From Electric-Magnetic Duality to Seiberg Duality

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Outline

* EM Duality
* Dirac Monopoles
* A Short Course on SUSY
* Seiberg Duality
* Electroweak Symmetry Breaking
* Conclusions

EM Duality

$$\vec{\nabla} \cdot \vec{E} = 0\,, \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0, \qquad \vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t}$$

$$\vec{E} \longrightarrow \vec{B}$$

$$\vec{B} \longrightarrow -\vec{E}$$

EM Duality

$$\vec{\nabla} \cdot \vec{E} \quad = \quad e\,\rho \qquad\qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} \quad = \quad 0 \qquad\qquad \vec{\nabla} \times \vec{B} = e\,\vec{J} + \frac{\partial \vec{E}}{\partial t}$$

EM Duality

$$\vec{\nabla} \cdot \vec{E} = e\,\rho \qquad \vec{\nabla} \times \vec{E} = -\frac{1}{e}\vec{J}_m - \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = \frac{1}{e}\rho_m \qquad \vec{\nabla} \times \vec{B} = e\,\vec{J} + \frac{\partial \vec{E}}{\partial t}$$

$$\vec{E} \rightarrow \vec{B} \qquad \vec{J} \rightarrow \vec{J}_m \qquad \rho \rightarrow \rho_m$$

$$\vec{B} \rightarrow -\vec{E} \qquad \vec{J}_m \rightarrow -\vec{J} \qquad \rho_m \rightarrow -\rho$$

$$e \rightarrow \frac{1}{e}$$

EM Duality

$$^*F^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

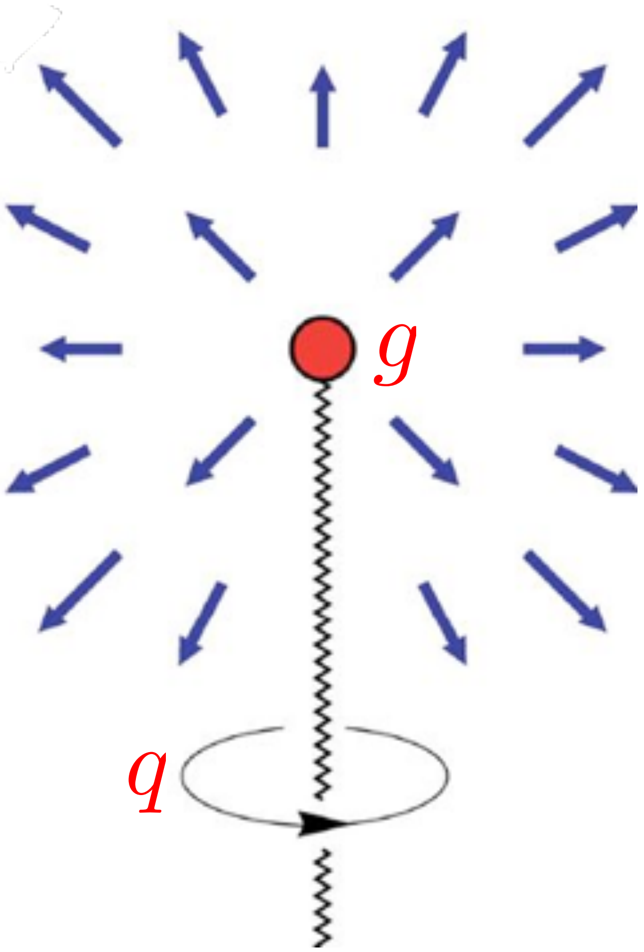
$$\partial_\mu F^{\mu\nu} = e\,J^\nu$$

$$\partial_\mu^* F^{\mu\nu} = \frac{1}{e}\,J_m^\nu$$

$$\begin{aligned} F^{\mu\nu} &\rightarrow \;^*F^{\mu\nu} \\ ^*F^{\mu\nu} &\rightarrow -F^{\mu\nu} \\ J^\nu &\rightarrow J_m^\nu \\ J_m^\nu &\rightarrow -J_m^\nu \end{aligned}$$

$$e \rightarrow \frac{1}{e}$$

Dirac



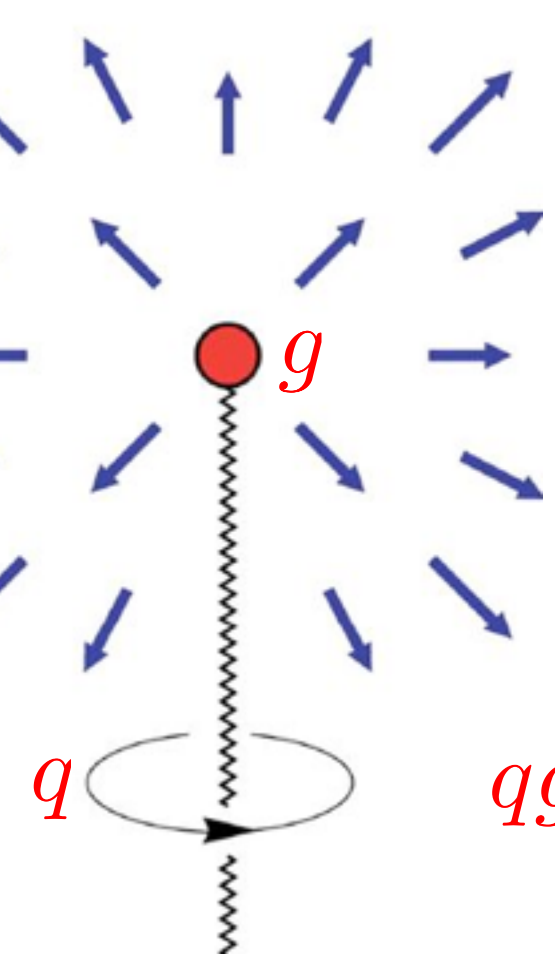
Proc. Roy. Soc. Lond. A133 (1931) 60

Dirac

$$\vec{B} = \frac{g}{e}\frac{\hat{r}}{r^2}$$

$$\vec{A} = \frac{g}{e}\frac{1-\cos\theta}{r\sin\theta}\,\hat{e}_\phi$$

$$\int_{loop} e\,q\,A^\mu dx_\mu = 4\pi qg = 2\pi n$$

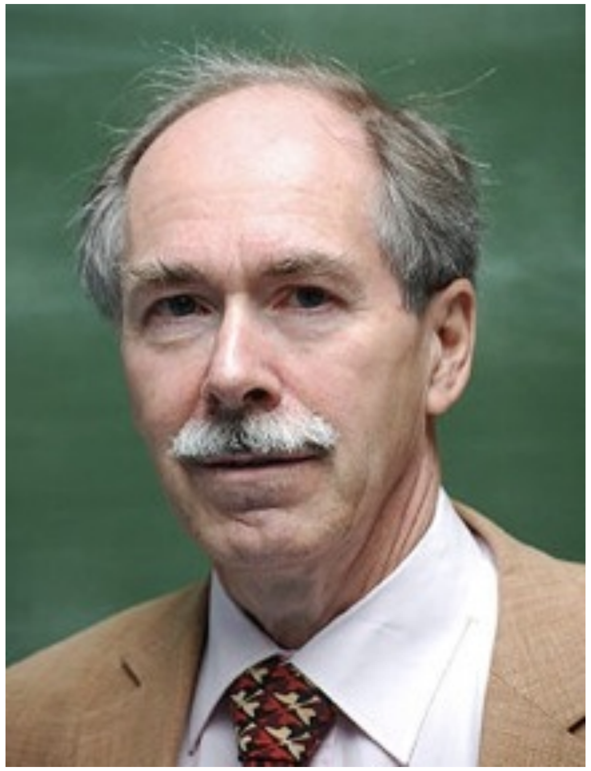


$$qg = \frac{n}{2}$$

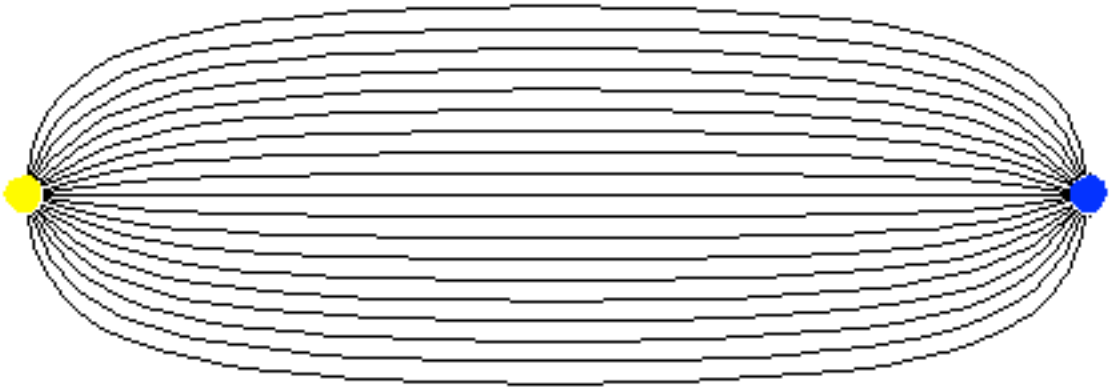
charge quantization

Proc. Roy. Soc. Lond. A133 (1931) 60

`t Hooft-Mandelstam



magnetic condensate
confines electric charge



High Energy Physics Ed. Zichichi, (1976) 1225
Phys. Rept. 23 (1976) 245

Phases of Gauge Theories

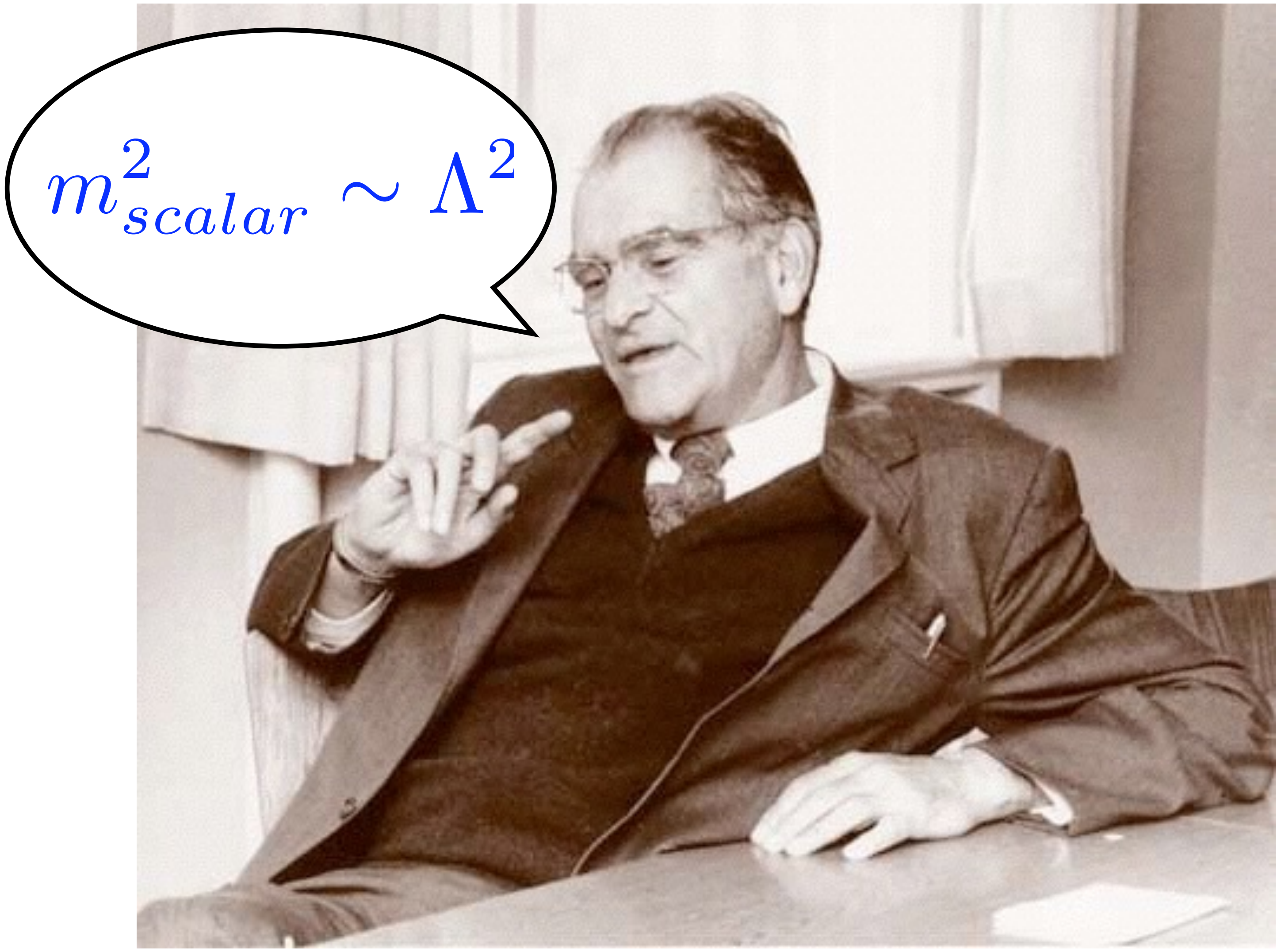
Coulomb :	$V(R) \sim \frac{1}{R}$
Free electric :	$V(R) \sim \frac{1}{R \ln(R\Lambda)}$
Free magnetic :	$V(R) \sim \frac{\ln(R\Lambda)}{R}$
Higgs :	$V(R) \sim \text{constant}$
Confining :	$V(R) \sim \sigma R$

Phases of Gauge Theories

	electron	⟷	monopole
EM Duality:	free electric	⟷	free magnetic
	Coulomb phase	⟷	Coulomb phase
`t Hooft-Mandelstam conjectured duality:	Higgs phase	⟷	confining phase

Seiberg found strongly coupled SUSY analogs where more precise tests can be made

SUSY and the problem with scalars



Weisskopf Phys. Rev.56 (1939) 72

Supersymmetry

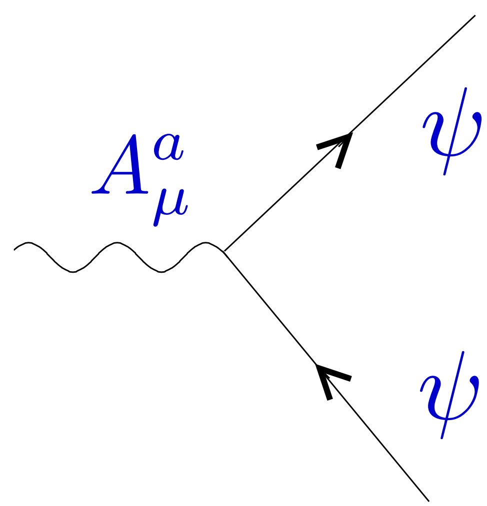
ψ	ϕ
fermion	boson
spinor	complex scalar
quark	squark
top	stop

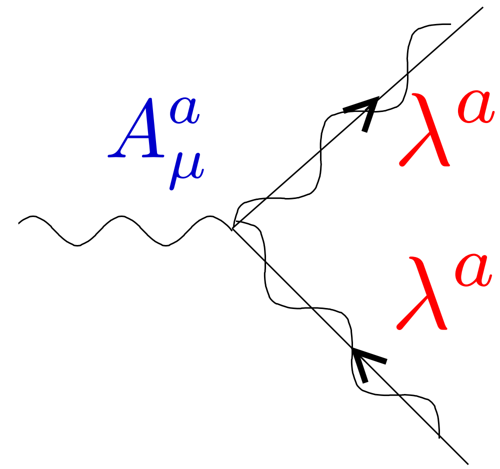
Supersymmetry

A^a_μ		λ^a
boson		fermion
spin 1	→	spinor
gluon		gluino
W		wino

Gauge Interactions

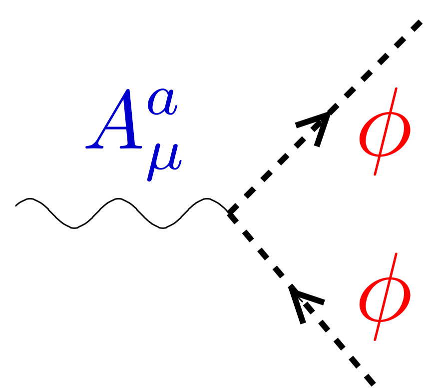
$g\,T^a$

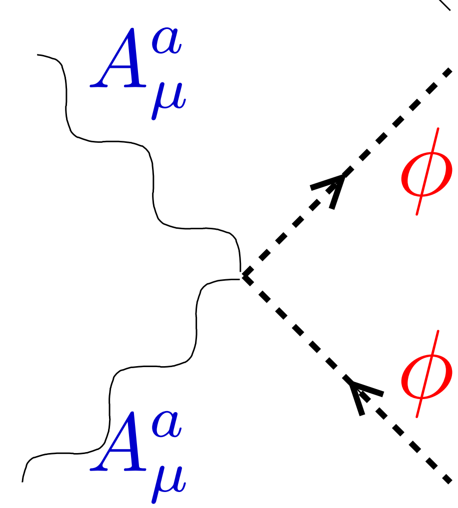




$g\,f^{abc}$

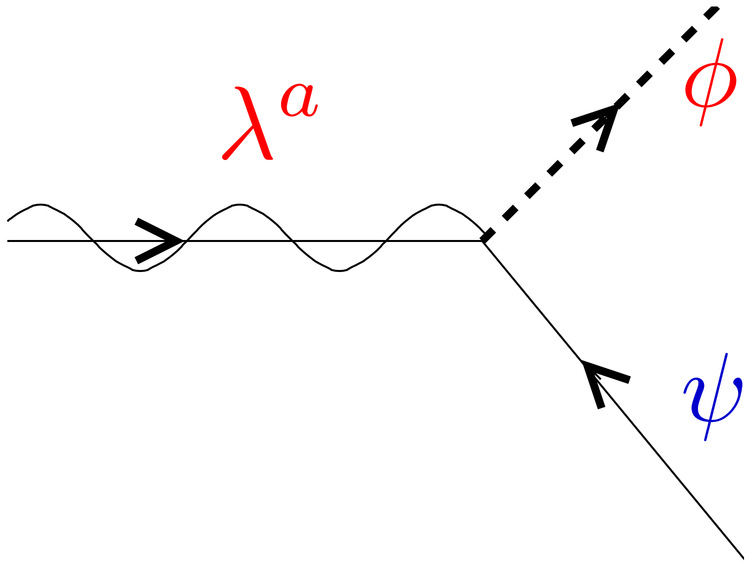
$g\,T^a$



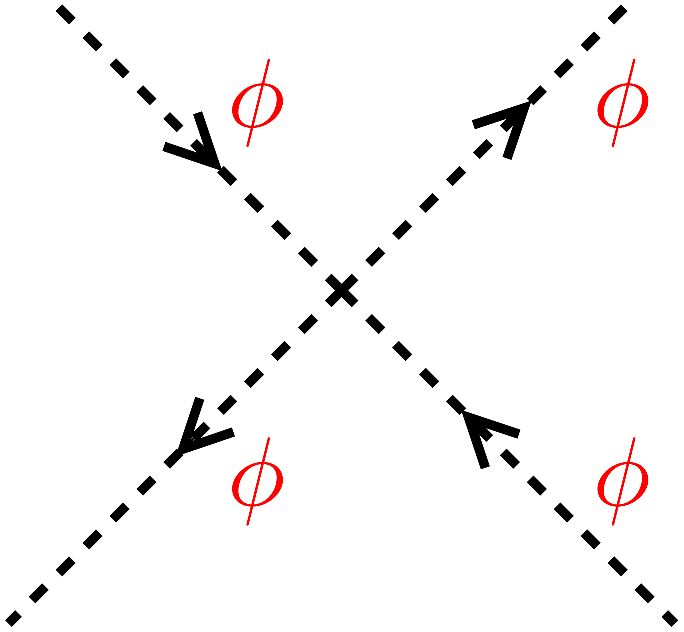


$g^2\left(T^aT^b + T^bT^a\right)$

Gauge Interactions

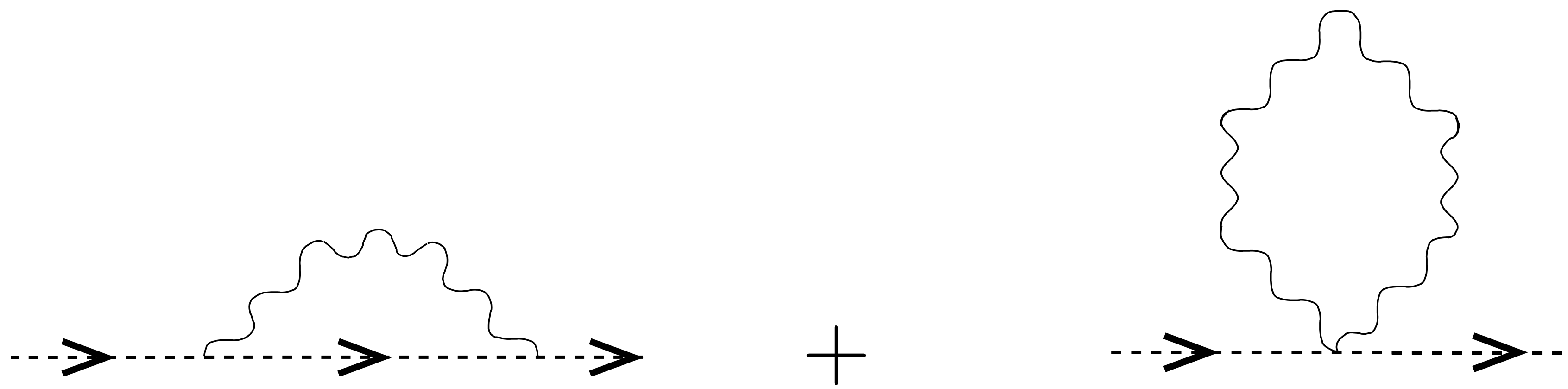


$\sqrt{2}\, g\, T^a$

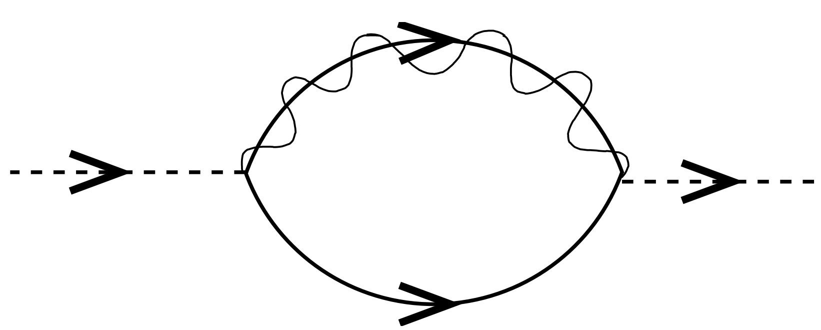


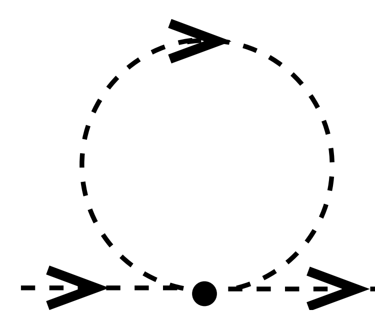
$g^2\, T^a T^a$

Scalar Mass



$-3\,g^2\Lambda^2$





$+4\,g^2\Lambda^2$

$-\,g^2\Lambda^2$

Superpotential Interactions

$$\begin{aligned}\mathcal{L} &= \partial^\mu \phi^{*j} \partial_\mu \phi_j + i\psi^{\dagger j} \overline{\sigma}^\mu \partial_\mu \psi_j \\ &\quad - \frac{1}{2}\left(W^{jk}\psi_j\psi_k + W^{*jk}\psi^{\dagger j}\psi^{\dagger k}\right) - W^j W^*_j\end{aligned}$$

$$W^j = \frac{\partial W}{\partial \phi_j} \qquad\qquad W^{ij} = \frac{\partial W}{\partial \phi_i \partial \phi_j}$$

W is a holomorphic function of ϕ_i

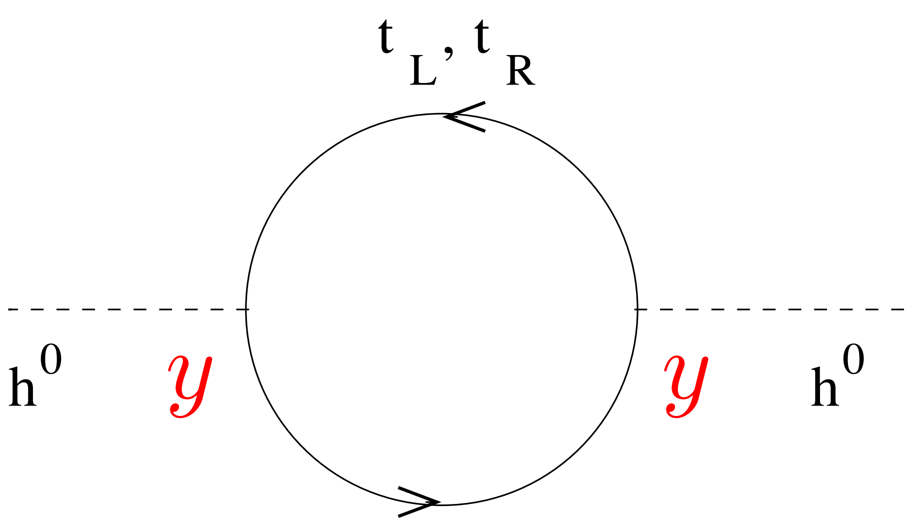
Superpotential

$$W = \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k$$

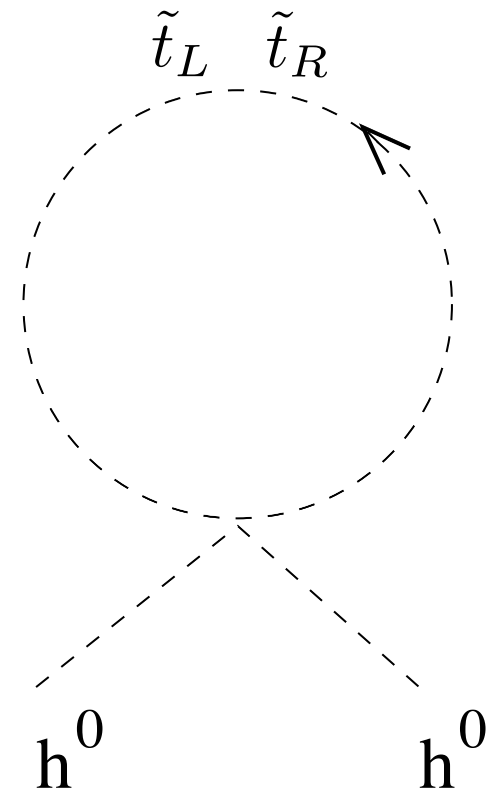
$$\mathcal{L}_{int} = -\frac{1}{2} y^{ijk} \phi_i \psi_j \psi_k - \frac{1}{2} y^*_{ijk} \phi^{*i} \psi^{\dagger j} \psi^{\dagger k}$$

$$-\frac{1}{4} y^{jkm} y^*_{npm} \phi_j \phi_k \phi^{*n} \phi^{*p}$$

Quadratic Cancellation



$-y^2$



$+y^2$

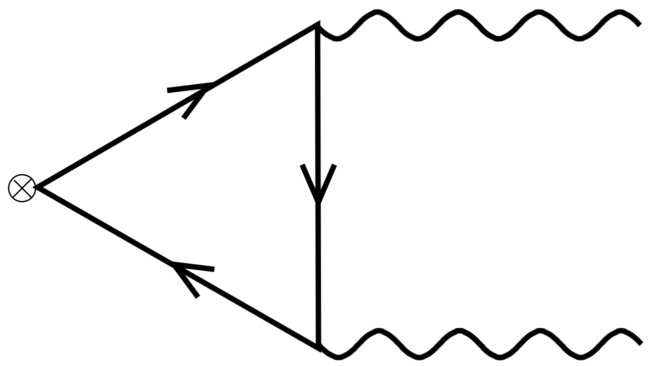
SUSY QCD

	$SU(N)$	$SU(F)$	$SU(F)$	$U(1)_B$	$U(1)_R$
$\mathcal{Q}$	$\Box$	$\Box$	$\mathbf{1}$	1	$\frac{F-N}{F}$
$\overline{\mathcal{Q}}$	$\overline{\Box}$	$\mathbf{1}$	$\overline{\Box}$	-1	$\frac{F-N}{F}$

$$\mathcal{Q}_i = (\phi_i, \psi_i) \qquad R_\psi = R_\phi - 1$$

$$\overline{\mathcal{Q}}_i = (\overline{\phi}_i, \overline{\psi}_i) \qquad R_{A_\mu} = R_\lambda - 1 = 0$$

$SU(N)^2U(1)_R$ Anomaly



$$R_\psi = R_\phi - 1$$

$$R_{A_\mu} = R_\lambda - 1 = 0$$

$$1 \cdot T(\mathbf{Ad}) + (R-1)T(\Box)\,2F = 0$$

$$R = \frac{F-N}{F}$$

Scalar Potential

$$D^a = g(\phi^{*in}(T^a)_n^m \phi_{mi} - \overline{\phi}^{in}(T^a)_n^m \overline{\phi}^*_{mi})$$

$$V = \frac{1}{2} D^a D^a \geq 0$$

Flat Directions F>N

$$D^a = 0$$

N x F matrix of VEVs

$$\langle \Phi \rangle = \begin{pmatrix} v_1 & & & 0 & \dots & 0 \\ & \ddots & & \vdots & & \vdots \\ & & v_N & 0 & \dots & 0 \end{pmatrix} \, , \quad \langle \overline{\Phi} \rangle = \begin{pmatrix} \overline{v}_1 & & \\ & \ddots & \\ & & \overline{v}_N \\ 0 & \dots & 0 \\ \vdots & & \vdots \\ 0 & \dots & 0 \end{pmatrix}$$

F

F

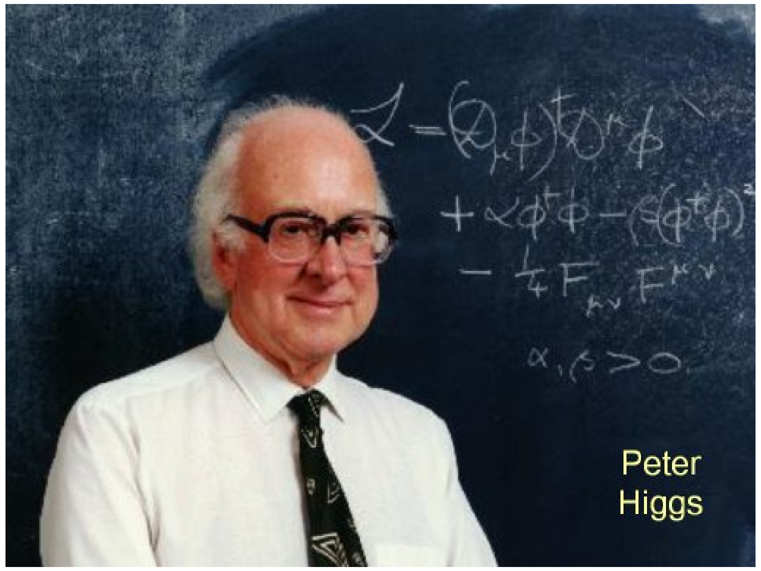
moduli space of inequivalent vacua

Flat Directions F>N

classically, can parameterize by gauge invariant "mesons" and "baryons"

$$
\begin{aligned}
M_i^j &= \overline{\Phi}^{jn}\Phi_{ni} \\
B_{i_1,\ldots,i_N} &= \Phi_{n_1 i_1}\ldots\Phi_{n_N i_N}\epsilon^{n_1,\ldots,n_N} \\
\overline{B}^{i_1,\ldots,i_N} &= \overline{\Phi}^{n_1 i_1}\ldots\overline{\Phi}^{n_N i_N}\epsilon_{n_1,\ldots,n_N}
\end{aligned}
$$

Super Higgs Mechanism



Fayet

$$\begin{matrix} W_\mu \\ \lambda \end{matrix} \quad + \quad \begin{matrix} \psi \\ h + i\,\pi \end{matrix} \quad \rightarrow \quad \begin{matrix} W_\mu \\ \lambda + \psi \\ h \end{matrix}$$

massless massless massive

$$\begin{matrix} 2 \\ 2 \end{matrix} \quad + \quad \begin{matrix} 2 \\ 2 \end{matrix} \quad \rightarrow \quad \begin{matrix} 3 \\ 2 + 2 \\ 1 \end{matrix}$$

Running Coupling

$$\beta_g \quad = \quad \mu \frac{dg}{d\mu} = -\frac{g^3}{16\pi^2}\left(\frac{11}{3}T(\mathrm{Ad}) - \frac{2}{3}T(F) - \frac{1}{3}T(S)\right) \equiv -\frac{g^3\, b}{16\pi^2} \ ,$$

$$b = 3N - F$$

F > 3N infrared free

Banks-Zaks Fixed Point

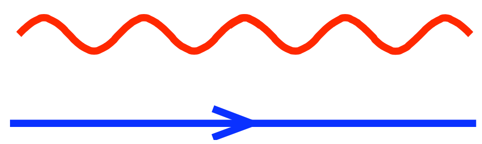
large N, with $F = 3N - \epsilon N$

$$16\pi^2\beta(g) \quad = \quad -g^3\epsilon N + \frac{g^5}{8\pi^2}\Big(3(N^2-1) + \mathcal{O}(\epsilon)\Big) + \mathcal{O}(g^7)$$

$$g_*^2 = \frac{8\pi^2}{3}\frac{N}{N^2-1}\,\epsilon$$

Conformal Field Theory

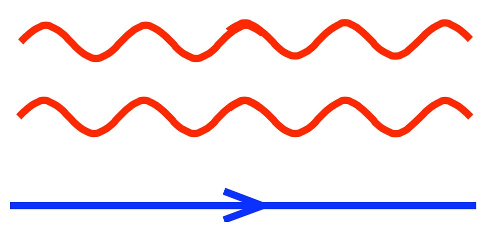
CFT 101



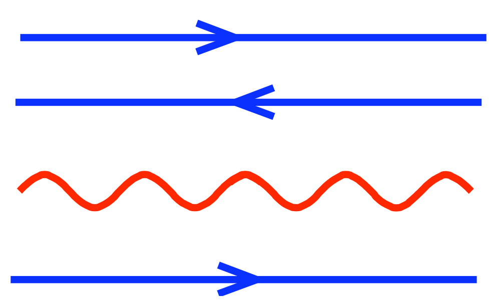
$$p_1^2 = 0$$

$$p_2^2 = 0$$

$$p^2 = (p_1 + p_2)^2 \neq 0$$



$$p_i^2 = 0$$



$$p^2 = \left(\sum_i p_i \right)^2 \neq 0$$

jets!

Enhanced Symmetry

SUSY + Conformal = super-conformal

for holomorphic gauge invariant operators

$$d = \frac{3}{2} R$$

$$d(Q\overline{Q}) = \frac{3(F-N)}{F} < 2$$

$$d = 1 \;\; at \;\; F = \frac{3}{2} N$$ free field!

Minwalla hep-th/9712074

Seiberg

SUSY QCD

	$SU(N)$	$SU(F)$	$SU(F)$	$U(1)_B$	$U(1)_R$
$\mathcal{Q}$	$\Box$	$\Box$	$\mathbf{1}$	1	$\frac{F-N}{F}$
$\overline{\mathcal{Q}}$	$\overline{\Box}$	$\mathbf{1}$	$\overline{\Box}$	-1	$\frac{F-N}{F}$

$$\mathcal{Q}_i = (\phi_i, \psi_i) \qquad R_\psi = R_\phi - 1$$

$$\overline{\mathcal{Q}}_i = (\overline{\phi}_i, \overline{\psi}_i) \qquad R_{A_\mu} = R_\lambda - 1 = 0$$

Dual Theory

	$SU(F-N)$	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$
q	$\square$	$\overline{\square}$	$\mathbf{1}$	$\frac{N}{F-N}$	$\frac{N}{F}$
$\bar{q}$	$\overline{\square}$	$\mathbf{1}$	$\square$	$-\frac{N}{F-N}$	$\frac{N}{F}$
M	$\mathbf{1}$	$\square$	$\overline{\square}$	0	$2\,\frac{F-N}{F}$

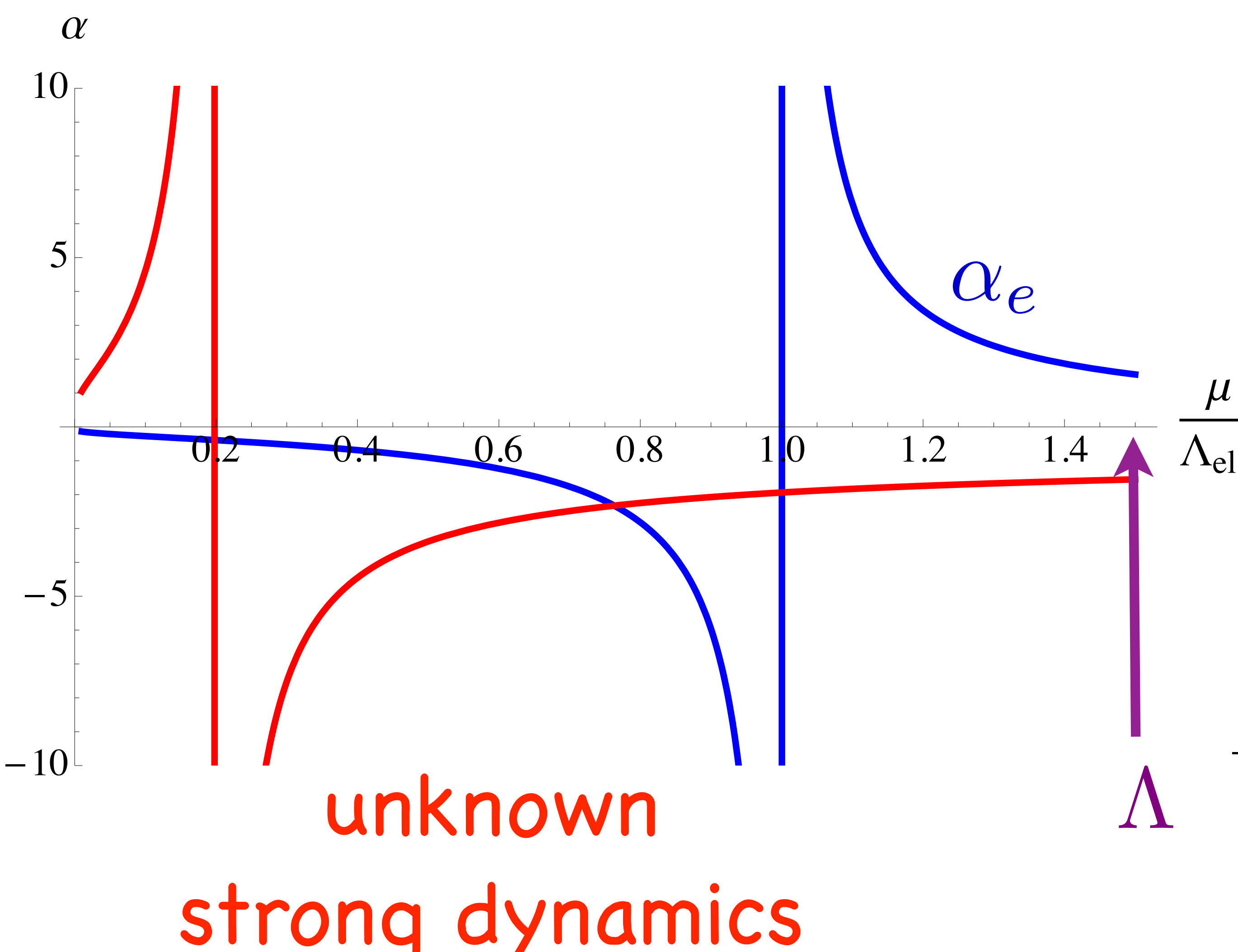
$$W = \frac{\tilde{M} q \bar{q}}{\Lambda} \qquad W = y\, M q \bar{q}$$

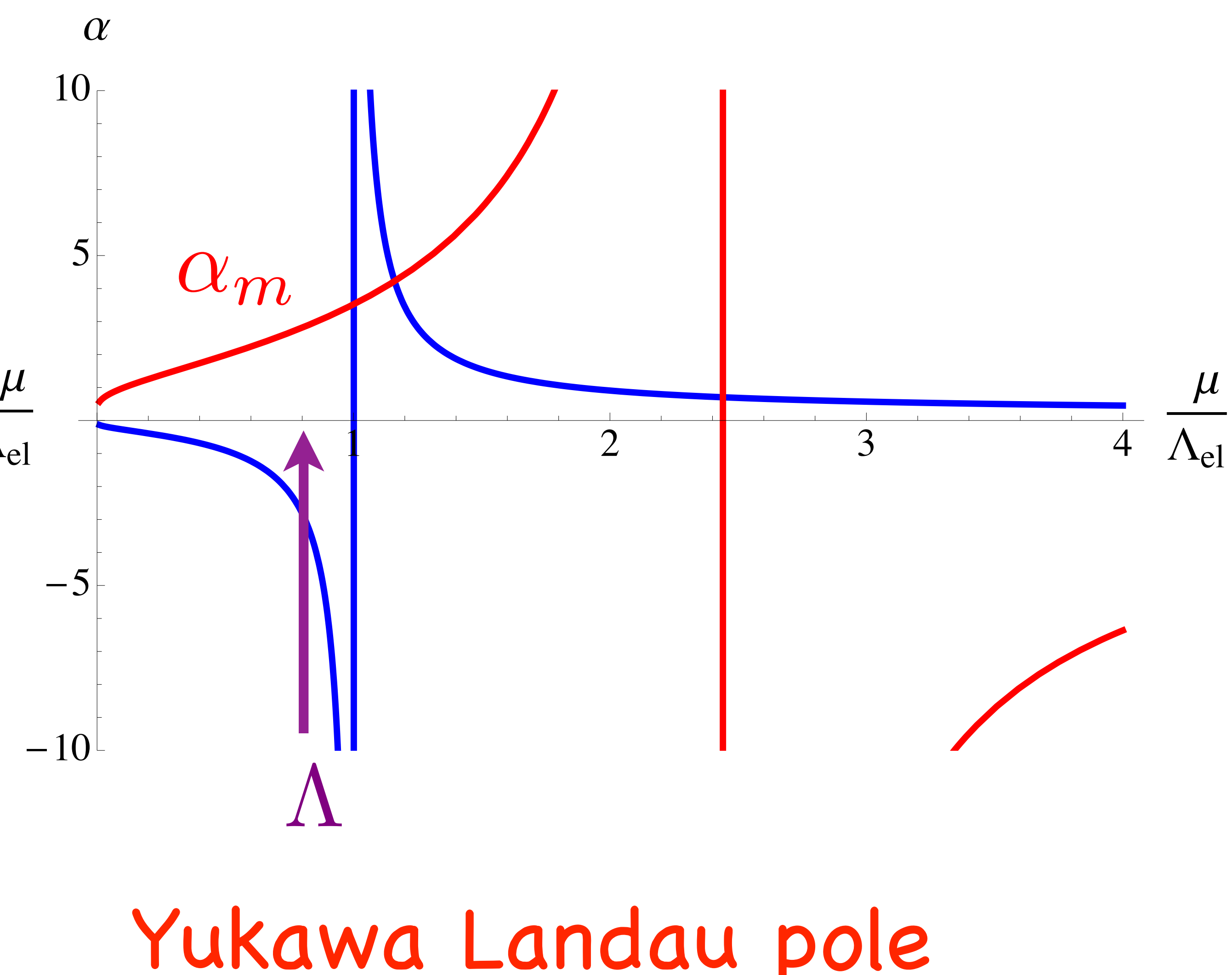
$$\Lambda_{\rm el}^{b_{\rm el}} \Lambda_{\rm mag}^{b_{\rm mag}} = (-1)^N \Lambda^{b_{\rm el}+b_{\rm mag}}$$

Seiberg hep-th/9411149

Dual Gauge Coupling

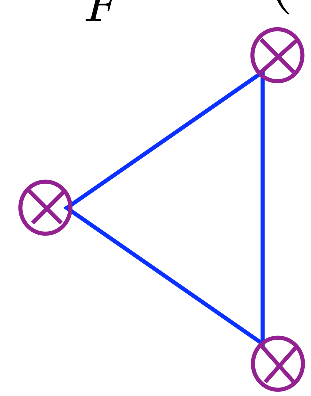
$$\frac{1}{g_{\rm el}^2(|\Lambda|)} = \frac{b_{\rm el}}{8\pi^2}\log\left(\frac{|\Lambda|}{\Lambda_{\rm el}}\right) = -\frac{b_{\rm mag}}{8\pi^2}\log\left(\frac{|\Lambda|}{\Lambda_{\rm mag}}\right) = -\frac{1}{g_{\rm mag}^2(|\Lambda|)}$$





Anomaly Matching

global symmetry	anomaly = dual anomaly
$SU(F)^3$	$-(F-N)+F=N$
$U(1)SU(F)^2$	$\frac{N}{F-N}(F-N)\frac{1}{2}=\frac{N}{2}$
$U(1)_R SU(F)^2$	$\frac{N-F}{F}(F-N)\frac{1}{2}+\frac{F-2N}{F}F\frac{1}{2}=-\frac{N^2}{2F}$
$U(1)^3$	$0=0$
$U(1)$	$0=0$
$U(1)U(1)_R^2$	$0=0$
$U(1)_R$	$\left(\frac{N-F}{F}\right)2(F-N)F+\left(\frac{F-2N}{F}\right)F^2+(F-N)^2-1$ $=-N^2-1$
$U(1)_R^3$	$\left(\frac{N-F}{F}\right)^3 2(F-N)F+\left(\frac{F-2N}{F}\right)^3 F^2+(F-N)^2-1$ $=-\frac{2N^4}{F^2}+N^2-1$
$U(1)^2U(1)_R$	$\left(\frac{N}{F-N}\right)^2\frac{N-F}{F}2F(F-N)=-2N^2$



Moduli Mapping

$$Q\overline{Q} \leftrightarrow M$$

$$B_{i_1,\ldots,i_N} \leftrightarrow \epsilon_{i_1,\ldots,i_N,j_1,\ldots j_{F-N}} b^{j_1,\ldots,j_{F-N}}$$

$$\overline{B}^{i_1,\ldots,i_N} \leftrightarrow \epsilon^{i_1,\ldots,i_N,j_1,\ldots j_{F-N}} \overline{b}_{j_1,\ldots,j_{F-N}}$$

Running Coupling

$$\beta_g \;=\; \mu\frac{dg}{d\mu} = -\frac{g^3}{16\pi^2}\left(\frac{11}{3}T(\mathrm{Ad}) - \frac{2}{3}T(F) - \frac{1}{3}T(S)\right) \equiv -\frac{g^3\, b}{16\pi^2}\;,$$

$$b = 3N - F$$

F > 3N infrared free

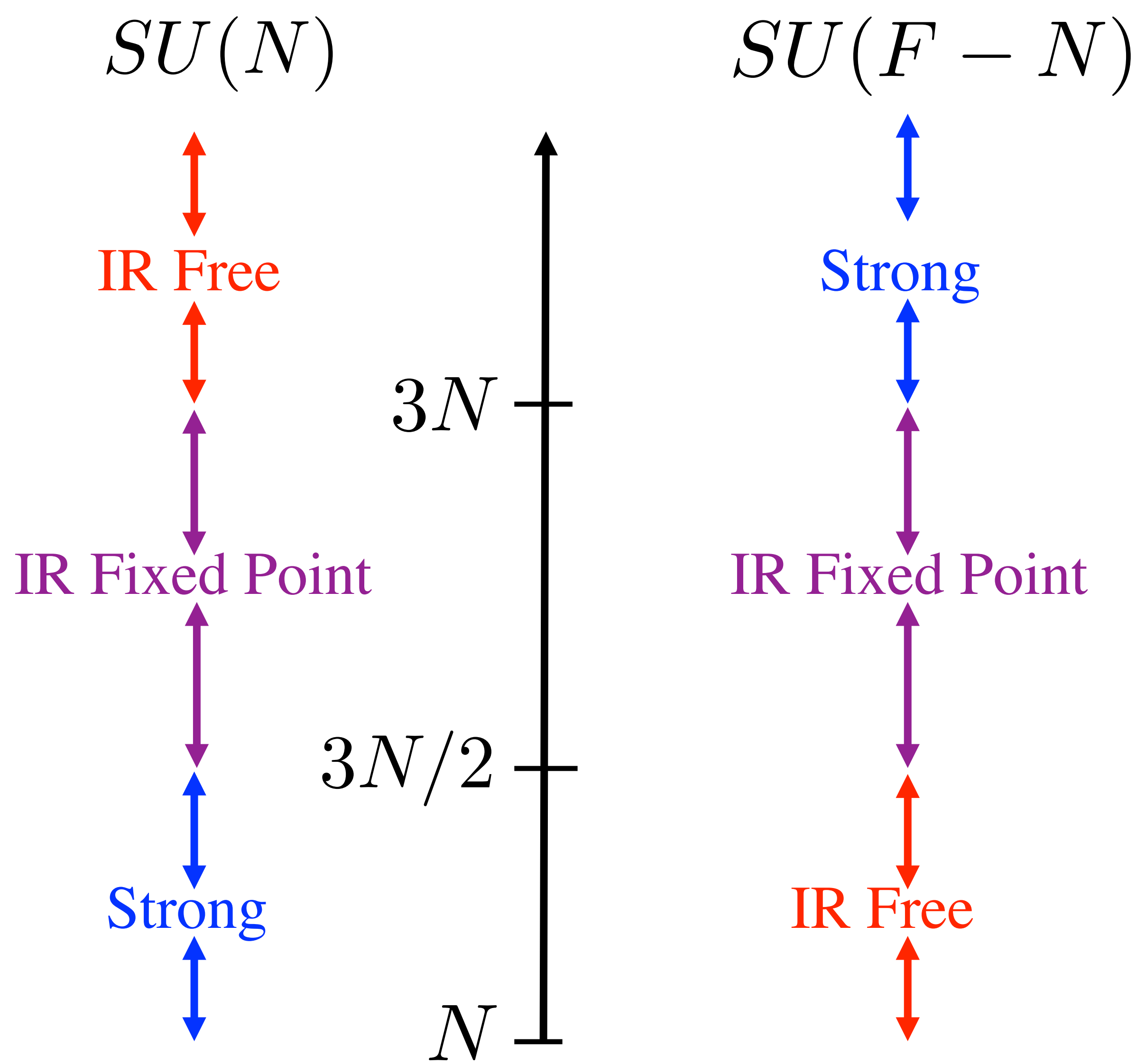
$$\tilde{b} = 3\tilde{N} - F = 3(F-N) - F = 2F - 3N$$

dual infrared free for F < 3N/2

weakly coupled Banks-Zaks fixed point

for F = 3N/2 + ϵ

Duality for SUSY QCD



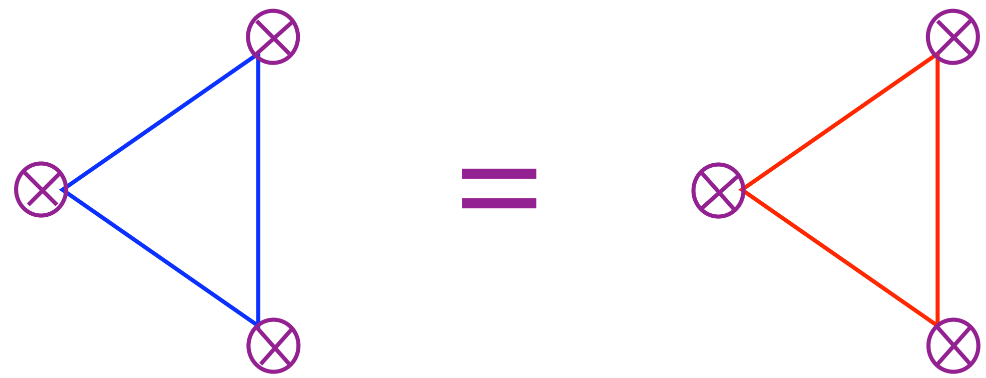
special cases:

F=N+1 → confinement without χSB

F=N → confinement with χSB

Duality Consistency Checks

Anomalies



$SU(N): \lambda, Q, \overline{Q}$ $\qquad$ $SU(F-N): \lambda, q, \overline{q}, M$

Identical Space of Vacua

$$Q\overline{Q} \longleftrightarrow M$$

$$Q^N, \overline{Q}^N \longleftrightarrow q^{F-N}, \overline{q}^{F-N}$$

Deformations

$$SU(N), F \qquad \longleftrightarrow \qquad SU(F-N), F$$

$$W = mQ_F\overline{Q}_F \qquad \longleftrightarrow \qquad W = Mq\overline{q} + mM_{FF}$$

$$\downarrow \qquad\qquad \downarrow \ \langle q \rangle \neq 0, \langle \overline{q} \rangle \neq 0$$

$$SU(N), F-1 \longleftrightarrow SU(F-1-N), F-1$$

Integrating Out

$$F = N+2$$

$SU(N)$ $\longrightarrow$ $SU(2)$

$$W = m\,\overline{Q}^F Q_F \qquad\qquad W = y\,\bar{q}_j M^j{}_i\, q^i + m\, M^F{}_F$$

$\downarrow$ $\qquad\qquad$ $\downarrow$ $\langle q^F \rangle \neq 0$, $\langle \bar{q}^F \rangle \neq 0$

$$F = N+1$$

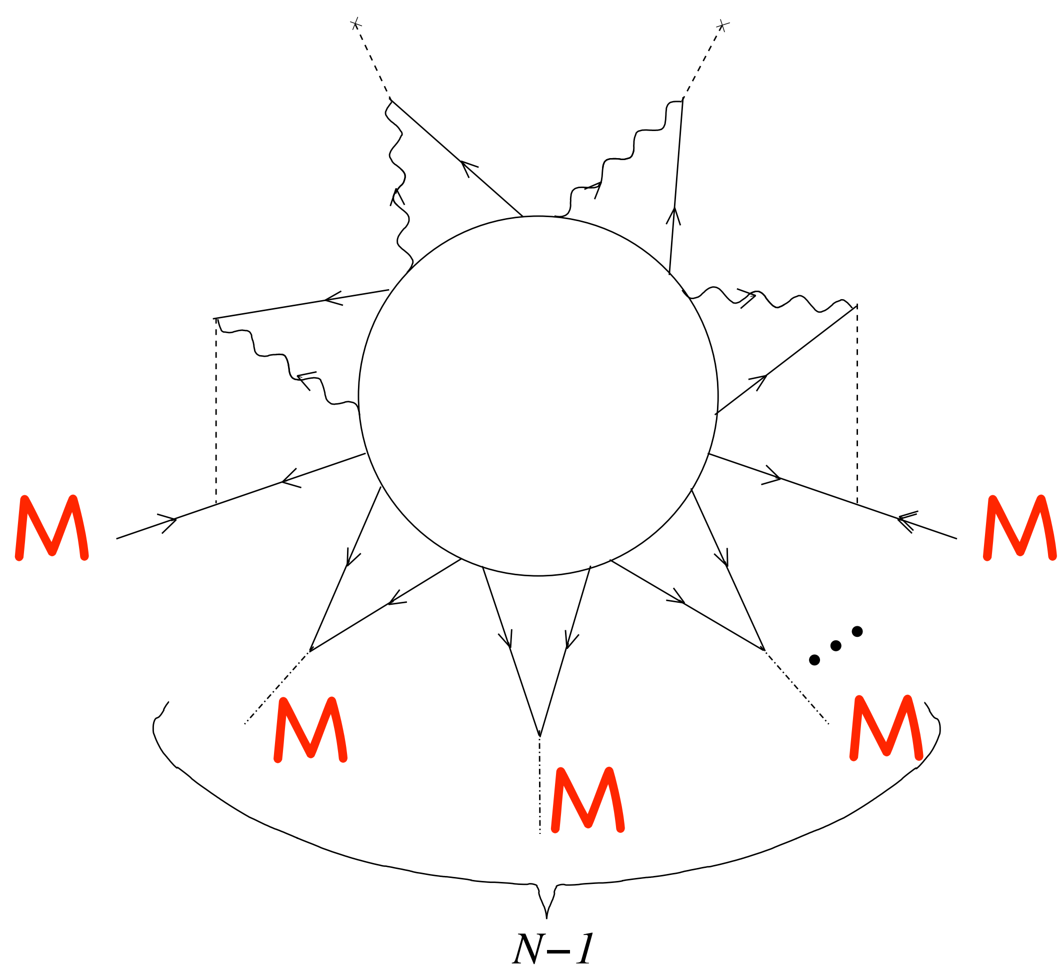
$SU(N)$ $\longrightarrow$ ~~$SU(2)$~~

$$W = 0 \qquad\qquad W = y\,\bar{q}_j M^j{}_i\, q^i - \frac{\det M}{\Lambda^{2N-1}}$$

↑ \`t Hooft instanton vertex

`t Hooft instanton vertex

$$W = y\,\bar{q}_j M^j{}_i\, q^i - \frac{\det M}{\Lambda^{2N-1}}$$



Integrating Out

$$SU(N) \xrightarrow{F = N+2} SU(2)$$

$$W = m\,\overline{Q}^{F} Q_F \qquad\qquad W = y\,\bar{q}_j M^j{}_i\, q^i + m\, M^F{}_F$$

$$\Big\downarrow \qquad\qquad \Big\downarrow\ \langle q^F \rangle \neq 0,\ \langle \bar{q}^F \rangle \neq 0$$

$$SU(N) \xrightarrow{F = N+1}$$ ~~$SU(2)$~~

$$W = 0 \qquad\qquad W = y\,\bar{q}_j M^j{}_i\, q^i - \frac{\det M}{\Lambda^{2N-1}}$$

$$B^i \leftrightarrow q^i$$

confinement

Integrating Out

$$SU(N) \xrightarrow{F = N+1} W = m\, M^F{}_F + \frac{1}{\Lambda^{2N-1}}\left[\overline{B}_j M^j{}_i B^i - \det M\right]$$

$$W = m\, \overline{Q}^F Q_F$$

$$SU(N) \xrightarrow{F = N}$$

$$W = 0$$

$$W = \frac{X}{\Lambda^{2N-1}}\left(B\overline{B} - \det M + \Lambda^{2N}_{N,N}\right)$$

confinement with chiral symmetry breaking

Chiral Symmetry Breaking

Phases of Gauge Theories

EM Duality:	electron	⟷	monopole
	free electric	⟷	free magnetic
	Coulomb phase	⟷	Coulomb phase
Seiberg duality:	quark, gluon	⟷	dual quark, gluon, meson
	strong electric	⟷	free magnetic
	Coulomb phase	⟷	Coulomb phase
	Higgs phase	⟷	confining phase

Discovering Hierarchies

SPS: W,Z --> gauge hierarchy
LEP: no light Higgs --> little hierarchy
Tevatron: top --> Yukawa hierarchy
LHC: no light SUSY --> squark mass hierarchy

Minimal Composite SSM
can resolve all these
hierarchy problems

Minimal Composite SSM

	$SU(4)$	$SU(6)_1$	$SU(6)_2$	$U(1)_V$	$U(1)_R$
$\mathcal{Q}$	$\square$	$\overline{\square}$	$\mathbf{1}$	1	$\frac{1}{3}$
$\bar{\mathcal{Q}}$	$\overline{\square}$	$\mathbf{1}$	$\overline{\square}$	-1	$\frac{1}{3}$

Csaki, Shirman, JT hep-ph/1106.3074

Minimal Composite SSM

	$SU(2)_{\text{mag}}$	$SU(6)_1$	$SU(6)_2$	$U(1)_V$	$U(1)_R$
q	$\square$	$\square$	$\mathbf{1}$	2	$\frac{2}{3}$
$\bar{q}$	$\overline{\square}$	$\mathbf{1}$	$\square$	-2	$\frac{2}{3}$
M	$\mathbf{1}$	$\overline{\square}$	$\overline{\square}$	0	$\frac{2}{3}$

$$W = y\, M q \bar{q}$$

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Minimal Composite SSM

	$SU(2)_{\rm mag}$	$SU(6)_1$	$SU(6)_2$	$U(1)_V$	$U(1)_R$
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M	$\mathbf{1}$	$\overline{\square}$	$\overline{\square}$	0	$\frac{2}{3}$

$$q = Q_3, \mathcal{H}, H_d$$
$$\bar{q} = X, \bar{\mathcal{H}}, H_u$$

$$M = \begin{pmatrix} V & U & \bar{t} \\ E & G+P & \phi_u \\ R & \phi_d & S \end{pmatrix}$$

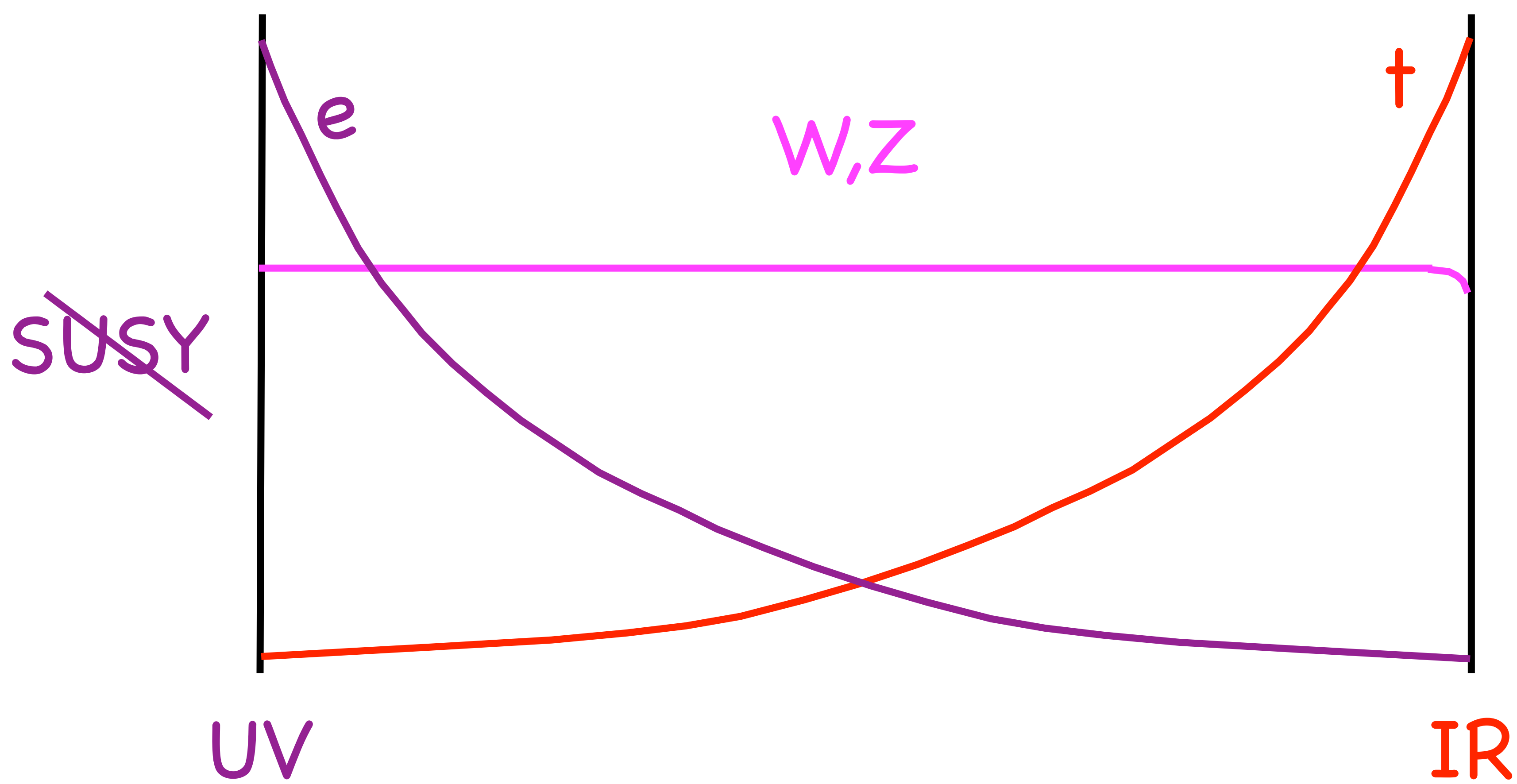
Minimal Composite SSM

	$SU(2)_{\mathrm{mag}}$	$SU(6)_1$	$SU(6)_2$	$U(1)_V$	$U(1)_R$
q	$\square$	$\square$	$\mathbf{1}$	2	$\frac{2}{3}$
$\bar{q}$	$\overline{\square}$	$\mathbf{1}$	$\square$	-2	$\frac{2}{3}$
M	$\mathbf{1}$	$\overline{\square}$	$\overline{\square}$	0	$\frac{2}{3}$

predicts stop much lighter
than other squarks

Csaki, Randall, JT hep-ph/1201.1293

Realistic SUSY RS



Conclusions

Seiberg duality allows us to obtain exact results for nonperturbative effects in SUSY QCD

this allows for composite SUSY models of electroweak symmetry breaking