A dispersive analysis of the decays $\tau \to \nu_\tau(2\pi/2K/3\pi)$

Sergi Gonzàlez-Solís

INDIANA UNIVERSITY

Department of Physics, Indiana University
Bloomington, IN 47405, USA

Based on:

S. Gonzàlez-Solís and P. Roig; Eur. Phys. J. C79 (2019) 436,

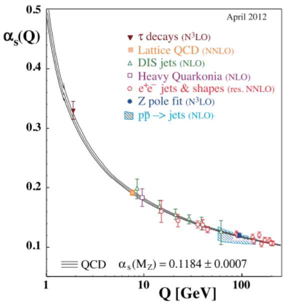
A. Strickland, S. Gonzàlez-Solís and E. Passemar; to appear soon.

Institut de Física d'Altes Energies, September 26, 2019

Outline

Introduction

- **asymptotic freedom**:
  "like QED", but only at high energies
- **confinement**:
  at low energies the gluons bind the quarks together to form the hadrons



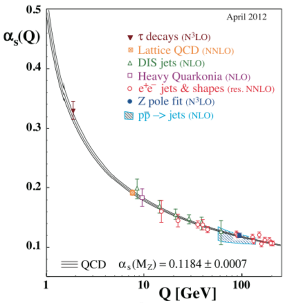
- Approaches to describe the Low-energy regime of QCD:
  1. Lattice QCD simulations: determination of SM fundamental parameters from first principles (quark masses, α_s)
  2. Chiral Perturbation theory
  3. *S*-matrix theory: based on analyticity and unitarity arguments (dispersion relations)

Test of QCD and ElectroWeak Interactions

- Inclusive decays: $\tau^- \to (\bar{u}d, \bar{u}s)\nu_\tau$

  Full hadron spectra (precision physics)

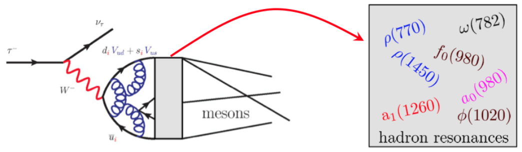
  Fundamental SM parameters:
  $\alpha_s(m_\tau), m_s, |V_{us}|$



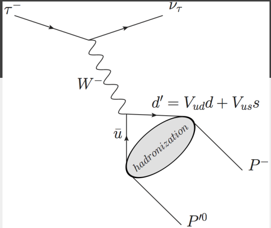
- Exclusive decays: $\tau^- \to (PP, PPP, \ldots)\nu_\tau$

  specific hadron spectrum (approximate physics)

  Hadronization of QCD currents, study of Form Factors, resonance parameters (M_R, Γ_R)



τ DECAYS INTO TWO MESONS



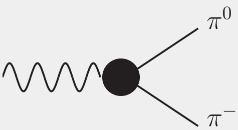
$$\frac{d\Gamma\left(\tau^- \to P^- P^0 \nu_\tau\right)}{ds} = \frac{G_F^2 |V_{ui}|^2 m_\tau^3}{768\pi^3} S_{EW}^{\mathrm{had}} C_{PP'}^2 \left(1 - \frac{s}{M_\tau^2}\right)^2$$

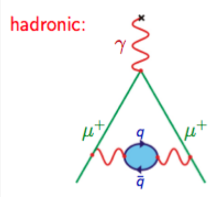
$$\left\{\left(1 + \frac{2s}{m_\tau^2}\right) \lambda_{P^- P^0}^{3/2}(s) |F_V^{P^- P^0}(s)|^2 + 3\frac{\Delta_{P^- P^0}^2}{s^2} \lambda_{P^- P^0}^{1/2}(s) |F_S^{P^- P^0}(s)|^2\right\}$$

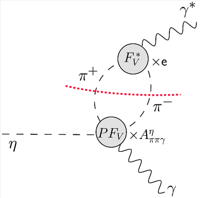
$$\Delta_{P^- P^0} = m_{P^-}^2 - m_{P^0}^2$$

- $\tau^- \to \pi^-\pi^0\nu_\tau$: Pion vector form factor, $\rho(770), \rho(1450), \rho(1700)$
- $\tau^- \to K^-K_S\nu_\tau$: Kaon vector form factor, $\rho(770), \rho(1450), \rho(1700)$
- $\tau^- \to K_S\pi^-\nu_\tau$: $K\pi$ form factor, $K^*(892), K^*(1410), K_{\ell 3}, V_{us}$
- $\tau^- \to K^-\eta^{(\prime)}\nu_\tau$: $K^*(1410), V_{us}$
- $\tau^- \to \pi^-\eta^{(\prime)}\nu_\tau$: isospin violation

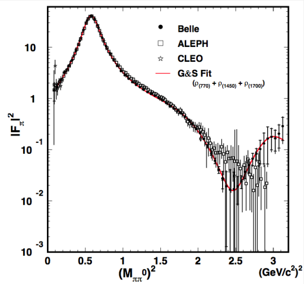
- Enters the description of many physical processes







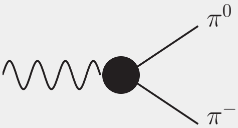
- Belle measurement of the pion vector form factor (0805.3773)

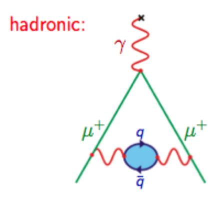


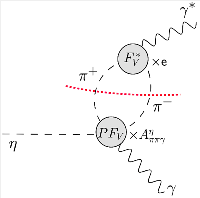
- high-statistics data until de τ mass
- sensitive to $\rho(1450)$ and $\rho(1700)$
- our aim: to improve the description of the $\rho(1450)$ and $\rho(1700)$ region

The pion vector form factor: Motivation

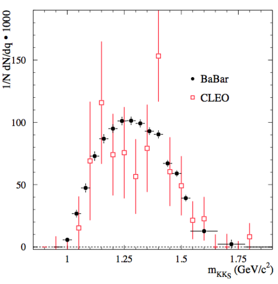
- Enters the description of many physical processes







- BaBar measurement of $\tau^- \to K^- K_S \nu_\tau$ (1806.10280)

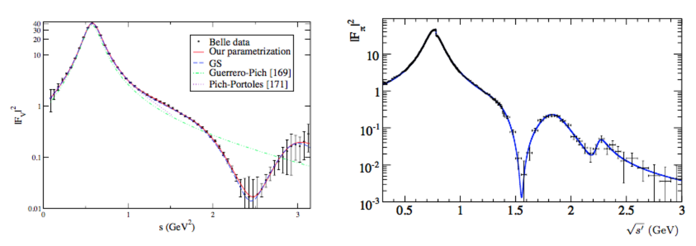


- good quality data
- sensitive to $\rho(1450)$ and $\rho(1700)$
- our aim: to improve the description of the $\rho(1450)$ and $\rho(1700)$ region

The Pion Vector Form Factor: a bottom-up approach

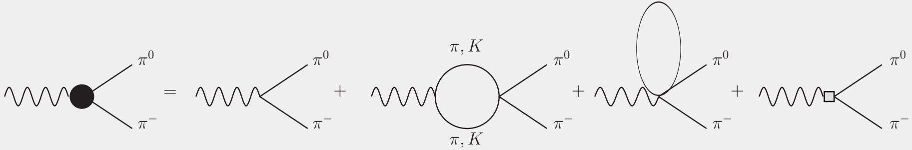
The Pion vector form factor $F_V^\pi(s)$

- How to determine $F_V^\pi(s)$ experimentally?
  - $\tau^- \to \pi^-\pi^0\nu_\tau$ and $e^+e^- \to \pi^+\pi^-$



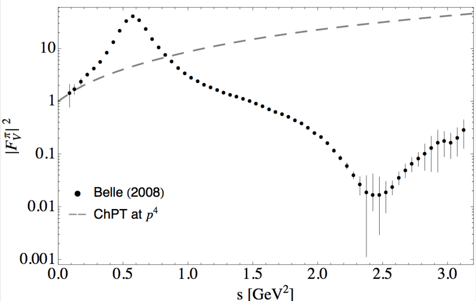
- What do we know theoretically on the form factor?
  - Its low-energy behaviour: given by ChPT (Gasser&Leutwyler'85)
  - Its high-energy behaviour ($\sim 1/s$): given by pQCD (Brodsky&Lepage'79)
  - For the intermediate energy region: models

Pion vector form factor: ChPT $\mathcal{O}(p^4)$



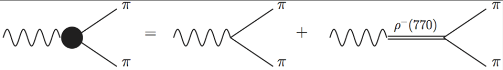
$$F_V^\pi(s)|_{\mathrm{ChPT}} = 1 + \frac{2L_9^r(\mu)}{F_\pi^2}s - \frac{s}{96\pi^2 F_\pi^2}\left(A_\pi(s,\mu^2) + \frac{1}{2}A_K(s,\mu^2)\right) \, ,$$

$$A_P(s,\mu^2) = \log\frac{m_P^2}{\mu^2} + \frac{8m_P^2}{s} - \frac{5}{3} + \sigma_P^3(s)\log\left(\frac{\sigma_P(s)+1}{\sigma_P(s)-1}\right) \, , \sigma_P(s) = \sqrt{1-4\frac{m_P^2}{s}}$$



Pion vector Form Factor: ChPT with resonances

- Resonace Chiral Th. (RχT)



$$F_V^{\pi}(s) = 1 + \frac{F_V G_V}{F_\pi^2}\frac{s}{M_\rho^2 - s} \overset{F_V G_V = F_\pi^2}{\Longrightarrow} \frac{M_\rho^2}{M_\rho^2 - s},$$

- Expansion in s and comparing ChPT and RχT

$$F_V^{\pi}(s) = 1 + \frac{2L_9^r(\mu)}{F_\pi^2}s - \frac{s}{96\pi^2 F_\pi^2}\left(A_\pi(s,\mu^2) + \frac{1}{2}A_K(s,\mu^2)\right)$$

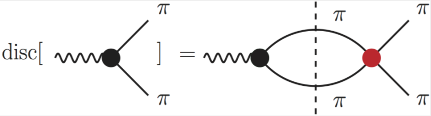
$$F_V^{\pi}(s) = 1 + \left(\frac{s}{M_\rho^2}\right) + \left(\frac{s}{M_\rho^2}\right)^2 + \cdots$$

- Chiral coupling estimate: $L_9^r(M_\rho) = \frac{F_V G_V}{2M_\rho^2} = \frac{F_\pi^2}{2M_\rho^2} \simeq 7.2 \times 10^{-3}$
- Combining ChPT and RχT

$$F_V^{\pi}(s) = \frac{M_\rho^2}{M_\rho^2 - s} - \frac{s}{96\pi^2 F_\pi^2}\left[A_\pi(s,\mu^2) + \frac{1}{2}A_K(s,\mu^2)\right],$$

Dispersive representation

- Dispersive approach

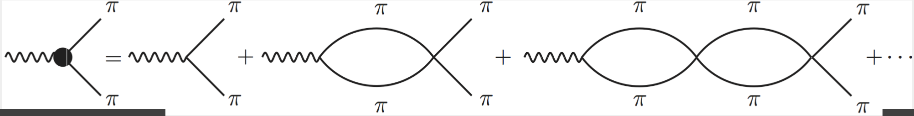


$$\mathrm{disc}F_V(s) = 2i\sigma_\pi(s)F_V(s)T_1^{1*}(s) = 2iF_V(s)\sin\delta_1^1(s)e^{-i\delta_1^1(s)}\,,$$

$$F_V(s) = \frac{1}{2i\pi}\int_{4M_\pi^2}^\infty ds'\,\frac{\mathrm{disc}F_V(s')}{s'-s-i\varepsilon}\,,$$

- Analytic solution (Omnès equation)

$$F_V(s) = P(s)\Omega(s)\,,\quad \Omega(s) = \exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^\infty ds'\,\frac{\delta_1^1(s')}{s'(s'-s-i\varepsilon)}\right\}\,,$$



RχT + Omnès: Exponential representation

- Resummation of final-state interactions to all orders (Omnès)

$$F_V^\pi(s) = P_n(s)\exp\left\{\frac{s^n}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{(s')^n}\frac{\delta_1^1(s')}{s'-s-i\varepsilon}\right\},$$

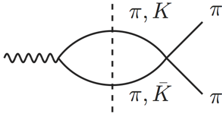
- Get a model for the phase from $\pi\pi\to\pi\pi$ scattering at $\mathcal{O}(p^2)$

$$T(s) = \frac{s-m_\pi^2}{F_\pi^2} \to T_1^1(s) = \frac{s\sigma_\pi^2(s)}{96\pi F_\pi^2} \to \delta_1^1(s) = \sigma_\pi(s)T_1^1(s) = \frac{s\sigma_\pi^3(s)}{96\pi F_\pi^2},$$

- Omnès exponentiation of the full loop function

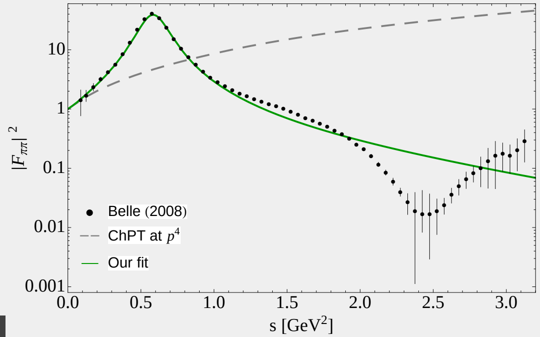
$$F_V^\pi(s) = \frac{M_\rho^2}{M_\rho^2-s}\exp\left\{-\frac{s}{96\pi^2F_\pi^2}A_\pi(s,\mu^2)\right\}.$$

- Incorporatation of the (off-shell) ρ width



$$\Gamma_\rho(s) = -\frac{M_\rho s}{96\pi^2 F_\pi^2}\mathrm{Im}\left[A_\pi(s) + \frac{1}{2}A_K(s)\right] = \frac{M_\rho s}{96\pi F_\pi^2}\left[\sigma_\pi(s)^3\theta(s-4m_\pi^2) + \sigma_K(s)^3\theta(s-4m_K^2)\right] .$$

$$F_V^\pi(s)\big|_{\mathrm{expo}}^{1\,\mathrm{res}} = \frac{M_\rho^2}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)}\exp\left\{-\frac{s}{96\pi^2 F_\pi^2}\mathrm{Re}\left[A_\pi(s,\mu^2) + \frac{1}{2}A_K(s,\mu^2)\right]\right\} .$$



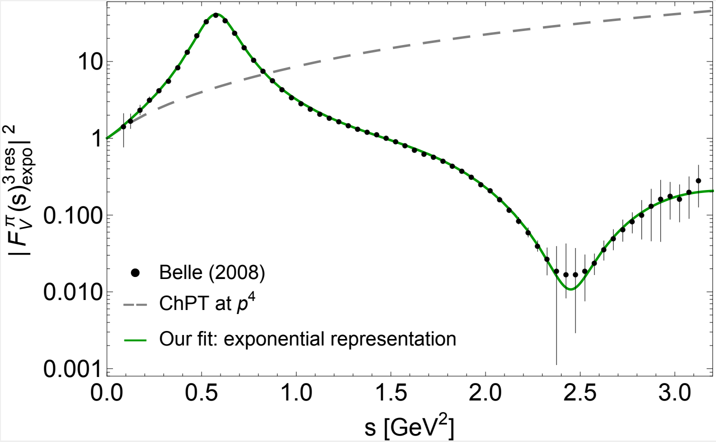
- Incorporation of the $\rho'(1450)$, $\rho''(1700)$

$$F_V^\pi(s)|_{\text{expo}}^{3\,\text{res}} = \frac{M_\rho^2 + s\left(\gamma e^{i\phi_1} + \delta e^{i\phi_2}\right)}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} \exp\left\{\mathrm{Re}\left[-\frac{s}{96\pi^2 F_\pi^2}\left(A_\pi(s) + \frac{1}{2}A_K(s)\right)\right]\right\}$$
$$-\gamma \frac{s\, e^{i\phi_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} \exp\left\{-\frac{s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)}\mathrm{Re}A_\pi(s)\right\}$$
$$-\delta \frac{s\, e^{i\phi_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)} \exp\left\{-\frac{s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)}\mathrm{Re}A_\pi(s)\right\},$$

where

$$\Gamma_{\rho',\rho''}(s) = \Gamma_{\rho',\rho''}\frac{M_{\rho',\rho''}}{\sqrt{s}}\frac{\sigma_\pi^3(s)}{\sigma_\pi^3(M_{\rho',\rho''}^2)}\,.$$

RχT + Omnès: Exponential representation

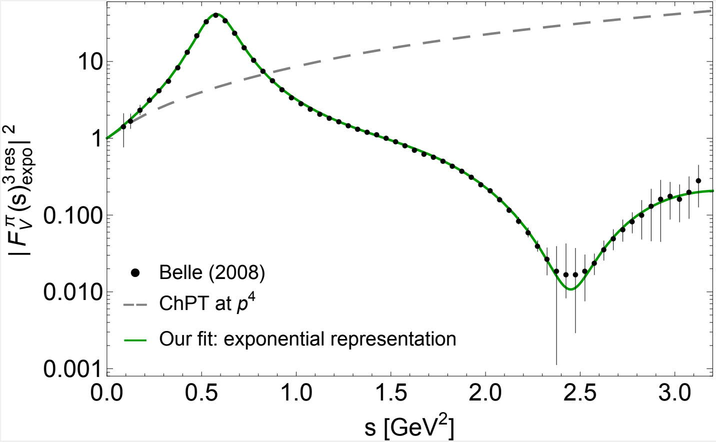


$$M_\rho = 775.2(4)\,\mathrm{MeV}\,, \quad \gamma = 0.15(4)\,, \quad \phi_1 = -0.36(24)\,,$$

$$M_{\rho'} = 1438(39)\,\mathrm{MeV}\,, \quad \Gamma_{\rho'} = 535(63)\,\mathrm{MeV}\,, \quad \delta = -0.12(4)\,, \phi_2\,, = -0.02(45)\,,$$

$$M_{\rho''} = 1754(91)\,\mathrm{MeV}\,, \quad \Gamma_{\rho''} = 412(102)\,\mathrm{MeV}\,, \quad \chi^2_{\mathrm{dof}} = 0.92$$

RχT + Omnès: Exponential representation



$$M_\rho^{\mathrm{pole}} = 762.0(3)\,\mathrm{MeV}\,, \quad \Gamma_\rho^{\mathrm{pole}} = 143.0(2)\,\mathrm{MeV}\,,$$

$$M_{\rho'}^{\mathrm{pole}} = 1366(38)\,\mathrm{MeV}\,, \quad \Gamma_{\rho'}^{\mathrm{pole}} = 488(48)\,\mathrm{MeV}\,,$$

$$M_{\rho''}^{\mathrm{pole}} = 1718(82)\,\mathrm{MeV}\,, \quad \Gamma_{\rho''}^{\mathrm{pole}} = 397(88)\,\mathrm{MeV}\,,$$

Dispersive representation

- Drawbacks: Constraints from analyticity, unitarity and chiral symmetry not fully respected
- Dispersion relation with subtractions:

$$F_V^\pi(s) = \exp\left[\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{s_{\rm cut}} ds' \frac{\phi(s')}{(s')^3(s'-s-i\varepsilon)}\right] ,$$

  - Low-energy observables

$$F_V^\pi(s) = 1 + \frac{1}{6}\langle r^2\rangle_V^\pi s + c_V^\pi s^2 + d_V^\pi s^3 + \cdots ,$$

$$\langle r^2\rangle_V^\pi\Big|_{\rm ChPT}^{\mathcal{O}(p^4)} = \frac{12L_9^r(\mu)}{F_\pi^2} - \frac{1}{32\pi^2F_\pi^2}\left[2\log\left(\frac{M_\pi^2}{\mu^2}\right) + \log\left(\frac{M_K^2}{\mu^2}\right) + 3\right] ,$$

$$\langle r^2\rangle_V^\pi = 6\alpha_1 , \quad c_V^\pi = \frac{1}{2}\left(\alpha_2 + \alpha_1^2\right) , \quad \alpha_k = \frac{k!}{\pi}\int_{4m_\pi^2}^{s_{\rm cut}} ds' \frac{\phi(s')}{s'^{k+1}} .$$

  - $s_{\rm cut}$: cut-off to check stability

Dispersive representation

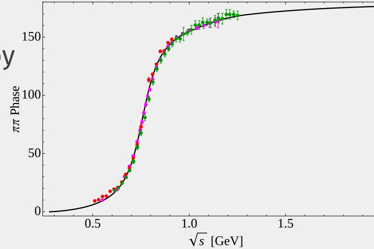
- Dispersion relation with subtractions:

$$F_V^\pi(s) = \exp\left[\alpha_1 s + \frac{\alpha_2}{2}s^2 + \frac{s^3}{\pi}\int_{4m_\pi^2}^{s_{\rm cut}} ds' \frac{\phi(s')}{(s')^3(s'-s-i\varepsilon)}\right] ,$$

- Form Factor phase $\phi(s)$
  - $4m_\pi^2 < s < 1$ GeV: $\pi\pi$ phase from Roy (García-Martín et.al PRD 83, 074004 (2011))
  - $1 < s < m_\tau^2$: "Pheno" phase shift

$$\tan\phi(s) = \frac{{\rm Im}F_V^\pi(s)|^{3\,\rm res}_{\rm expo}}{{\rm Re}F_V^\pi(s)|^{3\,\rm res}_{\rm expo}} ,$$

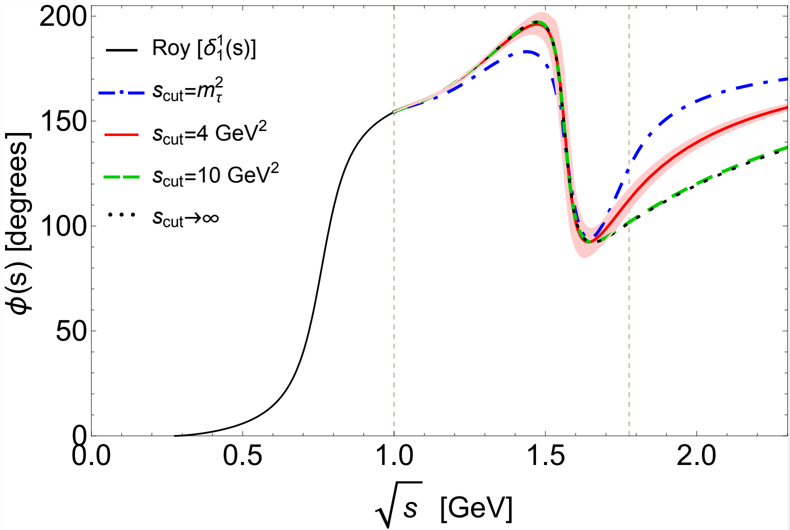
  - $m_\tau^2 < s$: phase guided smoothly to π



- Fits for different values of s_{cut} and matching at 1 GeV

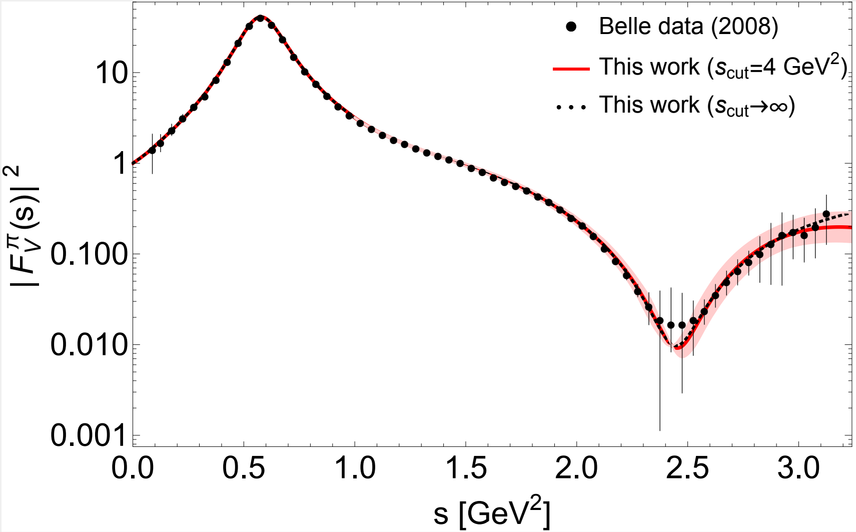
Fits	Parameter	s_{cut} [GeV²]			
		m_τ^2	4 (reference fit)	10	∞
Fit 1	α_1 [GeV^{-2}]	1.87(1)	1.88(1)	1.89(1)	1.89(1)
	α_2 [GeV^{-4}]	4.40(1)	4.34(1)	4.32(1)	4.32(1)
	m_ρ [MeV]	$=$ 773.6(9)	$=$ 773.6(9)	$=$ 773.6(9)	$=$ 773.6(9)
	M_ρ [MeV]	$= m_\rho$	$= m_\rho$	$= m_\rho$	$= m_\rho$
	$M_{\rho'}$ [MeV]	1365(15)	1376(6)	1313(15)	1311(5)
	$\Gamma_{\rho'}$ [MeV]	562(55)	603(22)	700(6)	701(28)
	$M_{\rho''}$[MeV]	1727(12)	1718(4)	1660(9)	1658(1)
	$\Gamma_{\rho''}$ [MeV]	278(1)	465(9)	601(39)	602(3)
	γ	0.12(2)	0.15(1)	0.16(1)	0.16(1)
	ϕ_1	$-$0.69(1)	$-$0.66(1)	$-$1.36(10)	$-$1.39(1)
	δ	$-$0.09(1)	$-$0.13(1)	$-$0.16(1)	$-$0.17(1)
	ϕ_2	$-$0.17(5)	$-$0.44(3)	$-$1.01(5)	$-$1.03(2)
	χ^2/d.o.f	1.47	0.70	0.64	0.64

- Form Factor phase shift for different values of s_{cut}



- The results can be found in tables provided as ancillary material in 1902.02273 [hep-ph]

- Modulus squared of the pion vector form factor



- The results can be found in tables provided as ancillary material in 1902.02273 [hep-ph]

Variant (I)

- Fits for different matching point and with $s_{\mathrm{cut}} = 4$ GeV

Fits	Parameter	Matching point [GeV]			
		0.85	0.9	0.95	1 (reference fit)
Fit I	α_1 [GeV^{-2}]	1.88(1)	1.88(1)	1.88(1)	1.88(1)
	α_2 [GeV^{-4}]	4.35(1)	4.35(1)	4.34(1)	4.34(1)
	m_ρ [MeV]	$= 773.6(9)$	$= 773.6(9)$	$= 773.6(9)$	$= 773.6(9)$
	M_ρ [MeV]	$= m_\rho$	$= m_\rho$	$= m_\rho$	$= m_\rho$
	$M_{\rho'}$ [MeV]	1394(6)	1374(8)	1351(5)	1376(6)
	$\Gamma_{\rho'}$ [MeV]	592(19)	583(27)	592(2)	603(22)
	$M_{\rho''}$[MeV]	1733(9)	1715(1)	1697(3)	1718(4)
	$\Gamma_{\rho''}$ [MeV]	562(3)	541(45)	486(7)	465(9)
	γ	0.12(1)	0.12(1)	0.13(1)	0.15(1)
	ϕ_1	$-0.44(3)$	$-0.60(1)$	$-0.80(1)$	$-0.66(1)$
	δ	$-0.13(1)$	$-0.13(1)$	$-0.13(1)$	$-0.13(1)$
	ϕ_2	$-0.38(3)$	$-0.51(2)$	$-0.62(1)$	$-0.44(3)$
	χ^2/d.o.f	0.75	0.74	0.68	0.70

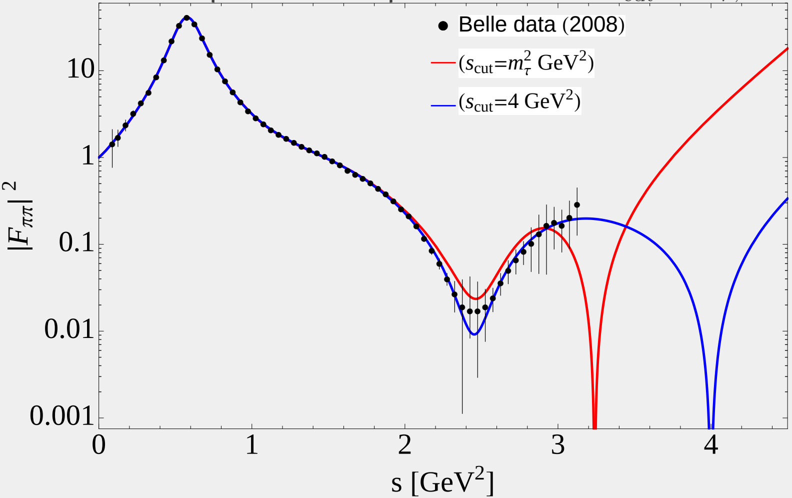
Variant (II): intermediate states other than $\pi\pi$

- Fit A: $\rho' \to K\bar{K}$ and $\rho'' \to K\bar{K}$
- Fit B: $\rho' \to K\bar{K}$ + $\rho' \to \omega\pi$

Parameter	$s_{\rm cut} = 4$ GeV²		
	Fit A	Fit B	reference fit
α_1 [GeV^{-2}]	1.87(1)	1.88(1)	1.88(1)
α_2 [GeV^{-4}]	4.37(1)	4.35(1)	4.34(1)
m_ρ [MeV]	$= 773.6(9)$	$= 773.6(9)$	$= 773.6(9)$
M_ρ [MeV]	$= m_\rho$	$= m_\rho$	$= m_\rho$
$M_{\rho'}$ [MeV]	1373(5)	1441(3)	1376(6)
$\Gamma_{\rho'}$ [MeV]	462(14)	576(33)	603(22)
$M_{\rho''}$[MeV]	1775(1)	1733(9)	1718(4)
$\Gamma_{\rho''}$ [MeV]	412(27)	349(52)	465(9)
γ	0.13(1)	0.15(3)	0.15(1)
ϕ_1	$-0.80(1)$	$-0.53(5)$	$-0.66(1)$
δ	$-0.14(1)$	$-0.14(1)$	$-0.13(1)$
ϕ_2	$-0.44(2)$	$-0.46(3)$	$-0.44(3)$
χ^2/d.o.f	0.93	0.70	0.70

Dispersive representation: singularities at $s = s_{\rm cut}$

- Modulus squared of the pion form factor $s_{\rm cut} = m_\tau, 4$ GeV²



- Dispersive representation of the pion vector form factor

$$F_V^\pi(s) = \exp\left[\frac{s}{\pi}\int_{4m_\pi^2}^{s_\mathrm{cut}} ds' \frac{\delta_1^1(s')}{(s')(s'-s-i\varepsilon)} + \frac{s}{\pi}\int_{s_\mathrm{cut}}^{\infty} ds' \frac{\delta_\mathrm{eff}(s')}{(s')(s'-s-i\varepsilon)}\right]\Sigma(s)$$

- Properties for $\delta_\mathrm{eff}(s)$
  - $\delta_\mathrm{eff}(s_\mathrm{cut}) = \delta_1^1(s_\mathrm{cut})$ and $\delta_\mathrm{eff}(s) \to \pi$ for large s to recover $1/s$

$$\delta_\mathrm{eff}(s) = \pi + \left(\delta_1^1(s_\mathrm{cut}) - \pi\right)\frac{s_\mathrm{cut}}{s}$$

  - Integrating the piece with $\delta_\mathrm{eff}(s)$

$$\begin{aligned} F_V^\pi(s) &= e^{1-\frac{\delta_1^1(s_\mathrm{cut})}{\pi}}\left(1-\frac{s}{s_\mathrm{cut}}\right)^{\left(1-\frac{\delta_1^1(s_\mathrm{cut})}{\pi}\right)\frac{s_\mathrm{cut}}{s}}\left(1-\frac{s}{s_\mathrm{cut}}\right)^{-1} \\ &\times \exp\left[\frac{s}{\pi}\int_{4m_\pi^2}^{s_\mathrm{cut}} ds' \frac{\delta_1^1(s')}{(s')(s'-s-i\varepsilon)}\right]\Sigma(s) \\ \Sigma(s) &= \sum^{\infty} a_i\omega^i(s)\,, \quad \omega(s) = \frac{\sqrt{s_\mathrm{cut}}-\sqrt{s_\mathrm{cut}-s}}{\sqrt{s_\mathrm{cut}}+\sqrt{s_\mathrm{cut}-s}} \end{aligned}$$

Variant (III)

The resulting fit parameters are found to be

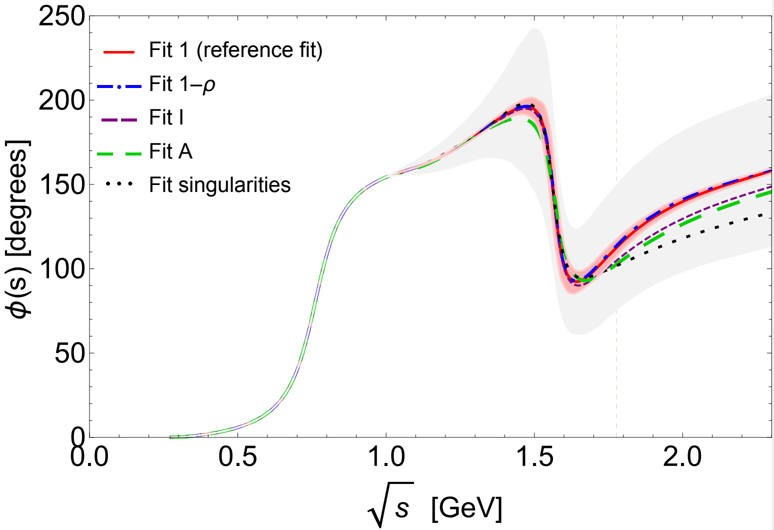
$$a_1 = 2.99(12)\,,$$
$$M_{\rho'} = 1261(7)\,\mathrm{MeV}\,,\quad \Gamma_{\rho'} = 855(15)\,\mathrm{MeV}\,,$$
$$M_{\rho''} = 1600(1)\,\mathrm{MeV}\,,\quad \Gamma_{\rho''} = 486(26)\,\mathrm{MeV}\,,$$
$$\gamma = 0.25(2)\,,\quad \phi_1 = -1.90(6)\,,$$
$$\delta = -0.15(1)\,,\quad \phi_2 = -1.60(4)\,,$$

with a χ^2/d.o.f $= 32.3/53 \sim 0.61$ for the one-parameter fit, and

$$a_1 = 3.03(20)\,,\quad a_2 = 1.04(2.10)\,,$$
$$M_{\rho'} = 1303(19)\,\mathrm{MeV}\,,\quad \Gamma_{\rho'} = 839(102)\,\mathrm{MeV}\,,$$
$$M_{\rho''} = 1624(1)\,\mathrm{MeV}\,,\quad \Gamma_{\rho''} = 570(99)\,\mathrm{MeV}$$
$$\gamma = 0.22(10)\,,\quad \phi_1 = -1.65(4)\,,$$
$$\delta = -0.18(1)\,,\quad \phi_2 = -1.34(14)\,,$$

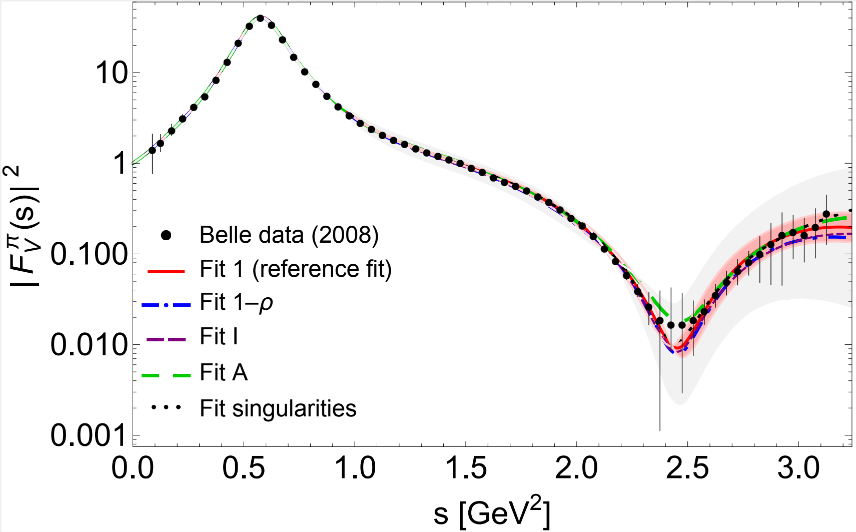
with a χ^2/d.o.f $= 35.6/52 \sim 0.63$ for the two-parameter fit.

- Form Factor phase shift for different parametrizations



- The results can be found in tables provided as ancillary material in 1902.02273 [hep-ph]

■ Modulus squared of the pion vector form factor



■ The results can be found in tables provided as ancillary material in 1902.02273 [hep-ph]

Central results

- Fit results (central value $\pm$ stat fit error $\pm$ syst th. error)

$$\alpha_1 = 1.88(1)(1)\ \mathrm{GeV}^{-2},\ \alpha_2 = 4.34(1)(3)\ \mathrm{GeV}^{-4},$$

$$M_\rho \doteq 773.6 \pm 0.9 \pm 0.3\ \mathrm{MeV}\,,$$

$$M_{\rho'} = 1376 \pm 6^{+18}_{-73}\ \mathrm{MeV}\,, \quad \Gamma_{\rho'} = 603 \pm 22^{+236}_{-141}\ \mathrm{MeV}\,,$$

$$M_{\rho''} = 1718 \pm 4^{+57}_{-94}\ \mathrm{MeV}\,, \quad \Gamma_{\rho''} = 465 \pm 9^{+137}_{-53}\ \mathrm{MeV}\,,$$

$$\gamma = 0.15 \pm 0.01^{+0.07}_{-0.03}\,, \quad \phi_1 = -0.66 \pm 0.01^{+0.22}_{-0.99}\,,$$

$$\delta = -0.13 \pm 0.01^{+0.00}_{-0.05}\,, \quad \phi_2 = -0.44 \pm 0.03^{+0.06}_{-0.90}\,,$$

- Physical pole mass and width

$$M_\rho^{\mathrm{pole}} = 760.6 \pm 0.8\ \mathrm{MeV}\,, \quad \Gamma_\rho^{\mathrm{pole}} = 142.0 \pm 0.4\ \mathrm{MeV}\,,$$

$$M_{\rho'}^{\mathrm{pole}} = 1289 \pm 8^{+52}_{-143}\ \mathrm{MeV}\,, \quad \Gamma_{\rho'}^{\mathrm{pole}} = 540 \pm 16^{+151}_{-111}\ \mathrm{MeV}\,,$$

$$M_{\rho''}^{\mathrm{pole}} = 1673 \pm 4^{+68}_{-125}\ \mathrm{MeV}\,, \quad \Gamma_{\rho''}^{\mathrm{pole}} = 445 \pm 8^{+117}_{-49}\ \mathrm{MeV}\,,$$

$\rho(1450)$ and $\rho(1700)$ resonance parameters

Reference	Model parameters $M_{\rho'}, \Gamma_{\rho'}$ [MeV]	Pole parameters $M^{\text{pole}}_{\rho'}, \Gamma^{\text{pole}}_{\rho'}$ [MeV]	Data
ALEPH	$1328 \pm 15, 468 \pm 41$	$1268 \pm 19, 429 \pm 31$	τ
ALEPH	$1409 \pm 12, 501 \pm 37$	$1345 \pm 15, 459 \pm 28$	τ & e
Belle (fixed $\lvert F^{\pi}_{V}(0)\rvert^2$)	$1446 \pm 7 \pm 28, 434 \pm 16 \pm 60$	$1398 \pm 8 \pm 31, 408 \pm 13 \pm 50$	τ
Belle (all free)	$1428 \pm 15 \pm 26, 413 \pm 12 \pm 57$	$1384 \pm 16 \pm 29, 390 \pm 10 \pm 48$	τ
Dumm et. al.	—	$1440 \pm 80, 320 \pm 80$	τ
Celis et. al.	$1497 \pm 7, 785 \pm 51$	$1278 \pm 18, 525 \pm 16$	τ
Bartos et. al.	—	$1342 \pm 47, 492 \pm 138$	e^+e^-
Bartos et. al.	—	$1374 \pm 11, 341 \pm 24$	τ
This work	$1376 \pm 6^{+18}_{-73}, 603 \pm 22^{+236}_{-141}$	$1289 \pm 8^{+52}_{-143}, 540 \pm 16^{+151}_{-111}$	τ

Reference	Model parameters $(M_{\rho''}, \Gamma_{\rho''})$ [MeV]	Pole parameters $(M^{\text{pole}}_{\rho''}, \Gamma^{\text{pole}}_{\rho''})$ [MeV]	Data
ALEPH	$= 1713, = 235$	$1700, 232$	τ
ALEPH	$1740 \pm 20, = 235$	$1728 \pm 20, 232$	τ & e
Belle (fixed $\lvert F^{\pi}_{V}(0)\rvert^2$)	$1728 \pm 17 \pm 89, 164 \pm 21^{+89}_{-26}$	$1722 \pm 18, 163 \pm 21^{+88}_{-27}$	τ
Belle (all free)	$1694 \pm 41, 135 \pm 36^{+50}_{-26}$	$1690 \pm 94, 134 \pm 36^{+49}_{-28}$	τ
Dumm et. al.	—	$1720 \pm 90, 180 \pm 90$	τ
Celis et. al.	$1685 \pm 30, 800 \pm 31$	$1494 \pm 37, 600 \pm 17$	τ
Bartos et. al.	—	$1719 \pm 65, 490 \pm 17$	e^+e^-
Bartos et. al.	—	$1767 \pm 52, 415 \pm 120$	τ
This work	$1718 \pm 4^{+57}_{-94}, 465 \pm 9^{+137}_{-53}$	$1673 \pm 4^{+68}_{-125}, 445 \pm 8^{+117}_{-49}$	τ

Low-energy observables

References	$\langle r^2 \rangle_V^\pi$ (GeV^{-2})	c_V^π (GeV^{-4})
Colangelo et al. [55]	11.07 ± 0.66	3.2 ± 1.03
Bijnens et al. [32]	11.22 ± 0.41	3.85 ± 0.60
Pich et al. [6]	11.04 ± 0.30	3.79 ± 0.04
Bijnens et al. [33]	11.61 ± 0.33	4.49 ± 0.28
de Troconiz et al. [56]	11.10 ± 0.03	3.84 ± 0.02
Masjuan et al. [57]	11.43 ± 0.19	3.30 ± 0.33
Guo et al. [58]	–	4.00 ± 0.50
Lattice [59]	10.50 ± 1.12	3.22 ± 0.40
Ananthanarayan et al. [60]	11.17 ± 0.53	[3.75, 3.98]
Ananthanarayan et al. [61]	[10.79, 11.3]	[3.79, 4.00]
Schneider et al. [48]	10.6	3.84 ± 0.03
Dumm et al. [7]	10.86 ± 0.14	3.84 ± 0.03
Celis et al. [8]	11.30 ± 0.07	4.11 ± 0.09
Ananthanarayan et al. [62]	11.10 ± 0.11	–
Hanhart et al. [63]	$11.34 \pm 0.01 \pm 0.01$	–
Colangelo et al. [39]	11.02 ± 0.10	–
PDG [42]	11.61 ± 0.28	–
This work	11.28 ± 0.08	3.94 ± 0.04

Sum rule	$s_{\rm cut}$ (GeV^2)			Fit Eq. (42)
	4	10	∞	
α_1	1.52	1.66	1.75	$1.88 \pm 0.01 \pm 0.01$
α_2	4.26	4.30	4.31	$4.34 \pm 0.01 \pm 0.03$

The Kaon Vector Form Factor

Kaon vector Form Factor

$$\frac{d\Gamma(\tau^- \to K^- K^0 \nu_\tau)}{d\sqrt{s}} = \frac{G_F^2 |V_{ud}|^2}{768\pi^3} M_\tau^3 \left(1 - \frac{s}{M_\tau^2}\right)^2 \left(1 + \frac{2s}{M_\tau^2}\right) \sigma_K^3(s) |F_V^K(s)|^2 \,,$$

- Chiral Perturbation Theory $\mathcal{O}(p^4)$

$$\begin{aligned} F_{K^+K^-}(s)|_{\mathrm{ChPT}} &= 1 + \frac{2L_9^r}{F_\pi^2} - \frac{s}{192\pi^2 F_\pi^2}\left[A_\pi(s,\mu^2) + 2A_K(s,\mu^2)\right] \,, \\ F_{K^0\bar{K}^0}(s)|_{\mathrm{ChPT}} &= -\frac{s}{192\pi^2 F_\pi^2}\left[A_\pi(s,\mu^2) - A_K(s,\mu^2)\right] \,. \end{aligned}$$

- Extract the $I = 1$ component

$$F_V^K(s)|_{\mathrm{ChPT}} = F_{K^+K^-}(s) - F_{K^0\bar{K}^0}(s) = 1 + \frac{2L_9^r}{F_\pi^2} - \frac{s}{96\pi^2 F_\pi^2}\left[A_\pi(s,\mu^2) + \frac{1}{2}A_K(s,\mu^2)\right]$$

- At $\mathcal{O}(p^4)$, the pion and kaon vector form factor are the same
- Assumption: we consider that both are also the same at higher energies

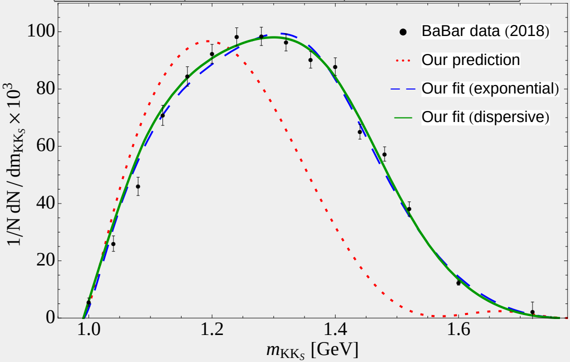
- Different resonance mixing contribution than $F_V^\pi(s)$

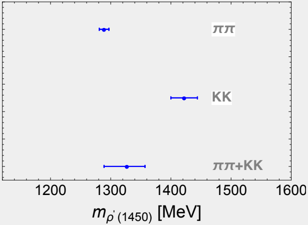
$$
\begin{aligned}
F_V^K(s) &= \frac{M_\rho^2 + s\left(\tilde{\gamma} e^{i\tilde{\phi}_1} + \tilde{\delta} e^{i\tilde{\phi}_2}\right)}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} \exp\left\{ \mathrm{Re}\left[-\frac{s}{96\pi^2 F_\pi^2}\left(A_\pi(s) + \frac{1}{2} A_K(s) \right) \right] \right\} \\
&\quad - \tilde{\gamma} \frac{s\, e^{i\tilde{\phi}_1}}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} \exp\left\{ -\frac{s\Gamma_{\rho'}(M_{\rho'}^2)}{\pi M_{\rho'}^3 \sigma_\pi^3(M_{\rho'}^2)} \mathrm{Re} A_\pi(s) \right\} \\
&\quad - \tilde{\delta} \frac{s\, e^{i\tilde{\phi}_2}}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)} \exp\left\{ -\frac{s\Gamma_{\rho''}(M_{\rho''}^2)}{\pi M_{\rho''}^3 \sigma_\pi^3(M_{\rho''}^2)} \mathrm{Re} A_\pi(s) \right\}, \\
\Gamma_{\rho',\rho''}(s) &= \Gamma_{\rho',\rho''} \frac{s}{M_{\rho',\rho''}^2} \frac{\sigma_\pi^3(s)}{\sigma_\pi^3(M_{\rho',\rho''}^2)} \theta(s - 4m_\pi^2)\,.
\end{aligned}
$$

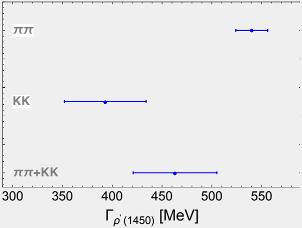
- Extract the phase $\tan \phi_{KK}(s) = \mathrm{Im} F_V^K(s) / \mathrm{Re} F_V^K(s)$
- Use a three-times subtracted dispersion relation

Fit results to BaBar $\tau^- \to K^- K_S \nu_\tau$ data

Parameter	Fit dispersive	Fit exponential
$\tilde{\alpha}_1$	$= 1.88(1)$	—
$\tilde{\alpha}_2$	$= 4.34(1)$	—
$M_{\rho'}$ [MeV]	1467(24)	1411(12)
$\Gamma_{\rho'}$ [MeV]	415(48)	394(35)
$\tilde{\gamma}$	0.10(2)	0.09(1)
$\tilde{\phi}_1$	$-1.19(16)$	$-1.88(9)$
χ^2/d.o.f.	2.9	3.3



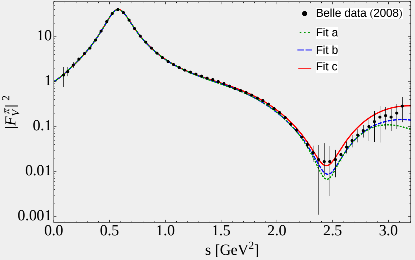


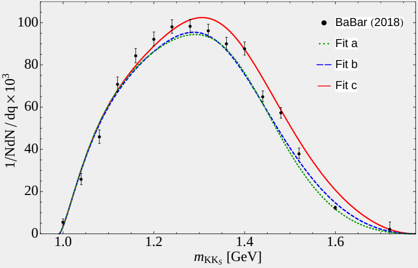


1902.02273 [hep-ph]

Combined analysis of $F_V^\pi(s)$ and $\tau^- \to K^- K_S \nu_\tau$

Parameter	$s_{\rm cut} = 4$ [GeV^2]		
	Fit *a*	Fit *b*	Fit *c*
α_1	1.88(1)	1.89(1)	1.87(1)
α_2	4.34(2)	4.31(2)	4.38(3)
$\tilde{\alpha}_1$	$= \alpha_1$	$= \alpha_1$	1.88(24)
$\tilde{\alpha}_2$	$= \alpha_2$	$= \alpha_2$	4.38(29)
m_ρ [MeV]	$= 773.6(9)$	$= 773.6(9)$	$= 773.6(9)$
M_ρ [MeV]	$= m_\rho$	$= m_\rho$	$= m_\rho$
$M_{\rho'}$ [MeV]	1396(19)	1453(19)	1406(61)
$\Gamma_{\rho'}$ [MeV]	507(31)	499(51)	524(149)
$M_{\rho''}$[MeV]	1724(41)	1712(32)	1746(1)
$\Gamma_{\rho''}$ [MeV]	399(126)	284(72)	413(362)
γ	0.12(3)	0.15(3)	0.11(11)
$\tilde{\gamma}$	0.11(2)	$= \gamma$	0.11(5)
ϕ_1	$-0.23(26)$	0.29(21)	$-0.27(42)$
$\tilde{\phi}_1$	$-1.83(14)$	$-1.48(13)$	$-1.90(67)$
δ	$-0.09(2)$	$-0.07(2)$	$-0.10(5)$
$\tilde{\delta}$	$= 0$	$= 0$	$-0.01(4)$
ϕ_2	$-0.20(31)$	0.27(29)	$-1.15(71)$
$\tilde{\phi}_2$	$= 0$	$= 0$	0.40(3)
χ^2/d.o.f	1.52	1.19	1.25





$\tau \to 3\pi\nu_\tau$ (PRELIMINARY RESULTS)

- Nucleon matrix element

$$\langle N(p')|J^{\mu}_{em}|N(p)\rangle = \bar{u}(p')\left[\gamma^{\mu}F_1(t) + \frac{i}{2m_N}\sigma^{\mu\nu}(p'-p)_{\nu}F_2(t)\right]u(p)\,,$$

$$\langle N(p')|J^{\mu a}_{A}|N(p)\rangle = \bar{u}(p')\frac{\tau^a}{2}\gamma_5\left[\gamma^{\mu}F_A(t) + \frac{(p'-p)^{\mu}}{2m_N}F_P(t)\right]u(p)\,,$$

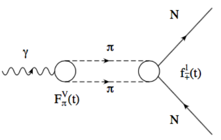
- Four Form Factor to determine ($t = (p'-p)^2$)
  - $F_1(t)$ and $F_2(t)$: Dirac and Pauli Form Factors
  - $G_E(t)$ and $G_M(t)$: electric and magnetic (Sachs) FFs, well-known

$$G_E(t) = F_1(t) + \frac{t}{4m_N^2}F_2(t)\,, \quad G_M(t) = F_1(t) + F_2(t)$$

  - $F_A(t)$: Axial Form Factor: main unkown
  - $F_P(t)$: Pseudo-scalar Form Factor. It can be related to the Axial FF using PCAC or pion-pole approximation

$$F_P(t) = \frac{2m_N^2}{m_\pi^2 - t}F_A(t)$$

- Dispersive parametrizations
  (Belushkin'06, Lorenz'12, Hoferichter'16, Leupold'17, Alarcon'18)



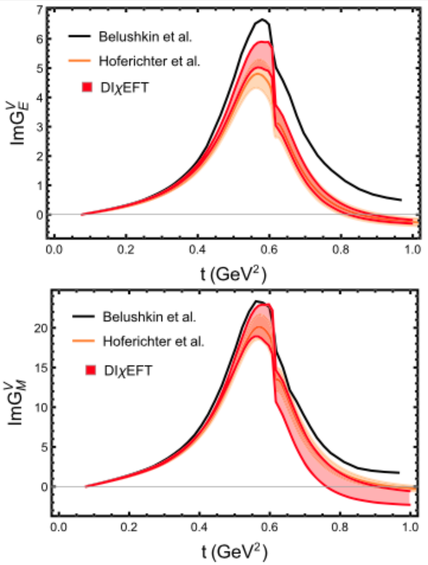
$$G_{E,M}^{p,n}(t) = \frac{1}{\pi}\int_{s_{\rm th}}^{\infty} ds \frac{{\rm Im}G_{E,M}^{p,n}(s)}{s-t-i\epsilon},$$

$$G_{E,M}^{V,S} = \frac{1}{2}\left(G_{E,M}^{p} - G_{E,M}^{n}\right),$$

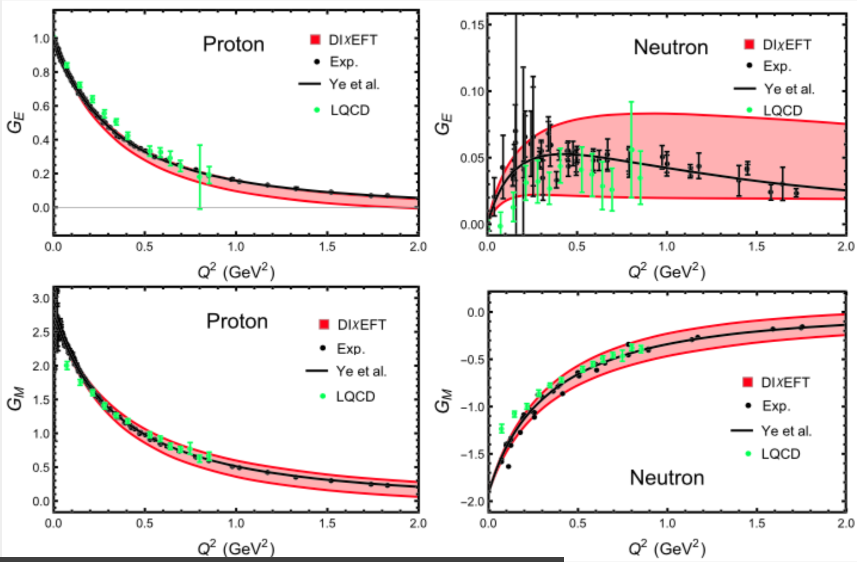
$$\mathrm{Im}G_{E}^{V}(t) = \frac{k_{\rm cm}^3}{m_N\sqrt{t}} f_{+}^{1}(t) F_{\pi}^{V*}(t)$$

$$\mathrm{Im}G_{M}^{V}(t) = \frac{k_{\rm cm}^3}{\sqrt{2t}} f_{-}^{1}(t) F_{\pi}^{V*}(t)$$

where $k_{\rm cm} = \sqrt{t/4 - m_\pi^2}$

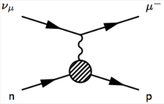


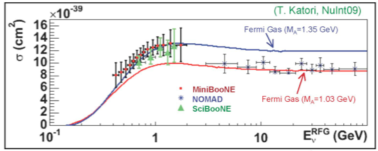
$Q^2 \equiv -t \geq 0$



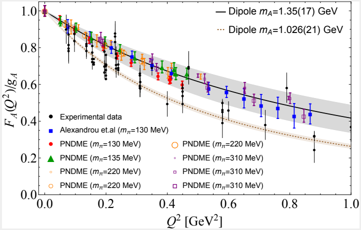
Axial Vector Form Factor

- $F_A(t)$: Main Unknown
- Motivation: How to determine $F_A(t)$ experimentally?
  - $\nu_\mu n \to \mu p$: $F_A(Q^2) = \frac{F_A(0)}{(1+Q^2/M_A^2)^2}$

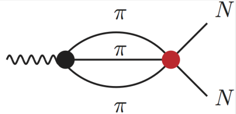




- Objective: Theoretical effort to improve $F_A(t)$
  - Lattice

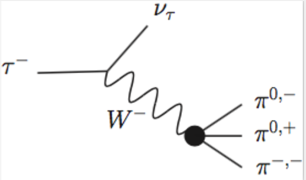


  - Analytically: 3π intermediate state



Our proposal:

- To investigate the axial-vector weak hadronic current through the $\tau \to 3\pi\nu_\tau$ axial-vector spectral function
- 3π system (predominantly) in $J^{PC} = 1^{++}$ produced through the $a_1(1260)$



- To see what can be learned for the low-Q^2 behavior of the axial form factor of the nucleon

- Generic Amplitude for a 3-meson decay of the τ

$$\mathcal{M}(\tau^- \to (PPP)^- \nu_\tau) = \frac{G_F}{\sqrt{2}} |V_{ij}| \bar{u}_{\nu_\tau} \gamma_\mu (1 - \gamma_5) u_\tau \langle (PPP)^- | (V-A)^\mu | 0 \rangle \,,$$

- Hadronic matrix element in terms of four form factors

$$\begin{aligned} \langle (P(p_1)P(p_2)P(p_3))^- | (V-A)^\mu | 0 \rangle &= V_1^\mu F_1^A(Q^2, s_1, s_2) + V_2^\mu F_2^A(Q^2, s_1, s_2) \,, \\ &+ Q^\mu F_3^A(Q^2, s_1, s_2) + i V_4^\mu F_4^V(Q^2, s_1, s_2) \,, \end{aligned}$$

where

$$\begin{aligned} V_1^\mu &= \left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} \right) (p_1 - p_3)_\nu \,, \quad V_2^\mu = \left(g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} \right) (p_2 - p_3)_\nu \,, \\ V_4^\mu &= \varepsilon^{\mu\alpha\beta\gamma} p_{1\alpha} p_{2\beta} p_{3\gamma} \,, \quad Q^\mu = (p_1 + p_2 + p_3)^\mu \,, \quad s_i = (Q - p_i)^2 \,, \end{aligned}$$

$\tau^- \to (PPP)^- \nu_\tau$: Basics

- Generic Amplitude for a 3-meson decay of the τ

$$\mathcal{M}(\tau^- \to (PPP)^- \nu_\tau) = \frac{G_F}{\sqrt{2}} |V_{ij}| \bar{u}_{\nu_\tau} \gamma_\mu (1 - \gamma_5) u_\tau \langle (PPP)^- | (V-A)^\mu | 0 \rangle \,,$$

- hadronic matrix element in terms of four form factors

$$\begin{aligned} \langle (P(p_1)P(p_2)P(p_3))^- | (V-A)^\mu | 0 \rangle &= V_1^\mu F_1^A(Q^2, s_1, s_2) + V_2^\mu F_2^A(Q^2, s_1, s_2)\,, \\ &+ Q^\mu F_3^A(Q^2, s_1, s_2) + i V_4^\mu F_4^V(Q^2, s_1, s_2)\,, \end{aligned}$$

  - $F_{1,2}^A(Q^2, s_1, s_2)$: $J^P = 1^+$ transition (axial-vector form factors)
  - $F_3^A(Q^2, s_1, s_2)$: $J^P = 0^-$ transition (pseudoscalar form factor)
  - $F_4^V(Q^2, s_1, s_2)$: $J^P = 1^-$ transition (vector form factor)

$\tau \to \pi^-\pi^+\pi^-\nu_\tau$

- Bose symmetry: $F_1^A(Q^2, s_1, s_2) = F_2^A(Q^2, s_2, s_1) \equiv F_A(Q^2, s_1, s_2)$
- Conservation of J_A^μ in the chiral limit: $F_3^A(Q^2, s_1, s_2)$ must vanish with m_π^2
- G-parity conservation: $F_4^V(Q^2, s_1, s_2) = 0$
- The axial-vector hadronic current takes the form

$$J_A^\mu = F_A(Q^2, s_1, s_2)V_1^\mu + F_A(Q^2, s_2, s_1)V_2^\mu \,,$$

- Decay rate

$$\frac{d\Gamma(\tau^- \to \pi^-\pi^-\pi^+\nu_\tau)}{dQ^2} = \frac{G_F^2|V_{ud}|^2}{32\pi^2 M_\tau}\left(M_\tau^2 - Q^2\right)^2\left(1 + \frac{2Q^2}{M_\tau^2}\right)a_1(Q^2)\,,$$

$\tau \to \pi^- \pi^+ \pi^- \nu_\tau$

- Spectral function

$$a_1(Q^2) \quad = \quad \frac{1}{768\pi^3} \frac{1}{Q^4} \int_{s_{1,\min}}^{s_1^{\max}} ds_1 \int_{s_{2,\min}}^{s_2^{\max}} ds_2 \, W_A \, .$$

where

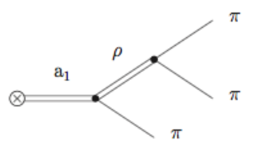
$$W_A \quad = \quad -\left[V_1^\mu F_A(Q^2, s_1, s_2) + V_2^\mu F_A(Q^2, s_2, s_1)\right] \times$$
$$\left[V_{1\mu} F_A(Q^2, s_1, s_2) + V_{2\mu} F_A(Q^2, s_2, s_1)\right] \, ,$$

- For comparison of theory and experiment

$$\frac{\Gamma(\tau^- \to (3\pi)^- \nu_\tau)}{\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} \frac{1}{N_{\text{events}}} \frac{dN_{\text{events}}}{dQ^2} = \frac{6\pi |V_{ub}|^2 S_{EW}}{M_\tau^2} \left(1 - \frac{Q^2}{M_\tau^2}\right)^2 \left(1 + 2\frac{Q^2}{M_\tau^2}\right) a_1(Q^2)$$

Evaluation of the axial-vector form factor

- Factorization ansatz: $\tau \to \nu_\tau a_1 \to \nu_\tau \rho\pi \to 3\pi$



$$F_A(Q^2, s_1, s_2) = F_{a_1}(Q^2) F_\rho(s_2) \,,$$

  - $F_{a_1}(Q^2)$ accounts for the $a_1(1260)$-resonance production
  - $F_\rho(s_i) = \Omega(s_i)$ (Omnès): line-shape of the ρ-meson ($\rho \to \pi\pi$)

$$F_\rho(s_i) \to 1 \,, \quad s_i \to 0 \,.$$

  - Axial-vector current

$$J_A^\mu = F_{a_1}(Q^2) \left[F_\rho(s_2) V_{1\mu} + F_\rho(s_1) V_{2\mu} \right] \,,$$

  - Isospin relation: same predictions for both modes (in the isospin limit)

$$F^A(Q^2, s_1, s_2) = -F^A_{\pi^0\pi^0\pi^-}(Q^2, s_1, s_2) \,,$$

Evaluation of the axial-vector form factor

- ChPT prediction at $\mathcal{O}(p^2)$

$$J_A^\mu|_{\mathrm{ChPT}}^{\mathcal{O}(p^2)} = -\frac{2\sqrt{2}}{3F_\pi}\left(V_{1\mu} + V_{2\mu}\right) ,$$

- Normalization of $F_{a_1}(Q^2)$

$$F_{a_1}(Q^2) = -\frac{2\sqrt{2}}{3F_\pi} f_{a_1}(Q^2) , \quad f_{a_1}(Q^2 \to 0) \to 1 ,$$

- In this framework, the axial spectral function reads:

$$a_1(Q^2) = \frac{1}{768\pi^3}\left(-\frac{2\sqrt{2}}{3F_\pi}\right)^2 |f_{a_1}(Q^2)|^2 \frac{g(Q^2)}{Q^2} ,$$

where

$$g(Q^2) = \frac{1}{Q^2}\int_{s_{1,\mathrm{min}}}^{s_1^{\mathrm{max}}} ds_1 \int_{s_{2,\mathrm{min}}}^{s_2^{\mathrm{max}}} ds_2 \left\{ - V_1^2|F_\rho(s_2)|^2 - V_2^2|F_\rho(s_1)|^2 - 2V_1V_2\mathrm{Re}\left[F_\rho(s_1)(F_\rho(s_2))^*\right] \right\} ,$$

Breit-Wigner evaluation of the axial-vector f.f.

- Breit-Wigner (non-dispersive): $a_1(1260)$

$$f_{a_1}(Q^2)|^{1\,\mathrm{res}}_{\mathrm{BW}} = \frac{m_{a_1}^2}{m_{a_1}^2 - Q^2 - i m_{a_1}\Gamma_{a_1}(Q^2)}\,, \quad \Gamma_{a_1}(Q^2) = \gamma_{a_1}\frac{g(Q^2)}{g(m_{a_1}^2)}$$

- Breit-Wigner (dispersive)

$$f_{a_1}(Q^2)|^{1\,\mathrm{res}}_{\mathrm{BW\,disp}} = \frac{m_{a_1}^2 + \mathrm{Re}\Pi_{a_1}(0)}{m_{a_1}^2 - Q^2 + \mathrm{Re}\Pi_{a_1}(Q^2) - i m_{a_1}\Gamma_{a_1}(Q^2)}\,,$$

$$\mathrm{Re}\Pi_{a_1}(Q^2) = \mathcal{H}_{a_1}(Q^2) - \mathcal{H}_{a_1}(m_{a_1}^2)\,, \quad \mathcal{H}_{a_1}(Q^2) = -\frac{Q^2}{\pi}\int_{9m_\pi^2}^{s_{\mathrm{cut}}} ds' \frac{m_{a_1}\Gamma_{a_1}(s')}{(s')(s'-Q^2)}\,,$$

- Breit-Wigner (dispersive): $a_1(1260) + a_1(1640)$

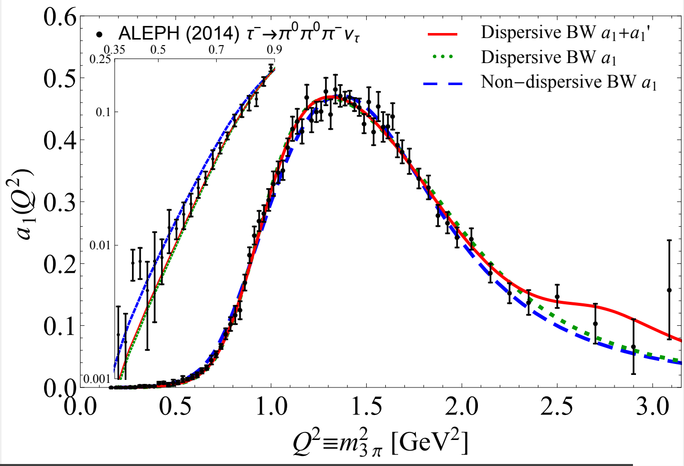
$$\begin{aligned} f_{a_1}(Q^2)|^{2\,\mathrm{res}}_{\mathrm{BW\,disp}} &= \frac{1}{1+|\kappa|e^{i\phi}}\left[\frac{m_{a_1}^2 + \mathrm{Re}\Pi_{a_1}(0)}{m_{a_1}^2 - Q^2 + \mathrm{Re}\Pi_{a_1}(Q^2) - i m_{a_1}\Gamma_{a_1}(Q^2)}\right. \\ &+ \left.|\kappa|e^{i\phi}\frac{m_{a_1'}^2 + \mathrm{Re}\Pi_{a_1'}(0)}{m_{a_1'}^2 - Q^2 + \mathrm{Re}\Pi_{a_1'}(Q^2) - i m_{a_1'}\Gamma_{a_1'}(Q^2)}\right]\,, \end{aligned}$$

Fits to the $\tau \to \pi^0\pi^0\pi^-\nu_\tau$ axial spectral function

Breit-Wigner $m_{a_1} = 1271(13)$ MeV, $\gamma_{a_1} = 523(18)$ MeV, $\mathcal{N} = 1.59(7)$, $\chi^2_{\rm dof} = 1.16$

$m_{a_1} = 1293(10)$ MeV, $\gamma_{a_1} = 501(11)$ MeV, $\mathcal{N} = 0.88(2)$, $\chi^2_{\rm dof} = 1.01$

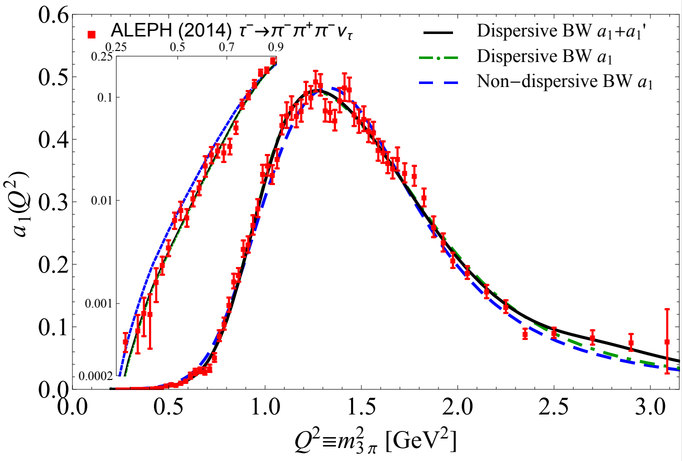
$m_{a_1} = 1293(6)$ MeV, $\gamma_{a_1} = 485(10)$ MeV, $|\kappa| = 0.12(4)$, $\mathcal{N} = 0.97(5)$, $\chi^2_{\rm dof} = 0.86$



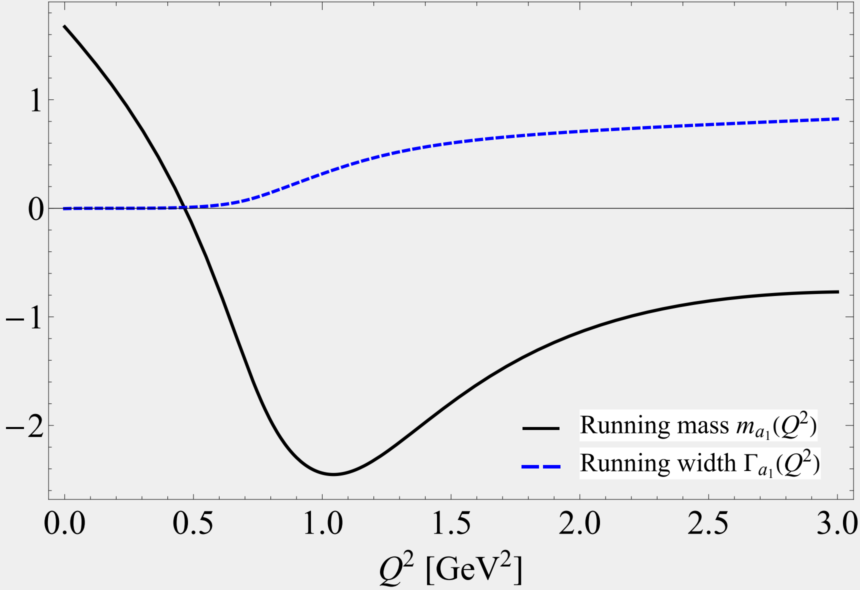
Breit-Wigner: $m_{a_1} = 1243(6)$ MeV, $\gamma_{a_1} = 480(8)$ MeV, $\mathcal{N} = 1.43(3)$, $\chi^2_{\rm dof} = 3.11$

$m_{a_1} = 1259(6)$ MeV, $\gamma_{a_1} = 474(8)$ MeV, $\mathcal{N} = 0.81(2)$, $\chi^2_{\rm dof} = 1.51$

$m_{a_1} = 1260(5)$ MeV, $\gamma_{a_1} = 467(8)$ MeV, $|\kappa| = 0.05(2)$, $\mathcal{N} = 0.85(2)$, $\chi^2_{\rm dof} = 1.43$



Running $a_1(1260)$ mass and width



- Dispersive description

$$f_{a_1}(Q^2) = \exp\left[\alpha_1 Q^2 + \frac{Q^4}{\pi}\int_{9m_\pi^2}^{s_{\mathrm{cut}}} ds' \frac{\delta(s')}{(s')^2(s'-Q^2-i\varepsilon)}\right],$$

$$\tan\delta(Q^2) = \frac{\mathrm{Im} f_{a_1}(Q^2)|^{1\,\mathrm{res}}_{\mathrm{BW\,disp}}}{\mathrm{Re} f_{a_1}(Q^2)|^{1\,\mathrm{res}}_{\mathrm{BW\,disp}}},$$

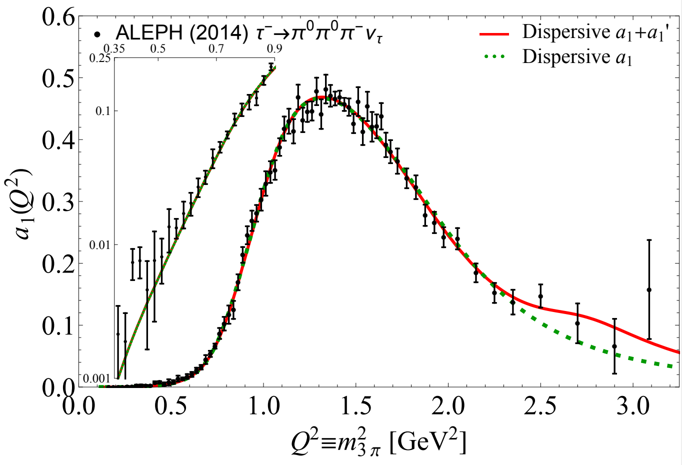
$$\alpha_k^{\mathrm{s.r.}} = \frac{k!}{\pi}\int_{9m_\pi^2}^{s_{\mathrm{cut}}} ds' \frac{\delta(s')}{s'^{k+1}}.$$

  - α_1: fit parameter that absorbs other production mechanism

Fits to the $\tau \to \pi^0\pi^0\pi^-\nu_\tau$ axial spectral function

Dispersive: $m_{a_1} = 1302(8)$ MeV, $\gamma_{a_1} = 493(11)$ MeV, $\alpha_1 = 0.59(1)$, $\alpha_1^{\text{s.r.}} = 0.64(1)$, $\chi^2_{\text{dof}} = 0.96$

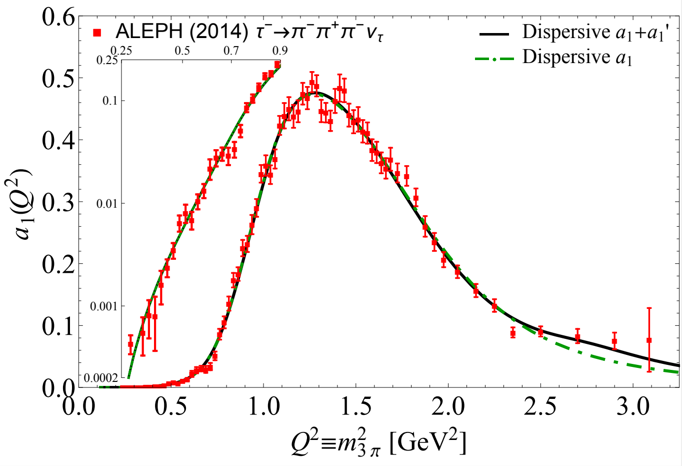
$m_{a_1} = 1296(6)$ MeV, $\gamma_{a_1} = 483(10)$ MeV, $\alpha_1 = 0.60(1)$, $\alpha_1^{\text{s.r.}} = 0.62(1)$, $|\kappa| = 0.10(4)$, $\chi^2_{\text{dof}} = 0.88$



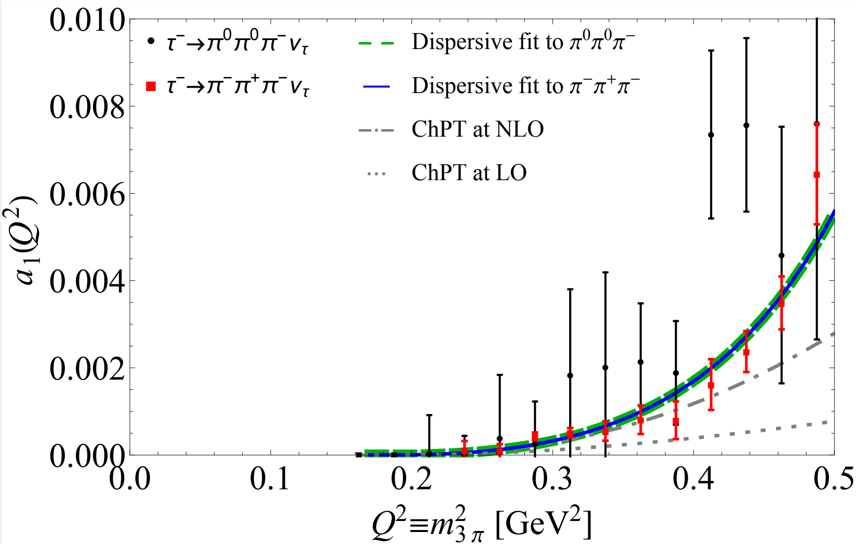
FITS TO THE $\tau \to \pi^+\pi^-\pi^-\nu_\tau$ AXIAL SPECTRAL FUNCTION

Dispersive: $m_{a_1} = 1277(5)$ MeV, $\gamma_{a_1} = 475(8)$ MeV, $\alpha_1 = 0.58(1)$, $\alpha_1 = 0.66(1)$, $\chi^2_{\rm dof} = 1.49$

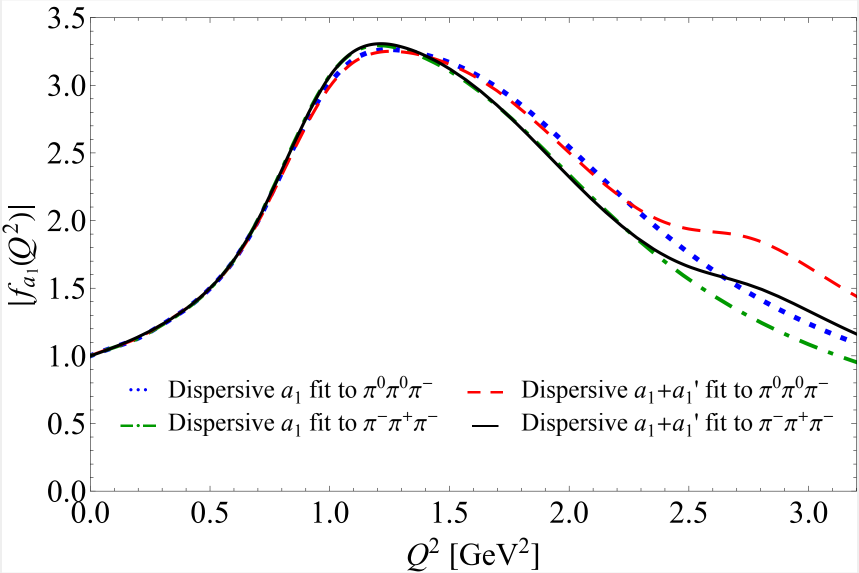
$m_{a_1} = 1273(5)$ MeV, $\gamma_{a_1} = 466(8)$ MeV, $\alpha_1 = 0.59(1)$, $\alpha_1^{\rm s.r.} = 0.64(1)$, $|\kappa| = 0.06(2)$, $\chi^2_{\rm dof} = 1.39$



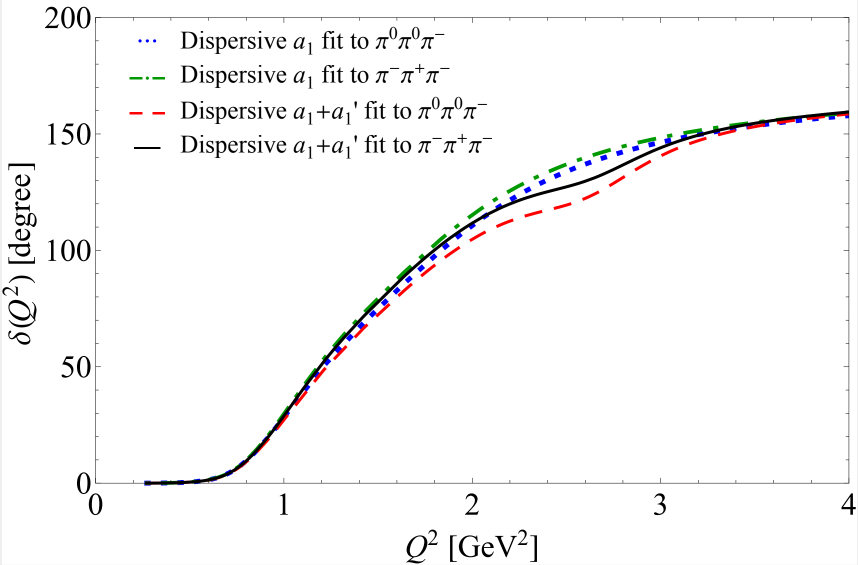
Fits to $\tau \to 3\pi\nu_\tau$: Low-Q^2 region



Axial-Vector Form Factor



Axial-Vector Form Factor



Outlook

Outlook

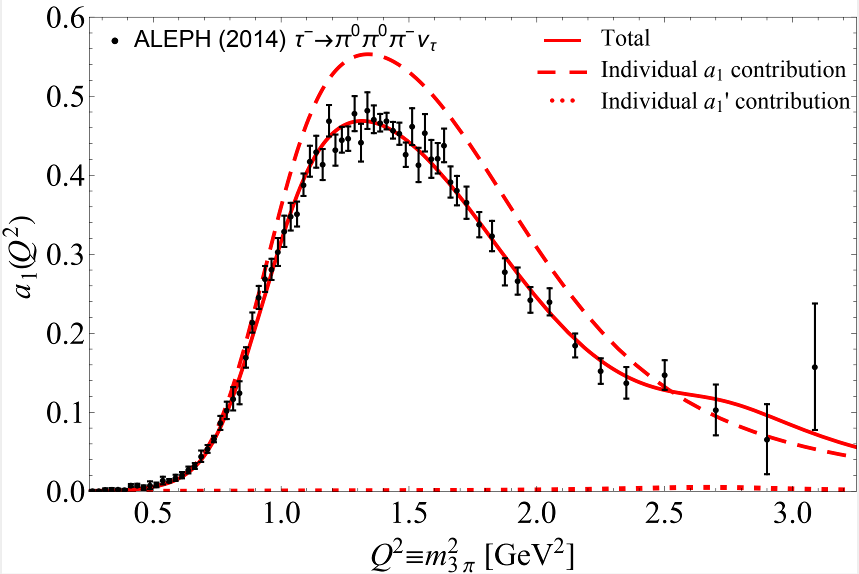
- Tau physics is a very rich field to test QCD and EW
- Important experimental activities: Belle (II), BaBar, LHCb, BESIII
- Form Factors from dispersion relations with subtractions
  - $F_V^\pi(s)$: important for testing QCD dynamics and the SM
  - Determination of the $\rho(1450)$ and $\rho(1700)$ properties limited by theoretical uncertainties
  - Higher-quality data on $\tau^- \to K_S K^- \nu_\tau$: improve $\rho(1450)$ and $\rho(1700)$ parameters
  - $\tau \to 3\pi\nu_\tau$ decays: input for the axial-vector form factor of the nucleon
- A lot of interesting physics to be done in the tau sector

Variant (IV)

- Fits for different $s_{\rm cut}$ and allowing the ρ-mass to float

Fits	Parameter	$s_{\rm cut}$ [GeV²]			
		m_τ^2	4 (reference fit)	10	∞
Fit 1-ρ	α_1 [GeV^{-2}]	1.88(1)	1.88(1)	1.89(1)	1.88(1)
	α_2 [GeV^{-4}]	4.37(3)	4.34(1)	4.31(3)	4.34(1)
	m_ρ [MeV]	773.9(3)	773.8(3)	773.9(3)	773.9(3)
	M_ρ [MeV]	$= m_\rho$	$= m_\rho$	$= m_\rho$	$= m_\rho$
	$M_{\rho'}$ [MeV]	1382(71)	1375(11)	1316(9)	1312(8)
	$\Gamma_{\rho'}$ [MeV]	516(165)	608(35)	728(92)	726(26)
	$M_{\rho''}$[MeV]	1723(1)	1715(22)	1655(1)	1656(8)
	$\Gamma_{\rho''}$ [MeV]	315(271)	455(16)	569(160)	571(13)
	γ	0.12(13)	0.16(1)	0.18(2)	0.17(1)
	ϕ_1	−0.56(35)	−0.69(1)	−1.40(19)	−1.41(8)
	δ	−0.09(3)	−0.13(1)	−0.17(4)	−0.17(3)
	ϕ_2	−0.19(69)	−0.45(12)	−1.06(10)	−1.05(11)
	χ^2/d.o.f	1.09	0.70	0.63	0.66

Spectral function: individual contributions



Spectral function: individual contributions