Quantum gravity and κ-deformation

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CA18108 Kick-off Meeting, Barcelona, October 3, 2019

Quantum gravity phenomenology problem

- Following the naïve logic of the last century development of particle physics, to measure an effect we should build an accelerator with the scattering energy of order of the relevant energy scale.
- But then to measure quantum gravity effects, we need to accelerate particles to Planck energies (10^{19} GeV) and squeeze them down to Planck distance (10^{-35} m). This clearly cannot be done in a foreseeable future (optimistically extrapolating current progress trends, it will take about 250 years). Thus QG is, and will remain, just a speculative theory.

Quantum gravity phenomenology problem

- But this reasoning is fallacious. Even within well established particle physics we do have successful measurements of effects whose characteristic energy scale is of order of 10^{15} GeV (proton lifetime). What we need is an amplifier.
- But we also need to know what we are looking for. We need a template, to compare observations with.
- There is no way we can observe Planckian scattering of two gravitons, but perhaps there are effects of quantum gravity origin that we CAN observe.

What are quantum gravity effects?

- Quantum gravity effects are those, in which both G and ħ are relevant.
- Planck units:

$$M_{Pl} = \left(\frac{\hbar}{G}\right)^{1/2}, \quad L_{Pl} = \left(\hbar G\right)^{1/2}$$

- Quantum gravity are relevant if the characteristic energy of the system is of order of Planck energy AND the characteristic distance is of order of Planck length. (Confirmed by the analysis of Planckian scattering)*

* S. Giddings, *The gravitational S-matrix: Erice lectures*, Subnucl. Ser. **48** (2013) 93 [arXiv:1105.2036 [hep-th]].

Limits

- GR limit: ħ → 0, G finite. The product of Planck length and Planck mass going to zero, but their ratio is finite. Meaning, for object (an astrophysical BH) of mass M and size L

$$\frac{M_{Pl}}{M}\frac{L_{Pl}}{L} \approx 10^{-38} \cdot 10^{-38}\text{, small, } \frac{M_{Pl}}{M} : \frac{L_{Pl}}{L} \approx 1\text{, finite}$$

- SR limit (no gravity) $L_{Pl} \to 0$, $M_{Pl} \to \infty$ (ħ → 0, G→ 0, with their ratio diverging), meaning (for an electron, for example)

$$\frac{L}{L_{Pl}} \text{ huge and } \frac{M}{M_{Pl}} \text{ small}$$

Limits

- Semiclassical limit (relative locality limit)*: $L_{Pl} \rightarrow 0$, M_{Pl} finite ($\hbar \rightarrow 0$, $G \rightarrow 0$, with their ratio fixed) , meaning

$$\frac{L_{Pl}}{L} \text{ small, } \frac{M_{Pl}}{M} \text{ finite}$$

- This might be the only limit with realistic QG phenomenology.
- The Question: How the dynamics of particles and fields is going to look like in this limit? Are there going to be any observable effects?

*G. Amelino-Camelia, L. Freidel, JKG and L. Smolin, ``The principle of relative locality,'' Phys. Rev. D **84** (2011) 084010 [arXiv:1101.0931 [hep-th]].

Semiclassical limit

- The semiclassical limit is characterized by one scale κ (= M_{Pl}) of dimension of mass. This is exactly the property of classical gravity in 2+1 dimensions.
- We can hope that 2+1 gravity will share some basic properties with the semiclassical limit.
- The action

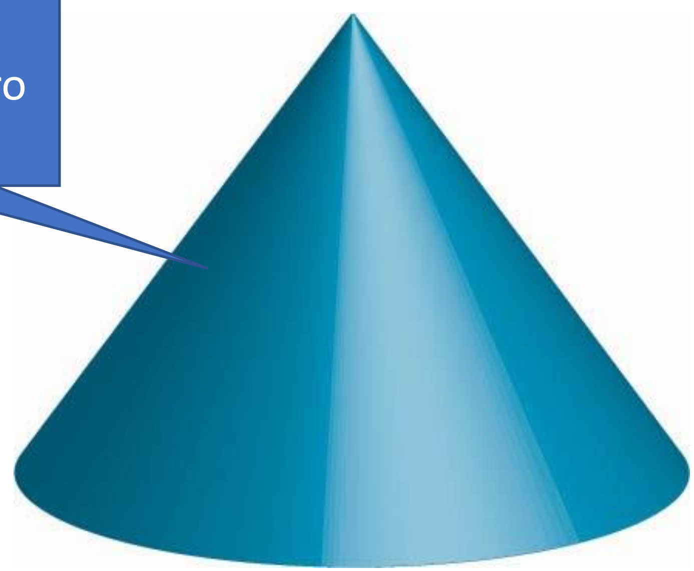
$$S_{grav} = \frac{\kappa}{4}\int d^3x \epsilon^{\mu\nu\rho} Tr\left(e_\mu F(\omega)_{\nu\rho}\right) + (\text{particle(s) coupling})$$

Field equations and their solutions

- Particle turns space into a cone, of opening $\alpha = 2m/\kappa$; the particle is placed at the tip of the cone.

Curvature has delta-like singularity

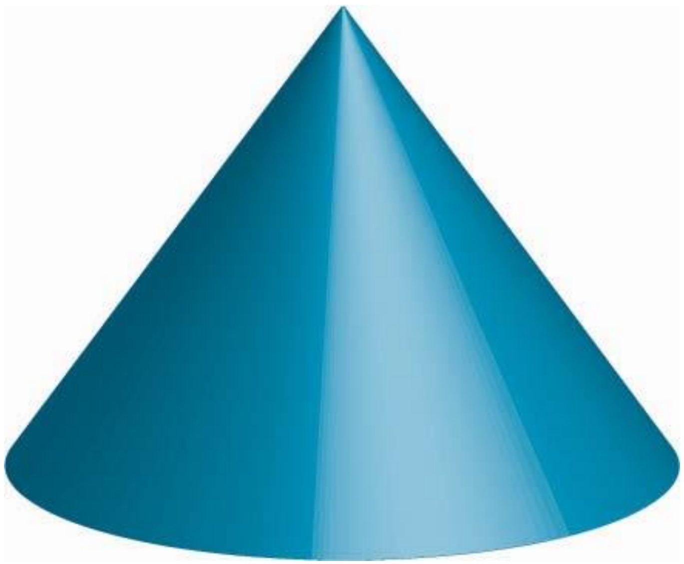
Curvature is zero



If there are no particles, space(time) is just flat

Field equations and their solutions

- A convenient alternative version*



$\Leftrightarrow$

$e_\phi=0$

Matching at $\varphi=\pi$ and $\varphi=-\pi$

Field equations: curvature and torsion vanish so that the (2+1)D Poincaré connection is a pure gauge (on a simply connected region).

*H. J. Matschull and M. Welling, *Quantum mechanics of a point particle in (2+1)-dimensional gravity*, Class. Quant. Grav. **15** (1998) 2981 [gr-qc/9708054].

Solution of field equation

- The frame field and Lorentz connection are gauge equivalent to zero, but as a result of nontrivial topology, they are essentially defined by a single, time dependent gauge group element

$$u(t) = \left(P(t)\mathbf{1} + \frac{p_\mu(t)}{\kappa} J^\mu \right), \quad P^2 - \frac{p_0^2}{4\kappa^2} + \frac{\vec{p}^{\,2}}{4\kappa^2} = 1$$

- This group element is interpreted as an element of momentum space that now becomes a curved manifold. p_μ are deformed or group-valued momenta. The momentum space is curved.

Effective action: why p is the momentum?

- We can solve for gravity (by symplectic reduction* or integrating out in path integral**) obtaining the effective, deformed particle action

$$S = -\int dt\, \dot{p}_\mu e_a^\mu(p) x^a - N\, C(p),$$

$$e_a^\mu(p) = \delta_a^\mu P - \epsilon^{\mu\nu}{}_a \frac{p_\nu}{2\kappa} - \frac{1}{P}\frac{p_\mu p_\nu}{4\kappa^2}\delta_\nu^a$$

$$C(p) = p^2 - 4\kappa^2 \sin^2 \frac{m}{2\kappa}$$

- In the limit $\kappa \to \infty$

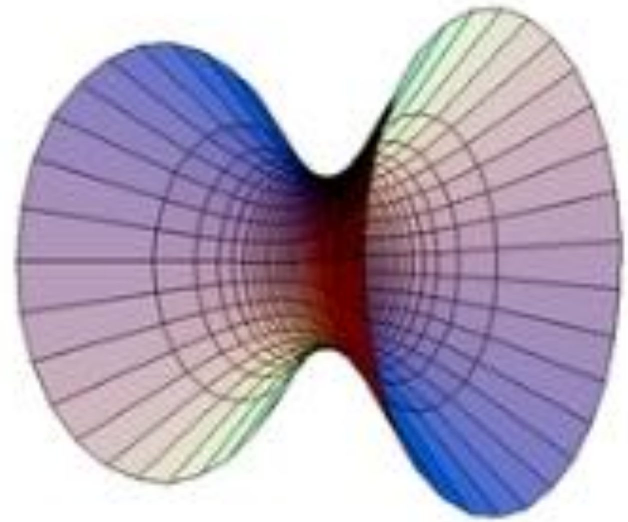
$$S = -\int dt\, \dot{p}_\mu\, x^a - N\,(p^2 - m^2),$$

- and this is the reason why we identify *p* with momentum.

*H-J. Matschull & M. Welling and C. Meusburger & B. Schroers **L. Freidel & E. Livine

Curved momentum space

- The momentum space of the effective, deformed particle is curved.
- The idea that geometry of momentum space can be non-trivial was first put forward by Max Born in the late 1930s*. There are many modern incarnations of this idea: Majid's dual gravity, relative locality, metastrings and metaparticle, Born geometry …



$$\pi^2 - \frac{p_0^2}{\kappa^2} + \frac{\vec{p}^{\,2}}{\kappa^2} = 1$$

*M. Born, *Quantised field theory and the mass of the proton* Nature **136** (1935) 952; *A Suggestion for Unifying Quantum Theory and Relativity* Proc. Roy. Soc. London A**165** (1938) 291.

Deformed momentum composition

- Since momentum is defined by SO(2,1) group element it is natural to define momentum composition with the help of group product:

$$u(p \oplus q) \equiv \mathrm{u(p)}u(q)$$

$$(p \oplus q)_\mu = q_\mu \sqrt{1 + \frac{p^2}{4\kappa^2}} + p_\mu \sqrt{1 + \frac{q^2}{4\kappa^2}} - \frac{1}{\kappa} \epsilon_\mu{}^{\nu\rho} p_\nu q_\rho$$

- One can also define an opposite momentum

$$(p \oplus (\ominus p)) = 0$$

- These follow from coproduct and antipode structures of Hopf algebra , which arises as a result of gravity induced deformation.

The big picture in (2+1)D

- After solving for gravity the effective theory of particle(s) and/or fields becomes deformed.
- Spacetime symmetries are deformed too and form Hopf algebra.
- Consequently, the conservation rules are altered.
- Spacetime becomes non-commutative; as a consequence of a non-trivial geometry of momentum space the Poisson bracket of positions becomes non-zero

$$\left\{x^a, x^b\right\} = \frac{1}{\kappa}\epsilon^{ab}{}_c\, x^c$$

(2+1)D vs (3+1)D

2+1 dimensions

- Gravity is a TFT with no local DOF;
- Spacetime is curved at the positions of particles only;

$$G_3 \sim 1/M_{Pl}$$

3+1 dimensions

- There are local DOF of gravity;
- Spacetime is curved everywhere;

$$G_4 \sim l_{Pl}/M_{Pl}$$

Therefore the 3d experience cannot be directly applied in 4d!

Possibilities

- If indeed there is a semiclassical quantum gravity, we can hope that the structure of the effective theory of particles and/or fields will be similar to that of (2+1)D gravity.
- Then we may have to do with deformation scale of dimension of mass, curved momentum space, deformed symmetries, altered composition laws, non-commutative spacetime, etc.
- A candidate: κ-Poincaré a deformed counterpart of Poincaré algebra.

Possibilities

- Another possibility is that the presence of a scale with dimension of mass indicates Lorentz symmetry breaking at some intermediate scale below Planck scale.
- Some argue that LIV is a prediction of string theory.

Kappa-Poincaré

- κ-Poincaré (Hopf) algebra is a single known non-trivial deformation of Poincaré algebra.
- There are three possible κ deformations, depending on spacetime direction the deformation acts:
  - Timelike deformation makes rotations undeformed, but deforming boosts;
  - Spacelike deformation deforms rotations and singles out one direction, therefore considered unphysical;
  - Lightlike deformation not well understood, but given by twist and therefore suspected to be trivial.

Kappa-Poincaré

- The momentum composition laws for κ-Poincaré are pretty complicated

$$(P \oplus Q)_i = P_i \frac{Q_0 + Q_4}{\kappa} + Q_i \quad \text{for large } \kappa \quad \approx P_i + Q_i + \frac{P_i Q_0}{\kappa}$$

$$(P \oplus Q)_0 = P_0 \frac{Q_0 + Q_4}{\kappa} + Q_0 \frac{\kappa}{P_0 + P_4} + \frac{P_i Q_i}{P_0 + P_4} \quad \text{for large } \kappa \quad \approx P_0 + Q_0 + \frac{P_i Q_i}{\kappa}$$

$$P_4 = \kappa \sqrt{1 + \frac{P_0^2}{\kappa^2} - \frac{P_i^2}{\kappa^2}}$$

- But dispersion relation is simple

$$P_0^2 - P_i^2 = m^2$$

Kappa-Poincaré

- It corresponds to the momentum space being a AN(3) group manifold (locally de Sitter space) and a non-commutative spacetime structure

$$\left\{x^0, x^i\right\} = \frac{1}{\kappa} x^i , \quad \left\{x^i, x^j\right\} = 0$$

- There is a long-standing claim that this algebra arises from quantum gravity and describes spacetime symmetries near Planck scale.

Phenomenological consequences

- Energy dependent time of flight from a distant source:

$$\Delta v \approx \frac{\Delta E}{\kappa} \text{ so that } \Delta t \approx \frac{\Delta E}{\kappa} D$$

- If the distance is huge it can act as an amplifier and effect might be detectable even for moderate energies.
- The experimental situation is unclear (meaning some people claim they may see the signal, but others do not believe them*).

* G. Amelino-Camelia, G. D'Amico, G. Rosati and N. Loret, In-vacuo-dispersion features for GRB neutrinos and photons, Nature Astron. **1** (2017) 0139 [arXiv:1612.02765 [astro-ph.HE]].

Phenomenological consequences

- Deformation of spacetime symmetries results in deformation of discrete symmetries, C, P, and T.
- For example, in the case of κ-Poincaré the decay rates for particles and antiparticles differ*

$$P_{part} = \frac{\Gamma E}{m} \mathrm{e}^{-\Gamma t \frac{E}{m}}, \quad P_{apart} = \Gamma\left(\frac{E}{m} - \frac{\mathbf{p}^2}{\kappa m}\right) e^{-\Gamma t\left(\frac{E}{m} - \frac{\mathbf{p}^2}{\kappa m}\right)}$$

- The effect is minute, but is amplified by extremely accurate measurements. For LHC muons we find $\kappa > 10^{14}$ GeV, but the effect might be, optimistically, observable in the next generation of accelerators (if somebody builds one)

* M. Arzano, JKG, W. Wiślicki, *A bound on Planck-scale deformation of CPT from muon lifetime* Phys.Lett. B794 (2019) 41-44, arXiv:1904.06754 [hep-ph]

Concluding remarks

- It would be extremely interesting to find an effective deformed theory directly from QG as a semiclassical limit.
- Alternatively one can analyze a planar subsystem in (3+1)D that can be described by (2+1) gravity. Then an argument can be made in favor of deformations playing a role in (3+1) dimensions*.
- It is worth investigating low energy/weak gravity approximations of QG because there might predict some observable effects, so that
- There might be rich phenomenology hiding there.

*L. Freidel, JKG & L. Smolin *2+1 gravity and doubly special relativity*, Phys. Rev. D**69** (2004) 044001 [hep-th/0307085].