Gravitational waves from ultra-supercooled first-order phase transitions

Ryusuke Jinno (DESY)

Mainly based on 1905.00899
with Hyeonseok Seong (IBS & KAIST), Masahiro Takimoto (Weizmann), Choong Min Um (KAIST)
and partly based on 1707.03111 with Masahiro Takimoto

15.11.2019 @ IFAE

TALK PLAN

1. Introduction

2. Difficulty in estimating GWs in ultra-supercooled transitions

3. An approach:

   Effective description of fluid propagation & Implications to GW production

4. Summary

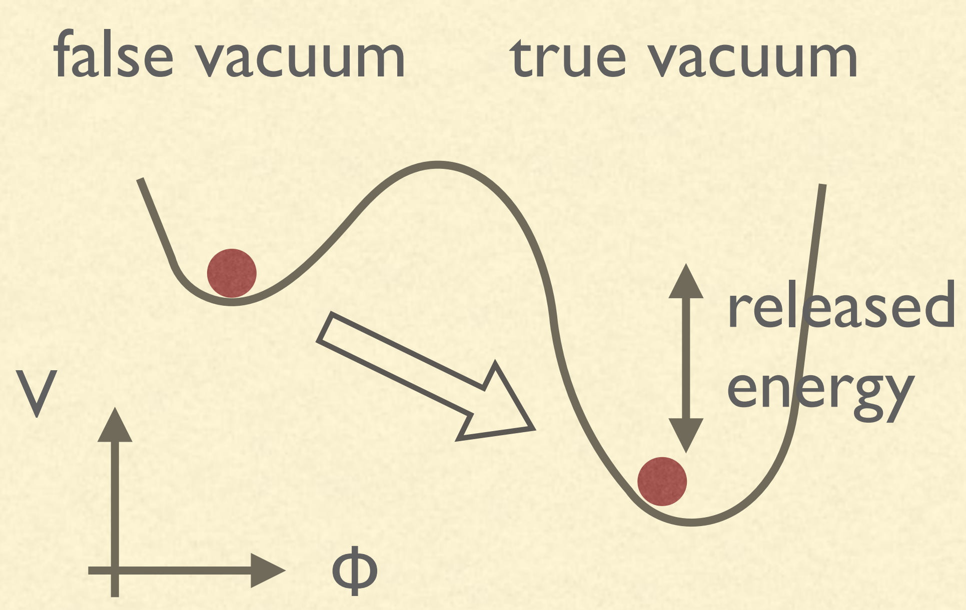
Introduction

FIRST-ORDER PHASE TRANSITION & GWS

- Rough sketch of 1st-order phase transition & GW production

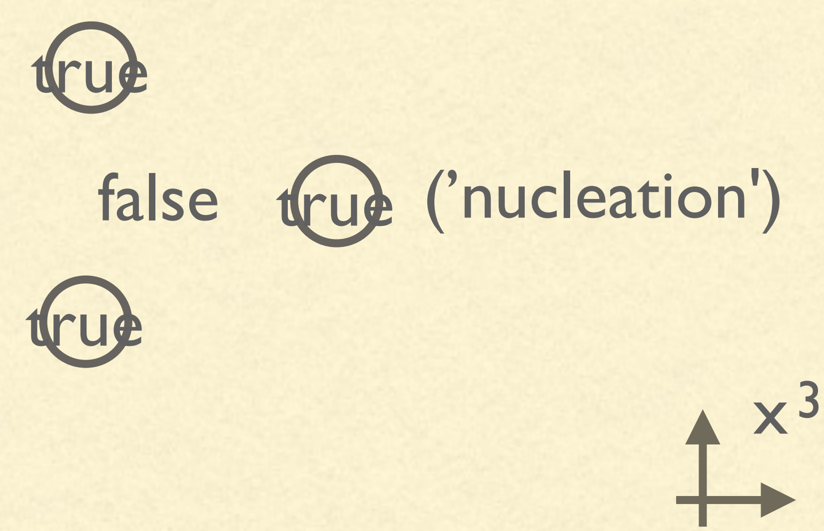
Bubbles nucleate, expand, collide and disappear, accompanying fluid dynamics

Field space



Quantum tunneling

Position space



Bubble formation & GW production

FIRST-ORDER PHASE TRANSITION & GWS

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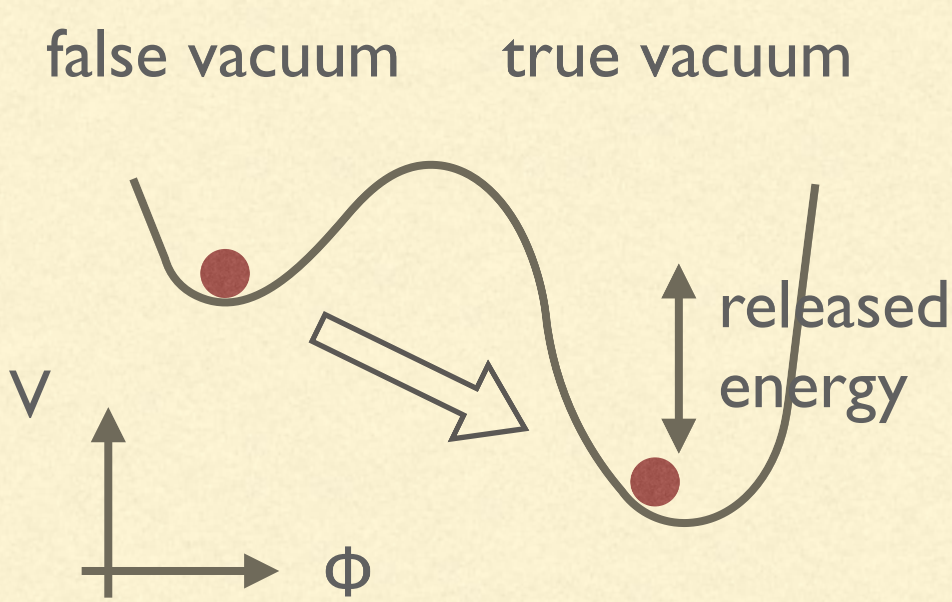

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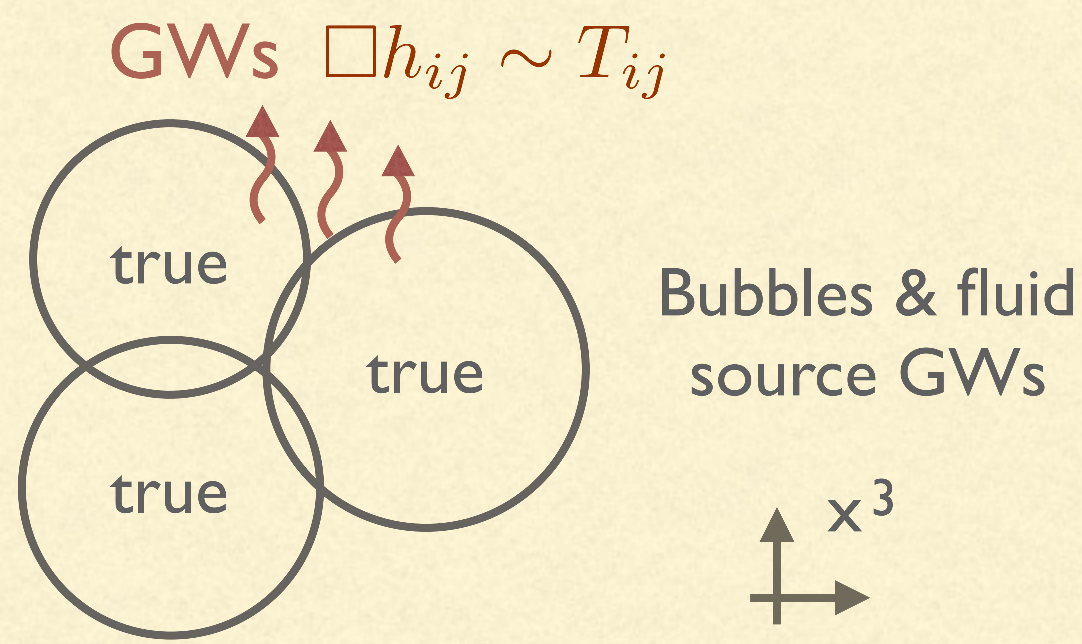
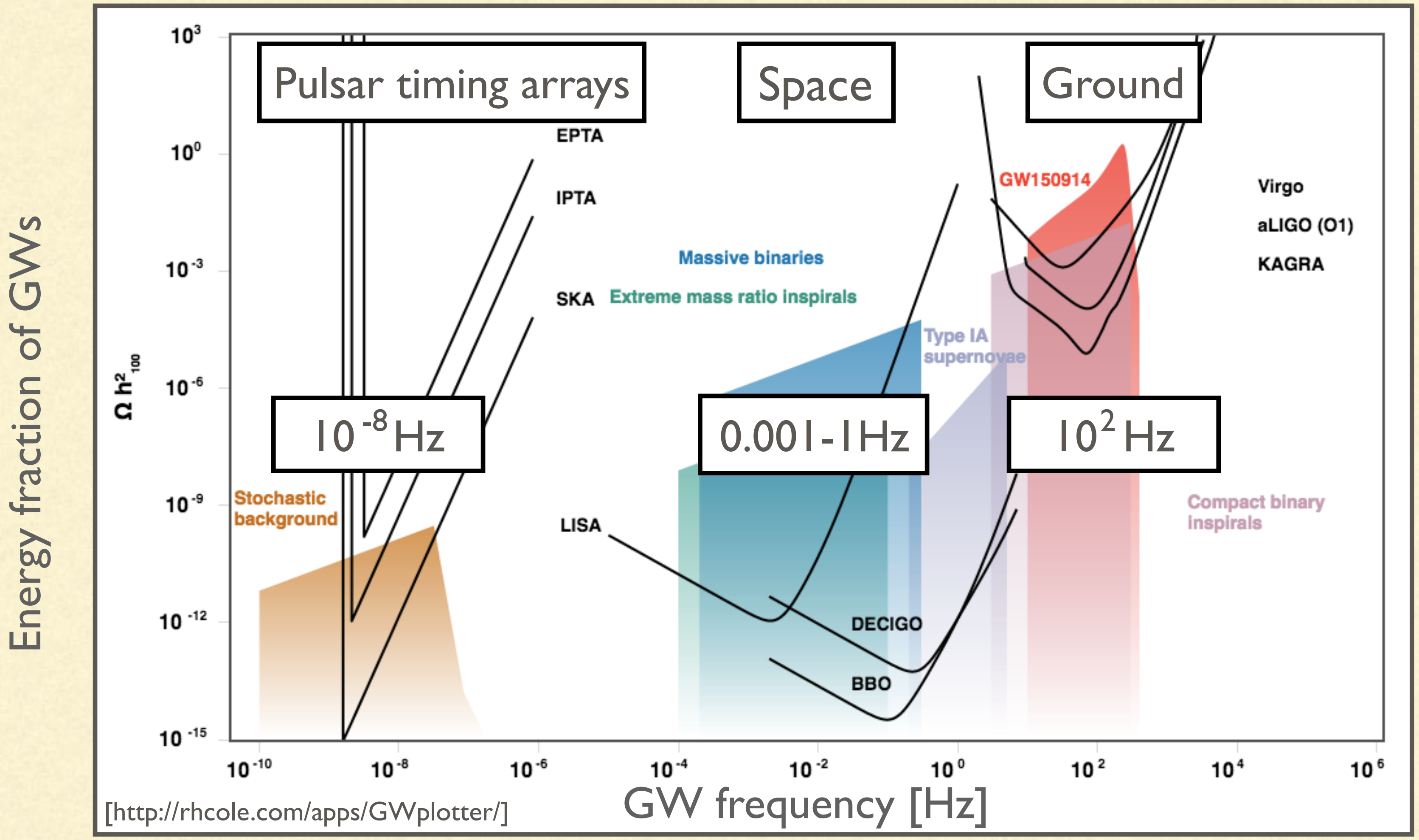


Bubble formation & GW production

FIRST-ORDER PHASE TRANSITION & GWS

- 10^{-3} ~ 1Hz GWs correpond to electroweak physics and beyond



Temperature of the Universe

@ transition time

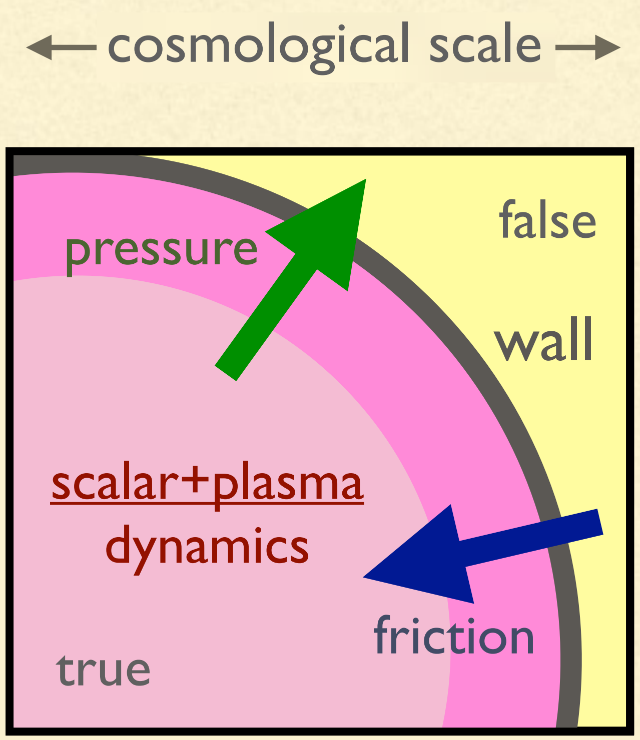
MeV GeV TeV PeV

Note :

$\beta / H_* \sim 10^3$

BUBBLE DYNAMICS BEFORE COLLISION

- "Pressure vs. friction" determines behavior of bubbles



- Two main players : **scalar field and plasma**

- Walls want to expand ("pressure")

  Parametrized by $\alpha \equiv \dfrac{\rho_{\rm vac}}{\rho_{\rm plasma}}$

- Walls are pushed back by plasma ("friction")

  Parametrized by coupling η btwn. scalar and plasma

- Let's see how bubbles behave for different α

  (with fixed coupling η)

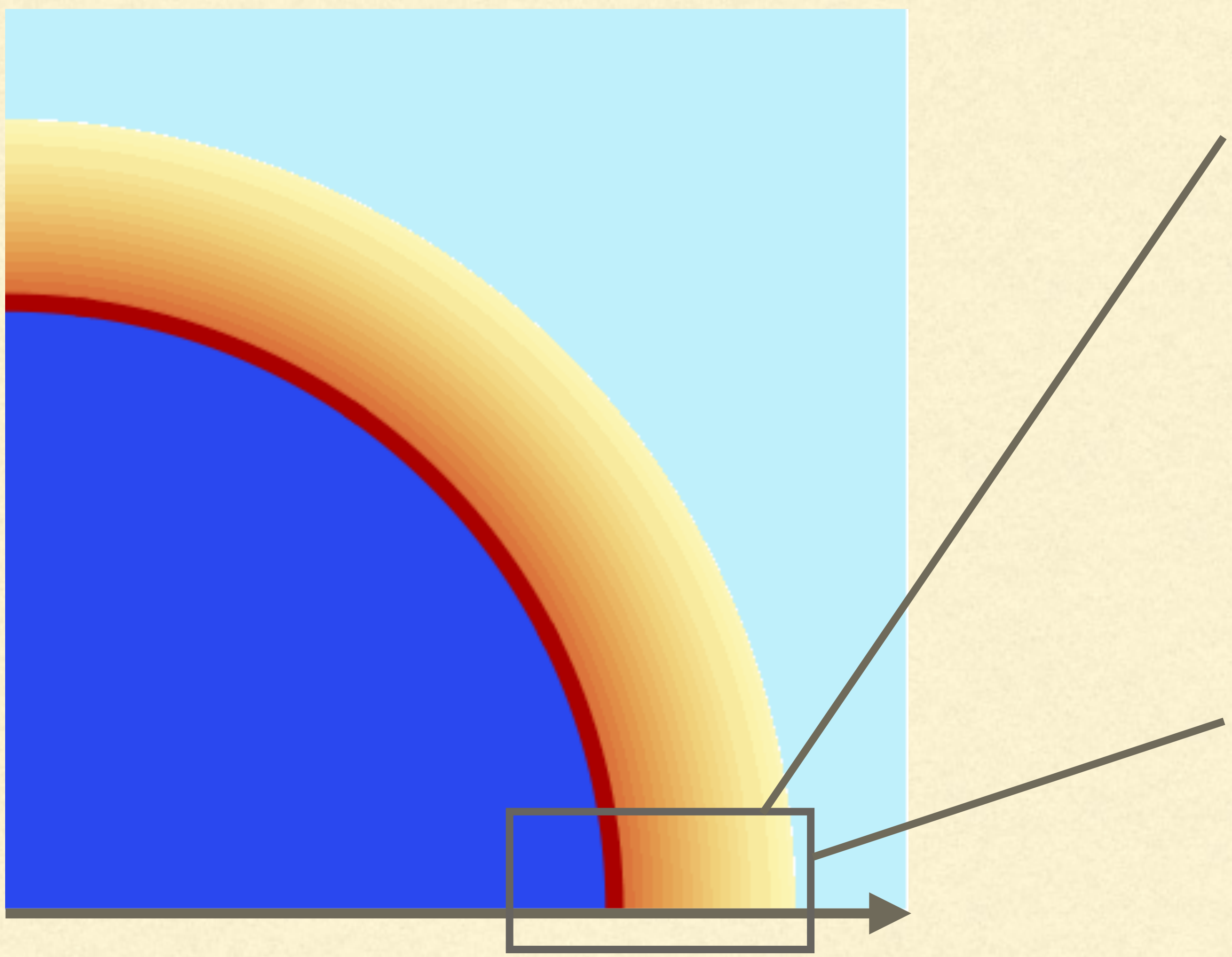
BUBBLE DYNAMICS BEFORE COLLISION

$$\alpha \equiv \frac{\rho_{\rm vac}}{\rho_{\rm plasma}}$$

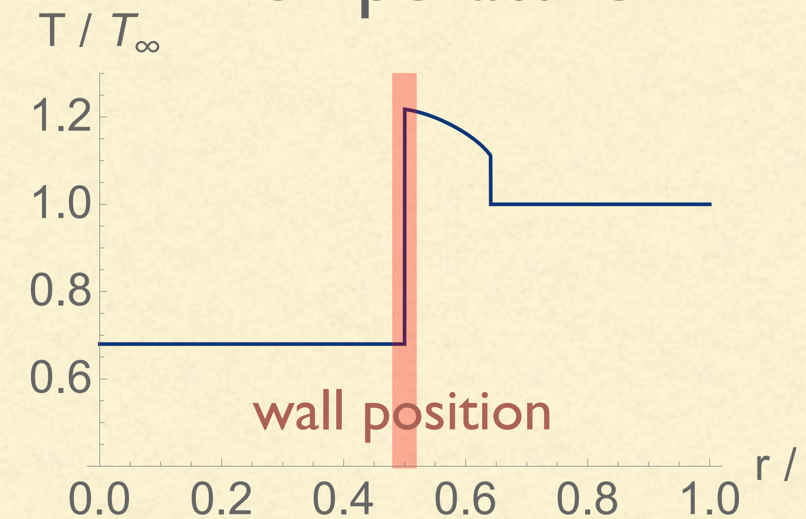
[Espinosa, Konstandin, No, Servant '10]

- Small α (say, $\alpha \lesssim \mathcal{O}(0.1)$)

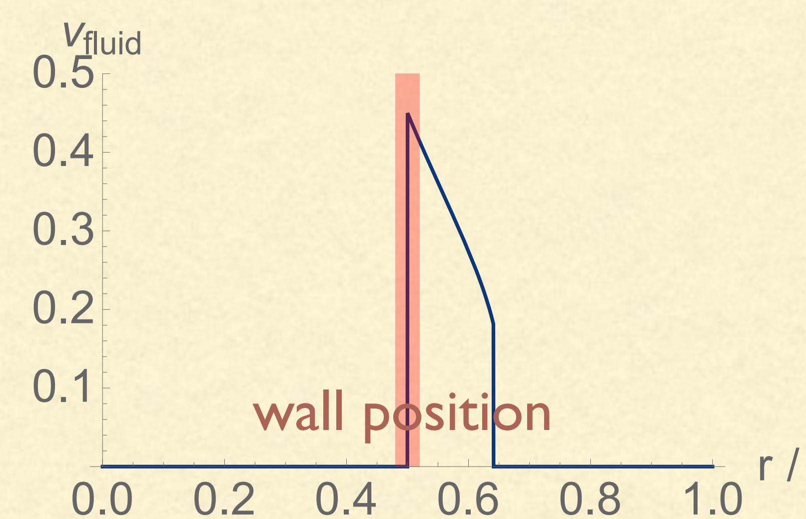
"deflagration"



Temperature



Fluid outward velocity



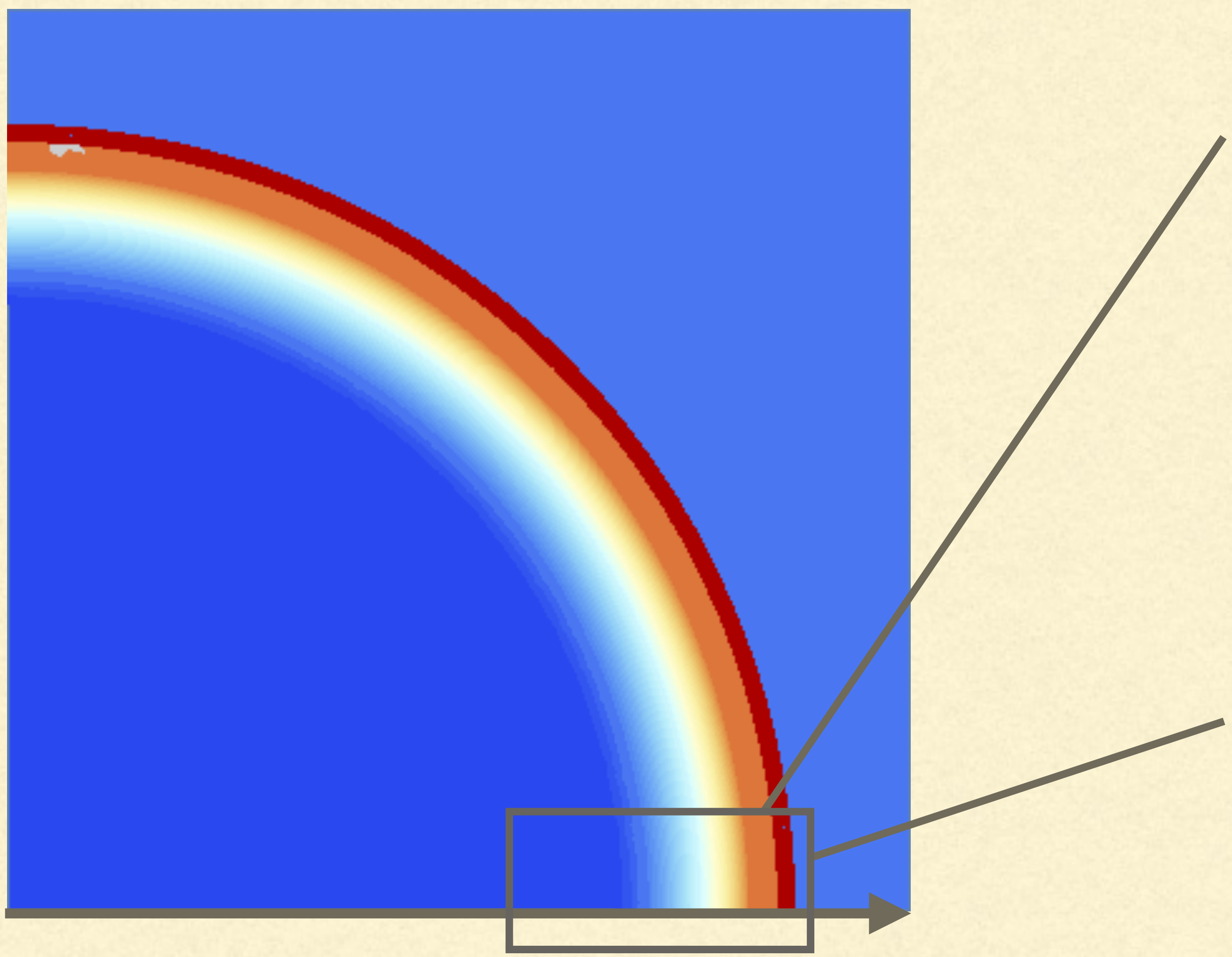
BUBBLE DYNAMICS BEFORE COLLISION

$$\alpha \equiv \frac{\rho_{\rm vac}}{\rho_{\rm plasma}}$$

[Espinosa, Konstandin, No, Servant '10]

- Small but slightly increased α

"detonation"



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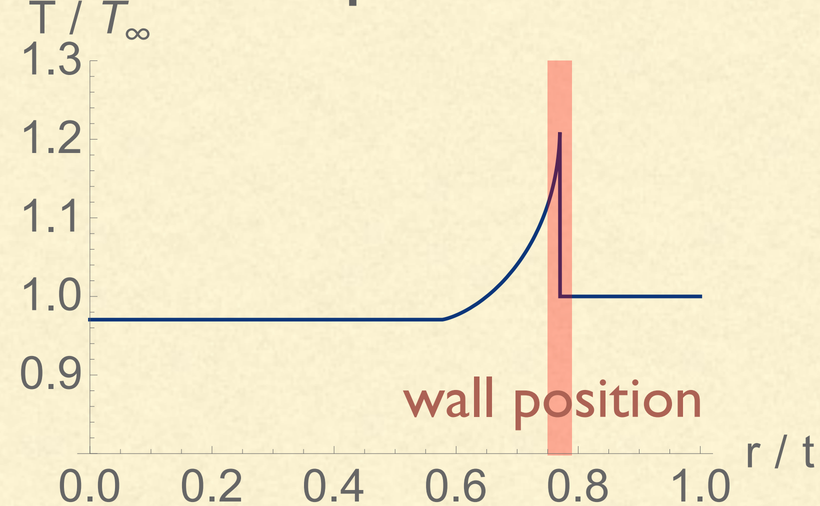

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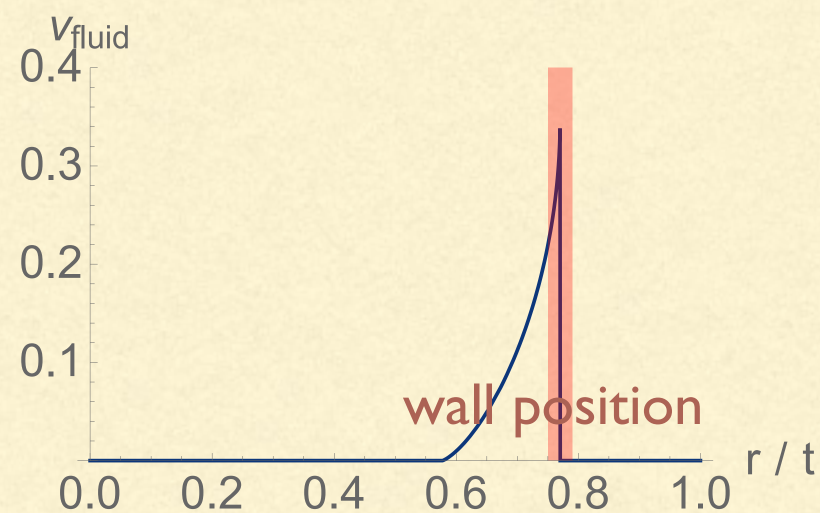
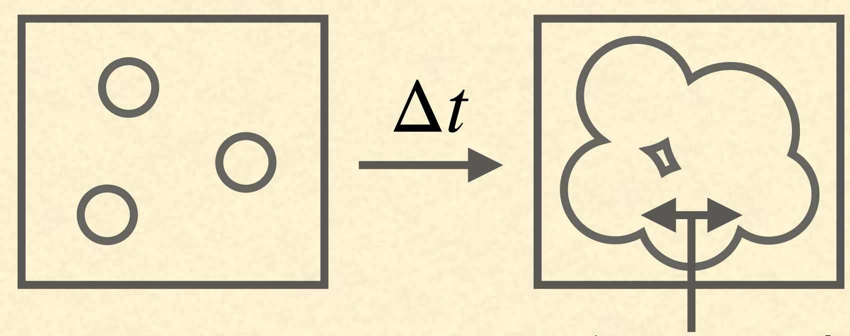
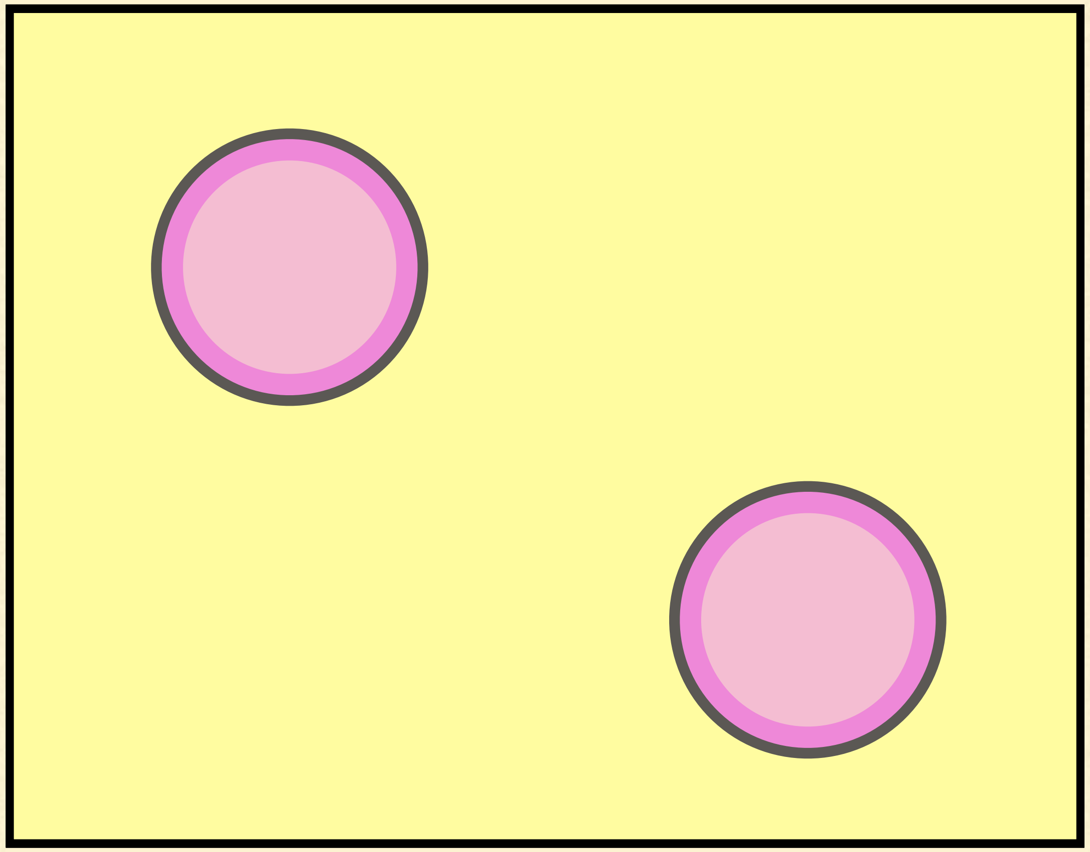


PARAMETERS CHARACTERIZING THE TRANSITION

	Definition	**Properties**
α	$\rho_{\mathrm{vac}}/\rho_{\mathrm{plasma}}$	Strength of the transition
β	Bubble nucleation rate Taylor-expanded around the transition time t_* $\Gamma(t) \propto e^{\beta(t-t_*)}$	Bubbles collide $\Delta t \sim 1/\beta$ after nucleation  Typical bubble size $\sim v_w \Delta t \sim v_w/\beta$
v_w	Wall velocity	Determined by the balance btwn. pressure & friction
T_*	Transition temperature	

DYNAMICS AFTER COLLISION

Bubbles nucleate & expand



- Nucleation rate (per unit time & vol)

$$\Gamma(t) \propto e^{\beta(t-t_*)}$$

- Released energy is mostly carried by fluid motion, not by the scalar field, unless α is extremely large [Bodeker & Moore '17]
- Collision occurs $\Delta t \sim 1/\beta$ after nucleation

DYNAMICS AFTER COLLISION

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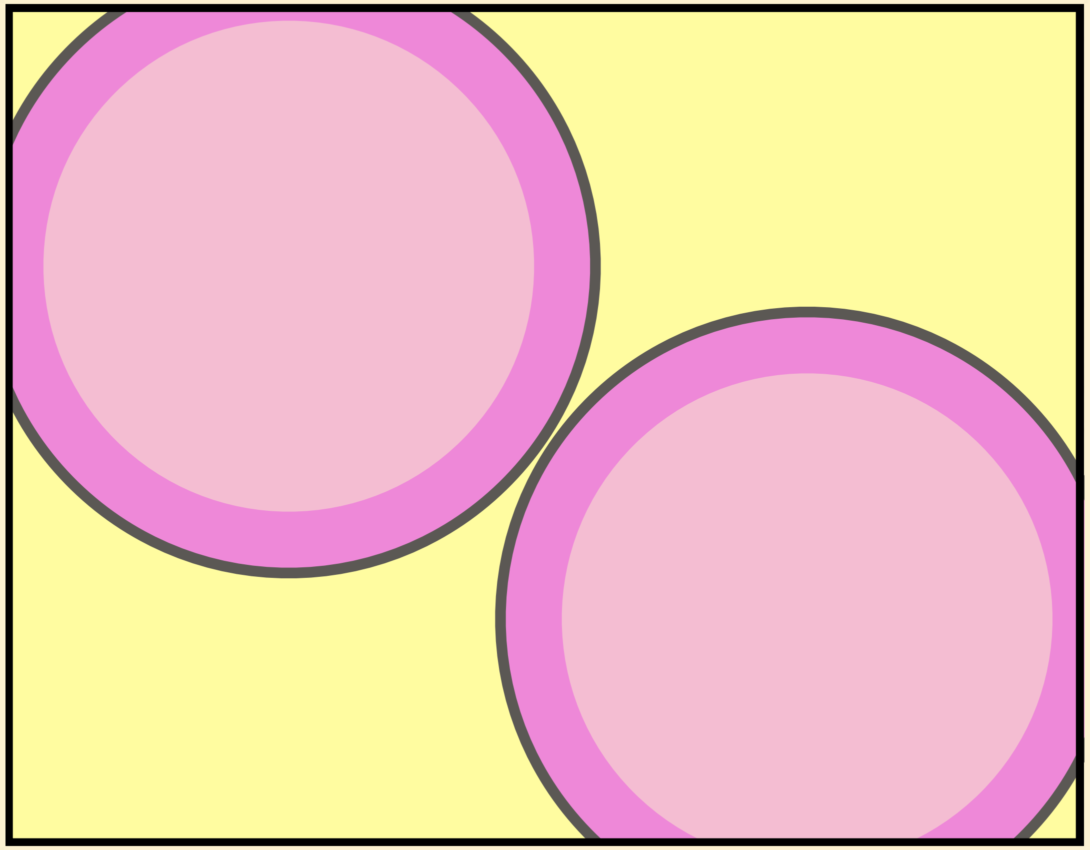

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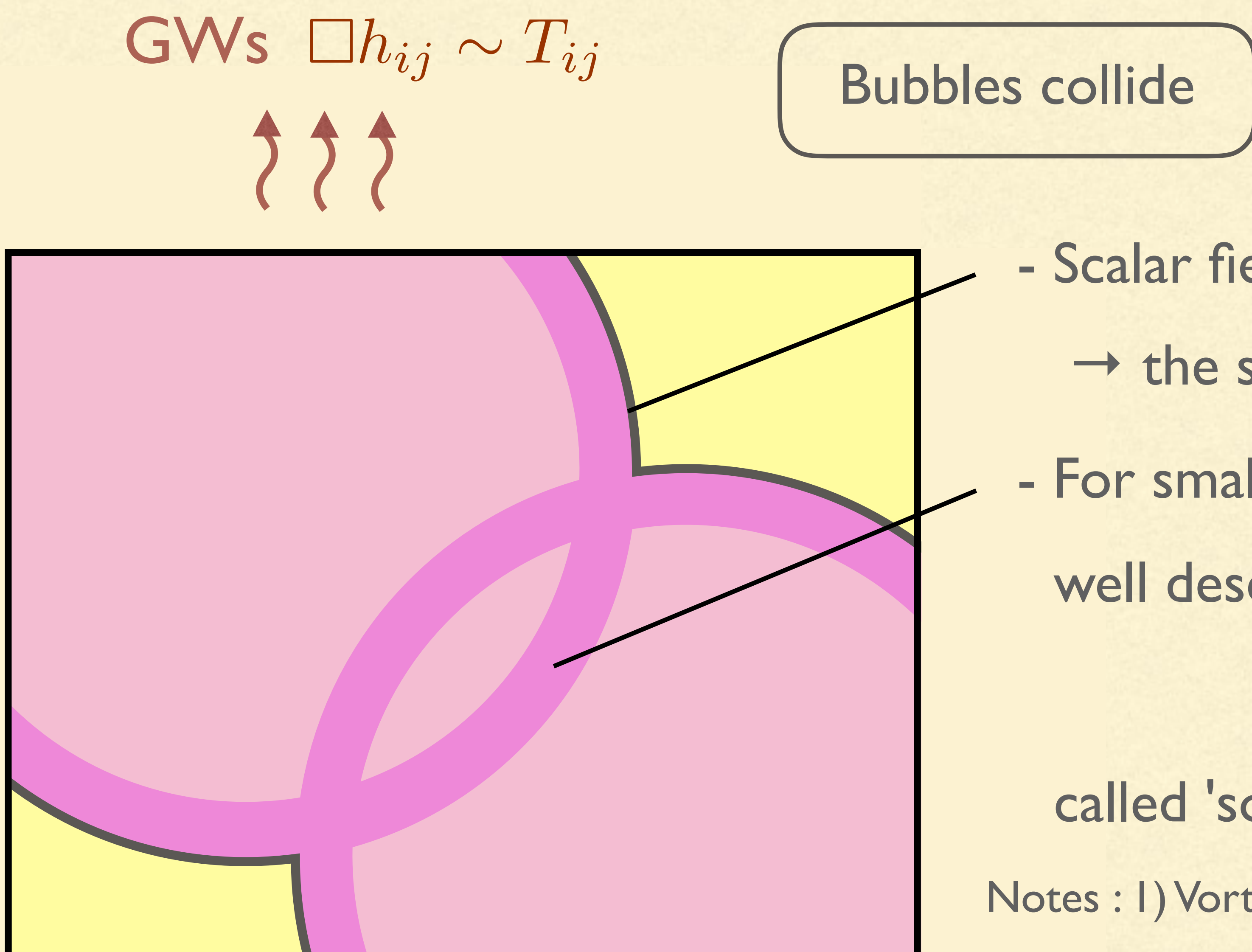

- Scalar field damps soon after collision

  → the system becomes fluid-only after this

- For small α ($\lesssim \mathcal{O}(0.1)$), plasma motion is well described by linear approximation:

$$(\partial_t^2 - c_s^2 \nabla^2)\overrightarrow{v}_{\text{fluid}} \simeq 0$$

  called 'sound waves' or 'compression waves'

Notes : 1) Vorticity is neglected

2) Bubbles deviate from sphere after collision

3) Fluid shell thickness is fixed at the time of collision

DYNAMICS AFTER COLLISION

GWs $\Box h_{ij} \sim T_{ij}$

Turbulence develops

- Fluid nonlinarity and magnetic field become important at late times

'turbulence'

- Different modeling of turbulence gives different GW spectrum

see e.g. Gogoberidze, Kahniashvili, Kosowsky PRD76 (2007)
Caprini, Durrer, Servant JCAP 0912(2009)
Niksa, Schlederer, Sigl CQG35(2018)
Mandal, Brandenburg, Kahniashvili, Kosowsky 1903.08585

SOURCES OF GWS IN FIRST-ORDER PHASE TRANSITION

- Time evolution of the system

Bubble nucleation & expansion → Collision → Sound waves → Turbulence

- Resulting GW spectrum is classified accordingly:

e.g. [Caprini et al., 1512.01236]
[Caprini et al. 1910.13125]

$$\Omega_{\rm GW} = \Omega_{\rm GW}^{\rm (coll)} + \Omega_{\rm GW}^{\rm (sw)} + \Omega_{\rm GW}^{\rm (turb)}$$

from scalar walls   from fluid motion

- Typically $\Omega_{\rm GW}^{\rm (sw)}$ is the largest (→ later)

[Hindmarsh, Huber, Rummukainen, Weir '13, '15, '17]

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SOURCES OF GWS IN FIRST-ORDER PHASE TRANSITION

- Time evolution & resulting GW spectrum

Bubble nucleation & expansion → Collision → Sound waves → Turbulence

$$\Omega_{\rm GW} = \Omega_{\rm GW}^{\rm (coll)} + \Omega_{\rm GW}^{\rm (sw)} + \Omega_{\rm GW}^{\rm (turb)}$$

- Typically $\Omega_{\rm GW}^{\rm (sw)}$ is the largest because of different parameter dependence:

$\Omega_{\rm GW}^{\rm (coll)}$ (from scalar walls) $\propto \left(\frac{\kappa_{\rm scalar}\,\alpha}{1+\alpha}\right)^2 \left(\frac{\beta}{H_*}\right)^{-2}$

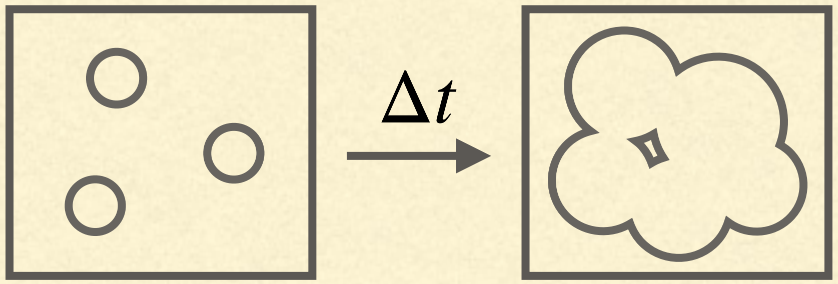
$\Omega_{\rm GW}^{\rm (sw)}$ (from sound waves) $\propto \left(\frac{\kappa_{\rm fluid}\,\alpha}{1+\alpha}\right)^2 \left(\frac{\beta}{H_*}\right)^{-1}$

Note : $\frac{\beta}{H_*} \sim 10^{1-5} \gg 1$

parametrizes the duration of the transition

GW ENHANCEMENT BY SOUND WAVES

- Reason for different dependence on β/H_*



Bubble collision

Bubbles collide and disappear within timescale $\Delta t \sim 1/\beta$

[However, see RJ, Takimoto 1707.03111, RJ, Takimoto, Konstandin 1906.02588]

GWs are sourced during this preiod $h_{ij} \propto \Delta t$

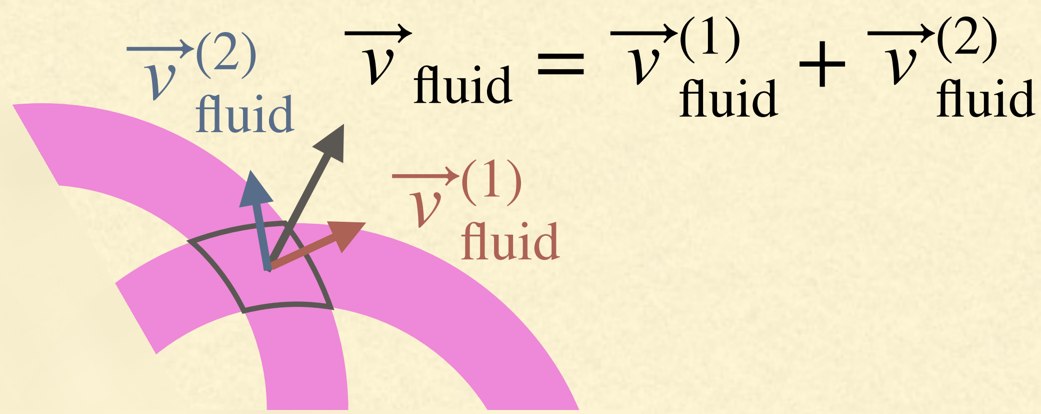
$$\Omega_{\rm GW} \propto h_{ij}^2 \propto \beta^{-2}$$

Sound waves [Hindmarsh PRL120(2018), Hindmarsh, Hijazi 1909.10040]

Shell overlap continuously creates

new velocity field during Hubble time

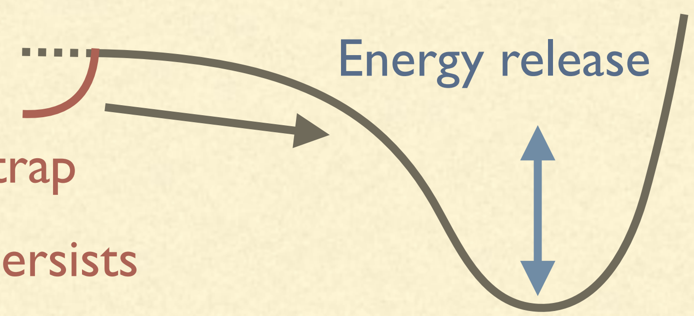
→ GW spectrum is enhanced by β/H_*



ULTRA-SUPERCOOLED TRANSITIONS

e.g. [Randall & Servant '07, Konstandin & Servant '11] [RJ, Takimoto '16]
[Harling & Servant '17, Bruggisser, Harling, Matsedonskyi, Servant '18]

- $\alpha \gg 1$ occurs in a certain class of models ('almost scale invariant' models)



- Thermal trap persists even at low temperatures → $\alpha \gg 1$

- These models often give small β/H_* (i.e. large bubbles)

- So, at least naively, large amplitude of GWs is expected

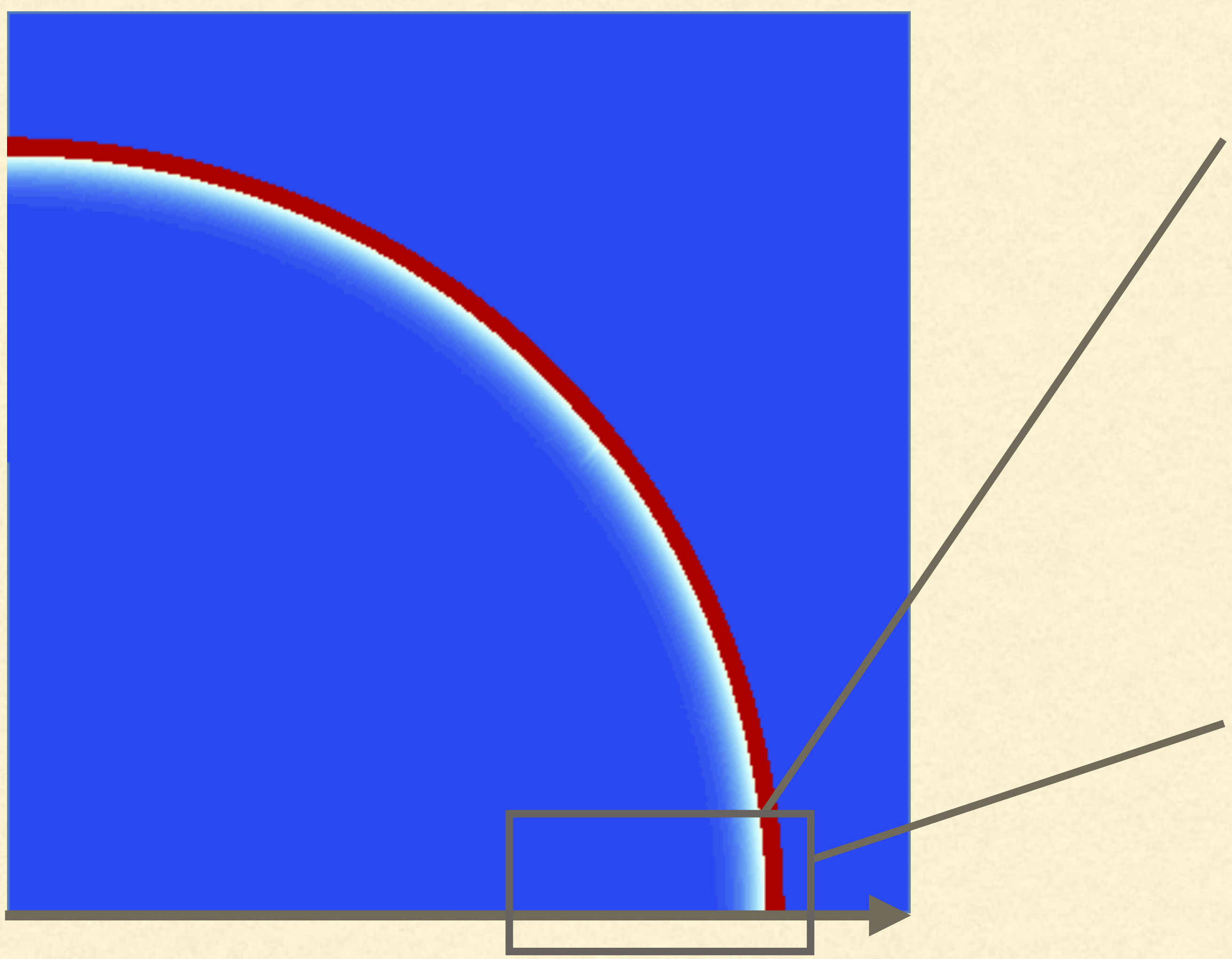
$$\Omega_{\rm GW}^{\rm (sw)} \propto \left(\frac{\alpha}{1+\alpha}\right)^2 \left(\frac{\beta}{H_*}\right)^{-1}$$

- However, the story is not so simple

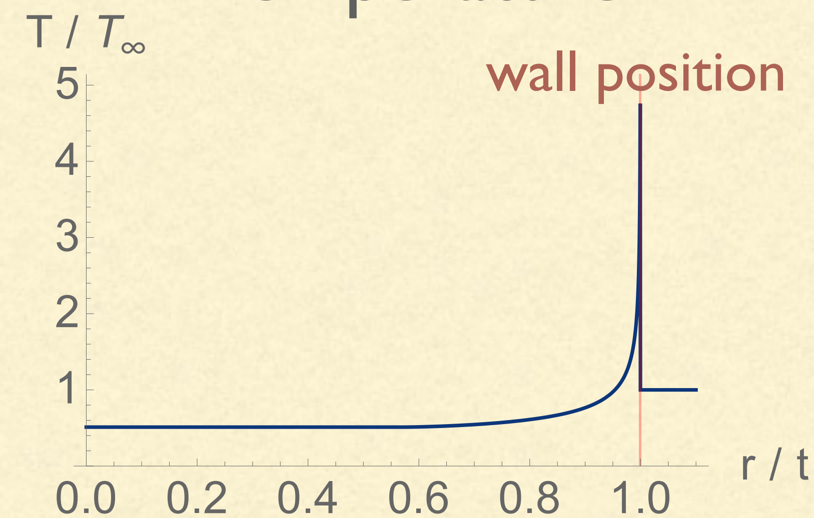
BUBBLE EXPANSION IN ULTRA-SUPERCOOLED TRANSITIONS

- Large α $(\gg 1)$

"strong detonation"



Temperature



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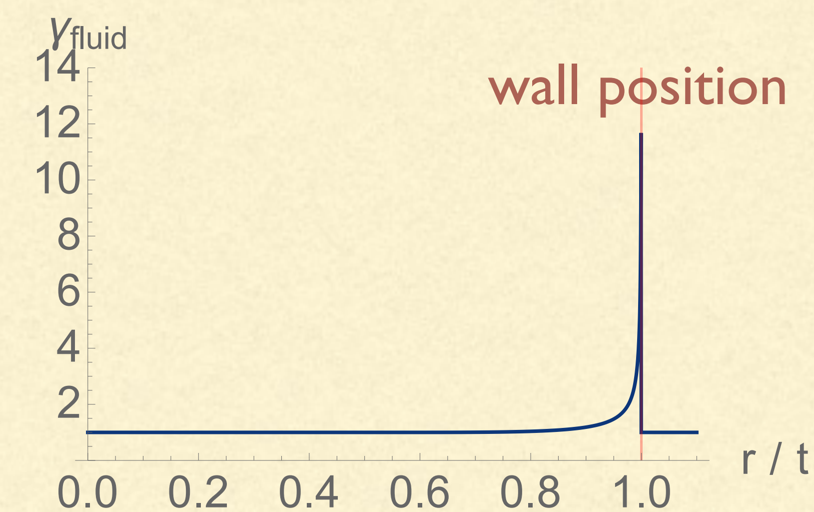
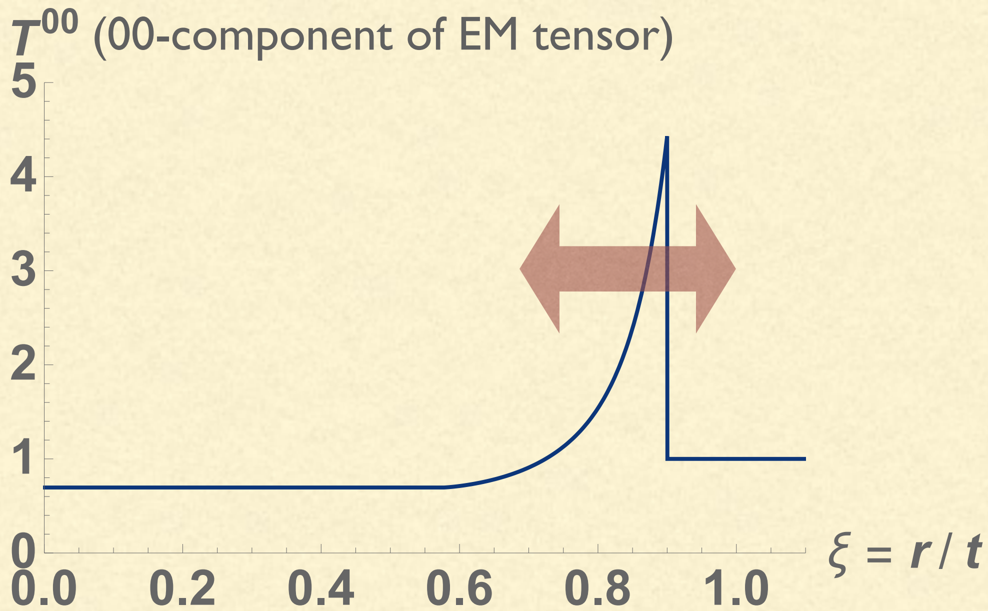
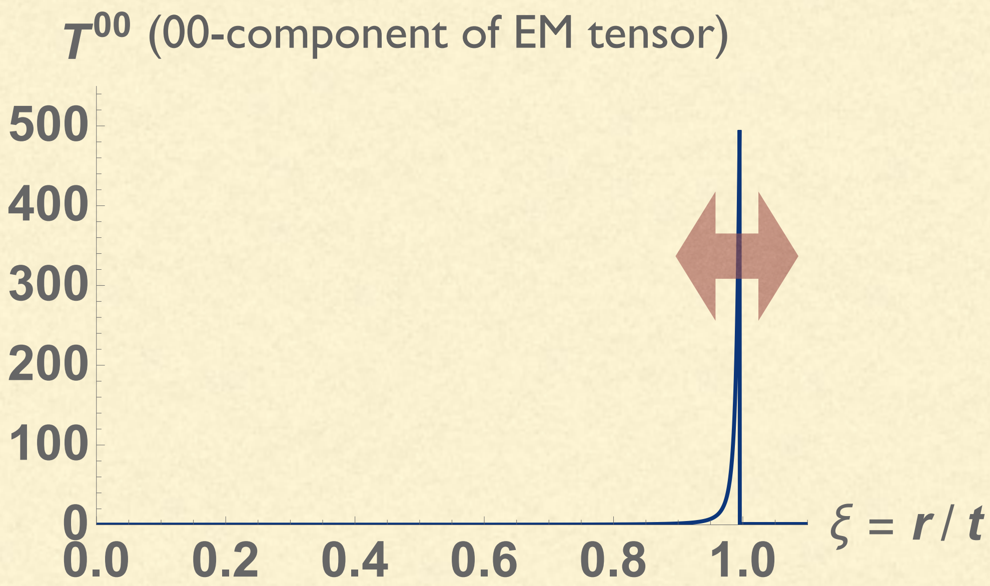


ENERGY LOCALIZATION IN ULTRA-SUPERCOOLED TRANSITIONS

- Energy profile before collision is sharply localized around the wall for $\alpha \gg 1$



$\alpha = 0.4, \quad v_w = 0.9$



$\alpha = 10, \quad v_w = 0.995$

- In realistic ultra-supercooled transitions, α can be much larger, e.g. $\alpha \sim 10^{12}$

- As a result, huge hierarchy appears between bubble size and energy localization

→ Hard to simulate fluid dynamics after bubble collisions numerically

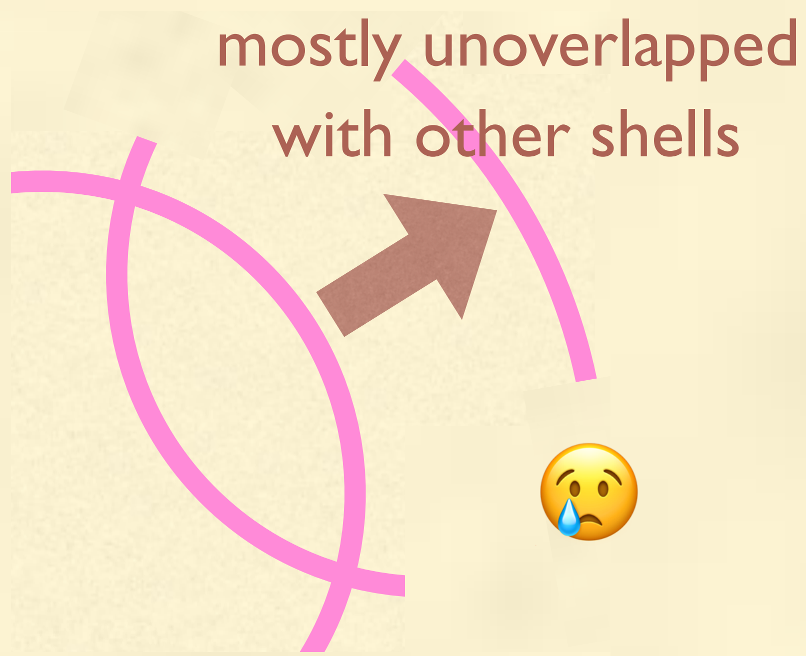
GW ENHANCEMENT CONDITION BY SOUND WAVES

- Necessary conditions to have GW enhancement by sound waves

  - Delayed onset of turbulence

  - Sound shell overlap

- In order to have shell overlap, the energy localization has to break up:



or

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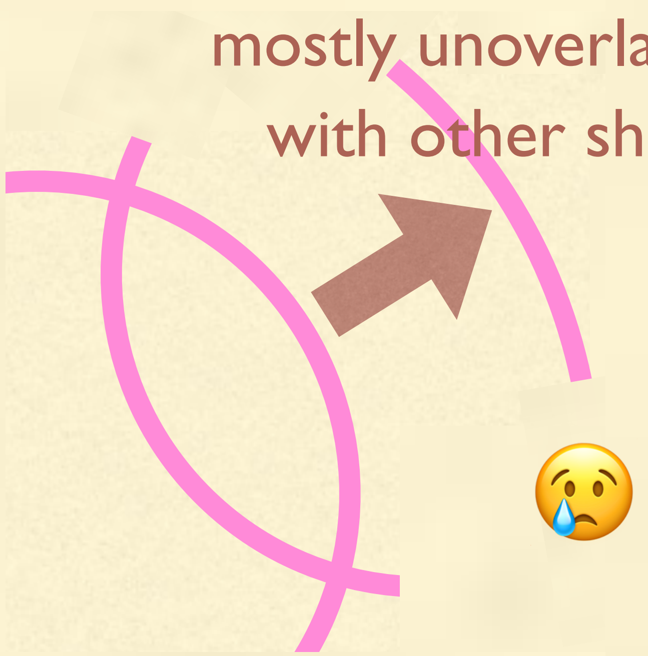


or

SUMMARY OF MOTIVATION

- Ultra-supercooled transitions ($\alpha \gg 1$) occur in a certain class of models,

  and they are interesting both theoretically and observationally

- Does GW enhancement by sound waves occur in these transitions?

  More precisely: When does the energy localization break up and shell overlap start?

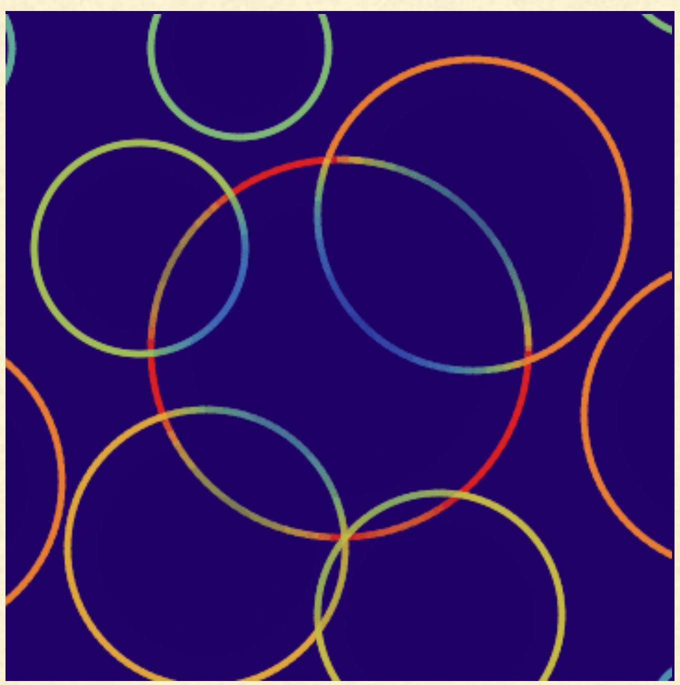
- Numerically difficult to study because of hierarchy in scales

What can we do?

BEFORE MOVING ON... GW SPECTRUM IN THIN-SHELL LIMIT

- If the fluid shells remain thin, the story is relatively simple

  because the GW spectrum in thin-shell limit is known analytically



→

$$\Omega_{\rm GW}(k) \propto \Delta^{(s)}(k) + \Delta^{(d)}(k)$$

$$\Delta^{(s)} = \int_{-\infty}^{\infty} dt_x \int_{-\infty}^{\infty} dt_y \int_{v|t_{x,y}|}^{\infty} dr \int_{-\infty}^{t_{\max}} dt_n \int_{t_n}^{t_x} dt_{xi} \int_{t_n}^{t_y} dt_{yi}$$
$$\frac{k^3}{3} \begin{bmatrix} e^{-I(x_i,y_i)}\,\Gamma(t_n)\,\dfrac{r}{r_{xn}^{(s)} r_{yn}^{(s)}} \\ \times \left[j_0(kr)\mathcal{K}_0(n_{xn\times}, n_{yn\times}) + \dfrac{j_1(kr)}{kr}\mathcal{K}_1(n_{xn\times}, n_{yn\times}) + \dfrac{j_2(kr)}{(kr)^2}\mathcal{K}_2(n_{xn\times}, n_{yn\times}) \right] \\ \times\, \partial_{txi}\left[r_B(t_{xi},t_n)^3 D(t_x,t_{xi}) \right] \partial_{tyi}\left[r_B(t_{yi},t_n)^3 D(t_y,t_{yi}) \right] \cos(kt_{x,y}) \end{bmatrix}$$

$$\Delta^{(d)} = \int_{-\infty}^{\infty} dt_x \int_{-\infty}^{\infty} dt_y$$
$$\int_0^{\infty} dr \int_{-\infty}^{t_x} dt_{xn} \int_{-\infty}^{t_y} dt_{yn} \int_{t_{xn}}^{t_x} dt_{xi} \int_{t_{yn}}^{t_y} dt_{yi} \int_{-1}^{1} dc_{xn} \int_{-1}^{1} dc_{yn} \int_0^{2\pi} d\phi_{xn,yn}$$
$$\frac{k^3}{3} \begin{bmatrix} \Theta_{\rm sp}(x_i,y_n)\Theta_{\rm sp}(x_n,y_i) e^{-I(x_i,y_i)}\Gamma(t_{xn})\Gamma(t_{yn}) \\ \times\, r^2 \left[j_0(kr)\mathcal{K}_0(n_{xn}, n_{yn}) + \dfrac{j_1(kr)}{kr}\mathcal{K}_1(n_{xn}, n_{yn}) + \dfrac{j_2(kr)}{(kr)^2}\mathcal{K}_2(n_{xn}, n_{yn}) \right] \\ \times\, \partial_{txi}\left[r_B(t_{xi},t_{xn})^3 D(t_x,t_{xi}) \right] \partial_{tyi}\left[r_B(t_{yi},t_{yn})^3 D(t_y,t_{yi}) \right] \cos(kt_{x,y}) \end{bmatrix}$$

[RJ, Takimoto 1707.03111]

BEFORE MOVING ON... GW SPECTRUM IN THIN-SHELL LIMIT

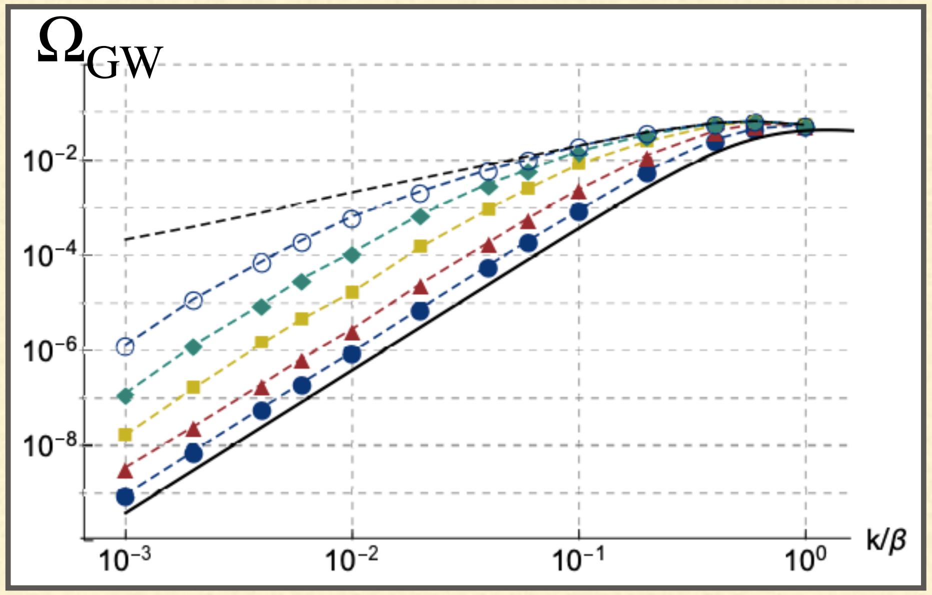
- If the fluid shells remain thin, the story is relatively simple

  because the GW spectrum in thin-shell limit is known analytically

- And indeed this spectrum does NOT have β/H_* enhancement,

  because shell overlapping does not occur

$$\Omega_{\rm GW} \propto \left(\frac{\alpha}{1+\alpha}\right)^2 \left(\frac{\beta}{H_*}\right)^{-2}$$



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REDUCING THE PROBLEM

- After collision, the system is fluid-only: $\partial_\mu T^{\mu\nu}_{\text{fluid}} = 0$

  (we assume relativistic ideal gas $T^{\mu\nu} = (\rho + p)u^\mu u^\nu + p\eta^{\mu\nu}$, $p = \rho/3$)

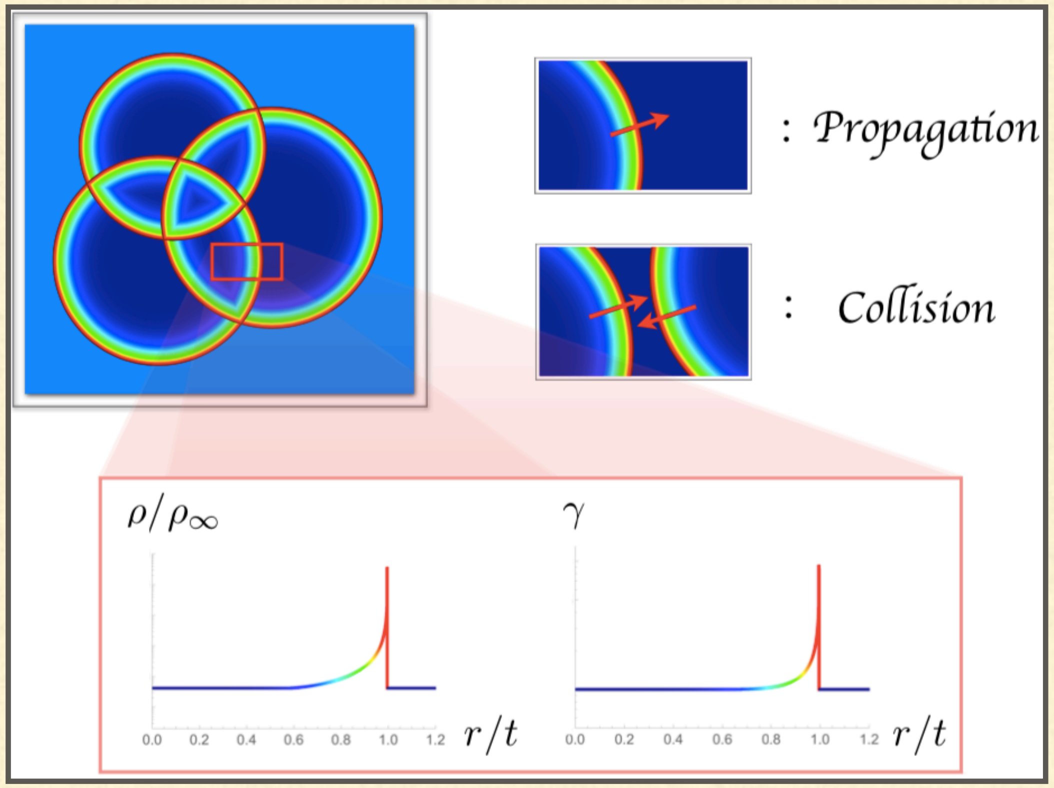
  However, nonlinearity and discontinuities (i.e. shocks → later) complicate the analysis

- Let's devide the problem into small pieces:

  (1) propagation of relativistic fluid

  (2) collision of relativistic fluid

  Even (1) is nontrivial. We study the effect of (1) on the deformation of fluid profile.



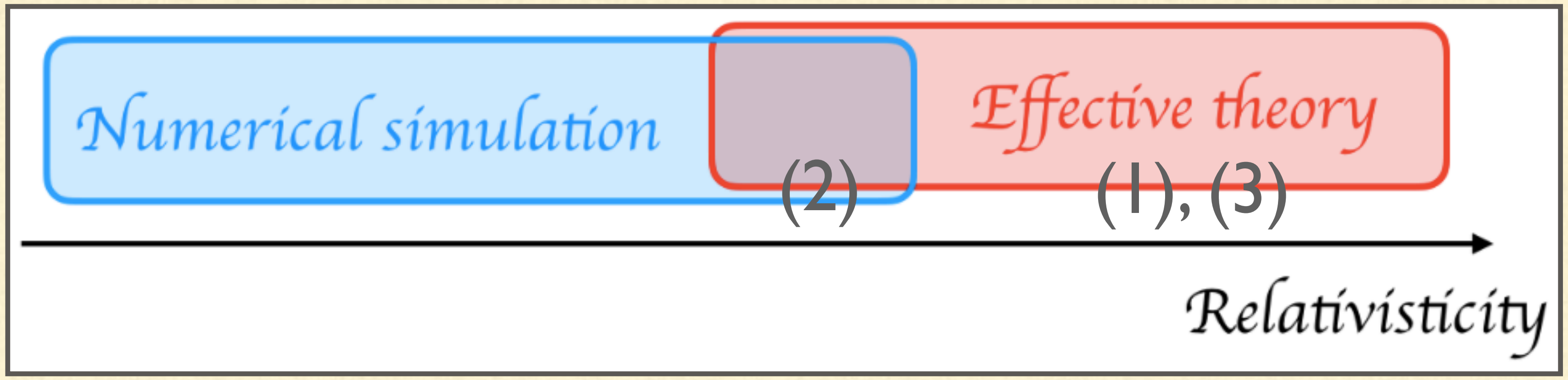
STRATEGY

- Our strategy:

(1) Develop an effective description of fluid propagation valid in highly relativistic regime

(2) Check the theory against simulation in mildly-relativistic regime

(3) Study implications to GW production

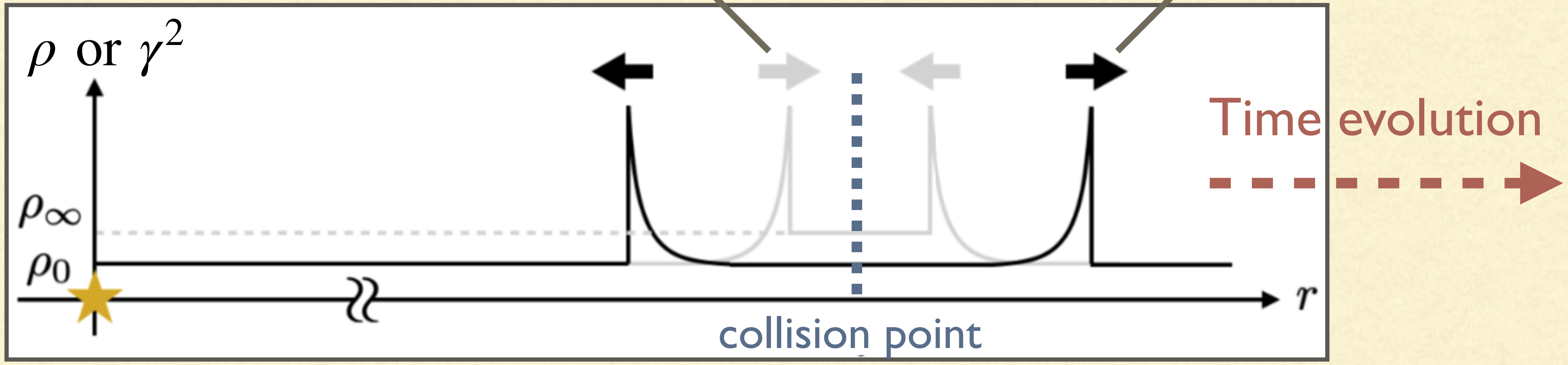


(or simply the transition strength α)

SETUP

- The setup we study : propagation of fluid profile after collision

⓪ nucleation ① just before collision ② just after collision

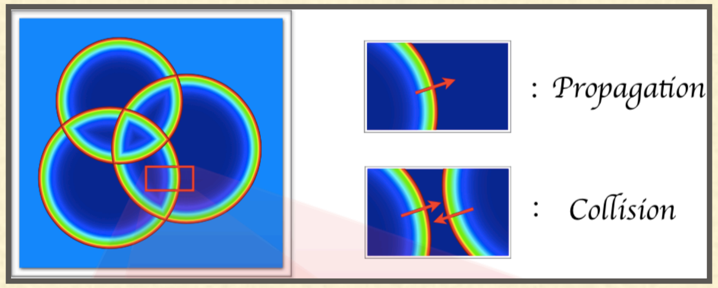


① Fluid profile just before collision: calculated from [Espinosa, Konstandin, No, Servant '10]

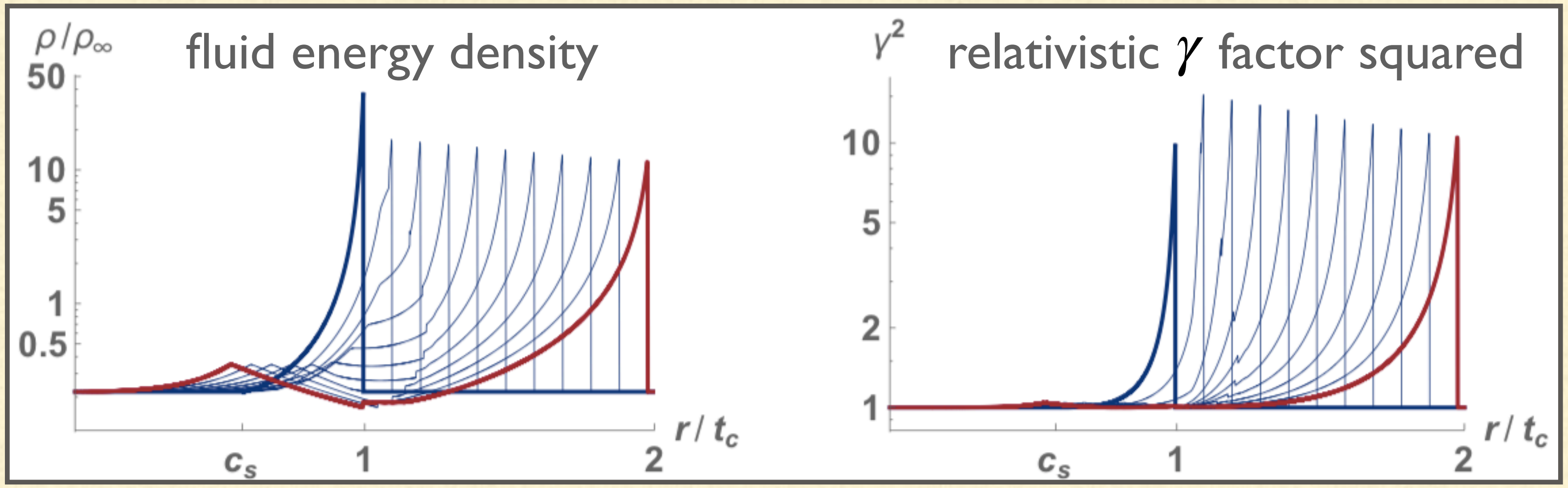
↓ Assumption: the first fluid collision does not change the profile significantly

② Fluid profile just after collision: our interest is in the time evolution from here

EFFECTIVE DESCRIPTION OF FLUID PROPAGATION

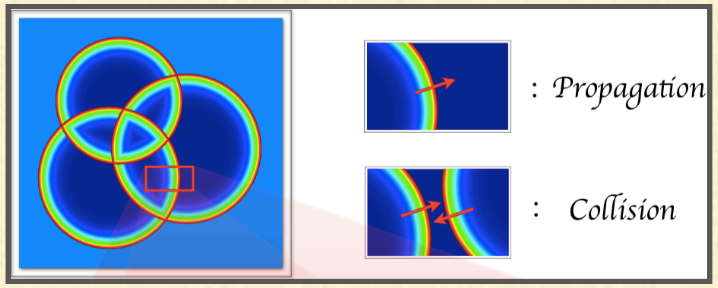


- Before constructing a theory, let's see the result of numerical simulation in the intermediate relativisticity



nucleation collision propagation

EFFECTIVE DESCRIPTION OF FLUID PROPAGATION



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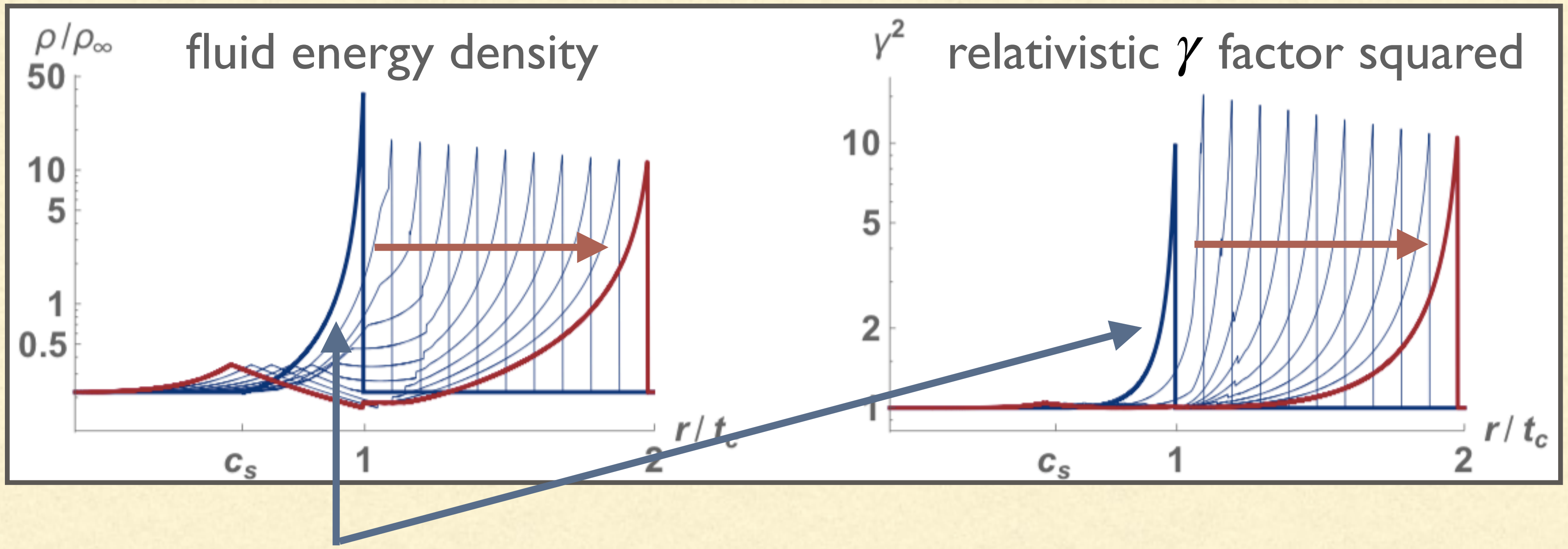

- Initial fluid profile (blue) propagates inside the other bubble (red)

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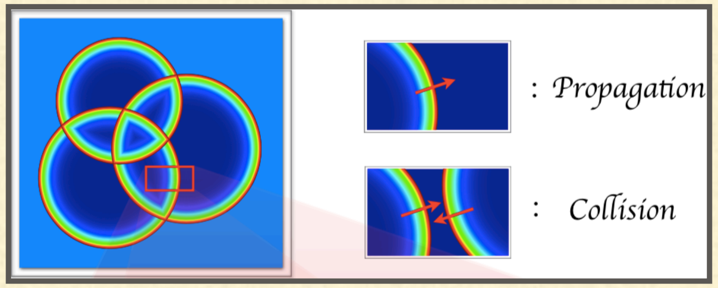

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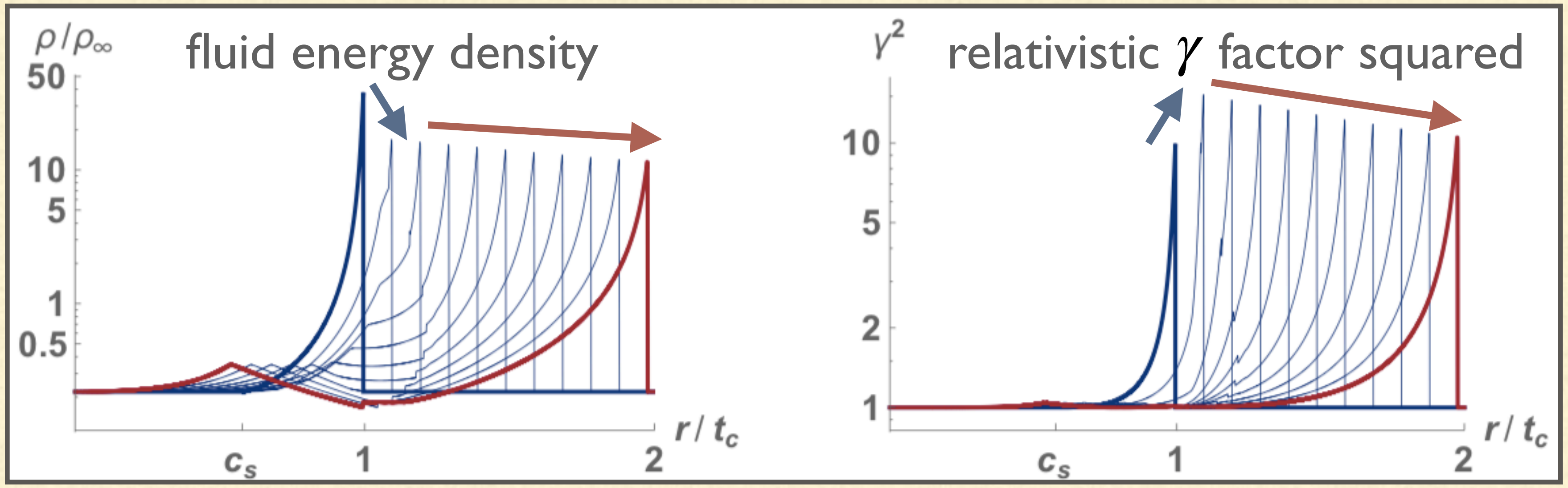

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- Peaks rearrange to new initial values, and gradually become less energetic

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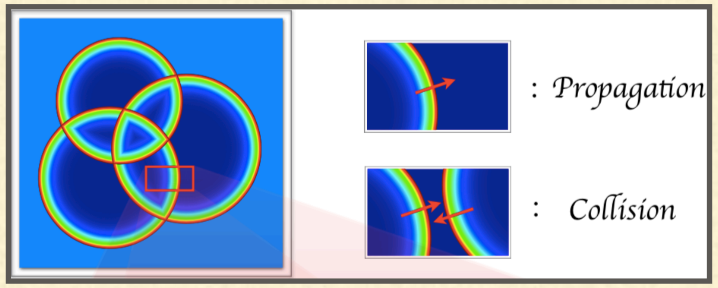

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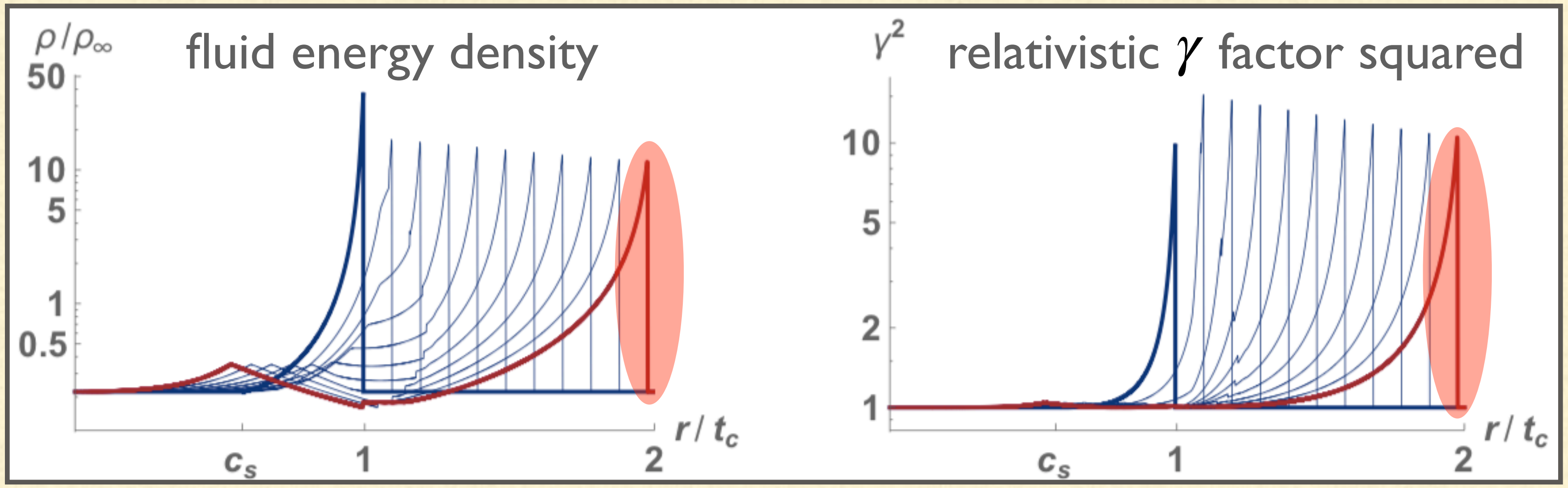
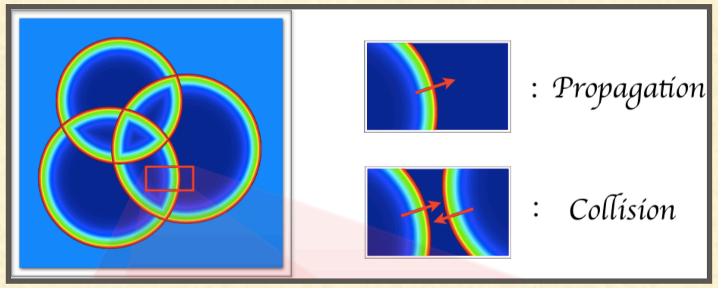


- Initial fluid profile (blue) propagates inside the other bubble (red)
- Peaks rearrange to new initial values, and gradually become less energetic
- Strong shocks (i.e. discontinuities) persist during propagation

EFFECTIVE DESCRIPTION OF FLUID PROPAGATION



- Can we construct an effective description?

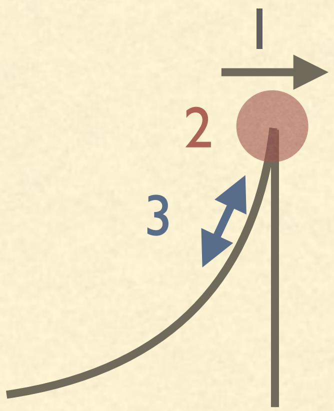
- From the viewpoint of GW production, we are interested only in PEAKS, not TAILS

- Then how about describing the system with peak-related quantities?

1) Shock velocity: v_s

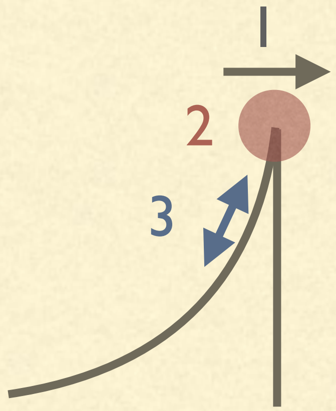
2) Peak values: $\rho_{\rm peak}, v_{\rm peak}$ (equivalently $\rho_{\rm peak}, \gamma^2_{\rm peak}$)

3) Derivatives at the peak: $\dfrac{d\rho_{\rm peak}}{dr}, \dfrac{dv_{\rm peak}}{dr}$ @ peak



- Can we construct a closed system for these quantities?

HOW TO CONSTRUCT A CLOSED SYSTEM



- Closed system for 5 quantities $\gamma_s^2,\ \rho_{\rm peak},\ \gamma_{\rm peak}^2,\ \dfrac{d\rho_{\rm peak}}{dr},\ \dfrac{d\gamma_{\rm peak}^2}{dr}$

  - First, there are strict equations:

    a) Rankine-Hugoniot conditions across the shock : 2 constraints

    (corresponding to energy and momentum conservation across the shock)

    b) Time evolution equations : 2 evolution equations

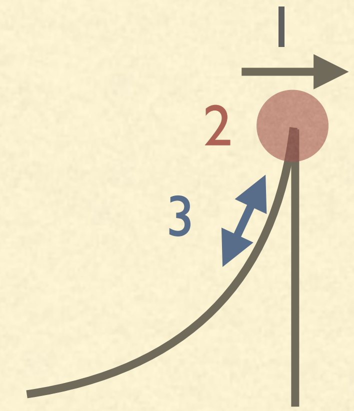
    (corresponding to temporal & spatial part of $\partial_\mu T^{\mu\nu}_{\rm fluid} = 0$ behind the shock)

  - Still we have less equations (4 eqs.) than the number of quantities (5).

    This is natural, because the original system has infinite # of dof (i.e. # of spacial grids),

    while we are trying to describe it with finite # of dof.

HOW TO CONSTRUCT A CLOSED SYSTEM



- The last equation?

  - The last equality will be an approximate relation which characterizes the system

  - What makes this system distinct from others is <u>energy domination by the peak</u>

- Imposing energy domination by the peak

  - Any relation like "(peak T^{00}) × (width of the peak) = const" will work.

    For example, approximating ρ and γ^2 to be exponential in r, we have

$$\sigma \simeq \begin{Bmatrix} 1 \\ t \\ t^2 \end{Bmatrix} \times \int dr\, \frac{4}{3}\rho\gamma^2 = \begin{Bmatrix} 1 \\ t \\ t^2 \end{Bmatrix} \times \frac{4}{3}\,\frac{\rho_{\rm peak}\gamma^2_{\rm peak}}{\ln\rho' + \ln\gamma^{2\prime}} \quad \text{for} \quad \begin{Bmatrix} d = 1 \\ d = 2 \\ d = 3 \end{Bmatrix}$$

: planar
: cylindical
: spherical

THEORY PREDICTION

- The resulting system can be solved analytically ($\delta = 10/13$)

1) Shock velocity:

$$\frac{1}{\gamma_s^2(t)} = \frac{8}{87}\left(\frac{\rho_0}{\sigma}\right)\left[t^3 - \left(\frac{t}{t_c}\right)^\delta t_c^3\right] + \frac{1}{\gamma_s^2(t_c)}\left(\frac{t}{t_c}\right)^\delta,$$

2) Peak values:

$$\frac{\rho_0}{\rho_{\rm peak}(t)} = \frac{1}{29}\left(\frac{\rho_0}{\sigma}\right)\left[t^3 - \left(\frac{t}{t_c}\right)^\delta t_c^3\right] + \frac{\rho_0}{\rho_{\rm peak}(t_c)}\left(\frac{t}{t_c}\right)^\delta,$$

$$\frac{1}{\gamma_{\rm peak}^2(t)} = \frac{16}{87}\left(\frac{\rho_0}{\sigma}\right)\left[t^3 - \left(\frac{t}{t_c}\right)^\delta t_c^3\right] + \frac{1}{\gamma_{\rm peak}^2(t_c)}\left(\frac{t}{t_c}\right)^\delta,$$

3) Derivatives at the peak:

$$\ln\rho'(t) = \frac{448}{117}\left(\frac{\rho_0}{\sigma}\right)t^2\gamma_{\rm peak}^4(t) + \frac{24}{13}\frac{\gamma_{\rm peak}^2(t)}{t},$$

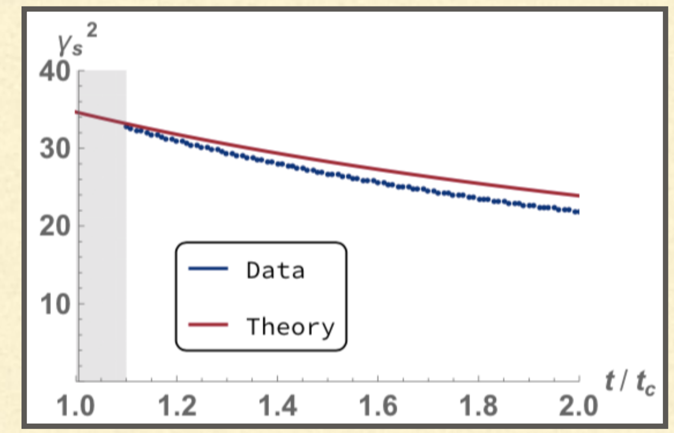
$$\ln\gamma^{2\prime}(t) = \frac{128}{39}\left(\frac{\rho_0}{\sigma}\right)t^2\gamma_{\rm peak}^4(t) - \frac{24}{13}\frac{\gamma_{\rm peak}^2(t)}{t},$$

COMPARISON WITH NUMERICAL SIMULATION

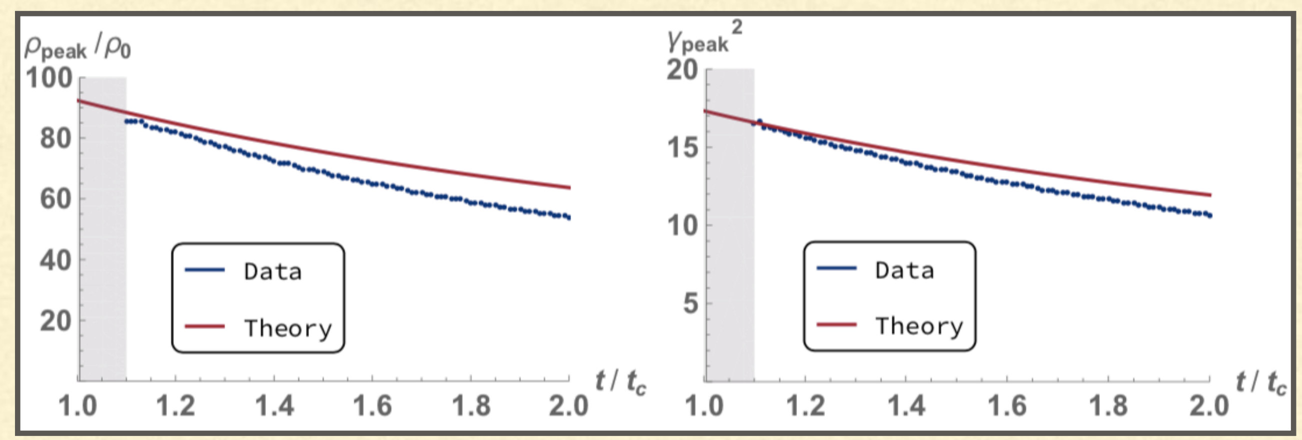
- Analytic (red) vs. Numerical (blue)

  with initial condition $\alpha = 10,\ \gamma_{\rm wall} = 10$

γ_s^2

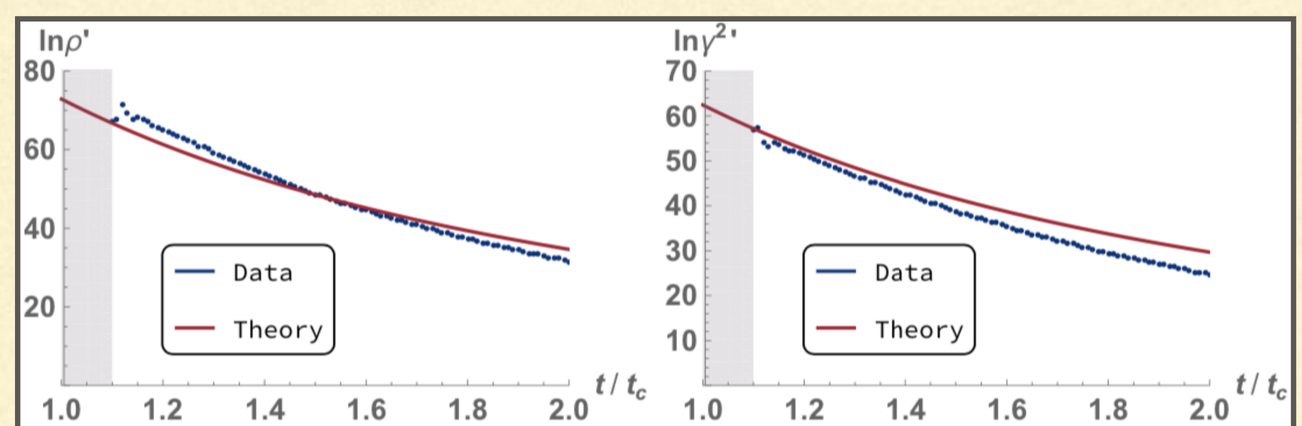


$\rho_{\rm peak},\ \gamma_{\rm peak}^2$



Qualitatively OK!

$$\frac{d\rho_{\rm peak}}{dr},\ \frac{d\gamma_{\rm peak}^2}{dr}$$



IMPLICATIONS OF THE EFFECTIVE DESCRIPTION

- What can we learn?

  - All quantities have time dependence like

$$\frac{\rho_0}{\rho_{\rm peak}(t)} = \frac{1}{29}\left(\frac{\rho_0}{\sigma}\right)\left[t^3 - \left(\frac{t}{t_c}\right)^{\delta} t_c^3\right] + \frac{\rho_0}{\rho_{\rm peak}(t_c)}\left(\frac{t}{t_c}\right)^{\delta} \qquad \delta = 10/13$$

effect of increase in the surface area

effect of nonlinearity in fluid equation

  - Surface area effect wins (3 > 10/13).

    In other words, nonlinearity is not effective in breaking up the energy localization.

  - Timescale for breaking up is controlled by $\tau \equiv \left(\sigma/\rho_0\right)^{1/3}$

IMPLICATIONS TO GW PRODUCTION

- Fluid profile remains to be thin, if we consider fluid propagation alone

- So, we have to see fluid collisions next:

  1) If the fluid profile still remains thin,

  the thin-shell spectrum will apply,

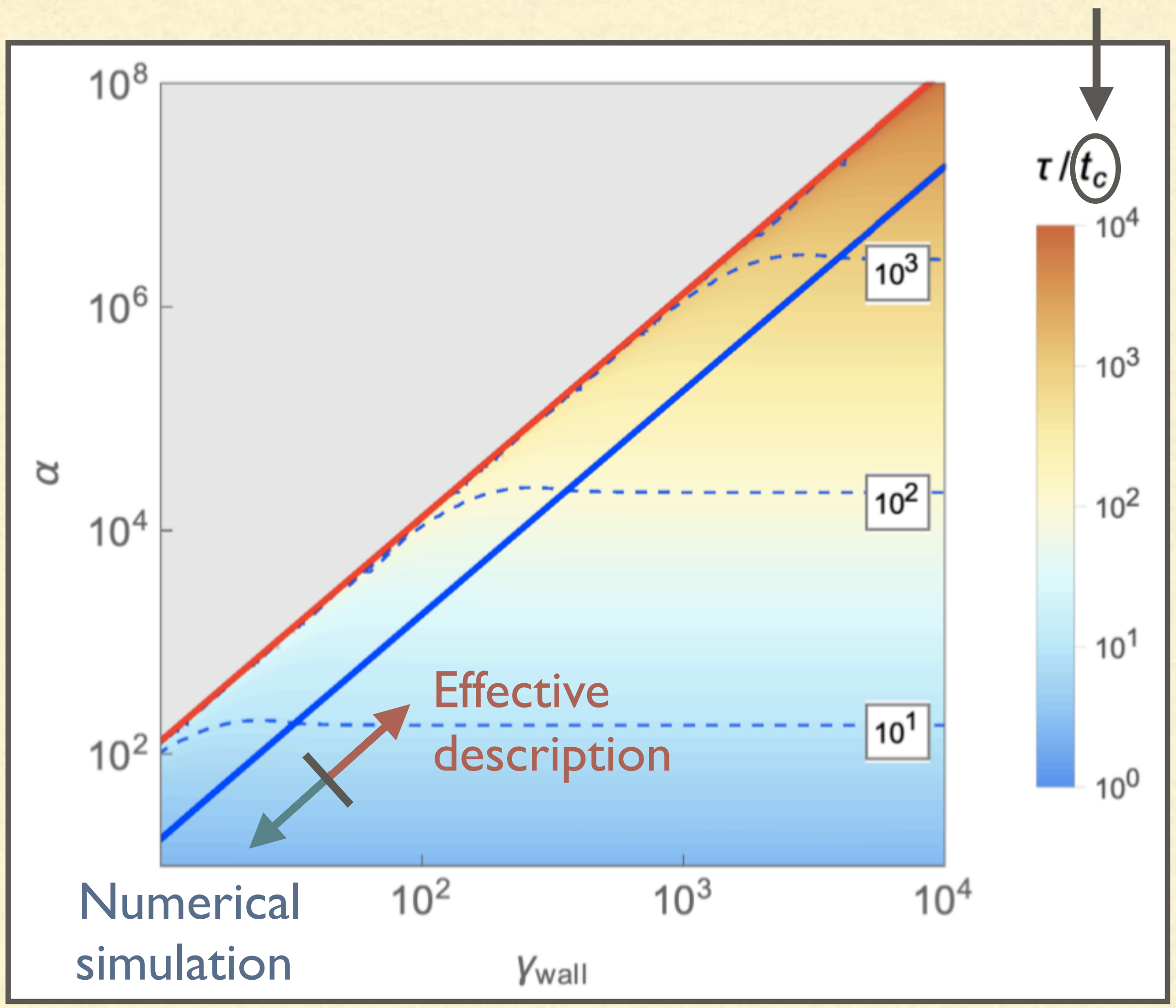
  and there will be no β/H_* enhancement

  2) If the fluid profile successfully breaks up,

  β/H_* enhancement will occur

  (until the onset of turbulence)

Time from bubble nucleation to collision



TALK PLAN

✔ 1. Introduction

✔ 2. Difficulty in estimating GWs in ultra-supercooled transitions

✔ 3. An approach:

Effective description of fluid propagation & Implications to GW production

4. Summary

SUMMARY

- GW production in ultra-supercooled transitions $\alpha \gg 1$ is interesting

  both theoretically and observationally, but they are hard to simulate numerically

- We reduced the problem into (1) propagation and (2) collision, and studied (1):

  - We constructed an effective description of fluid propagation

  - We discussed implications to GW production:

    The fluid profile remains to be thin, so GW enhancement might be somewhat delayed

- Still we have to address: Effect of fluid collision / Effect of turbulence

Back up

MORE DETAIL ON GW ENHANCEMENT BY SOUND WAVES

- GW spectrum is convoluted unequal-time correlator <T T>

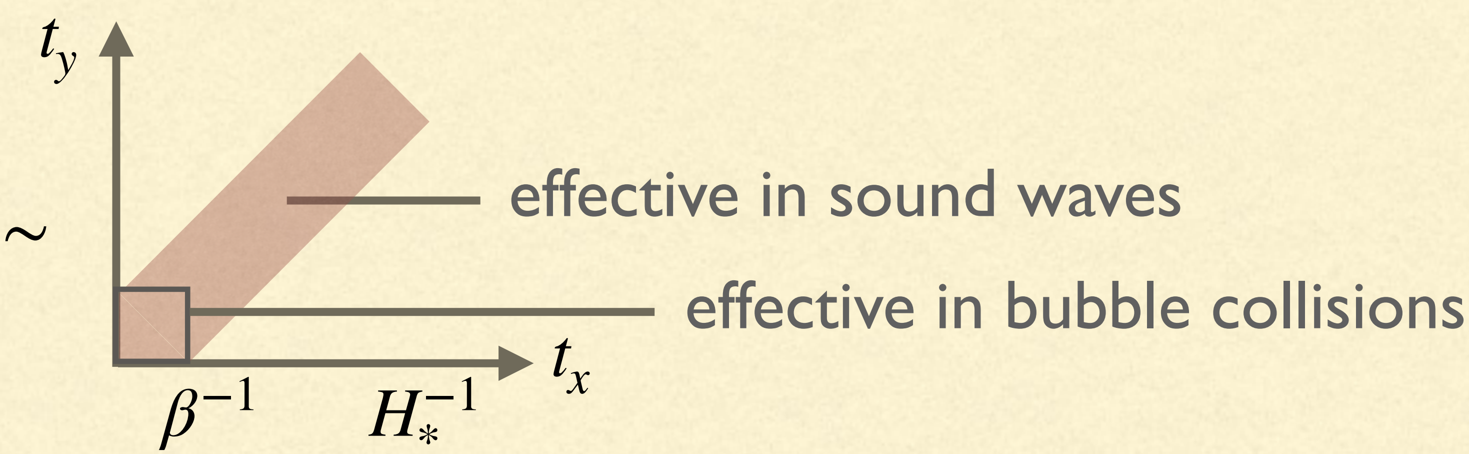
$$\Omega_{\rm GW}(k) \sim \int dt_x \int dt_y \; \cos(k(t_x - t_y))\; \Pi(t_x, t_y, \vec{k}) \quad = (\text{projection}) \times \left\langle T_{ij}(t_x, \vec{x})\; T_{kl}(t_y, \vec{y}) \right\rangle_{\rm ens}$$

Why? 1) GW is given by Green function $h_{ij}(t) \sim \int dt' \; \text{Green}(t, t')\; T_{ij}(t')$

2) GW spectrum is two-point correlator of $h_{ij}(t)$

- Shell overlap creates correlation in the diagonal direction

$\Pi(t_x, t_y, \vec{k}) \sim$



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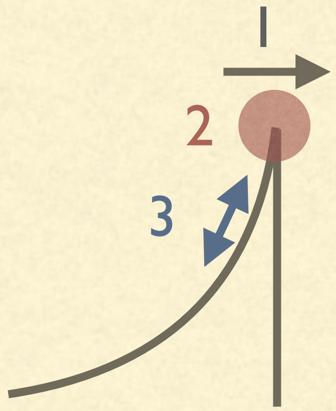


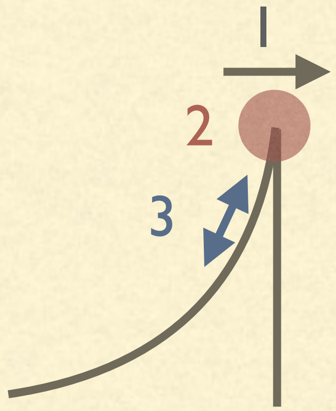
- Closed system for 5 quantities $\gamma_s^2, \rho_{\rm peak}, \gamma_{\rm peak}^2, \dfrac{d\rho_{\rm peak}}{dr}, \dfrac{d\gamma_{\rm peak}^2}{dr}$

  - Rankine-Hugoniot conditions across the shock : 2 constraints

    (corresponding to energy and momentum conservation across the shock)

$$p_{\rm peak} = \frac{p_0 + \rho_0 v_{\rm peak} v_s}{1 - v_{\rm peak} v_s}, \qquad v_s = \frac{(p_{\rm peak} + \rho_{\rm peak}) v_{\rm peak}}{p_{\rm peak} v_{\rm peak}^2 + \rho_{\rm peak} - \rho_0 (1 - v_{\rm peak}^2)}$$

HOW TO CONSTRUCT A CLOSED SYSTEM



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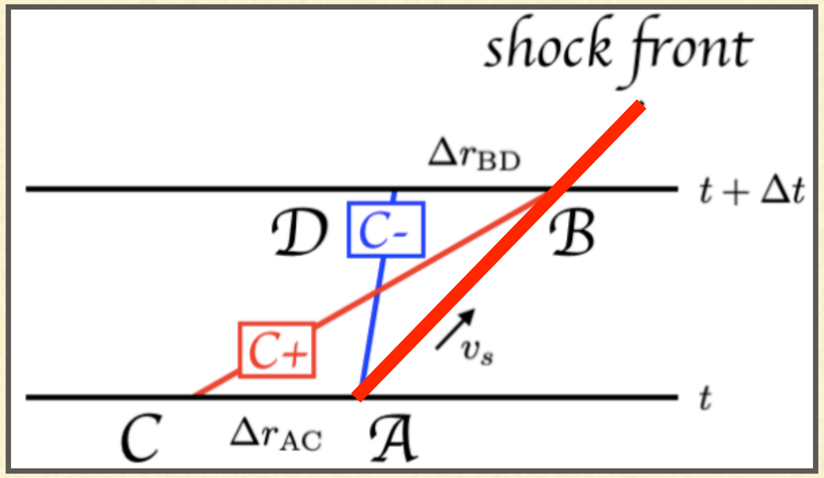
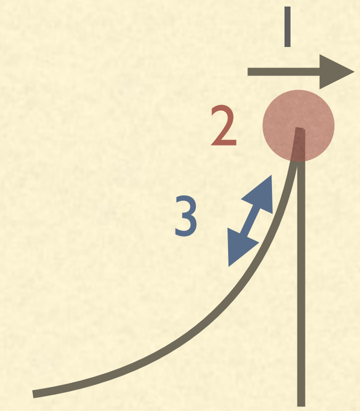


<u>Advanced note</u>

Easily derived from the conservation of Riemann invariants along C_+ & C_-

HOW TO CONSTRUCT A CLOSED SYSTEM



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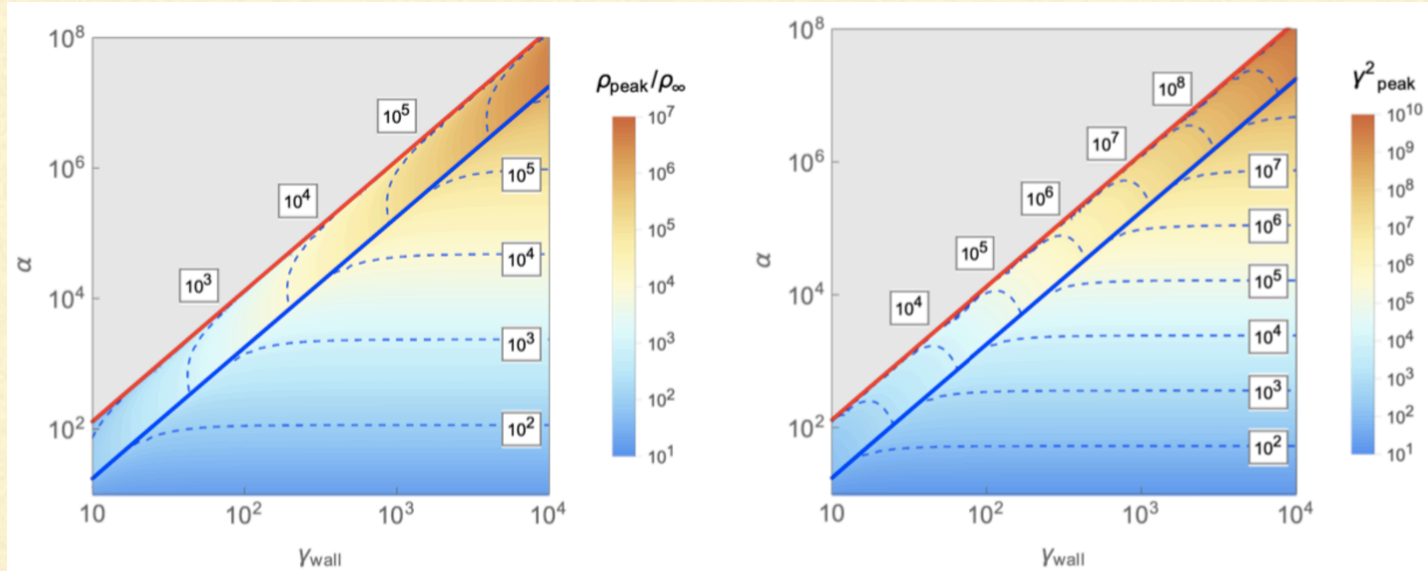
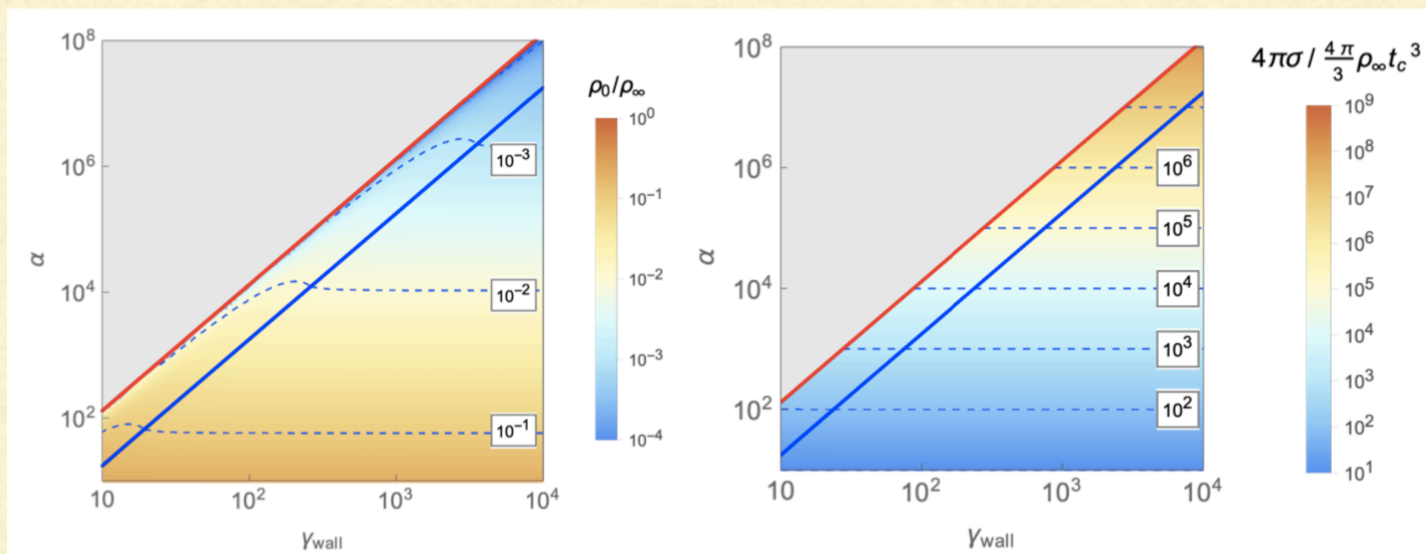
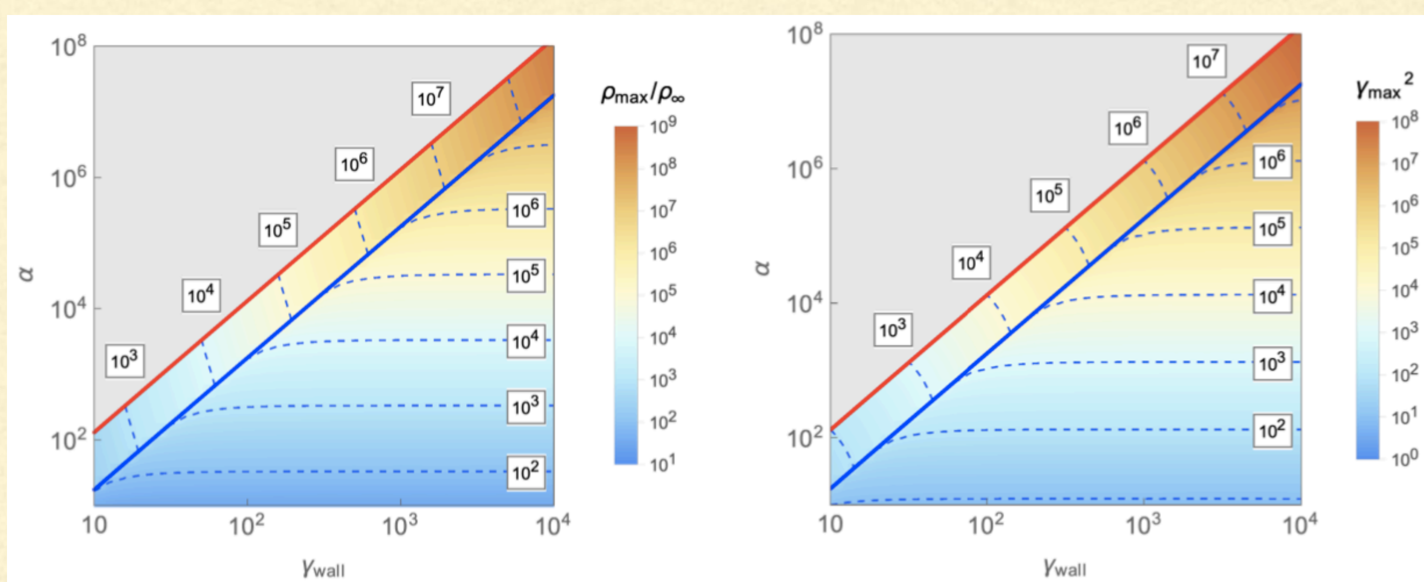
$$\frac{\sqrt{3}}{2}\partial_t \ln \rho_{\rm peak} + \partial_t \ln \gamma_{\rm peak}^2 = -\frac{2\sqrt{3}-3}{4}\frac{1}{\gamma_{\rm peak}^2}\left[\frac{\sqrt{3}}{2}\ln \rho' + \ln \gamma^{2'}\right] - \frac{(\sqrt{3}-1)(d-1)}{t}$$

$$-\frac{\sqrt{3}}{2}\partial_t \ln \rho_{\rm peak} + \partial_t \ln \gamma_{\rm peak}^2 = \frac{2\sqrt{3}+3}{4}\frac{1}{\gamma_{\rm peak}^2}\left[-\frac{\sqrt{3}}{2}\ln \rho' + \ln \gamma^{2'}\right] + \frac{(\sqrt{3}+1)(d-1)}{t}$$


ρ_{peak}/ρ_∞
$\gamma^2{}_{peak}$
α
γ_{wall}


ρ_0/ρ_∞
$4\pi\sigma / \frac{4\pi}{3}\rho_\infty t_c{}^3$
α
γ_{wall}


ρ_{max}/ρ_∞
$\gamma_{max}{}^2$
α
γ_{wall}