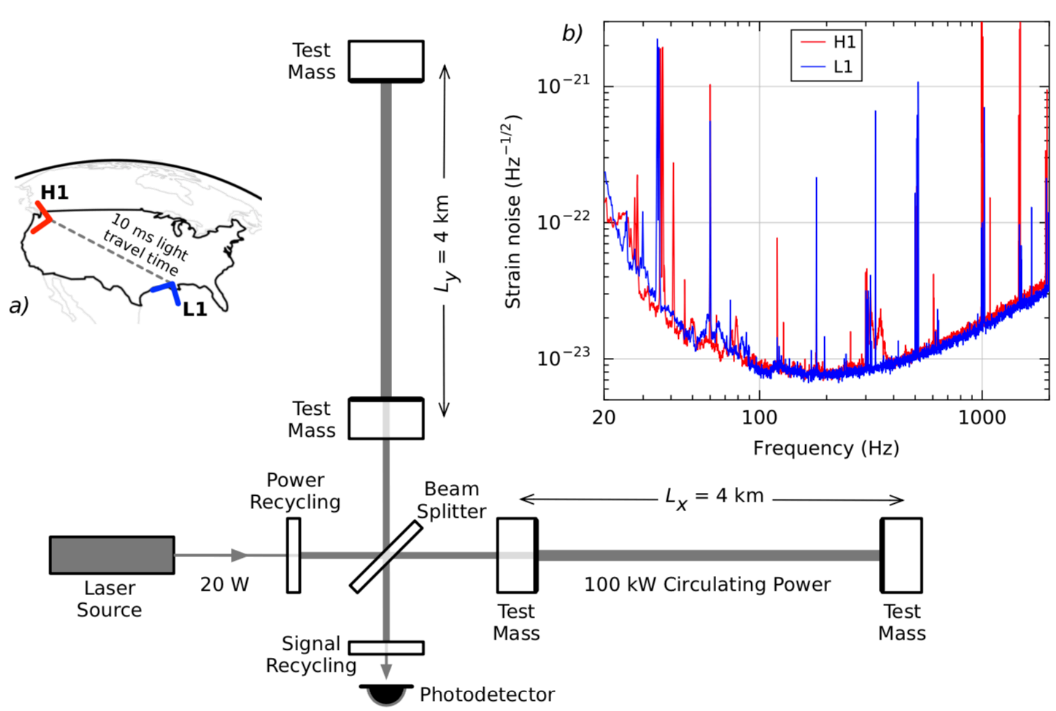
Evolution of detector sensitivity



Angular response of IFO



(a) Plus (+) (b) Cross (×)



$$h = h_A F^A \qquad F^A(\hat{n}, \psi) = D^{ab} e^A_{ab}$$

$$A = +\,,\times\,,\ldots$$

$$D^{ab} = \frac{1}{2}\left(d^a_x d^b_x - d^a_y d^b_y\right)$$

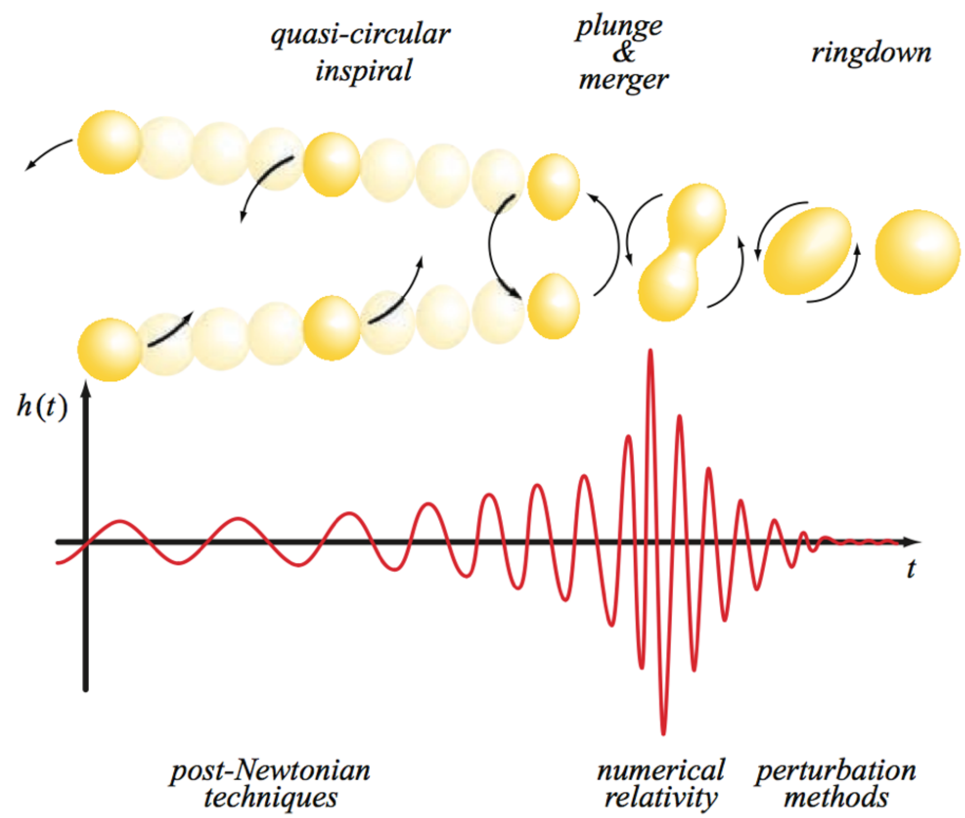
What do we need to model?

- A **waveform model** is a **parametrized function** of the waveform **polarizations** $h_{+,\times}(t;\vec{\lambda})$ or complex **modes** $h_{lm}(t;\vec{\lambda})$:

$$(h_+ - ih_\times)(t;\vec{\lambda}) = \sum_{l,m} h_{lm}(t;\vec{\lambda})\,{}^{-2}Y_{lm}(\theta,\phi)$$

- Need to model the **inspiral, merger and ringdown** stages in binary black hole coalescence.

BBH coalescence



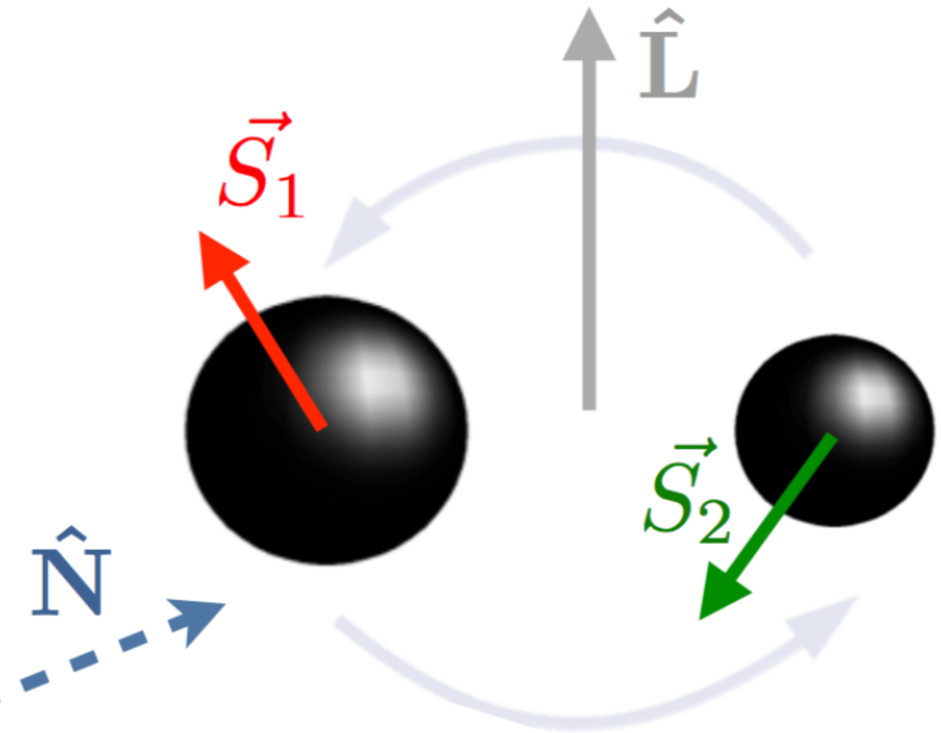
[Baumgarte & Shapiro, Numerical Relativity]

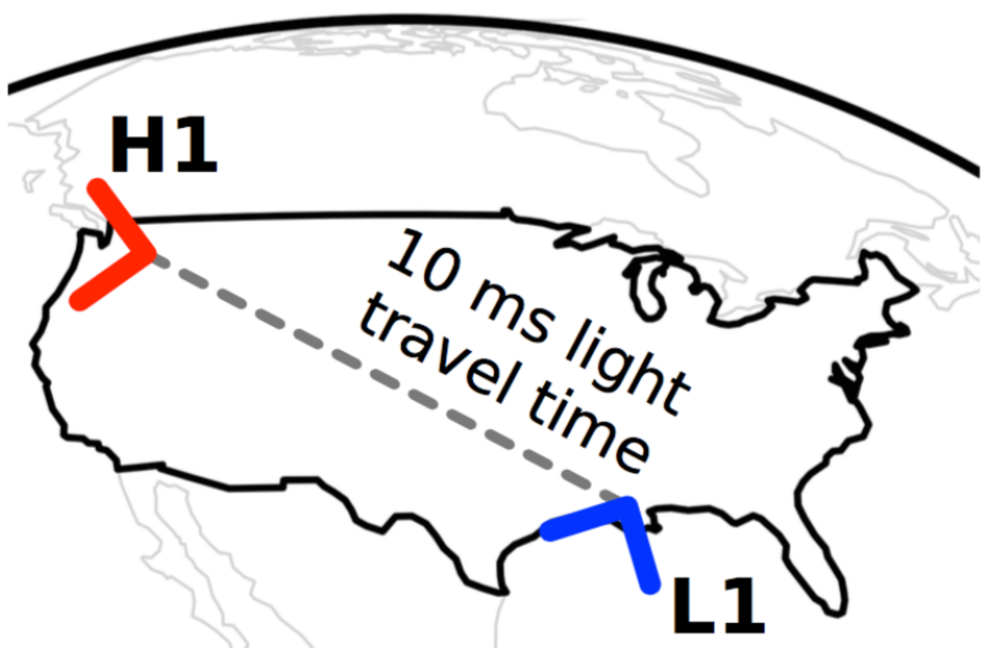
- GW detectors record **GW strain**:

$$h(t;\vec{\theta}) = h_+(t;\vec{\lambda})F_+(\hat{n},\psi) + h_\times(t;\vec{\lambda})F_\times(\hat{n},\psi)$$

Model parameters

Intrinsic parameters:
masses, spins, eccentricity, tidal deformability

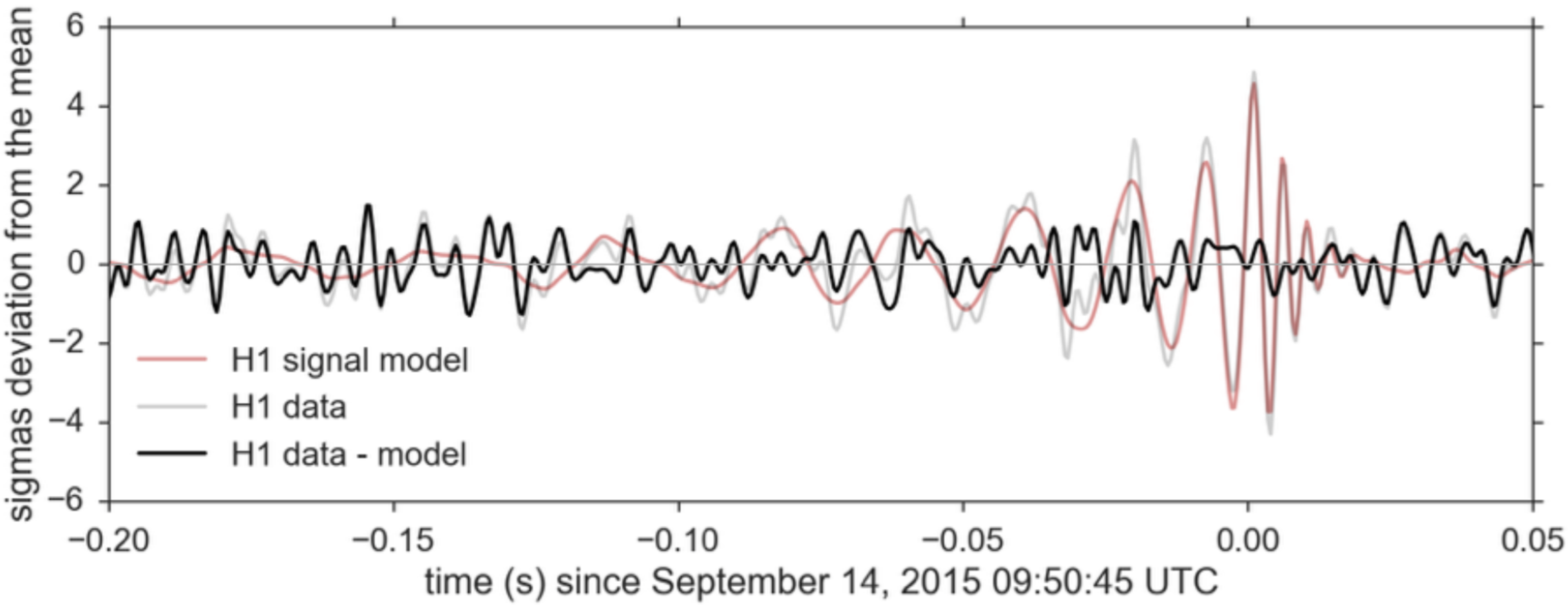




Extrinsic parameters:
time, sky position, distance, orientation, reference phase

Credit: LIGO/Virgo

To find the signal: Likelihood



- Subtract **signal model** from **data** $d - h(\vec{\lambda})$, where $d = n + h_{\rm true}$

- Assumptions: noise is **Gaussian** (zero mean) and **stationary**

$$\mathscr{L}(d\,|\,\vec{\lambda}) \propto \exp\left[-\frac{1}{2}\left(d - h(\vec{\lambda}), d - h(\vec{\lambda})\right)\right]$$

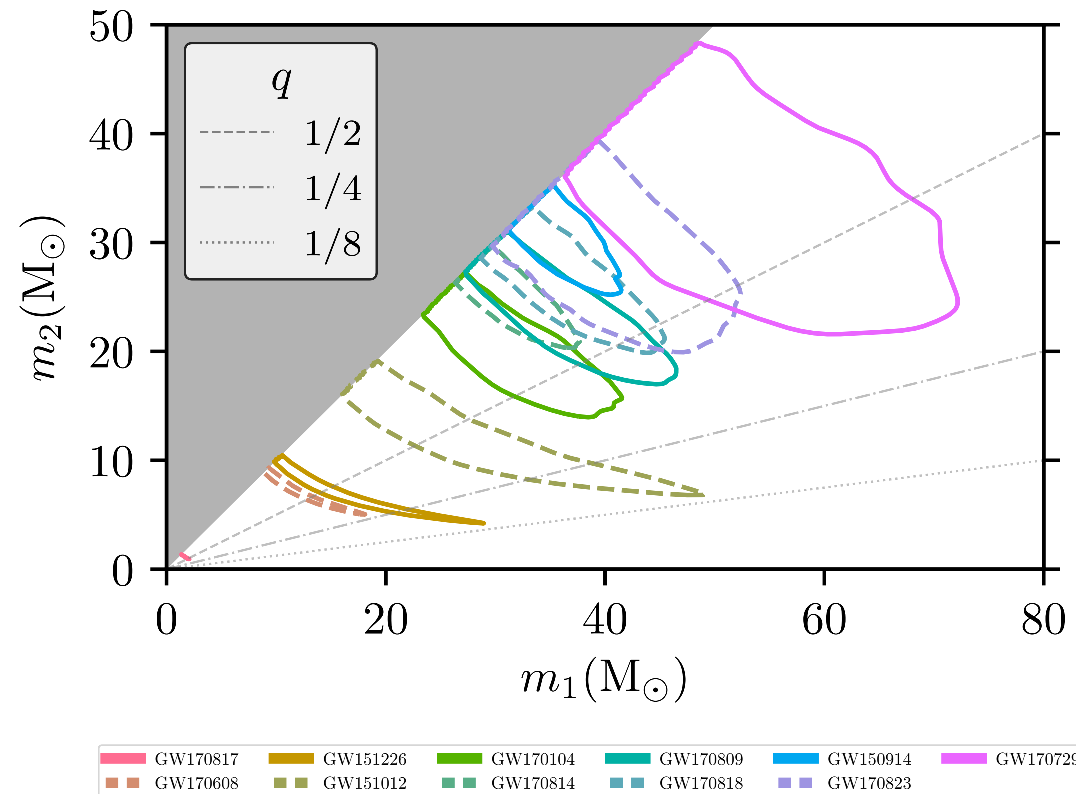
Parameter Estimation

- **Posterior probability** of model parameters $\vec{\lambda}$ given the data $\vec{d}$ (Bayes' Theorem):

$$\boxed{p(\vec{\lambda} \mid \vec{d}) \propto p(\vec{\lambda})\mathscr{L}(\vec{d} \mid \vec{\lambda})}$$

- Need **Models** for **signal** and **noise** & specify **prior knowledge**
- Numerically **sample** the posterior distribution

GWTC-1 Catalog



LVC, PRX 9, 031040, 2019

Tests of GR with GWs

See **LVC, arXiv:1903.04467**
Results on GWTC-1 Catalog

Residuals test

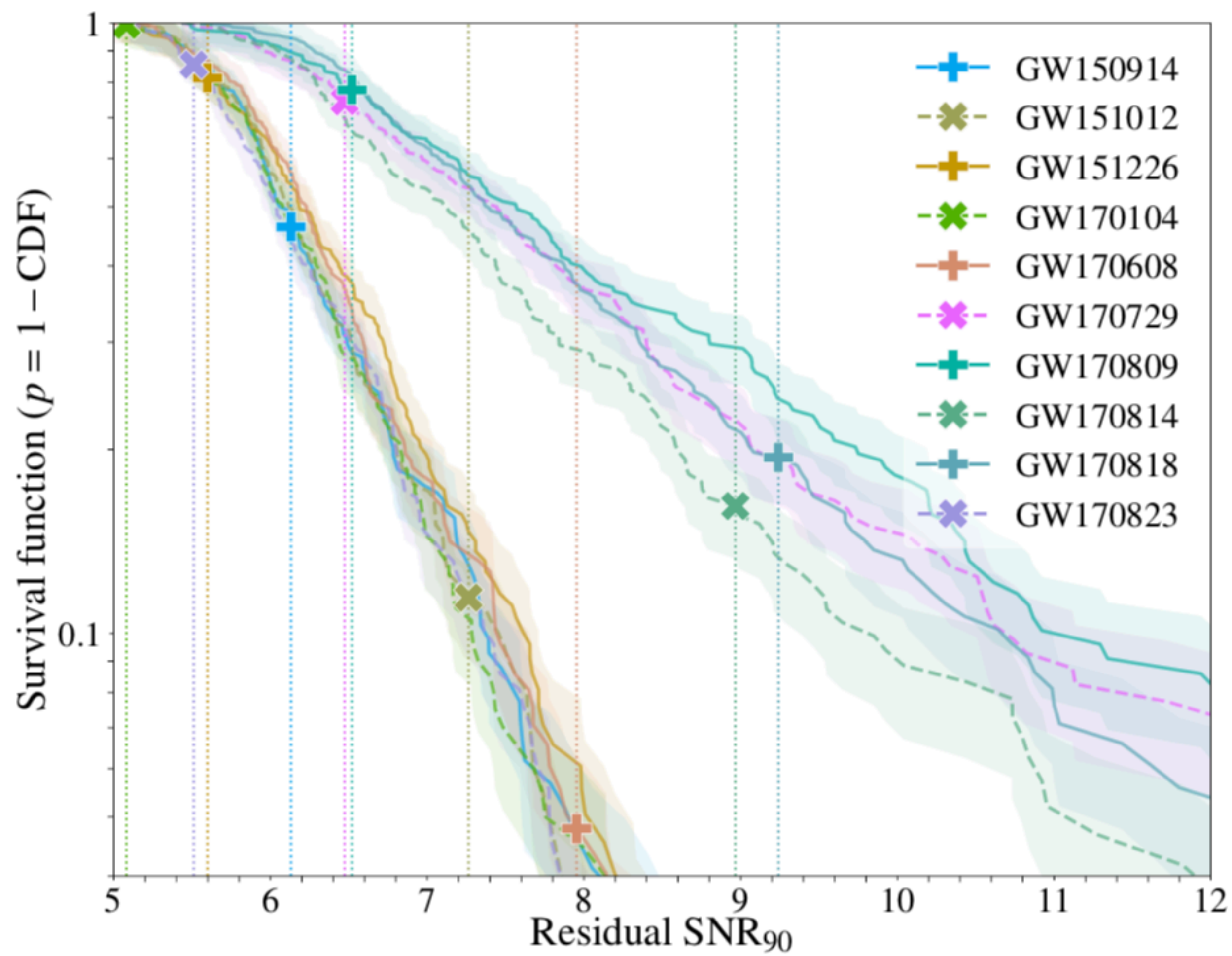
- **Residual:** detector data - maximum likelihood template

$$\mathscr{R}(t) := d(t) - h(\overrightarrow{\lambda}_{\max \mathscr{L}}; t)$$

- **Compute 90% upper limit of SNR** of residuals $\rho_{90}(\mathscr{R})$
  - describe signal as Gaussian noise + coherent signal

- Repeat analysis for **noise-only detector data** many times: $p(\rho_{90}(\mathscr{R}_{\text{noise}}))$

- **At which percentile does $\rho_{90}(\mathscr{R})$ lie in $p(\rho_{90}(\mathscr{R}_{\text{noise}}))$?**



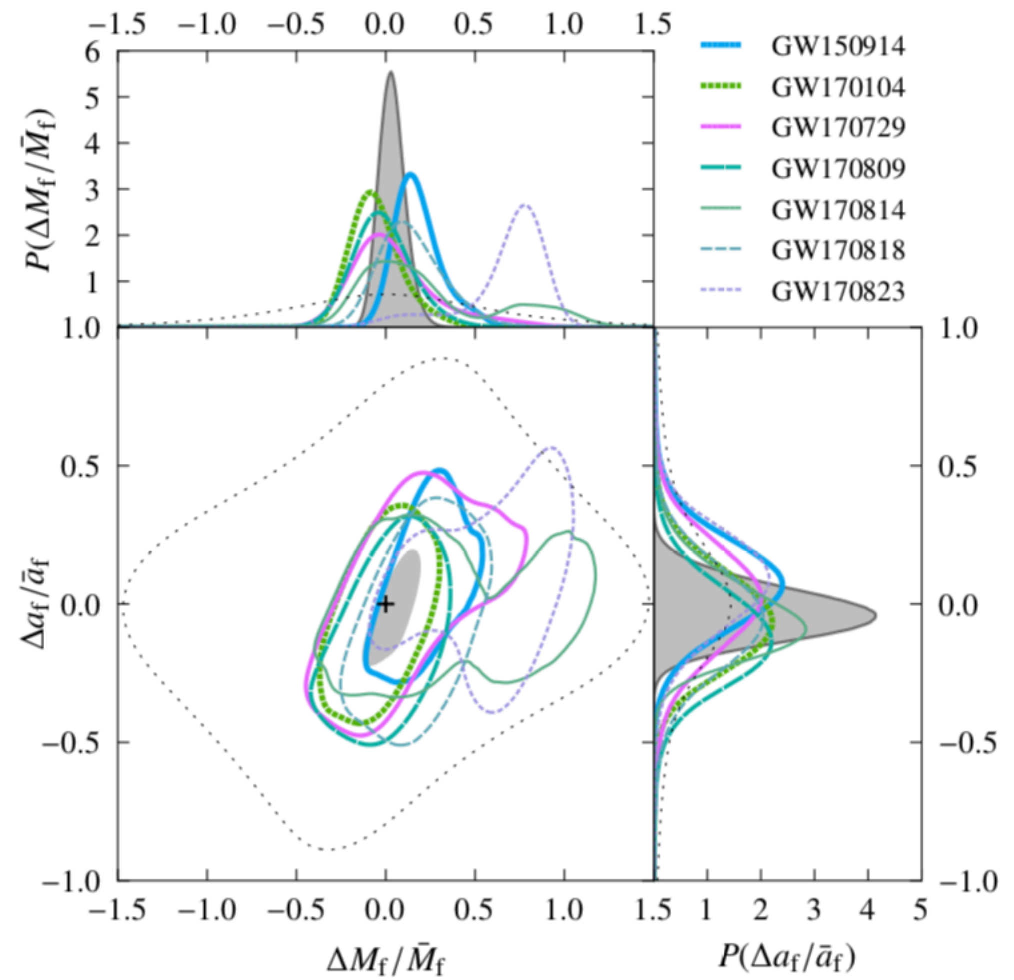
- p-values > 0.05
- Meta p-value = 0.4

No statistically significant evidence for deviations from GR

Inspiral-merger-ringdown consistency test

- Check whether **final mass** M_f and **final spin** a_f inferred from the **low** and **high frequency parts** of the signal are consistent.
- Compute **posterior distributions** with different **frequency bounds** for likelihood integral:
  - **Transition point:** innermost stable circular orbit (ISCO)
  - **Normalization:**

$$\Delta M_f / \bar{M}_f := \frac{M_f^{\mathrm{I}} - M_f^{\mathrm{MR}}}{(M_f^{\mathrm{I}} + M_f^{\mathrm{MR}})/2}$$



- lines = 90% credible regions of $p(\Delta M_f / \bar{M}_f, \Delta a_f / \bar{a}_f | d)$
- Plus sign = GR value
- GR value recovered at < 80% credible level

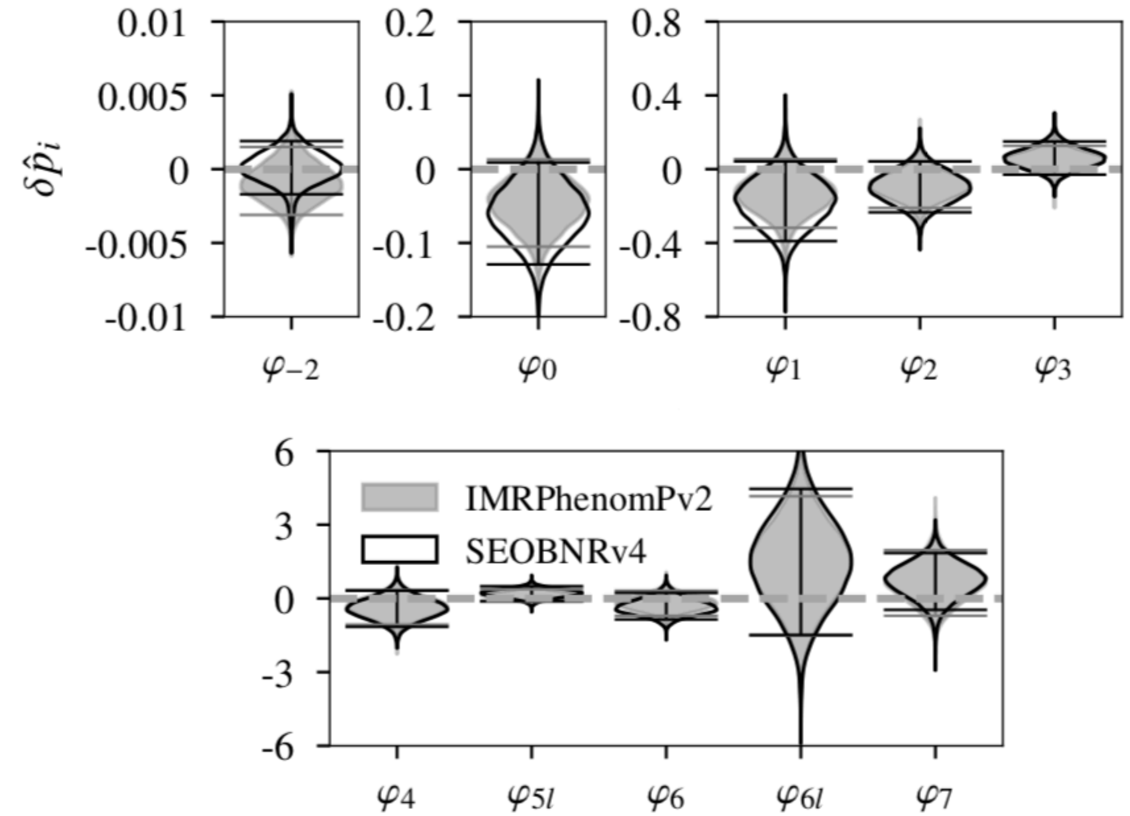
All events consistent with GR

Parametrized test of GW generation

- Compute **posteriors** $p(\delta\hat{p}_n | d)$ of **deviations in coefficients** p_n of the **phase of waveform** models
- Templates have to be **phase coherent** with the signal to a fractions of a radian to capture the SNR of an event
- post-Newtonian **phase** up to 3.5PN or $\mathcal{O}((v/c)^7)$

$$\Phi(\overrightarrow{\theta}; f) = 2\pi f t_c - \varphi_c - \pi/4$$
$$+ \frac{3}{128\eta}(\pi M f)^{-5/3} \sum_{i=0}^{7} \varphi_i(\overrightarrow{\theta})(\pi M f)^{i/3}$$

- **Fractional deviations**, $\delta\hat{\varphi}_n := (\varphi_n - \varphi_n^{\mathrm{GR}})/\varphi_n^{\mathrm{GR}}$ except for -1PN and 0.5PN which are *zero* in GR



- Combined posteriors for $\delta\hat{\varphi}_n$ for the most significant events
- Horizontal lines: 90% CR
- GR value = 0

All events consistent with GR

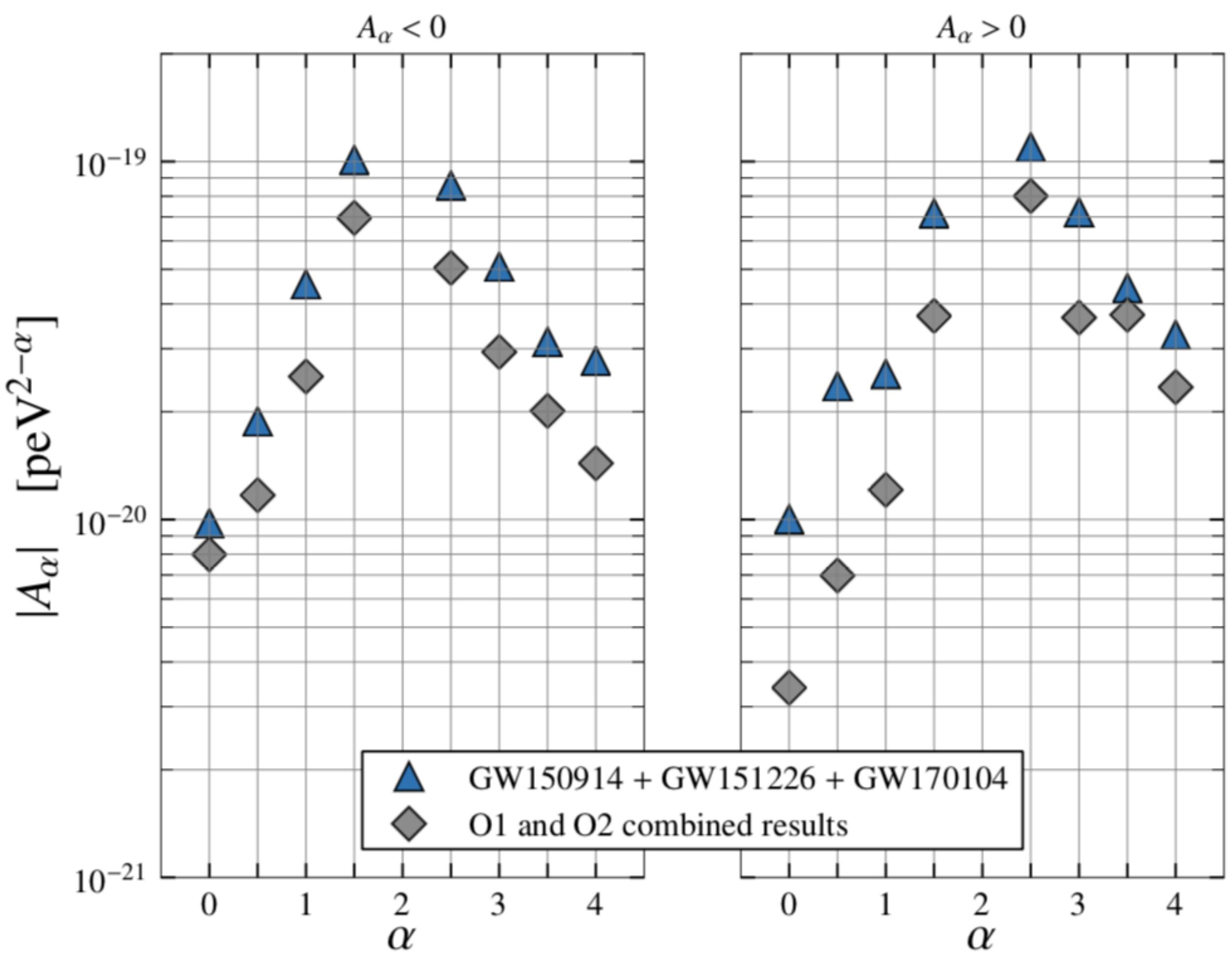
Parametrized test of GW propagation

- Assume a **phenomenological dispersion relation**

$$E^2 = p^2c^2 + A_\alpha p^\alpha c^\alpha$$

where **GR** has $A_\alpha = 0$

- **Massive graviton** $(\alpha = 0, A_\alpha > 0)$
- **Lorentz-violating theories**
- **Dispersive propagation of GWs** except for $\alpha = 2$
- Use GR waveforms and only modify propagation



- 90% credible upper bounds on $|A_\alpha|$
- $1\text{peV} \simeq h \times 250\text{Hz}$
- $m_g \leq 4.7 \times 10^{-23} eV/c^2$

No indication for dispersive propagation of GWs.

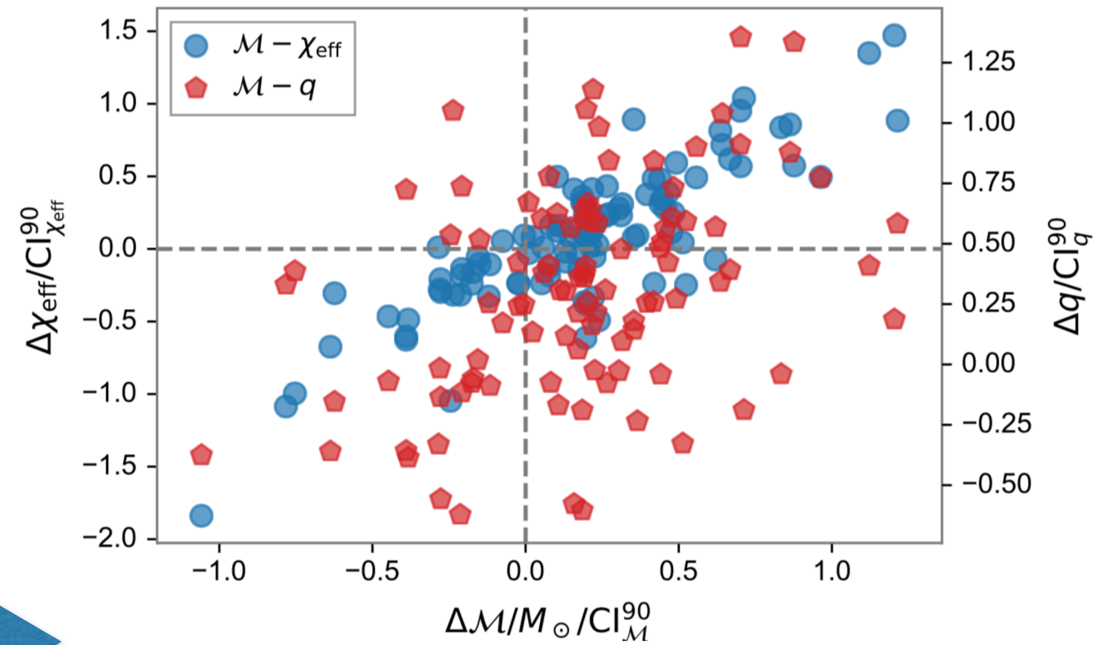
Constraints on Alternative Polarizations

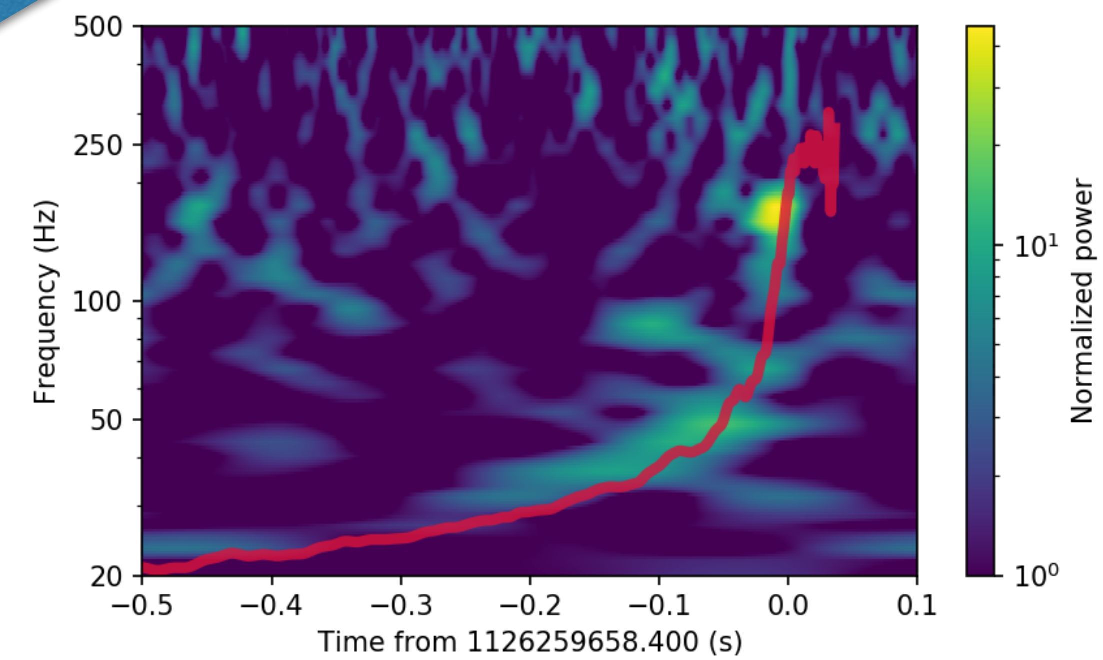
- For events observed with three or more GW detectors:
  - is possible to **distinguish purely tensor** from **purely vector** or **purely scalar perturbations**
  - detectors have **different antenna patterns** for different polarizations

- Compute **Bayes factors** between options:
  - GW170817: **BF > 10^{20}** in favor of **pure tensor polarizations**
  - Weaker constraints from binary black hole events (SNR and sky localizations play a role)

Conclusion & Outlook

- Will have **many more events** during O3 and future observing runs: **statistical errors** of the combined results will soon **decrease significantly**
- Potential sources of **systematic errors**:
  - **imperfect modeling of GR waveforms**
    & missing physics
    problem for 2G design & 3G detectors (ET)
  - **calibration uncertainties**
  - **noise artifacts**
- Goal: **inspiral-merger-ringdown waveforms for alternative theories of gravity**
  - This is hard: need to solve **initial value problem** for 2-body problem: well-posedness?
  - Obtain waveform in the **inspiral** regime
  - Develop **numerical relativity** codes for alternative theories



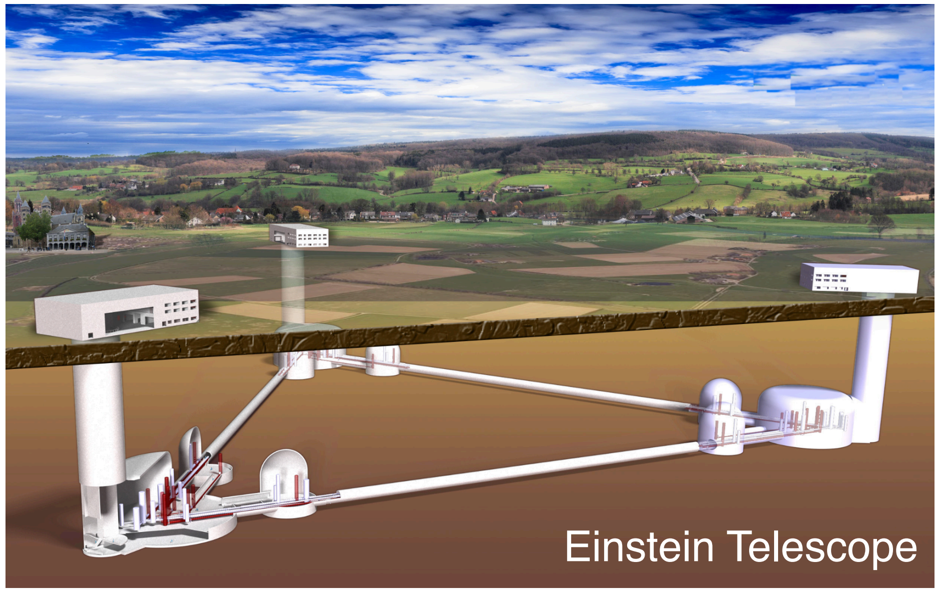


Biases / stat error and power in residual from population study
[MP & C. Haster, in prep]

Einstein Telescope and 3G Science

Science Targets for 3G GW Detectors

- *determine* the properties of the **hottest and densest matter** in the Universe
- *reveal* the **merging BH population** throughout the Universe; seeds
- *investigate* the **particle physics** of the **primeval Universe** and probe its dark sectors



- *3G Science Case Document, led subgroup on waveforms & data analysis*
- *3G White paper on Extreme Gravity and Fundamental Physics, arXiv:1903.09221*
- *3G network sensitivity ~ 20 times better than aLIGO/Virgo:* **precision measurements!**

Extra slides

Phase contributions from dispersion relation

- Treat the GW as a **stream of gravitons**, which travel at the **particle velocity** $v_p/c = pc/E = 1 - A_\alpha E^{\alpha-2}/2 + O(A_\alpha^2)$
- **Length scale** of modifications to the Newtonian potential $\lambda_A \;:=\; hc|A_\alpha|^{1/(\alpha-2)}$
- Fourier domain **dephasing** due to modified dispersion relation:

$$\delta\Phi_\alpha(f) = \mathrm{sign}(A_\alpha)\begin{cases}\dfrac{\pi D_\mathrm{L}}{\alpha-1}\lambda_{\mathrm{A,eff}}^{\alpha-2}\left(\dfrac{f}{c}\right)^{\alpha-1}, & \alpha \neq 1\\[2ex] \dfrac{\pi D_\mathrm{L}}{\lambda_{\mathrm{A,eff}}}\ln\left(\dfrac{\pi G\mathcal{M}^{\mathrm{det}} f}{c^3}\right), & \alpha = 1\end{cases}$$

$$\lambda_{\mathrm{A,eff}} := \left[\frac{(1+z)^{1-\alpha}D_\mathrm{L}}{D_\alpha}\right]^{1/(\alpha-2)}\lambda_A \qquad\qquad D_\alpha = \frac{(1+z)^{1-\alpha}}{H_0}\int_0^z \frac{(1+\bar{z})^{\alpha-2}}{\sqrt{\Omega_\mathrm{m}(1+\bar{z})^3+\Omega_\Lambda}}d\bar{z}\,,$$

Mirshekari et al, PRD 85, 024041 (2012)