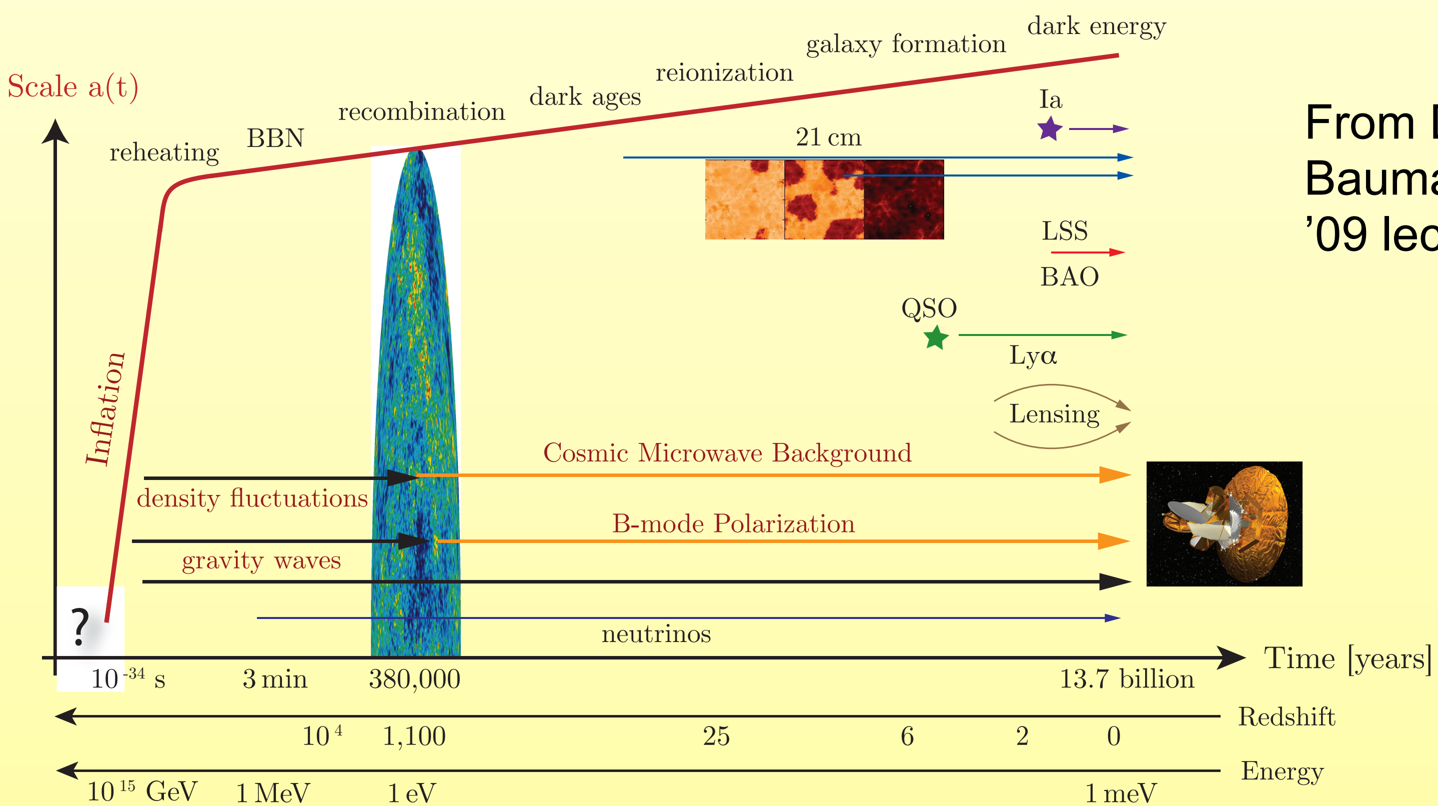
Inflation from Broken Scale Invariance

Csaba Csáki (Cornell)
with
Nemanja Kaloper (Davis)
Javi Serra (Cornell/Padua)
John Terning (Davis)

Theory Seminar, IFAE, UA Barcelona, Oct. 17, 2014

Introduction

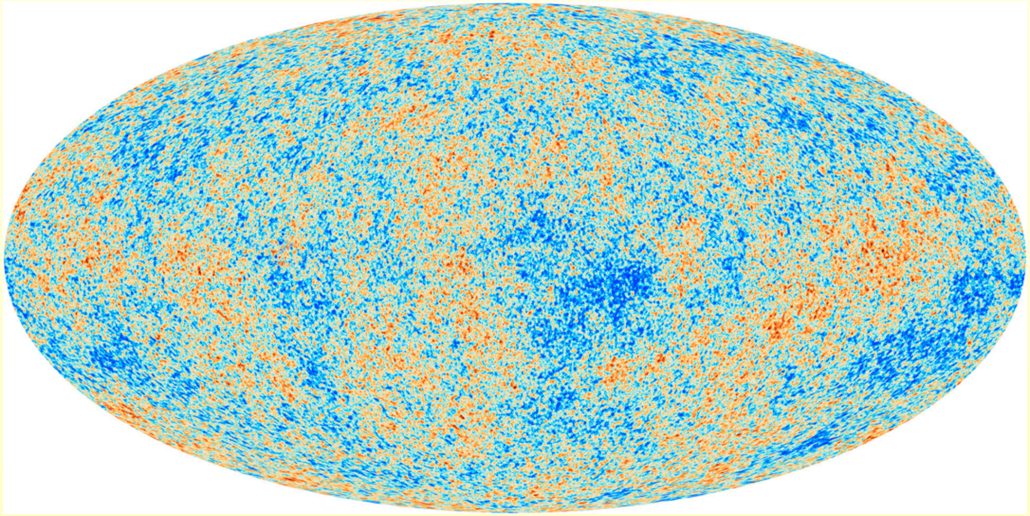
- Inflation leading candidate for theory of early Universe
- Explains why Universe is big, old and smooth
- Gives prediction of initial spectrum of density fluctuations, almost scale invariant

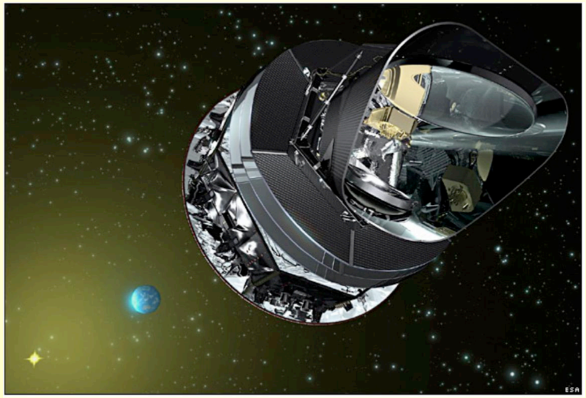


From Daniel Baumann's TASI '09 lectures

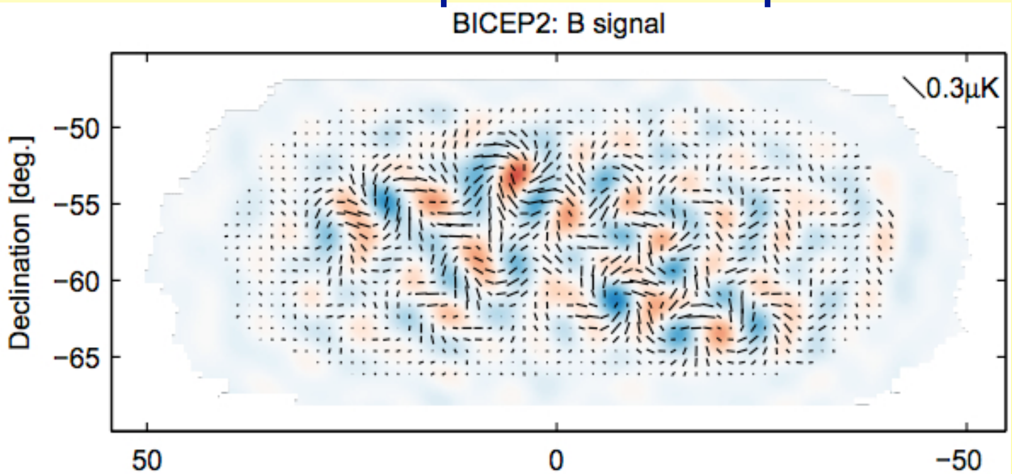
Introduction

•Tested by WMAP and Planck



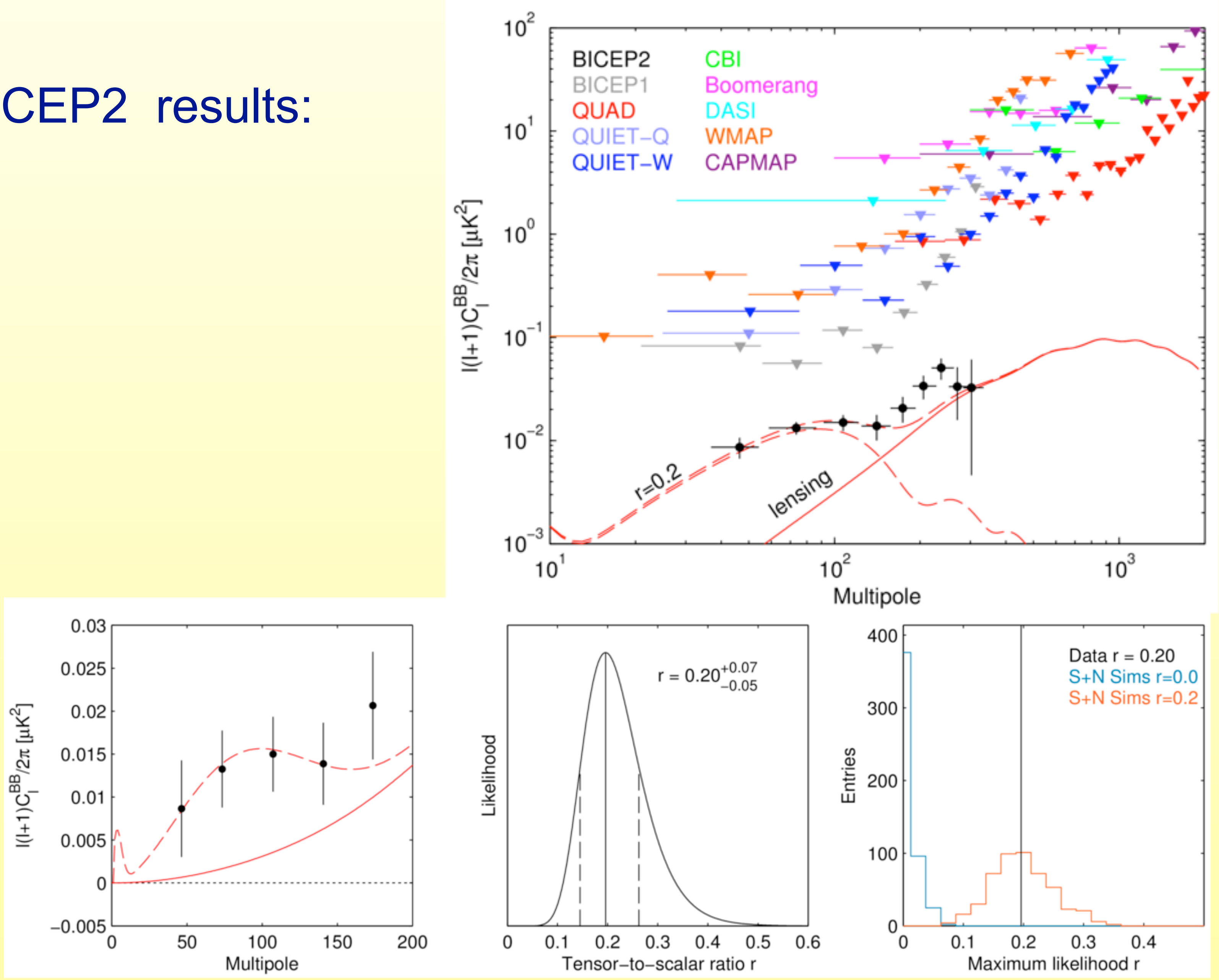


•Recently BICEP2 also claimed observation of B-mode polarization of CMB explained via primordial gravitational waves



BICEP2 results

- BICEP2 results:



Large field inflation?

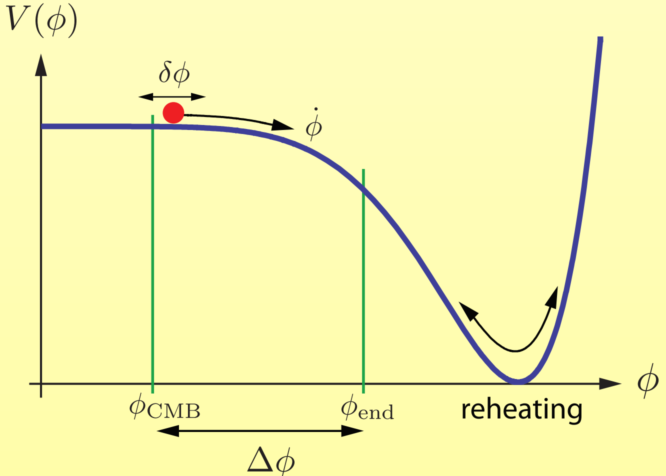
•If BICEP2 results hold, suggests large field inflation $\Delta\varphi > M_{Pl}$

$$\frac{\Delta\phi}{M_{\rm pl}} = \mathcal{O}(1) \times \left(\frac{r_\star}{0.01}\right)^{1/2}$$

• Potential should remain very flat during inflation, but difficult to do since potentially large quantum corrections

•One potential approach: inflaton is pseudo-Goldstone boson
Inflaton ~ axion, shift symmetry protects potential

•Not so easy to get large field displacements (inflaton is a phase, need to wind around many times).



Dilaton as inflaton?

- Could the inflaton be a scalar (rather than pseudo-scalar)?
- Should be Goldstone not of an internal symmetry (phase) but rather of a space-time symmetry (real exponential)
- Scale transformations provide such a candidate, dilaton
- Goldstone boson of broken scale invariance
- Non-compact symmetry - accommodates large $\Delta\phi$
- Scale invariance $\rightarrow$ shift symmetry $\rightarrow$ protection of potential like for axion case

Dilaton basics

- Scale transformations $x \to x' = e^{-\alpha} x$
- Operators transform $\mathcal{O}(x) \to \mathcal{O}'(x) = e^{\alpha\Delta}\mathcal{O}(e^{\alpha}x)$
- Δ is full dimension, classical plus quantum corrections
- Change in action:

$$S = \sum_i \int d^4x\, g_i \mathcal{O}_i(x) \longrightarrow S' = \sum_i \int d^4x e^{\alpha(\Delta_i - 4)} g_i \mathcal{O}_i(x)$$

- Assume spontaneous breaking of scale inv. (SBSI)

$$\langle \mathcal{O} \rangle = f^n$$

Dilaton basics

- Dilaton: Goldstone of SBSI, σ, transforms non-linearly under scale transf.:

$$\sigma(x) \to \sigma(e^\alpha x) + \alpha f$$

- Restore scale invariance by replacing VEV

$$f \to f\,\chi \equiv f\,e^{\sigma/f}$$

- Effective dilaton Lagrangian is then (using NDA for coeffs)

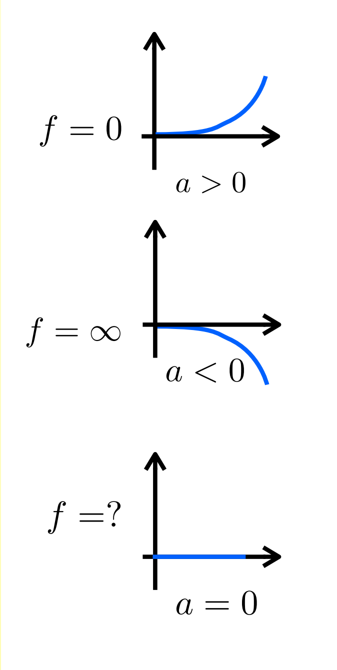
$$\begin{aligned}\mathcal{L}_{eff} &= \sum_{n,m\geqslant 0} \frac{a_{n,m}}{(4\pi)^{2(n-1)}\,f^{2(n-2)}} \frac{\partial^{2n}\chi^m}{\chi^{2n+m-4}} \\ &= -a_{0,0}\,(4\pi)^2 f^4\chi^4 + \frac{f^2}{2}(\partial_\mu\chi)^2 + \frac{a_{2,4}}{(4\pi)^2}\frac{(\partial\chi)^4}{\chi^4} + \dots\end{aligned}$$

Dilaton dynamics

- Main point of dilaton: effective action can have non-derivative χ^4 term - just the cosmological constant in the composite sector

$$S = \int d^4x \frac{f^2}{2}(\partial\chi)^2 - af^4\chi^4 + \text{higher derivatives}$$

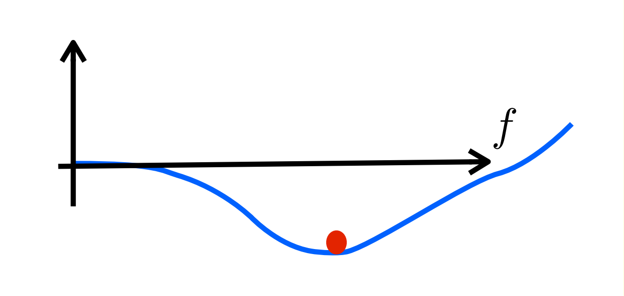
- Generically $a \neq 0$. Will make SBSI difficult:
  - $a>0$: VEV at $f=0$, no SBSI
  - $a<0$: runaway vacuum $f \to \infty$
  - $a=0$ arbitrary f



- Need to add additional almost-marginal operator to generate dilaton potential

Dilaton dynamics

•Perturbation: $$\delta S = \int d^4x \lambda(\mu)\mathcal{O}$$



$$af^4 \to f^4 F(\lambda(f))$$

•Dilaton potential: $V(\chi) = f^4 F(\lambda(f))$ vacuum energy in units of f

•To have a VEV: $$V' = f^3\left[4F(\lambda(f)) + \beta F'(\lambda(f))\right] = 0$$

$$\beta = \frac{d\lambda}{d\log\mu}$$

•Dilaton mass:

$$m^2_{dil} = f^2\beta\left[\beta F'' + 4F' + \beta' F'\right] \simeq 4f^2\beta F'(\lambda(f)) = -16 f^2 F(\lambda(f))$$

Dilaton-graviton system

- Assume scale invariant Lagrangian for Φ dilaton and $g_{\mu\nu}$ graviton

- Dilaton VEV will set Planck scale (EH term would violate scale invariance)

- Assume dilaton initially displaced far from VEV, rolling to minimum will lead to inflation

- Lagrangian:

$$\mathcal{L} = \sqrt{-g}\left[\tilde{\xi}\Phi^2 R - \frac{1}{2}(\nabla\Phi)^2 - V(\Phi)\right] + \Delta\mathcal{L}(g_{\mu\nu}, \Phi) + \mathcal{L}_M(g_{\mu\nu}, \Phi, \Psi)$$

- Scale invariant potential just a quartic $V(\Phi) = \alpha^2\Phi^4$

Dilaton-graviton system

- We treat this as a low-energy effective theory, won’t try to UV complete

- Since M_{Pl} appears dynamically, suspect that graviton dynamics also arises dynamically

- Possibilities: induced gravity? Start with no kinetic term for graviton...

- Composite gravity? Weinberg-Witten thm. says can not have a globally conserved stress tensor that becomes local in IR.

- May not have conserved stress tensor in UV
- Composite sector has its own conserved stress tensor and all matter couples to that (like Seiberg duality for gauge groups)

Dilaton-graviton system

- Conformal invariance would fix $\tilde{\xi} = 1/12$. Will not fix that, instead assume NDA size $\tilde{\xi} = O(16\pi^2)$
- Dilaton VEV will set Planck scale (EH term would violate scale invariance) $\langle \Phi \rangle^2 = M_{Pl}^2 / 2\tilde{\xi}$
- Assume dilaton initially displaced far from VEV, rolling to minimum will lead to inflation
- $\Delta \mathcal{L}(g_{\mu\nu}, \Phi)$ contains additional derivative interactions that are scale invariant (for example R^2 would be there)
- $\mathcal{L}_M(g_{\mu\nu}, \Phi, \Psi)$ contains interactions with matter (could violate scale invariance)

In Einstein Frame

- To understand dynamics, rescale metric to move from Jordan frame to Einstein frame: $g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}$

- This is determined by $\Omega^2 \tilde{\xi} \Phi^2 = \frac{M_P^2}{2}$ Field dependent rescaling of metric

- Rescaled Lagrangian:

$$\mathcal{L} = \sqrt{-g}\left[\frac{M_{Pl}^2}{2} R - \frac{1}{2}(\nabla\varphi)^2 - V(\varphi)\right] + \Delta\mathcal{L}\big(\Omega^2(\varphi) g_{\mu\nu}, \Phi(\varphi)\big) + \mathcal{L}_M\big(\Omega^2(\varphi) g_{\mu\nu}, \Phi(\varphi), \Psi\big)$$

- Potential in Einstein frame: $V(\varphi) = \frac{M_{Pl}^4}{4\tilde{\xi}^2} \frac{V(\Phi(\varphi))}{\Phi^4(\varphi)}$

Einstein frame

- Einstein frame inflaton:

$$\Phi(\varphi) = \langle\Phi\rangle \exp\left(\frac{\sqrt{\xi}\varphi}{M_{Pl}}\right) , \quad \frac{1}{\xi} = \frac{1}{2\tilde{\xi}} + 6 \, .$$

- Will have shift symmetry $\varphi \to \bar{\varphi} = \varphi + \frac{M_{Pl}}{\sqrt{\xi}}\lambda$

- Given the relation $\varphi = (M_{Pl}/\sqrt{\xi}) \log(\Phi/\langle\Phi\rangle)$

- If start out at $\Phi_0 \sim 10^{-15}\langle\Phi\rangle \sim \text{TeV}$ roll to $\langle\Phi\rangle \sim M_{Pl}$

- We obtain $|\Delta\varphi| \sim 15 M_{Pl}$ a seemingly super-Planckian field excursion in Einstein frame (though never left effective theory)

Einstein frame

- The potential: $\alpha^2\Phi^4$ will become completely flat in Einstein frame - consequence of scale invariance/shift symmetry.

- Need small explicit breaking terms to fix dilaton VEV

- Can come from interaction with matter

- As long as breaking terms small approximate shift symmetry will remain

Approximately scale invariant potentials

- Will add small explicit breaking terms
- Require that cosmological constant at minimum vanishes
- This is added by hand - scale invariance does not tell anything about cc
- CC is vacuum energy created during the phase transition
- To exit inflation will need to tune this to zero
- Nothing new to say about the cc

A single relevant operator

- Add one marginally relevant operator of dimension $4-\epsilon$

- Typical in warped extra dimension (Goldberger-Wise)

- Full potential: $V(\Phi) = \Phi^4 \left(\alpha + \beta \Phi^{-\epsilon}\right)^2$

- Minimized at $\langle \Phi \rangle = -\left(\frac{\alpha}{\beta}\right)^{\frac{1}{\epsilon}}$

- In Einstein frame: $V(\varphi) = \frac{M_{Pl}^4}{4} \frac{\alpha^2}{\tilde{\xi}^2} \left(1 - e^{-\epsilon \sqrt{\xi} \varphi / M_{Pl}}\right)^2$

- Same potential as for Starobinsky model

- Exponent controlled by small breaking parameter ϵ

- Starobinsky special case: if R^2 dominates also scale invariant. But here don't need to assume very large coefficient of higher dim. operator

A single relevant operator

- The slow-roll parameters:

$$\epsilon_V = \frac{M_{Pl}^2}{2}\left(\frac{V'(\varphi)}{V(\varphi)}\right)^2 = \frac{2\epsilon^2\xi}{(1-e^{\epsilon\sqrt{\xi}\varphi/M_{Pl}})^2}\ , \qquad \eta_V = M_{Pl}^2\frac{V''(\varphi)}{V(\varphi)} = \epsilon_V\left(2-e^{\epsilon\sqrt{\xi}\varphi/M_{Pl}}\right)$$

- Number of e-folds of inflation:

$$N \simeq \frac{1}{M_{Pl}^2}\int_0^{\varphi_0}\frac{V'(\varphi)}{V(\varphi)}d\varphi = \frac{1}{2\epsilon^2\xi}\left[\left(e^{\epsilon\sqrt{\xi}\varphi_0/M_{Pl}}-1\right)-\frac{\varphi_0}{\sqrt{2}M_{Pl}}\right]$$

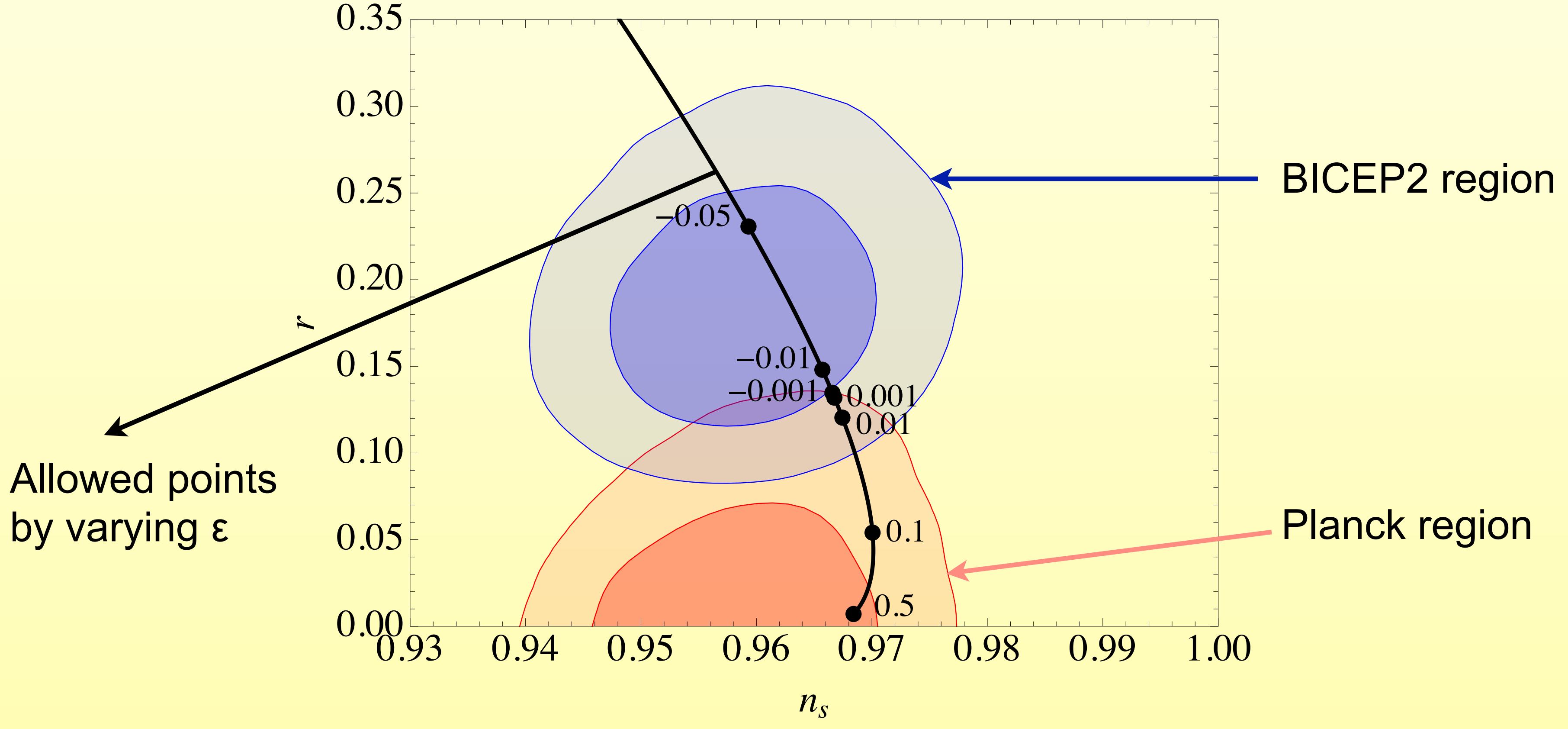
- Parameters of power spectrum:

$$\mathcal{P}_s \simeq \frac{1}{24\pi^2}\frac{V^\star}{M_{Pl}^4}\frac{1}{\epsilon_V^\star}\ ,$$

$$r = \frac{\mathcal{P}_t}{\mathcal{P}_s} \simeq 16\epsilon_V^\star\ , \qquad n_s = 1+\frac{d\ln\mathcal{P}_s}{d\ln k} \simeq 1+2\eta_V^\star-6\epsilon_V^\star$$

A single relevant operator

- The region of r vs. ns:



A single relevant operator

- COBE normalization:

$$\mathcal{P}_s = \frac{\alpha^2}{24\pi^2\tilde{\xi}^2}\frac{\sinh^4(\epsilon\sqrt{\xi}\varphi_{cmb}/2M_{Pl})}{\epsilon^2\xi} \quad \sim 10^{-9}$$

- Increases with ε

- How large ε expected? $1/\ln(M_{Pl}/\Lambda_\epsilon)$

- Example numbers: $\Lambda_\epsilon \sim 10^{\pm 3} M_{Pl}$ yields $\epsilon \sim 0.1$,

  $\Lambda_\epsilon \sim 10^{\pm 17} M_{Pl}$ gives $\epsilon \sim 0.01$

- Taking $\epsilon\sqrt{\xi} \sim 0.01, \tilde{\xi} \sim 16\pi^2$

$$\mathcal{P}_s \simeq \left(\frac{\alpha}{0.1}\right)^2 \times 10^{-9}$$

A cosh potential

- One marginally relevant, one marginally irrelevant $4 \pm \epsilon,$
- Full potential: $V(\Phi) = -\alpha^2 \Phi^4 + \beta^2 \Phi^{4-\epsilon} + \gamma^2 \Phi^{4+\epsilon}$
- Minimized at $\langle \Phi \rangle = \left(\frac{2\alpha^2 + \sqrt{4\alpha^4 + \beta^2 \gamma^2 (4-\epsilon)(4+\epsilon)}}{\gamma^2 (4+\epsilon)} \right)^{1/\epsilon}$
- In Einstein frame: $V(\varphi) = \frac{M_{Pl}^4}{4} \frac{\alpha^2}{\tilde{\xi}^2} \left(\cosh(\epsilon \sqrt{\xi} \varphi / M_{Pl}) - 1 \right)$
- Non-compact analog of generic axion potentials
- Analog of axion decay constant $M_{Pl}/\epsilon\sqrt{\xi}$
- Can be $\gg M_{Pl}$ for small ϵ

A cosh potential

- The slow-roll parameters:

$$\epsilon_V = \frac{1}{2}\epsilon^2\xi \coth^2(\epsilon\sqrt{\xi}\varphi/2M_{Pl}) \ , \qquad \eta_V = \frac{\epsilon_V}{\cosh(\epsilon\sqrt{\xi}\varphi/M_{Pl})} \ ,$$

- Number of e-folds of inflation:

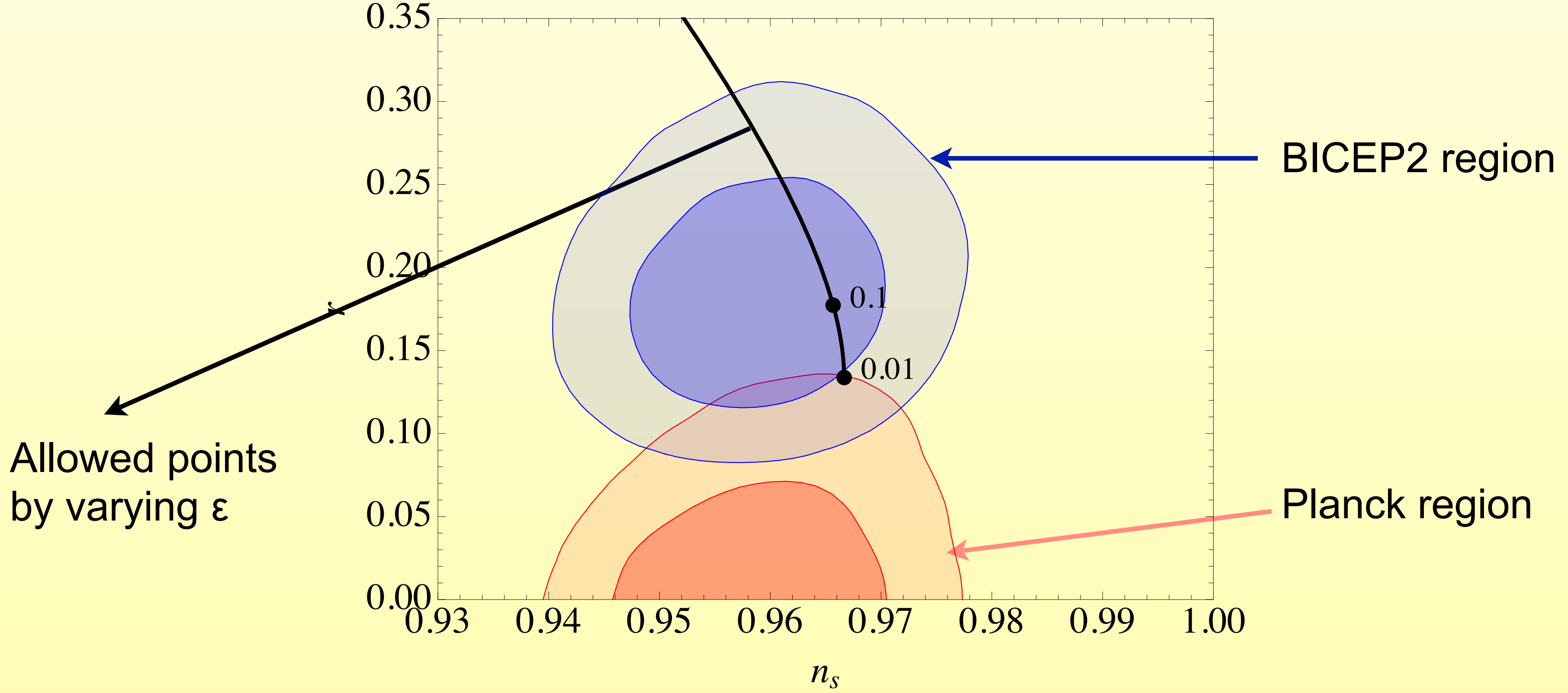
$$N \simeq \frac{2}{\epsilon^2\xi} \log\left[\cosh(\epsilon\sqrt{\xi}\varphi/2M_{Pl})\right]$$

- Parameters of power spectrum: $\mathcal{P}_s = \frac{\alpha^2}{12\pi^2\tilde{\xi}^2}\frac{\sinh^2(\epsilon^2\xi N_{cmb})}{\epsilon^2\xi}$

$$\mathcal{P}_s \simeq \left(\frac{\alpha}{0.1}\right)^2 \times 10^{-9} \quad \text{for} \quad \epsilon\sqrt{\xi} \sim 0.01, \tilde{\xi} \sim 16\pi^2$$

A cosh potential

- The region of r vs. ns:



Potential with matter induced CC

- CC from matter and one marginally irrelevant op dim $4-\epsilon$

- Full potential: $$V(\Phi) = \alpha^2\Phi^4 - \beta^2\Phi^{4-\epsilon} + \Lambda_M^4$$

- Minimized at $$\langle\Phi\rangle = \left(\frac{\beta^2(4-\epsilon)}{4\alpha^2}\right)^{1/\epsilon}$$

- In Einstein frame:

$$V(\varphi) = \frac{M_{Pl}^4}{4(4-\epsilon)}\frac{\alpha^2}{\tilde{\xi}^2}\left[4\left(1-e^{-\epsilon\sqrt{\xi}\varphi/M_{Pl}}\right) - \epsilon\left(1-e^{-4\sqrt{\xi}\varphi/M_{Pl}}\right)\right]$$

- Example of a racetrack inflation model

Potential with matter induced CC

- The slow-roll parameters:

$$\epsilon_V = \frac{\epsilon^2\xi\left(1 - e^{(\epsilon-4)\sqrt{\xi}\varphi/M_{Pl}}\right)^2}{2\left(1 - \frac{4-\epsilon}{4}e^{\epsilon\sqrt{\xi}\varphi/M_{Pl}} - \frac{\epsilon}{4}e^{(\epsilon-4)\sqrt{\xi}\varphi/M_{Pl}}\right)^2}\,, \quad \eta_V = \frac{\epsilon\xi(\epsilon - 4e^{(\epsilon-4)\sqrt{\xi}\varphi/M_{Pl}})}{1 - \frac{4-\epsilon}{4}e^{\epsilon\sqrt{\xi}\varphi/M_{Pl}} - \frac{\epsilon}{4}e^{(\epsilon-4)\sqrt{\xi}\varphi/M_{Pl}}}$$

- Number of e-folds complicated Hypergeometric, to simplify expand potential for large values:

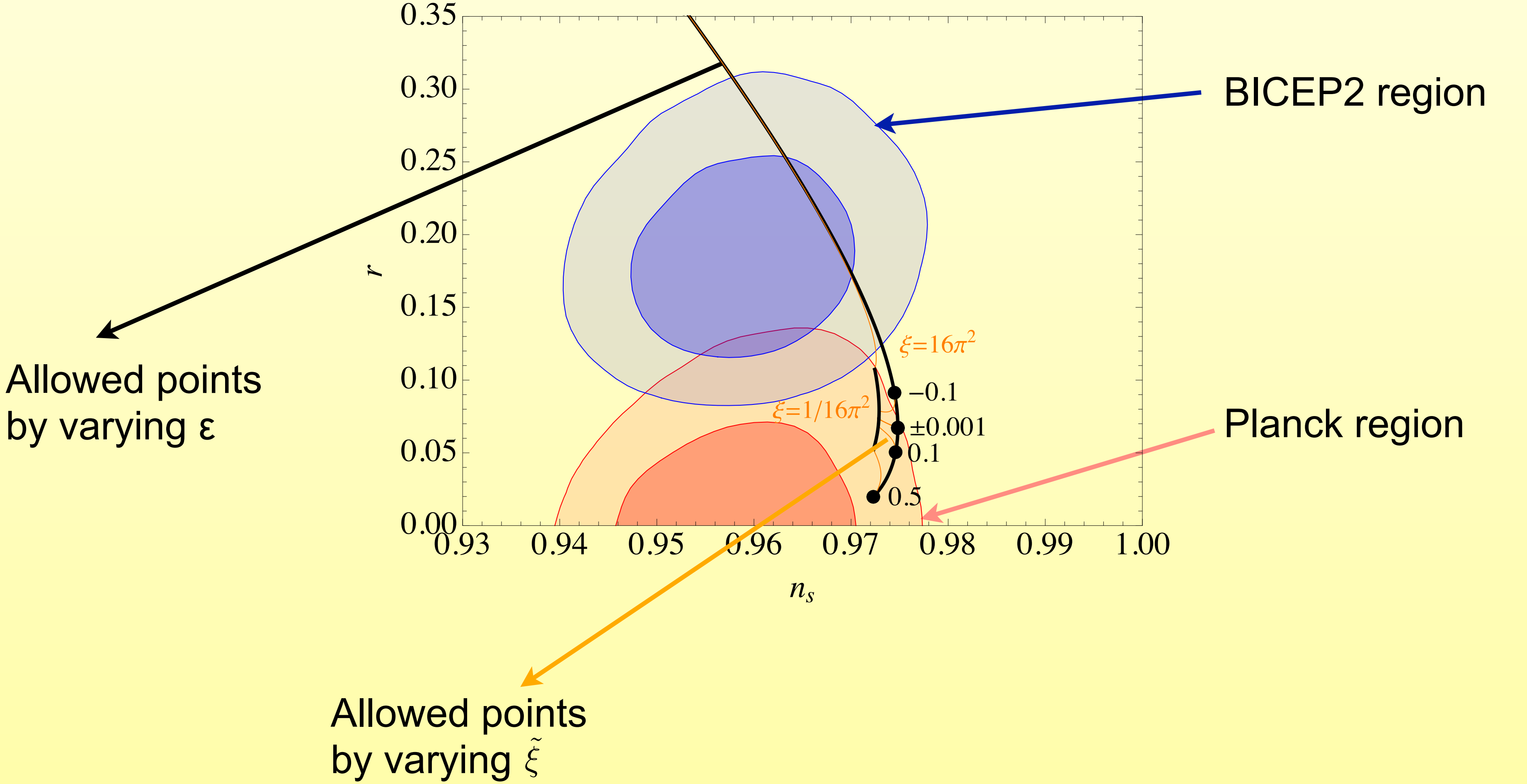
$$V(\varphi) = M_{Pl}^4\frac{\alpha^2}{\tilde{\xi}^2}\left[1 - e^{-\epsilon\sqrt{\xi}\varphi/M_{Pl}}\right]$$

- Approximate slow-roll parameters

$$\epsilon_V = \frac{\epsilon^2\xi}{2\left(1 - e^{\epsilon\sqrt{\xi}\varphi/M_{Pl}}\right)^2}\,, \qquad \eta_V = \frac{\epsilon^2\xi}{1 - e^{\epsilon\sqrt{\xi}\varphi/M_{Pl}}}\,.$$

Potential with matter induced CC

- The region of r vs. ns:



Power law potentials

- Most commonly used inflationary potentials simple power laws

- In fact n-loop logarithmic corrections give exactly those!

- Effect of n-loop in Jordan frame: $\beta_n \Phi^4 [\log(\Phi/\Lambda_\epsilon)]^n$,

- In Einstein frame: $V_n \sim \beta_n \langle\Phi\rangle^4 \left(\frac{\sqrt{\xi}\varphi}{M_{Pl}}\right)^n \sim \beta_n \frac{\varphi^n}{M_{Pl}^{n-4}}$

- $\beta_n \sim O(1/16\pi^2)^n$ n-loop beta-function, shift symmetry ensures it is small

- Could be leading order term, or lead to interesting potentially measurable sub-leading corrections

Cutoff scale and higher orders

- Easiest to analyze in Einstein frame
- The dilaton decay constant: $f = \frac{M_{Pl}}{\sqrt{\xi}}$
- Cutoff expected at or below $\Lambda_{UV} = \frac{4\pi}{\sqrt{\xi}} M_{Pl}$
- For example the R^2 term (in Einstein frame gets corrections):

$$\frac{1}{g_R^2} R^2 \to \frac{1}{g_R^2} \left[R + 6 \left(\frac{\sqrt{\xi}}{M_{Pl}} \nabla^2 \varphi - \frac{\xi}{M_{Pl}^2} (\nabla \varphi)^2 \right) \right]^2$$

- Can think of this as integrating out a scalar with mass

$$M_R^2 \simeq g_R^2 M_{Pl}^2$$

- NDA would $g_R \sim 4\pi$ giving rise to expected cutoff above.

Cutoff scale and higher orders

- Another example:

$$\frac{1}{g_\Phi^4}\frac{[(\nabla\Phi)^2]^2}{\Phi^4} \to \frac{1}{g_\Phi^4}\frac{\xi^2}{M_{Pl}^4}[(\nabla\Phi)^2]^2$$

- For $g_\Phi \sim 4\pi$ we again get same expression for cutoff

- All symmetric terms derivatively coupled $\nabla\varphi$

- Field excursions beyond Λ_{UV} not a problem since potential very flat and all terms contain derivatives

- Explicit breaking terms may not contain derivatives, but they are small by assumption

Dynamics of matter fields & reheating

- Very UV dependent. Assume SM fields still good degrees of freedom.

- All couplings classically marginal except Higgs mass term, which causes small explicit breaking $O(m_H^2/M_{Pl}^2)$

- Tree-level Higgs-dilaton quartic OK for dilaton (does not generate dilaton mass if cutoff done properly), but probematic for Higgs

- Loop level SM couplings run but small $O(1/16\pi^2)$ beta functions

- Could be small parameters β_n for loop induced polynomial potential

Dynamics of matter fields & reheating

- Coupling to SM fields: usually treat dimensionful parameters as spurions and dress them with $\Phi/\langle\Phi\rangle = e^{\varphi/f}$

- Even if absent in Jordan frame derivative couplings will be generated from $\sqrt{g}$

- For example Higgs-dilaton coupling $-\sqrt{\xi}|H|^2\partial^2\varphi/M_{Pl}$ in Einstein frame

- Gives rise to $\varphi \to WW, ZZ, hh$ via longitudinal modes

- Decay rate: $\Gamma_{\varphi\to WW,ZZ,hh} \simeq \frac{4\xi}{32\pi}\frac{m_\varphi^3}{M_{Pl}^2} \simeq 0.5\,\mathrm{GeV}\,\left(\frac{\xi}{1/12}\right)\left(\frac{m_\varphi}{10^{13}\,\mathrm{GeV}}\right)^3$

- Expression of mass in simplest model:

$$m_\varphi = M_{Pl}\frac{\alpha\epsilon\sqrt{\xi}}{\tilde{\xi}} \simeq 10^{13}\left(\frac{\alpha}{0.1}\right)\left(\frac{\epsilon\sqrt{\xi}}{0.01}\right)\left(\frac{16\pi^2}{\tilde{\xi}}\right)\,\mathrm{GeV}\ .$$

Dynamics of matter fields & reheating

- Resulting reheat temperature:

$$T_{RH} \sim g_*^{-1/4} (\Gamma M_{Pl})^{1/2} \sim 3 \times 10^8 \,\mathrm{GeV}$$

- Assuming $g_* \sim O(100)$

- High enough for EW baryogenesis, but sufficiently low to avoid GUT defects

- Note: $\varphi \to 2g$ suppressed compared to massive GB's

- Coupling given by $\frac{\alpha_{SM}}{8\pi} (b_{IR} - b_{UV}) F^{\mu\nu} F_{\mu\nu} \sqrt{\xi} \frac{\varphi}{M_{Pl}}$

- Much smaller width

$$\Gamma_{\varphi \to 2g} \simeq \frac{\alpha_s^2}{256\pi^3} \Delta b_s^2 \xi \frac{m_\varphi^3}{M_{Pl}^2} \simeq 3\,\mathrm{keV} \left(\frac{\xi}{1/12}\right) \left(\frac{m_\varphi}{10^{13}\,\mathrm{GeV}}\right)^3 (\Delta b_s)^2$$

Summary

- Few simple assumptions lead to a robust inflationary model:

    1. Underlying gravity theory scale invariant
    2. Scale symmetry spontaneously broken generating Planck scale and giving rise to inflation
    3. Explicit breaking small

- Low energy theory of dilaton has very flat potential protected by shift symmetry

- Since Goldstone corresponding to non-compact direction can easily get large field range

- Underlies many scale-invariant models

- Can give large field inflation, r can be sizable (or smallish), successful phenomenology