From neutrinos to flavour and back

C. Hagedorn

CP^3-Origins, University of Southern Denmark

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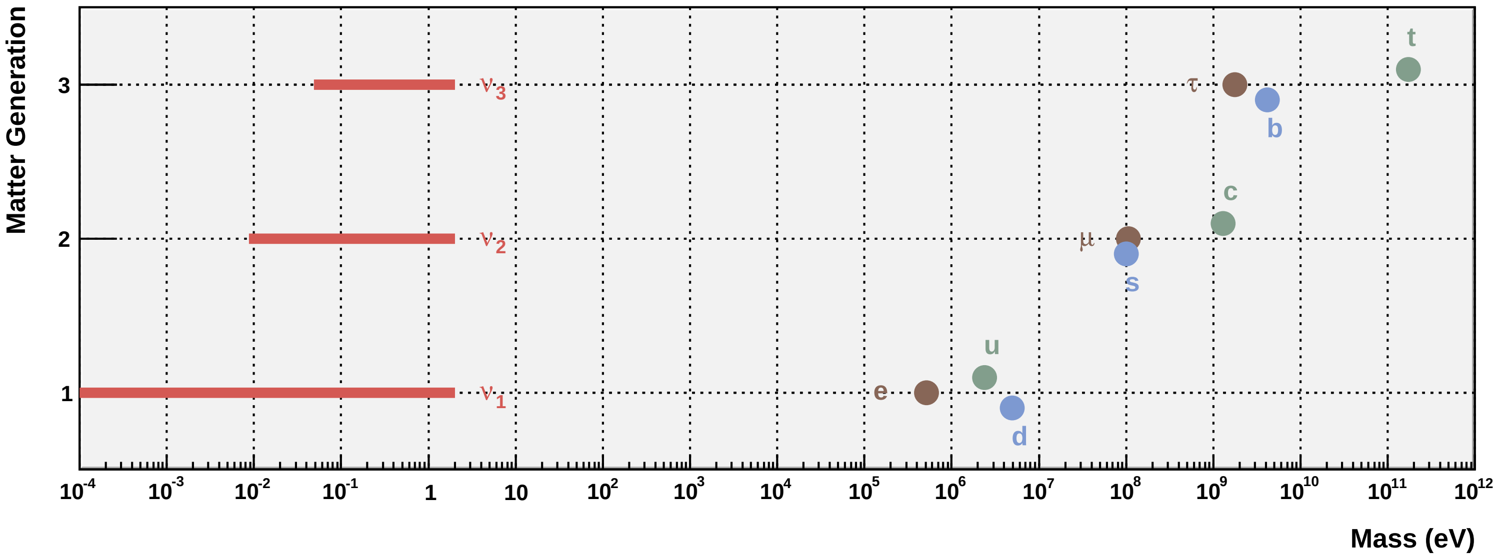
Outline

- introduction
  - current knowledge about neutrinos
  - experimental constraints on lepton flavour
- two examples
  - 1-loop model for neutrino masses
  - supersymmetric model with flavour and CP symmetry
- outlook

Introduction

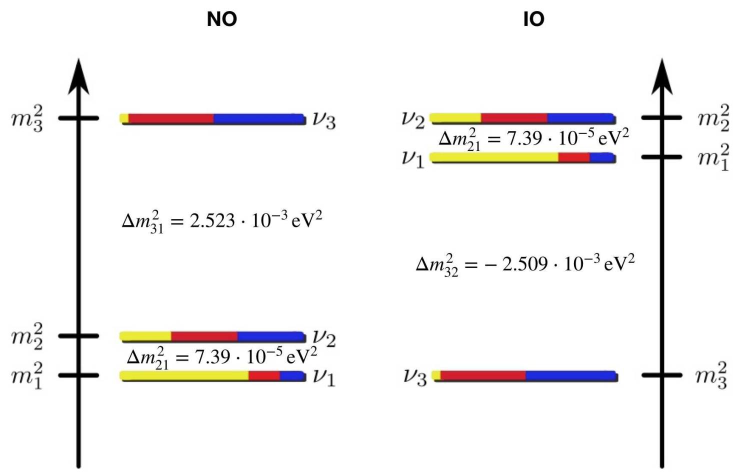


log scale

- three generations of neutrinos
- neutrino masses are very small
- mass hierarchy among neutrinos (possibly) much milder

Introduction

Summary of current knowledge about neutrino masses.

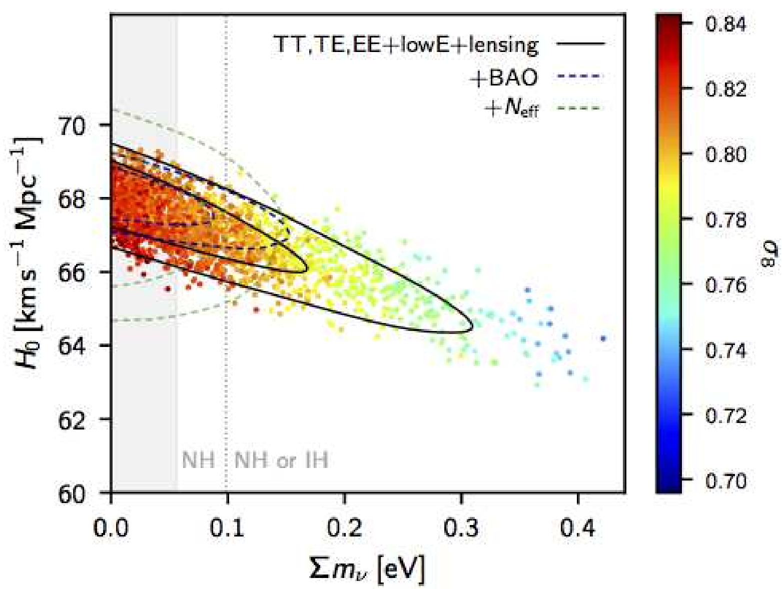


Their ordering is unknown, although NO seems preferred.

(NuFIT ('19))

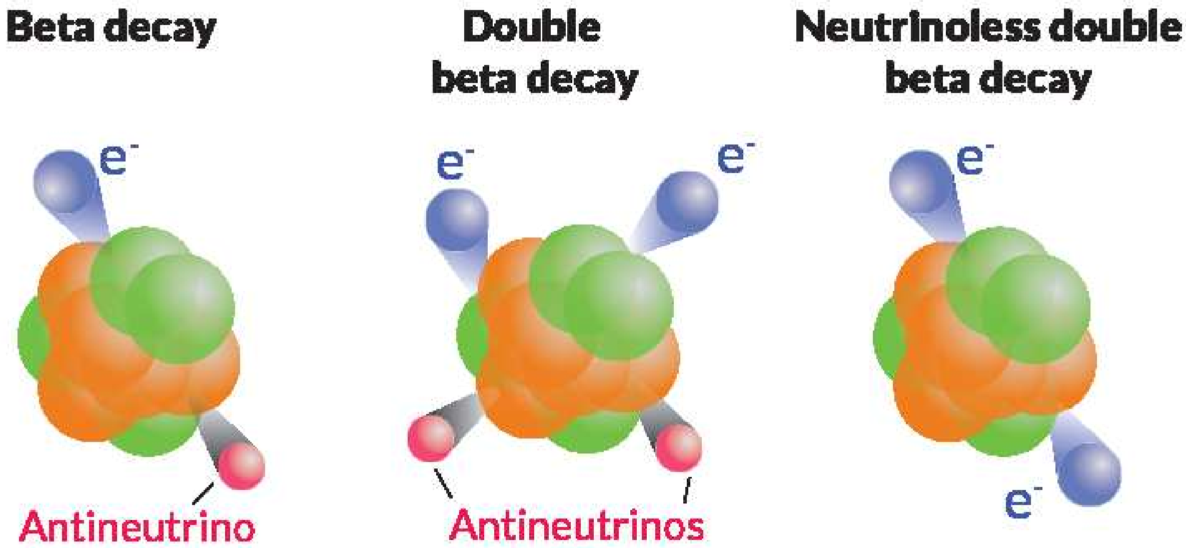
Their absolute scale is also unknown.

(Planck ('18))



Introduction

Neutrinos could be their own antiparticles.

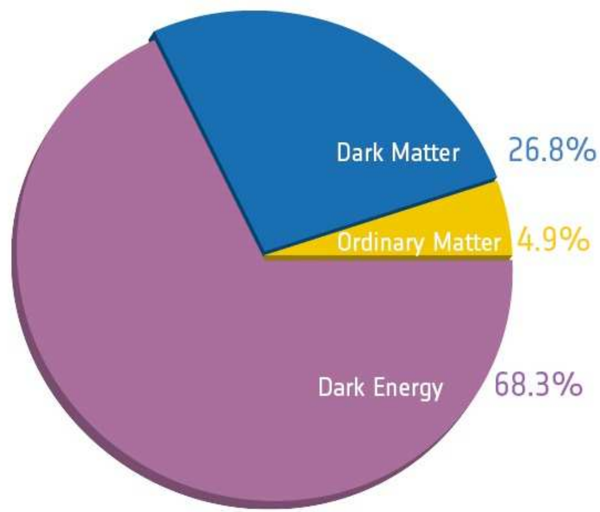


Introduction

Dark Matter is another important component of our Universe.

Observations show

- the known particles cannot account for Dark Matter
- there is five times more Dark Matter than ordinary matter
- Dark Matter shares several properties with neutrinos: no electric charge, no strong interaction



(Planck ('13))

Summary of current knowledge about lepton mixing.

NuFIT 4.1 (2019)

without SK atmospheric data		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.2$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
	$\sin^2\theta_{12}$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$
	$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$
	$\sin^2\theta_{23}$	$0.558^{+0.020}_{-0.033}$	$0.427 \to 0.609$	$0.563^{+0.019}_{-0.026}$	$0.430 \to 0.612$
	$\theta_{23}/^\circ$	$48.3^{+1.1}_{-1.9}$	$40.8 \to 51.3$	$48.6^{+1.1}_{-1.5}$	$41.0 \to 51.5$
	$\sin^2\theta_{13}$	$0.02241^{+0.00066}_{-0.00065}$	$0.02046 \to 0.02440$	$0.02261^{+0.00067}_{-0.00064}$	$0.02066 \to 0.02461$
	$\theta_{13}/^\circ$	$8.61^{+0.13}_{-0.13}$	$8.22 \to 8.99$	$8.65^{+0.13}_{-0.12}$	$8.26 \to 9.02$
	$\delta_{CP}/^\circ$	222^{+38}_{-28}	$141 \to 370$	285^{+24}_{-26}	$205 \to 354$

Introduction

- form of PMNS mixing matrix at best fit point

(NuFIT ('19))

$$||U_{\text{PMNS}}|| \approx \begin{pmatrix} 0.82 & 0.55 & 0.15 \\ 0.31 & 0.60 & 0.74 \\ 0.48 & 0.58 & 0.66 \end{pmatrix} \quad \text{[NO]}$$

and hint for CP violation: $\delta \approx 222^\circ$, $\alpha =$? , $\beta =$?

- lepton mixing is thus strikingly different from quark mixing

Introduction

Flavour violation among charged leptons ...

- ... is instead strongly constrained experimentally

Observable	Upper bound
BR $(\mu \to e\gamma)$	$2.55 \cdot 10^{-13}$
BR $(\tau \to \mu\gamma)$	$4.4 \cdot 10^{-8}$
BR $(\tau \to e\gamma)$	$3.3 \cdot 10^{-8}$
CR_{conv} (Au)	$7 \cdot 10^{-13}$
CR_{conv} (Ti)	$4.3 \cdot 10^{-12}$

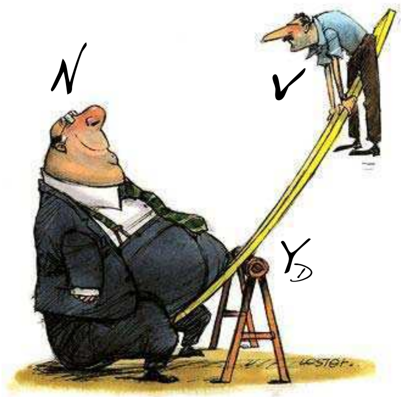
- ... is expected to arise in many beyond SM theories

Also dipole moments of charged leptons are interesting.

Introduction

One can understand the smallness of neutrino masses ...

- ... by invoking a large new physics scale.



Introduction

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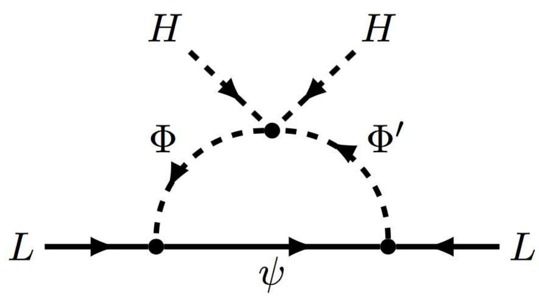
- ... by protecting them with an approximate symmetry.

Introduction

One can understand the smallness of neutrino masses ...

- ... by invoking a large new physics scale.
- ... by protecting them with an approximate symmetry.
- ... by generating them at loop level.



Introduction

About the flavour structure ...

- ... you can be agnostic.
- ... you can rely on some symmetry.

$U(2)_{fl}$

$SU(3)_{fl}$

$U(1)_{FN}$

$SO(3)_N$

A_4

Z_N

Γ_5

S_4

$\Delta(384)$

Example: 1-loop model

Let's consider a "bottom-up" model

(H/Herrero-Garcia/Molinaro/Schmidt ('18))

- starting point: SM extended by
  - global dark symmetry $U(1)_{\mathrm{DM}}$ (could be gauged)
  - two Higgs doublets Φ ($Y = 1/2$) and Φ' ($Y = -1/2$)
  - one Dirac fermion ψ that is a gauge singlet
  - Φ, Φ' and ψ are charged under $U(1)_{\mathrm{DM}}$

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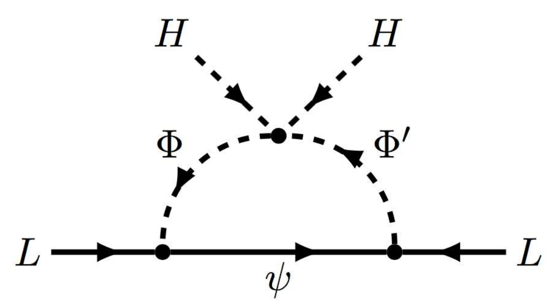
- purpose:
  - generation of two neutrino masses (third one is massless)
  - fermionic DM



Example: 1-loop model

Lagrangian

$$\mathcal{L}_\psi = i\,\overline{\psi}\,\partial\!\!\!/\,\psi - m_\psi\,\overline{\psi}\,\psi - \left(y_\Phi^\alpha\,\overline{\psi}\,\tilde{\Phi}^\dagger\,L_\alpha + (y_{\Phi'}^\alpha)^*\;\overline{\psi}\,\tilde{\Phi}'^\dagger\tilde{L}_\alpha + \text{H.c.}\right)$$

and

$$\begin{aligned}\mathcal{V} =& - m_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + m_\Phi^2 \Phi^\dagger \Phi + \lambda_\Phi (\Phi^\dagger\Phi)^2 + m_{\Phi'}^2 \Phi'^\dagger \Phi' + \lambda_{\Phi'} (\Phi'^\dagger \Phi')^2 \\ &+ \lambda_{H\Phi}(H^\dagger H)(\Phi^\dagger\Phi) + \lambda_{H\Phi'}(H^\dagger H)(\Phi'^\dagger\Phi') + \lambda_{\Phi\Phi'}(\Phi^\dagger\Phi)(\Phi'^\dagger\Phi') \\ &+ \lambda_{H\Phi,2}(H^\dagger\Phi)(\Phi^\dagger H) + \lambda_{H\Phi',2}(H^\dagger\tilde{\Phi}')(\tilde{\Phi}'^\dagger H) + \lambda_{\Phi\Phi',2}(\Phi^\dagger\tilde{\Phi}')(\tilde{\Phi}'^\dagger\Phi) \\ &+ \lambda_{H\Phi\Phi'}\left[(H^\dagger\tilde{\Phi}')(H^\dagger\Phi) + \text{H.c.}\right]\end{aligned}$$

Example: 1-loop model

Lagrangian

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and

mass eigenstates of scalars

- h with mass m_h
- two charged scalars $\eta^+ \equiv \phi^+$ and $\eta'^+ \equiv \phi'^+$
- two neutral (complex) scalars

$$\eta_0 = s_\theta\,\phi_0 \,+\, c_\theta\,\phi'_0\,,\;\; \eta'_0 = -\,c_\theta\,\phi_0 \,+\, s_\theta\,\phi'_0$$

with $m_{\eta_0} \geq m_{\eta'_0}$ and

$$\tan 2\theta \propto \lambda_{H\Phi\Phi'}$$

Example: 1-loop model

Neutrino mass matrix

$$(\mathcal{M}_\nu)_{\alpha\beta} = \frac{\sin 2\theta\, m_\psi}{32\,\pi^2} \left(y^\alpha_\Phi\, y^\beta_{\Phi'} + y^\alpha_{\Phi'}\, y^\beta_\Phi \right) F(m_{\eta_0}, m_{\eta'_0}, m_\psi)$$

with loop function

$$F(x, y, z) \;\equiv\; \frac{x^2}{x^2 - z^2} \,\ln \frac{x^2}{z^2} - \frac{y^2}{y^2 - z^2} \,\ln \frac{y^2}{z^2}$$

Example: 1-loop model

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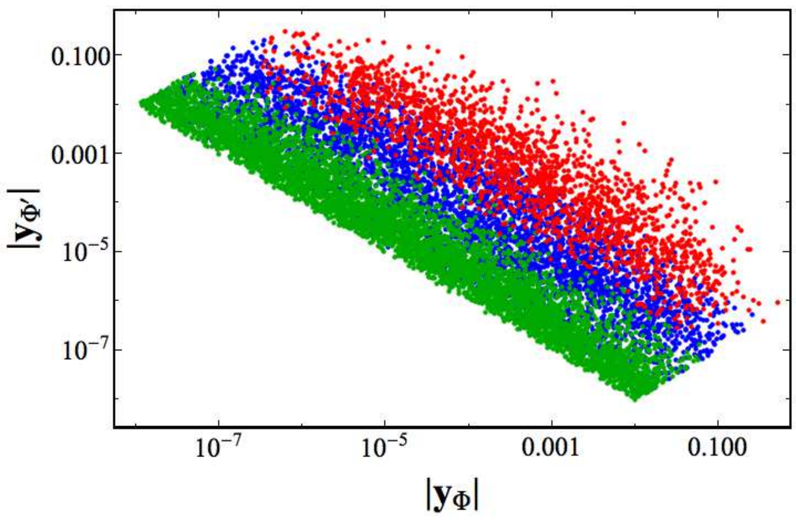

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- two non-zero neutrino masses only
- both neutrino mass hierarchies, NO and IO, possible
- Yukawa couplings y^α_Φ and $y^\alpha_{\Phi'}$, $\alpha = e, \mu, \tau$, are traded for lepton mixing angles θ_{ij},
  one Majorana phase γ and Dirac phase δ

Example: 1-loop model

Neutrino mass matrix

$$(\mathcal{M}_\nu)_{\alpha\beta} = \frac{\sin 2\theta\, m_\psi}{32\,\pi^2}\left(y^\alpha_\Phi\, y^\beta_{\Phi'} + y^\alpha_{\Phi'}\, y^\beta_\Phi\right)\, F(m_{\eta_0}, m_{\eta'_0}, m_\psi)$$

with loop function

$$F(x,y,z) \;\equiv\; \frac{x^2}{x^2-z^2}\,\ln\frac{x^2}{z^2} - \frac{y^2}{y^2-z^2}\,\ln\frac{y^2}{z^2}$$

- for NO

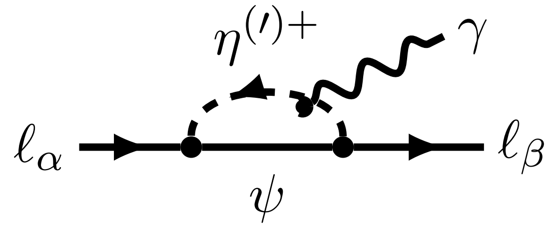
$$y^\mu_\Phi \approx y^\tau_\Phi \;,\;\; y^\mu_{\Phi'} \approx y^\tau_{\Phi'} \;\text{ and }\; y^e_\Phi, y^e_{\Phi'} \text{ smaller}$$

- for IO

$$y^\mu_\Phi \approx -y^\tau_\Phi \;,\; y^\mu_{\Phi'} \approx -y^\tau_{\Phi'} \;\text{ and }\; y^e_\Phi, y^e_{\Phi'} \text{ similar}$$

Example: 1-loop model

Charged lepton flavour violation: $\ell_\alpha \to \ell_\beta \gamma$



$$\text{BR}(\ell_\alpha \to \ell_\beta\,\gamma) \;=\; \frac{48\,\pi^3\,\alpha_{\text{em}}}{G_F^2}\left[\left|A_2^L\right|^2 + \left|A_2^R\right|^2\right] \times \text{BR}\left(\ell_\alpha \to \ell_\beta\,\nu_\alpha\,\overline{\nu_\beta}\right)$$

with

$$A_2^L = 0 \;\text{ and }\; A_2^R \;=\; -\frac{1}{32\,\pi^2}\left[\frac{y_\Phi^{\beta *}\,y_\Phi^\alpha}{m_{\eta^+}^2}\,f\left(\frac{m_\psi^2}{m_{\eta^+}^2}\right) + \frac{y_{\Phi'}^{\beta *}\,y_{\Phi'}^\alpha}{m_{\eta'^+}^2}\,f\left(\frac{m_\psi^2}{m_{\eta'^+}^2}\right)\right]$$

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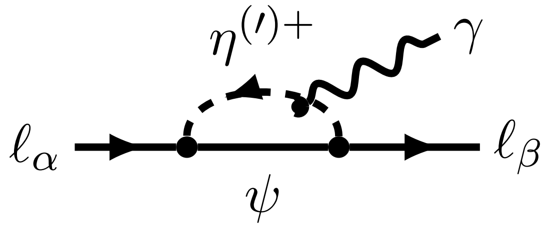


Use estimates for Yukawa couplings to estimate BRs

- for NO we find

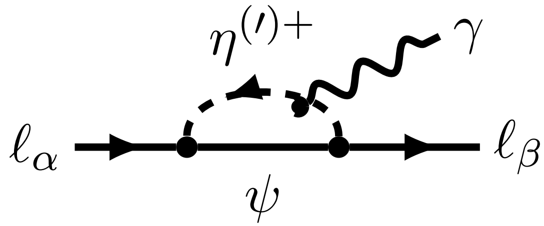
$$\frac{\text{BR}(\tau \to e\,\gamma)}{\text{BR}(\mu \to e\,\gamma)} \approx 0.2 \quad \text{and} \quad \frac{\text{BR}(\tau \to \mu\,\gamma)}{\text{BR}(\mu \to e\,\gamma)} \approx 5$$

- for IO we get

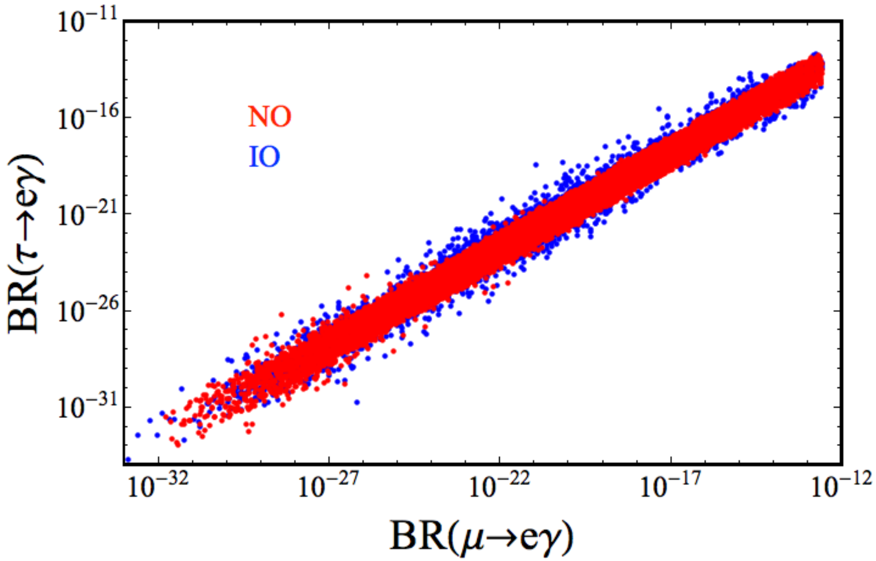
$$\frac{\text{BR}(\tau \to e\,\gamma)}{\text{BR}(\mu \to e\,\gamma)} \approx \frac{\text{BR}(\tau \to \mu\,\gamma)}{\text{BR}(\mu \to e\,\gamma)} \approx 0.2$$

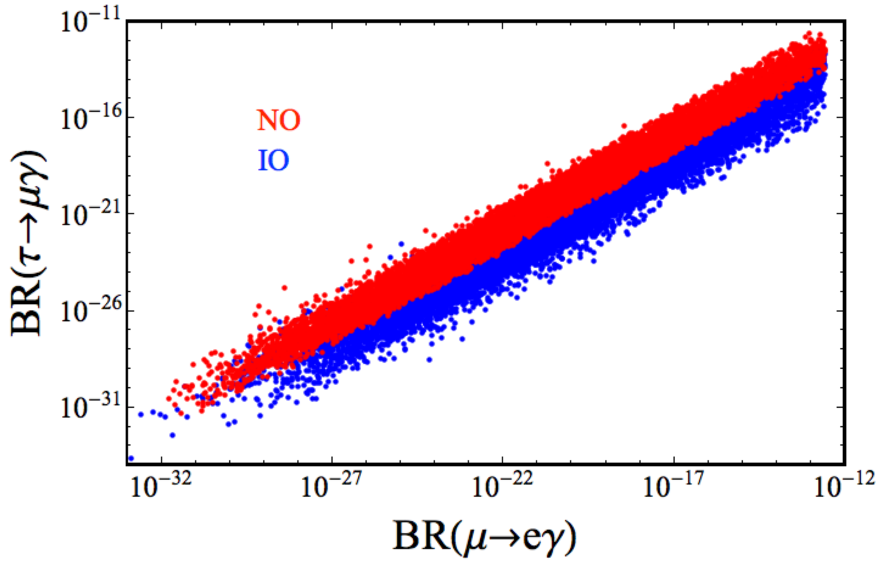
Example: 1-loop model

Charged lepton flavour violation: $\ell_\alpha \to \ell_\beta \gamma$



Numerical analysis

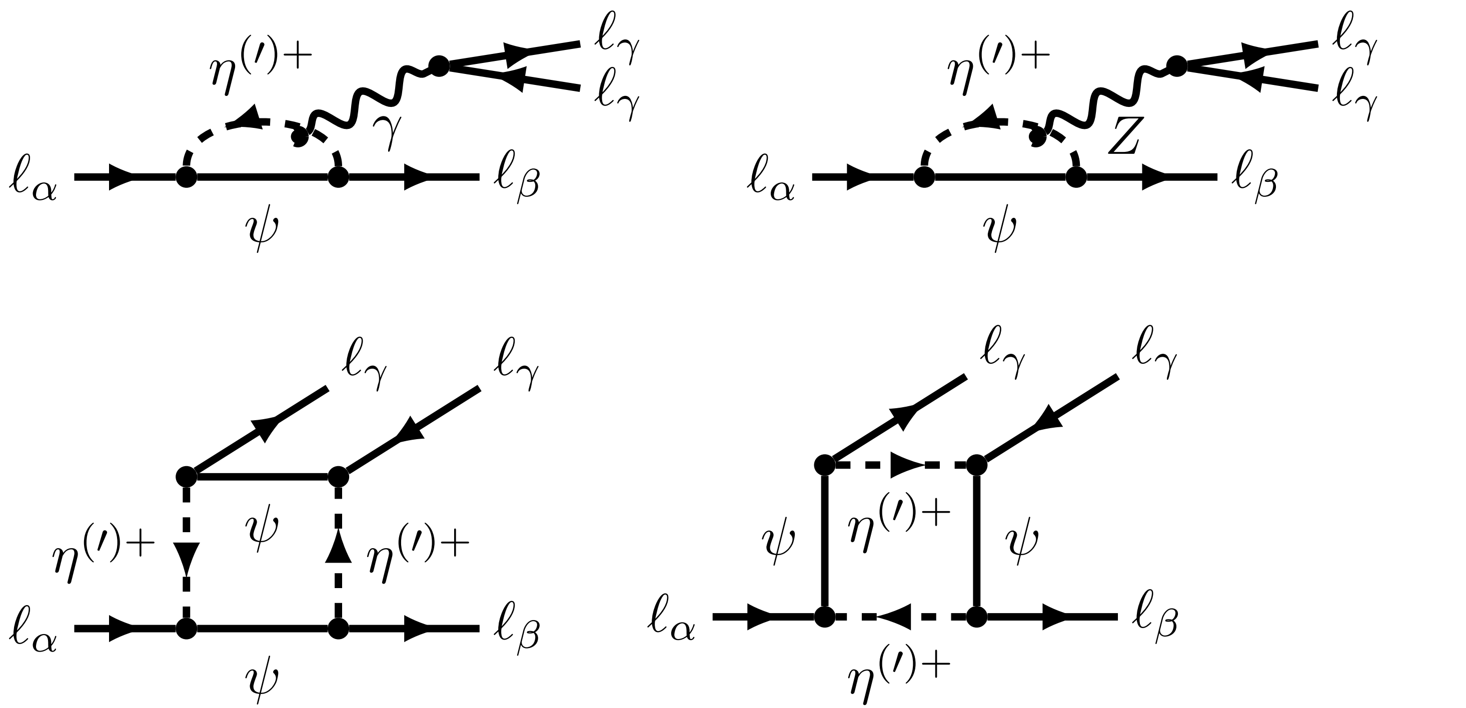




Example: 1-loop model

Charged lepton flavour violation: $\ell_\alpha \to \ell_\beta \ell_\gamma \ell_\gamma$

This decay can have different contributions



Example: 1-loop model

Charged lepton flavour violation: $\ell_\alpha \to \ell_\beta \, \overline{\ell}_\beta \, \ell_\beta$

We get

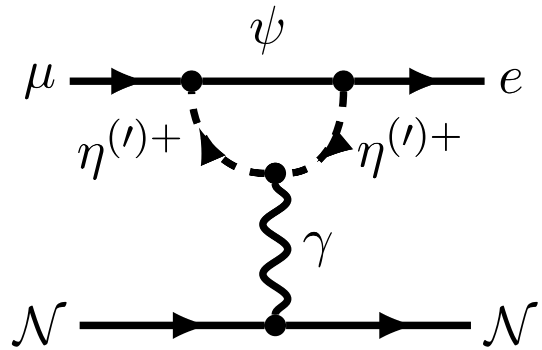
$$\mathrm{BR}(\ell_\alpha \to \ell_\beta \, \overline{\ell}_\beta \, \ell_\beta) \; = \frac{6\pi^2 \alpha_{\mathrm{em}}^2}{G_F^2} \Bigg[\left|A_1^L\right|^2 + \left|A_2^R\right|^2 \left(\frac{16}{3} \ln \frac{m_\alpha}{m_\beta} - \frac{22}{3} \right) + \frac{1}{6} \left|B\right|^2 - 4 \, \mathrm{Re} \left(A_1^{L*} A_2^R - \frac{1}{6} \left(A_1^L - 2 A_2^R \right) B^* \right) \Bigg] \times \mathrm{BR} \left(\ell_\alpha \to \ell_\beta \, \nu_\alpha \, \overline{\nu_\beta} \right)$$

However, we find in the numerical analysis that

$$\mathrm{BR}(\mu \to 3e) \; \approx \frac{\alpha_{\mathrm{em}}}{8\pi} \left(\frac{16}{3} \ln \frac{m_\mu}{m_e} - \frac{22}{3} \right) \times \mathrm{BR}(\mu \to e \, \gamma) \approx 0.006 \; \times \mathrm{BR}(\mu \to e \, \gamma)$$

Example: 1-loop model

Charged lepton flavour violation: $\mu - e$ conversion in nuclei



Other diagrams (with Z or SM Higgs) are suppressed.
In this case

$$\omega_{\mathrm{conv}} \; = \; 4 \left| \frac{e}{8} A_2^R D + \tilde{g}_{LV}^{(p)} V^{(p)} + \tilde{g}_{LV}^{(n)} V^{(n)} \right|^2$$

with

$$\tilde{g}_{LV}^{(p)} \approx e^2 A_1^L \text{ and } \tilde{g}_{LV}^{(n)} \approx 0$$

Example: 1-loop model

Charged lepton flavour violation: $\mu - e$ conversion in nuclei

In addition, A_1^L has a form similar to A_2^R

$$A_1^L \approx \frac{2}{3}\, r_{g/f}\, A_2^R \;\; \text{with} \;\; 1 \lesssim r_{g/f} \lesssim 1.5$$

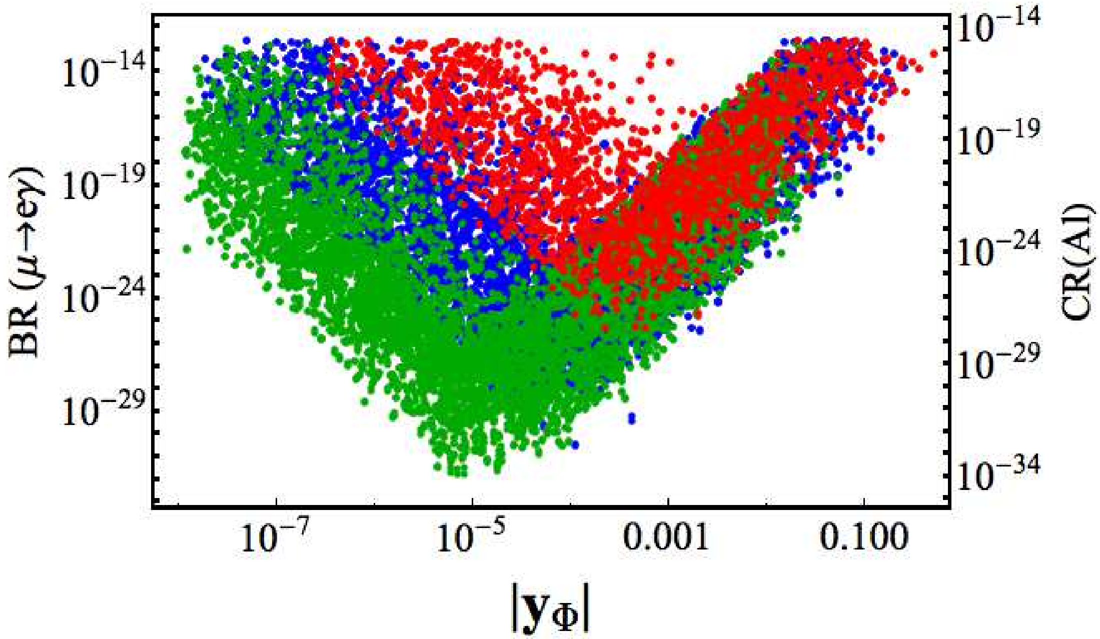
So, we get

$$\begin{aligned} \mathrm{CR}_{\mathrm{conv}} &\equiv \frac{\omega_{\mathrm{conv}}}{\omega_{\mathrm{capt}}} \\ &\approx \frac{G_F^2}{192\,\pi^2\,\omega_{\mathrm{capt}}} \left| D + \frac{16}{3}\, r_{g/f}\, e\, V^{(p)} \right|^2 \times \mathrm{BR}(\mu \to e\,\gamma) \\ &\approx [0.0077, 0.011]\,([0.010, 0.015])\,\{[0.013, 0.019]\} \times \mathrm{BR}(\mu \to e\,\gamma) \end{aligned}$$

for Al (Au) {Ti}

Example: 1-loop model

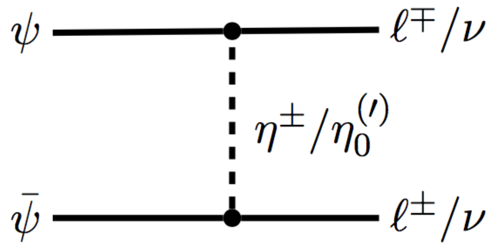
Charged lepton flavour violation: $\mu - e$ conversion in nuclei



Example: 1-loop model

DM relic abundance: Different channels

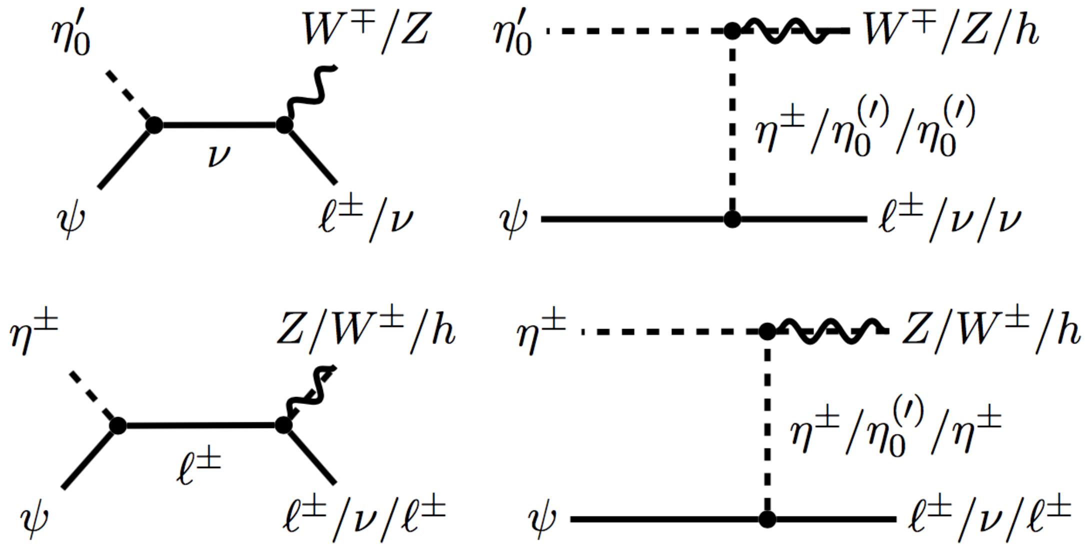
annihilation



Example: 1-loop model

DM relic abundance: Different channels

annihilation – constrained by charged lepton flavour violation
coannihilation



Example: 1-loop model

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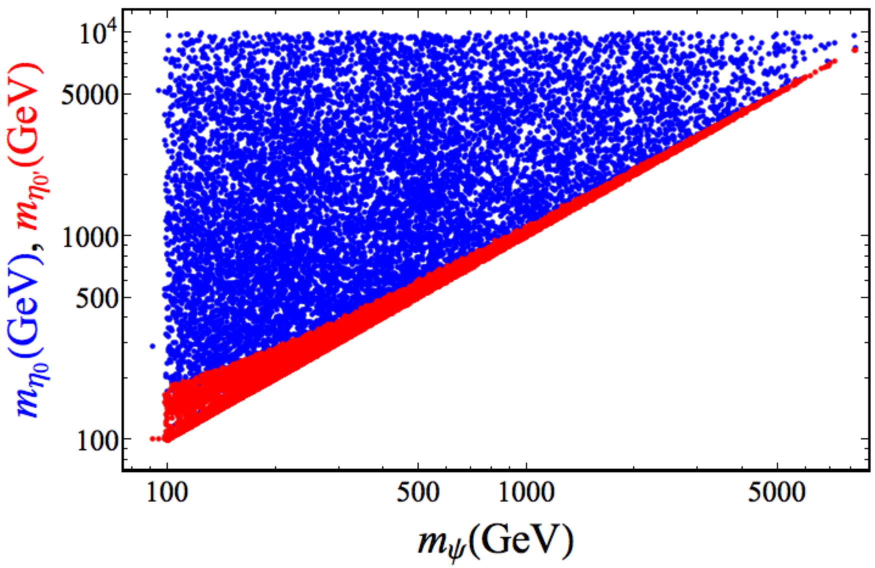
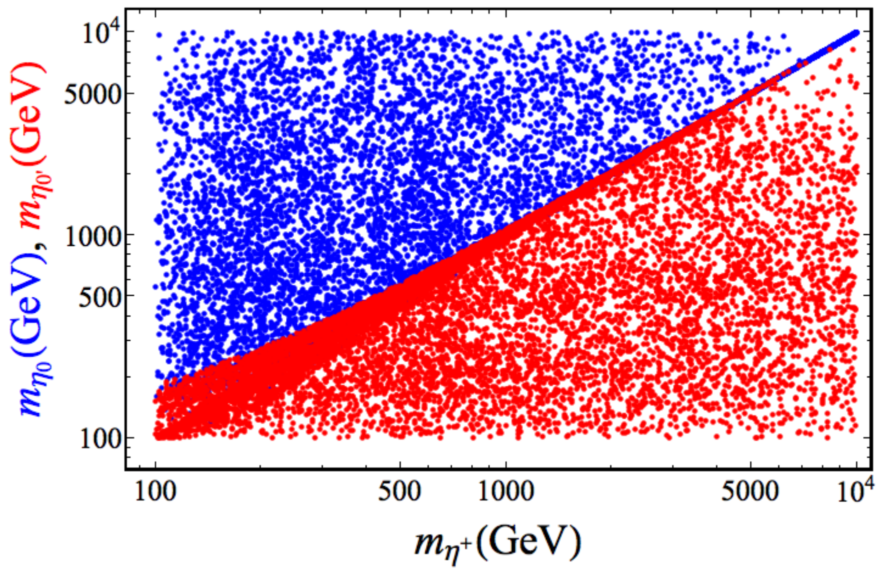


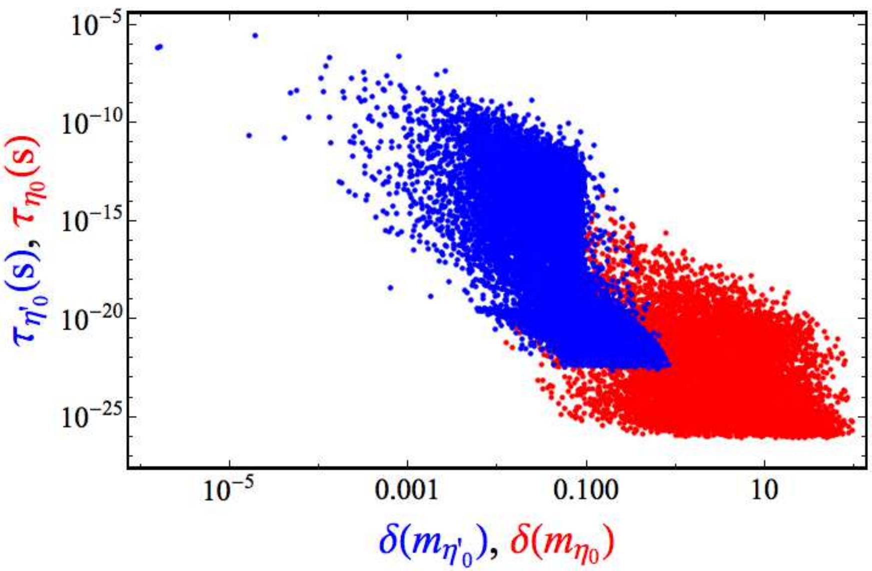


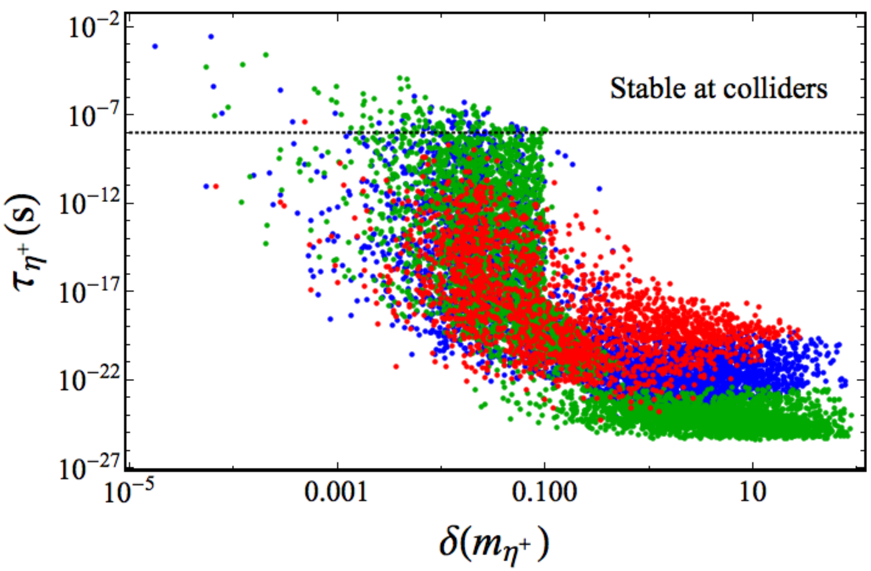
Example: 1-loop model

DM relic abundance: Different channels

annihilation – constrained by charged lepton flavour violation

coannihilation – DM fermion and scalar/s masses are close
such a compressed spectrum affects the scalars' lifetime





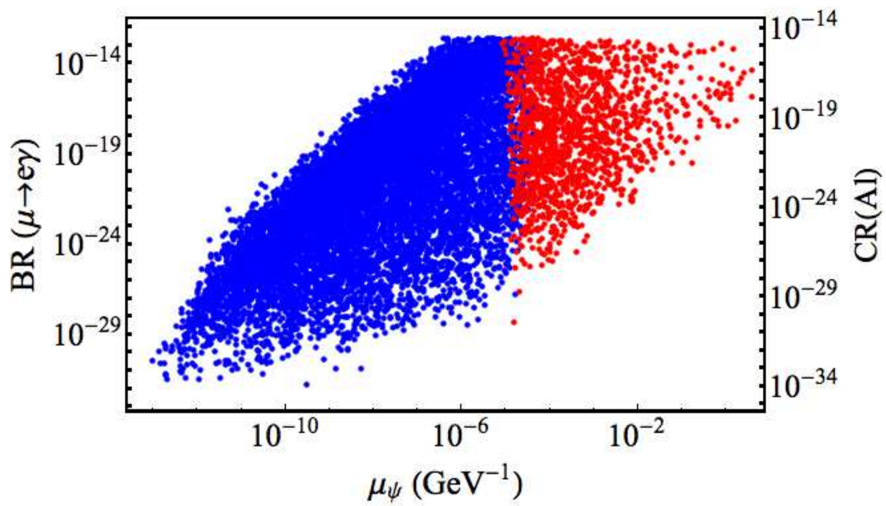
Example: 1-loop model

DM direct detection

- occurs at 1-loop level
- can be parametrised by magnetic (and electric) dipole interactions

$$\mathcal{L}_{\rm DD} = \mu_\psi \frac{e}{8\pi^2} \bar{\psi}\sigma_{\mu\nu}\psi F^{\mu\nu} + d_\psi \frac{e}{8\pi^2} \bar{\psi}\sigma_{\mu\nu} i\gamma_5 \psi F^{\mu\nu}$$

with $d_\psi = 0$ at 1-loop



Example: 1-loop model

Further phenomenology considered

- lepton dipole moments
- electroweak precision tests
- production and decay of new scalars at colliders
- decays of Higgs boson
- decays of Z boson
- other regions of parameter space for DM
- variants of the model

Example: model with $\Delta(384)$ and CP

Let's come to a "top-down" model of flavour *(H/König ('18))*

- starting point: supersymmetric extension of SM with three RH neutrinos
- impose flavour and CP symmetry on this theory which are both broken spontaneously
- choice of flavour (and CP) symmetry is driven by
  - good agreement of lepton mixing angles and the CP phase δ with global fit
  - capturing of main features of quark mixing: Cabibbo angle $\theta_C \approx 0.2$

  see preceding studies

  de Adelhart Toorop/Feruglio/H ('11), H/Meroni/Molinaro ('14)

Example: model with $\Delta(384)$ and CP

Flavour and CP symmetry

- we choose in the following $\Delta(384)$
- furthermore, we use a CP symmetry which acts non-trivially on flavour space

Example: model with $\Delta(384)$ and CP

Flavour and CP symmetry

- we choose in the following $\Delta(384)$
- features of this symmetry
  - it is a subgroup of $SU(3)$
  - it has several (complex) irreps of dimension 3
  - it is contained in the finite modular group Γ_{16}
  - it can be described with 4 generators a, b, c and d

$$a^3 = e \; , \;\; b^2 = e \; , \;\; c^8 = e \; , \;\; d^8 = e \; ,$$
$$(a\,b)^2 = e \; , \;\; c\,d = d\,c \; ,$$
$$a\,c\,a^{-1} = c^{-1}d^{-1} \; , \;\; a\,d\,a^{-1} = c \; ,$$
$$b\,c\,b^{-1} = d^{-1} \; , \;\; b\,d\,b^{-1} = c^{-1}$$

Example: model with $\Delta(384)$ and CP

Flavour and CP symmetry

- we choose in the following $\Delta(384)$
- furthermore, we use a CP symmetry which acts non-trivially on flavour space ***(Grimus/Rebelo ('95))***
  - imagine a set of scalar fields ϕ_i

$$\phi_i \to X_{ij}\, \phi_j^\star$$

    with

$$X X^\dagger = X X^\star = 1$$

  - the most known example in neutrino model building is the so-called

$\mu - \tau$ reflection symmetry

exchanging muon neutrino with a tau antineutrino

Example: model with $\Delta(384)$ and CP

Flavour and CP symmetry

- we choose in the following $\Delta(384)$
- furthermore, we use a CP symmetry which acts non-trivially on flavour space ***(Grimus/Rebelo ('95))***
- when considering such a theory, certain conditions have to be fulfilled (ρ irrep of $\Delta(384)$)

$$\left(X^{-1}\rho(g)X\right)^{\star} = \rho(g') \quad \text{with} \quad g, g' \in \Delta(384) \; , \quad \text{in general} \quad g \neq g'$$

(Feruglio/H/Ziegler ('12,'13), Holthausen et al. ('12), Chen et al. ('14))

- we use the CP symmetry that corresponds to the automorphism
  $a \;\rightarrow\; a \;\; , \;\; b \;\rightarrow\; b \;\; , \;\; c \;\rightarrow\; c^{-1}$ and $d \;\rightarrow\; d^{-1}$
  conjugated with group transformation $c^{4+s}\, d^{2\,s}$ with $s = 7$

Example: model with $\Delta(384)$ and CP

Assignment of fermion generations

- LH fields are in 3-dim. irreps
  (unification of generations, predictive power of approach)
- RH charged fermions are singlets
  (mass hierarchy)
- RH neutrinos are in 3-dim. irrep
  (relevant for lepton mixing)

Example: model with $\Delta(384)$ and CP

Breaking of flavour and CP symmetry

- spontaneously via gauge singlet fields/flavons
  (disentangle flavour and electroweak symmetry breaking, technically easier)
- to different residual symmetries
  (predictive power of model, interpretation of mixing as mismatch of residual groups)
- in different steps
  (motivation for e.g. smallness of θ_{13})

Example: model with $\Delta(384)$ and CP

Breaking of flavour and CP symmetry

$\Delta(384) \times Z_2^{(\mathrm{ext})} \times Z_3^{(\mathrm{ext})} \times Z_{16}^{(\mathrm{ext})}$ and CP

TB mixing

$\theta_C = \sin \pi/16$

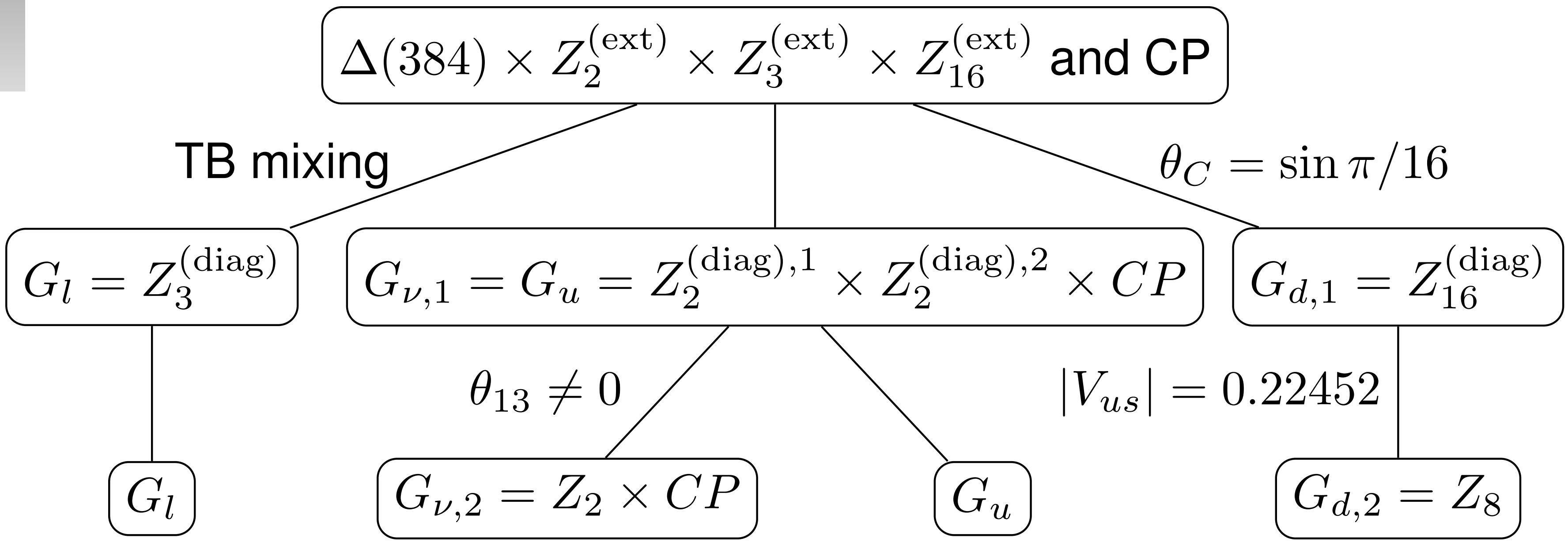
$G_l = Z_3^{(\mathrm{diag})}$

$G_{\nu,1} = G_u = Z_2^{(\mathrm{diag}),1} \times Z_2^{(\mathrm{diag}),2} \times CP$

$G_{d,1} = Z_{16}^{(\mathrm{diag})}$

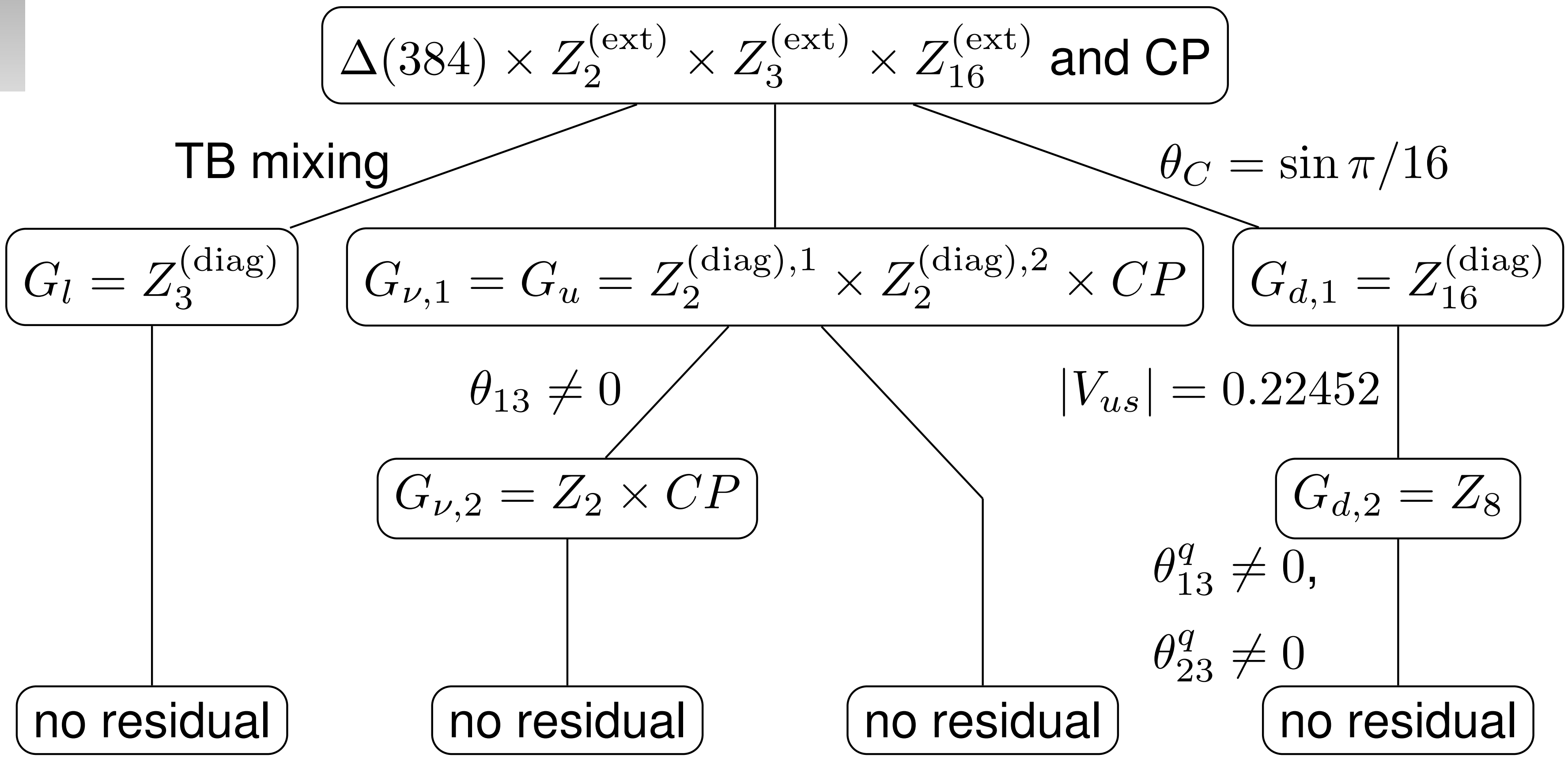
Example: model with $\Delta(384)$ and CP

Breaking of flavour and CP symmetry



Example: model with $\Delta(384)$ and CP

Breaking of flavour and CP symmetry



Example: model with $\Delta(384)$ and CP

To get some idea: look at charged leptons

- superpotential with Yukawa terms at leading order

$$
\begin{aligned}
w_l^{l.o.} &= \frac{1}{\Lambda} L\,\tau^c\,h_d\,\phi_l \\
&+ \frac{\omega^2}{\Lambda^2} L\,\mu^c\,h_d\,\chi_l\,\phi_l + \frac{1}{\Lambda^2} L\,\mu^c\,h_d\,\phi_l^2 \\
&+ \frac{i\,\omega^2}{\Lambda^3} L\,e^c\,h_d\,\chi_l^2\,\phi_l + \frac{i\,\omega^2}{\Lambda^3} L\,e^c\,h_d\,\chi_l\,\phi_l^2 + \frac{1}{\Lambda^3} L\,e^c\,h_d\,\phi_l^3
\end{aligned}
$$

where real couplings are suppressed $(\omega = e^{2\,\pi\,i/3})$

Example: model with $\Delta(384)$ and CP

To get some idea: look at charged leptons

- superpotential with Yukawa terms at leading order
- VEVs for flavons χ_l and ϕ_l

$$\langle \chi_l \rangle = x_{\chi_l} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \langle \phi_l \rangle = x_{\phi_l} \begin{pmatrix} \omega^2 \\ \omega \\ 1 \end{pmatrix}$$

with x_{χ_l} and x_{ϕ_l} complex
and orders of magnitude

$$\frac{|x_{\chi_l}|}{\Lambda}, \; \frac{|x_{\phi_l}|}{\Lambda} \approx \lambda^2 \quad \text{with} \quad \lambda \approx 0.2$$

Example: model with $\Delta(384)$ and CP

To get some idea: look at charged leptons

- superpotential with Yukawa terms at leading order
- VEVs for flavons χ_l and ϕ_l
- charged lepton mass matrix

$$m_l^{l.o.} = \begin{pmatrix} c_l\,\lambda^4 & \omega\, b_l\,\lambda^2 & \omega^2\, a_l \\ c_l\,\lambda^4 & \omega^2\, b_l\,\lambda^2 & \omega\, a_l \\ c_l\,\lambda^4 & b_l\,\lambda^2 & a_l \end{pmatrix} \lambda^2\,\langle h_d \rangle$$

with a_l, b_l and c_l complex

Example: model with $\Delta(384)$ *and CP*

To get some idea: look at charged leptons

- superpotential with Yukawa terms at leading order
- VEVs for flavons χ_l and ϕ_l
- contribution to lepton mixing

$$U_l^{l.o.} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \\ 1 & 1 & 1 \end{pmatrix}$$

- charged lepton masses

$$m_e^{l.o.} = \sqrt{3}\,|c_l|\,\lambda^6\,\langle h_d \rangle \;, \;\; m_\mu^{l.o.} = \sqrt{3}\,|b_l|\,\lambda^4\,\langle h_d \rangle \;, \;\; m_\tau^{l.o.} = \sqrt{3}\,|a_l|\,\lambda^2\,\langle h_d \rangle$$

Example: model with $\Delta(384)$ and CP

To get some idea: look at charged leptons

- superpotential with Yukawa terms at leading order
- VEVs for flavons χ_l and ϕ_l
- superpotential for VEV alignment

$$w_{fl,l}^{l.o.} = \alpha_l \, \sigma_l^0 \, \chi_l^2 + \beta_l \, \tilde{\sigma}_l^0 \, \phi_l^2 + \gamma_l \, \chi_l^0 \, \chi_l^2 + \omega \, \delta_l \, \chi_l^0 \, \phi_l^2$$

  with real couplings

- assume breaking of flavour and CP symmetry at high energy scale

$$\frac{\partial w_{fl,l}^{l.o.}}{\partial \sigma_l^0} = 0 \, , \quad \frac{\partial w_{fl,l}^{l.o.}}{\partial \tilde{\sigma}_l^0} = 0 \, , \quad \frac{\partial w_{fl,l}^{l.o.}}{\partial \chi_{l,i}^0} = 0$$

Example: model with $\Delta(384)$ and CP

To get some idea: look at charged leptons

- superpotential with Yukawa terms beyond leading order

$$
\begin{aligned}
& \frac{1}{\Lambda^4} L\,\tau^c\,h_d\,\phi_l\,\eta_u^3 + \frac{1}{\Lambda^5} L\,\tau^c\,h_d\,\chi_l\,\eta_u^4 + \frac{1}{\Lambda^5} L\,\tau^c\,h_d\,\phi_l\,\eta_u^4 + \frac{1}{\Lambda^4} L\,\tau^c\,h_d\,\eta_u^2\,\zeta^2 \\
+ \quad & \frac{1}{\Lambda^5} L\,\mu^c\,h_d\,\chi_l\,\phi_l\,\eta_u^3 + \frac{1}{\Lambda^5} L\,\mu^c\,h_d\,\phi_l^2\,\eta_u^3 + \frac{1}{\Lambda^4} L\,\mu^c\,h_d\,\phi_l\,\eta_u\,\zeta^2 \\
+ \quad & \frac{1}{\Lambda^5} L\,e^c\,h_d\,\phi_d\,\eta_u^3\,\xi_u + \frac{1}{\Lambda^6} L\,e^c\,h_d\,\phi_d\,\kappa_u^2\,\eta_u^3 + \frac{1}{\Lambda^5} L\,e^c\,h_d\,\chi_d\,\phi_u\,\kappa_u\,\eta_u\,\xi_u \\
+ \quad & \frac{1}{\Lambda^6} L\,e^c\,h_d\,\chi_d\,\phi_u\,\kappa_u^3\,\eta_u
\end{aligned}
$$

Example: model with $\Delta(384)$ and CP

To get some idea: look at charged leptons

- superpotential with Yukawa terms beyond leading order
- corrections to the VEV alignment

$$\langle \chi_l \rangle = \begin{pmatrix} \delta x_{\chi_l,1} \\ x_{\chi_l} + \delta x_{\chi_l,2} \end{pmatrix} \;, \quad \langle \phi_l \rangle = \begin{pmatrix} \omega^2 \left(x_{\phi_l} + \delta x_{\phi_l,1} \right) \\ \omega \left(x_{\phi_l} + \delta x_{\phi_l,2} \right) \\ x_{\phi_l} \end{pmatrix}$$

with

$$\frac{|\delta x_{\chi_l,i}|}{\Lambda} \,, \; \frac{|\delta x_{\phi_l,j}|}{\Lambda} \approx \lambda^5$$

Example: model with $\Delta(384)$ and CP

To get some idea: look at charged leptons

- superpotential with Yukawa terms beyond leading order
- corrections to the VEV alignment
- charged lepton mass matrix with corrections

$$\begin{pmatrix} c_l\,\lambda^4 & \omega\, b_l\,\lambda^2 & \omega^2\, a_l \\ c_l\,\lambda^4 & \omega^2\, b_l\,\lambda^2 & \omega\, a_l \\ c_l\,\lambda^4 & b_l\,\lambda^2 & a_l \end{pmatrix} \lambda^2\,\langle h_d\rangle + \begin{pmatrix} x_{l,11}\,\lambda^5 & x_{l,12}\,\lambda^5 & x_{l,13}\,\lambda^3 \\ -x_{l,11}\,\lambda^5 & x_{l,22}\,\lambda^5 & x_{l,23}\,\lambda^3 \\ x_{l,31}\,\lambda^5 & 0 & 0 \end{pmatrix} \lambda^2\,\langle h_d\rangle$$

Example: model with $\Delta(384)$ and CP

To get some idea: look at charged leptons

- superpotential with Yukawa terms beyond leading order
- corrections to the VEV alignment
- charged lepton mass matrix with corrections

$$\begin{pmatrix} c_l\,\lambda^4 & \omega\,b_l\,\lambda^2 & \omega^2\,a_l \\ c_l\,\lambda^4 & \omega^2\,b_l\,\lambda^2 & \omega\,a_l \\ c_l\,\lambda^4 & b_l\,\lambda^2 & a_l \end{pmatrix} \lambda^2\,\langle h_d\rangle + \begin{pmatrix} x_{l,11}\,\lambda^5 & x_{l,12}\,\lambda^5 & x_{l,13}\,\lambda^3 \\ -x_{l,11}\,\lambda^5 & x_{l,22}\,\lambda^5 & x_{l,23}\,\lambda^3 \\ x_{l,31}\,\lambda^5 & 0 & 0 \end{pmatrix} \lambda^2\,\langle h_d\rangle$$

- charged lepton masses get slightly corrected
- contribution to lepton mixing changes by

$$\theta^{h.o.}_{l,12} \approx \mathcal{O}(\lambda^3)\;,\quad \theta^{h.o.}_{l,13} \approx \mathcal{O}(\lambda^3)\;\text{ and }\;\theta^{h.o.}_{l,23} \approx \mathcal{O}(\lambda^3)$$

Example: model with $\Delta(384)$ and CP

Instead of going through all details of the model ...

- ... let's summarise the neutrino sector
  - at leading order

$$w_{\nu,D}^{l.o.} = \frac{1}{\Lambda} L \, \nu^c \, h_u \, \zeta \quad \text{and} \quad w_{\nu^c}^{l.o.,1} = \nu^c \, \nu^c \, \xi_u + \frac{1}{\Lambda} \nu^c \, \nu^c \, \kappa_u^2$$

giving rise to

$$m_\nu^{l.o.,1} = \omega_{16}^{10} \begin{pmatrix} a_\nu & c_\nu & 0 \\ c_\nu & a_\nu & 0 \\ 0 & 0 & \omega_{16}^{10} \, b_\nu \end{pmatrix} \lambda^2 \, \frac{\langle h_u \rangle^2}{\Lambda}$$

with real parameters and $\omega_{16} = e^{2\pi \, i/16}$

Example: model with $\Delta(384)$ and CP

Instead of going through all details of the model ...

- ... let's summarise the neutrino sector
  - at leading order

$$w_{\nu,D}^{l.o.} = \frac{1}{\Lambda} L\, \nu^c\, h_u\, \zeta \quad \text{and} \quad w_{\nu^c}^{l.o.,1} = \nu^c\, \nu^c\, \xi_u + \frac{1}{\Lambda} \nu^c\, \nu^c\, \kappa_u^2$$

giving rise to light neutrino masses

$$\begin{aligned} m_1^{l.o.,1} &= |a_\nu + c_\nu|\, \lambda^2\, \frac{\langle h_u \rangle^2}{\Lambda}\ , \\ m_2^{l.o.,1} &= |b_\nu|\, \lambda^2\, \frac{\langle h_u \rangle^2}{\Lambda}\ , \\ m_3^{l.o.,1} &= |a_\nu - c_\nu|\, \lambda^2\, \frac{\langle h_u \rangle^2}{\Lambda} \end{aligned}$$

Example: model with $\Delta(384)$ and CP

Instead of going through all details of the model ...

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$$w_{\nu,D}^{l.o.} = \frac{1}{\Lambda}\, L\, \nu^c\, h_u\, \zeta \quad \text{and} \quad w_{\nu^c}^{l.o.,1} = \nu^c\, \nu^c\, \xi_u + \frac{1}{\Lambda}\, \nu^c\, \nu^c\, \kappa_u^2$$

giving rise to light neutrino masses and a contribution to lepton mixing

$$U_\nu^{l.o.,1} = \frac{\omega_{16}^{10}}{\sqrt{2}} \begin{pmatrix} \omega_{16} & 0 & -\omega_{16} \\ \omega_{16} & 0 & \omega_{16} \\ 0 & \sqrt{2} & 0 \end{pmatrix}$$

Example: model with $\Delta(384)$ and CP

Instead of going through all details of the model ...

- ... let's summarise the neutrino sector
  - at leading order

$$w_{\nu,D}^{l.o.} = \frac{1}{\Lambda}\, L\, \nu^c\, h_u\, \zeta \quad \text{and} \quad w_{\nu^c}^{l.o.,1} = \nu^c\, \nu^c\, \xi_u + \frac{1}{\Lambda}\, \nu^c\, \nu^c\, \kappa_u^2$$

  - at next-to-leading order

$$w_{\nu^c}^{l.o.,2} = \frac{1}{\Lambda}\, \nu^c\, \nu^c\, \eta_u\, \xi_u + \frac{1}{\Lambda^2}\, \nu^c\, \nu^c\, \kappa_u^2\, \eta_u$$

Example: model with $\Delta(384)$ and CP

Instead of going through all details of the model ...

- ... let's summarise the neutrino sector
  - at leading order

$$w_{\nu,D}^{l.o.} = \frac{1}{\Lambda} L \, \nu^c \, h_u \, \zeta \quad \text{and} \quad w_{\nu^c}^{l.o.,1} = \nu^c \, \nu^c \, \xi_u + \frac{1}{\Lambda} \nu^c \, \nu^c \, \kappa_u^2$$

  - at next-to-leading order

$$w_{\nu^c}^{l.o.,2} = \frac{1}{\Lambda} \nu^c \, \nu^c \, \eta_u \, \xi_u + \frac{1}{\Lambda^2} \nu^c \, \nu^c \, \kappa_u^2 \, \eta_u$$

giving rise to

$$m_\nu^{l.o.} = \omega_{16}^{10} \begin{pmatrix} \tilde{a}_\nu & \tilde{c}_\nu & \omega_{16} \, d_\nu \, \lambda \\ \tilde{c}_\nu & \tilde{a}_\nu & -\omega_{16} \, d_\nu \, \lambda \\ \omega_{16} \, d_\nu \, \lambda & -\omega_{16} \, d_\nu \, \lambda & \omega_{16}^{10} \, \tilde{b}_\nu \end{pmatrix} \lambda^2 \, \frac{\langle h_u \rangle^2}{\Lambda}$$

Example: model with $\Delta(384)$ and CP

Instead of going through all details of the model ...

- ... let's summarise the neutrino sector
- we find for the lepton mixing angles

$$\sin^2\theta_{13} = \frac{1}{3}\,\sin^2\theta_\nu \quad \text{and} \quad \sin^2\theta_{12} = \frac{\cos^2\theta_\nu}{2+\cos^2\theta_\nu} = \frac{1}{3}\left(\frac{1-3\,\sin^2\theta_{13}}{1-\sin^2\theta_{13}}\right)$$

$$\sin^2\theta_{23} = \frac{1}{2}\left(1+\left(\frac{2\sqrt{6}\,\sin 2\,\theta_\nu}{5+\cos 2\,\theta_\nu}\right)\sin\left(\frac{\pi}{8}\right)\right)$$

with

$$\tan 2\,\theta_\nu = -\frac{2\sqrt{2}\,d_\nu}{\tilde{a}_\nu+\tilde{b}_\nu-\tilde{c}_\nu}\,\lambda$$

Example: model with $\Delta(384)$ and CP

Instead of going through all details of the model ...

- ... let's summarise the neutrino sector
- we find for the lepton mixing angles

$$\sin^2\theta_{13} = \frac{1}{3}\,\sin^2\theta_\nu \quad \text{and} \quad \sin^2\theta_{12} = \frac{\cos^2\theta_\nu}{2+\cos^2\theta_\nu} = \frac{1}{3}\left(\frac{1-3\,\sin^2\theta_{13}}{1-\sin^2\theta_{13}}\right)$$

$$\sin^2\theta_{23} = \frac{1}{2}\left(1+\left(\frac{2\sqrt{6}\,\sin 2\,\theta_\nu}{5+\cos 2\,\theta_\nu}\right)\sin\left(\frac{\pi}{8}\right)\right)$$

for $\theta_\nu \approx 0.26$ we get

$$\sin^2\theta_{13} \approx 0.022\;, \quad \sin^2\theta_{12} \approx 0.318 \quad \text{and} \quad \sin^2\theta_{23} \approx 0.579$$

Example: model with $\Delta(384)$ and CP

Instead of going through all details of the model ...

- ... let's summarise the neutrino sector
- for $\theta_\nu \approx 0.26$ we get

$$\sin^2\theta_{13} \approx 0.022 \;, \quad \sin^2\theta_{12} \approx 0.318 \;\text{ and }\; \sin^2\theta_{23} \approx 0.579$$

- we get predictions for all leptonic CP phases

$$\sin\delta \approx -0.936 \;, \quad \sin\alpha = \sin\beta = -\frac{1}{\sqrt{2}}$$

Example: model with $\Delta(384)$ and CP

Instead of going through all details of the model ...

- ... a summary of the up quark sector
  - at leading order

$$w_u^{l.o.} = \frac{1}{\Lambda} Q\, t^c\, h_u\, \phi_u + \frac{1}{\Lambda^3} Q\, c^c\, h_u\, \phi_u\, \kappa_u\, \xi_u$$

giving rise to

$$m_u^{l.o.} = \omega_{16}^5 \begin{pmatrix} 0 & b_u\, \lambda^3 & 0 \\ 0 & b_u\, \lambda^3 & 0 \\ 0 & 0 & \omega_{16}\, a_u \end{pmatrix} \lambda \langle h_u \rangle$$

Example: model with $\Delta(384)$ *and CP*

Instead of going through all details of the model ...

- ... a summary of the up quark sector
  - at leading order

$$w_u^{l.o.} = \frac{1}{\Lambda} Q\, t^c\, h_u\, \phi_u + \frac{1}{\Lambda^3} Q\, c^c\, h_u\, \phi_u\, \kappa_u\, \xi_u$$

  - we find for the up quark masses

$$m_u^{l.o.} = 0\ , \quad m_c^{l.o.} = \sqrt{2}\ |b_u|\ \lambda^4\ \langle h_u \rangle \ \text{ and } \ m_t^{l.o.} = |a_u|\ \lambda\ \langle h_u \rangle$$

and as contribution to quark mixing

$$U_u^{l.o.} = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}$$

Example: model with $\Delta(384)$ and CP

Instead of going through all details of the model ...

- ... a summary of the up quark sector
  - at leading order

$$w_u^{l.o.} = \frac{1}{\Lambda} Q\, t^c\, h_u\, \phi_u + \frac{1}{\Lambda^3} Q\, c^c\, h_u\, \phi_u\, \kappa_u\, \xi_u$$

  - at higher order there are many more terms, but phenomenologically relevant is

$$\frac{1}{\Lambda^4} Q\, u^c\, h_u\, \chi_d\, \phi_l\, \xi_u\, \psi$$

which generates the up quark mass of order

$$m_u^{h.o.} \propto \lambda^8 \langle h_u \rangle$$

Example: model with $\Delta(384)$ and CP

Instead of going through all details of the model ...

- ... a summary of the down quark sector
  - at leading order

$$w_d^{l.o.,1} = \frac{1}{\Lambda}\, Q\, b^c\, h_d\, \phi_d + \frac{1}{\Lambda^2}\, Q\, s^c\, h_d\, \phi_d\, \chi_d$$

giving rise to

$$m_d^{l.o.,1} = \begin{pmatrix} 0 & b_d\,\lambda^2 & 0 \\ 0 & \omega_{16}\, b_d\, \lambda^2 & 0 \\ 0 & 0 & a_d \end{pmatrix} \lambda^2\, \langle h_d \rangle$$

Example: model with $\Delta(384)$ and CP

Instead of going through all details of the model ...

- ... a summary of the down quark sector
  - at leading order

$$w_d^{l.o.,1} = \frac{1}{\Lambda} Q\, b^c\, h_d\, \phi_d + \frac{1}{\Lambda^2} Q\, s^c\, h_d\, \phi_d\, \chi_d$$

  - we find for the down quark masses

$$m_d^{l.o.,1} = 0\,, \quad m_s^{l.o.,1} = \sqrt{2}\, |b_d|\, \lambda^4\, \langle h_d \rangle \text{ and } m_b^{l.o.,1} = |a_d|\, \lambda^2\, \langle h_d \rangle$$

and as contribution to quark mixing

$$U_d^{l.o.,1} = \frac{1}{\sqrt{2}} \begin{pmatrix} -\omega_{16} & \omega_{16} & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}$$

Example: model with $\Delta(384)$ and CP

Instead of going through all details of the model ...

- ... a summary of the down quark sector
  - at leading order

$$w_d^{l.o.,1} = \frac{1}{\Lambda} Q\, b^c\, h_d\, \phi_d + \frac{1}{\Lambda^2} Q\, s^c\, h_d\, \phi_d\, \chi_d$$

- quark mixing at leading order is

$$\left| V_{\text{CKM}}^{l.o.,1} \right| = \left| \left(U_u^{l.o.}\right)^\dagger U_d^{l.o.,1} \right| = \begin{pmatrix} \cos\pi/16 & \sin\pi/16 & 0 \\ \sin\pi/16 & \cos\pi/16 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Example: model with $\Delta(384)$ *and CP*

Instead of going through all details of the model ...

- ... a summary of the down quark sector
  - at leading order

$$w_d^{l.o.,1} = \frac{1}{\Lambda} Q\, b^c\, h_d\, \phi_d + \frac{1}{\Lambda^2} Q\, s^c\, h_d\, \phi_d\, \chi_d$$

  - at next-to-leading order

$$w_d^{l.o.,2} = \frac{1}{\Lambda^3} Q\, s^c\, h_d\, \chi_d\, \psi^2$$

giving rise to

$$m_d^{l.o.,2} = \omega_{16}^6 \begin{pmatrix} 0 & \omega_{16}\, c_d & 0 \\ 0 & c_d & 0 \\ 0 & 0 & 0 \end{pmatrix} \lambda^6 \langle h_d \rangle$$

Example: model with $\Delta(384)$ *and CP*

Instead of going through all details of the model ...

- ... a summary of the down quark sector
  - at leading order

$$w_d^{l.o.,1} = \frac{1}{\Lambda} Q\, b^c\, h_d\, \phi_d + \frac{1}{\Lambda^2} Q\, s^c\, h_d\, \phi_d\, \chi_d$$

  - at next-to-leading order

$$w_d^{l.o.,2} = \frac{1}{\Lambda^3} Q\, s^c\, h_d\, \chi_d\, \psi^2$$

- this mainly corrects the result for the Cabibbo angle by

$$\tan 2\,\theta_d \approx 2\, \sin\left(\frac{\pi}{8}\right) \left|\frac{c_d}{b_d}\right| \lambda^2$$

Example: model with $\Delta(384)$ and CP

Instead of going through all details of the model ...

- ... a summary of the down quark sector
  - at leading order

$$w_d^{l.o.,1} = \frac{1}{\Lambda} Q\, b^c\, h_d\, \phi_d + \frac{1}{\Lambda^2} Q\, s^c\, h_d\, \phi_d\, \chi_d$$

  - at next-to-leading order

$$w_d^{l.o.,2} = \frac{1}{\Lambda^3} Q\, s^c\, h_d\, \chi_d\, \psi^2$$

  - at even higher order there are many terms, but only a few are of phenomenological relevance

Example: model with $\Delta(384)$ and CP

- at even higher order there are many terms, but only a few are of phenomenological relevance
  - ... the term

$$\frac{1}{\Lambda^3} Q\, d^c\, h_d\, \phi_l\, \zeta\, \psi$$

    leads to non-zero down quark mass of order $\lambda^6 \langle h_d \rangle$
  - ... the terms

$$\frac{1}{\Lambda^2} Q\, b^c\, h_d\, \psi^2 + \frac{1}{\Lambda^3} Q\, b^c\, h_d\, \eta_u\, \psi^2$$

    generate the quark mixing angles θ^q_{23} and θ^q_{13}, respectively

$$\theta^q_{23} \approx \sqrt{2} \left| \frac{d_d}{a_d} \right| \lambda^2 \text{ and } \theta^q_{13} \approx \sqrt{2} \left| \frac{e_d}{a_d} \right| \lambda^3$$

Example: model with $\Delta(384)$ and CP

- at even higher order there are many terms, but only a few are of phenomenological relevance
  - ... the term

$$\frac{1}{\Lambda^3}\, Q\, d^c\, h_d\, \phi_l\, \zeta\, \psi$$

  - ... the terms

$$\frac{1}{\Lambda^2}\, Q\, b^c\, h_d\, \psi^2 + \frac{1}{\Lambda^3}\, Q\, b^c\, h_d\, \eta_u\, \psi^2$$

- we find for the Jarlskog invariant

$$(J^q_{\mathrm{CP}})^{l.o.} \approx \sin\left(\frac{\pi}{8}\right)\, \frac{d_d\, e_d}{|a_d|^2}\, \lambda^5$$

Example: model with $\Delta(384)$ and CP

- at even higher order there are many terms, but only a few are of phenomenological relevance
  - ... the term
$$\frac{1}{\Lambda^3} \, Q \, d^c \, h_d \, \phi_l \, \zeta \, \psi$$
  - ... the terms
$$\frac{1}{\Lambda^2} \, Q \, b^c \, h_d \, \psi^2 + \frac{1}{\Lambda^3} \, Q \, b^c \, h_d \, \eta_u \, \psi^2$$
  - ... the term
$$\frac{1}{\Lambda^4} \, Q \, b^c \, h_d \, \chi_d \, \phi_u^2 \, \psi$$

  corrects the Jarlskog invariant at order λ^6

Outlook

Future directions for loop models

- endow them with flavour and CP symmetries
- embed them into (partially) unified theories
- consider ways to generate baryon asymmetry of the Universe
- chart them according to phenomenology

Outlook

Future directions for flavour models

- embed them into (partially) unified theories
- realise them in alternatives to MSSM-like models
- use modular invariance
- chart them according to phenomenology
- extend flavour systematically to dark sector
- study impact of flavour on different types of leptogenesis

Outlook

Surprises might be around the corner ...

- ... anomaly in $g-2$ of muon persists
- ... one of the B meson anomalies is confirmed
- ... charged lepton flavour violation is observed
- ... positive signal of neutrinoless double beta decay
- ... sterile neutrino is found
- ... searches for long-lived particles are successful
- ... lepton number and/or flavour violation at next run of LHC

Thank you for your attention.