Understanding the LSS of the Universe through Symmetries

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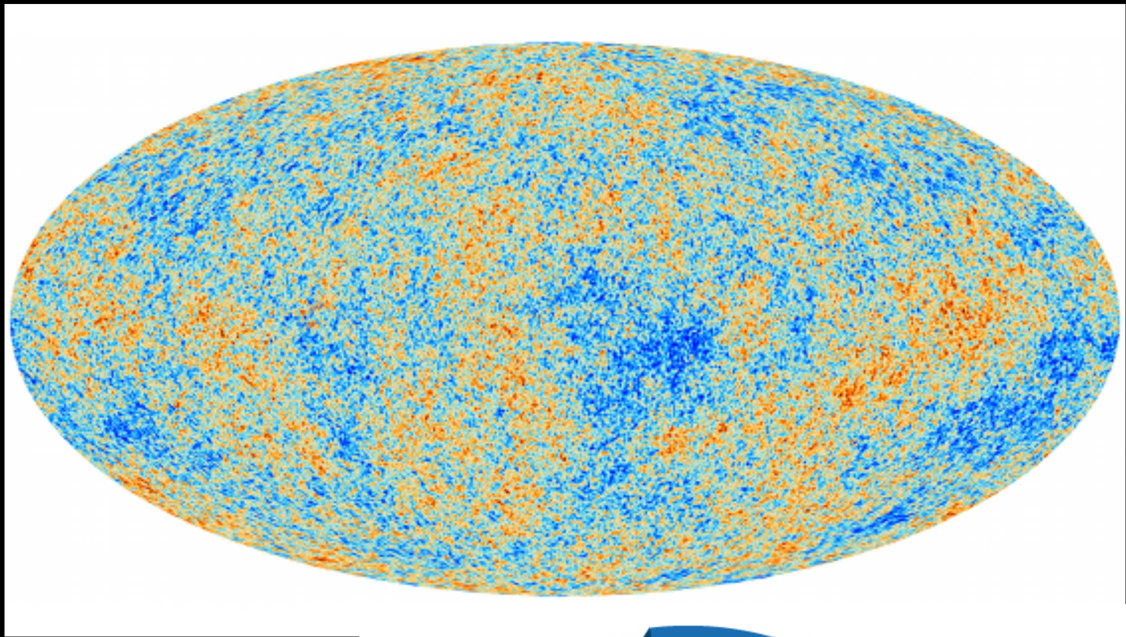
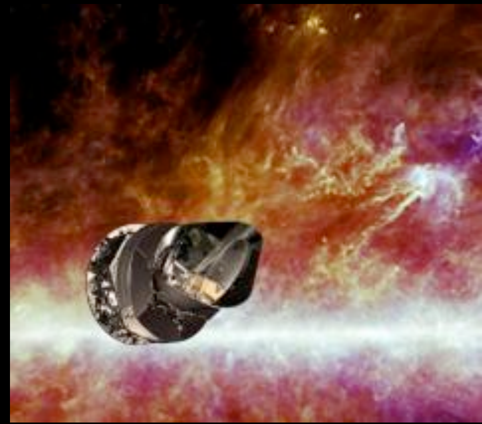
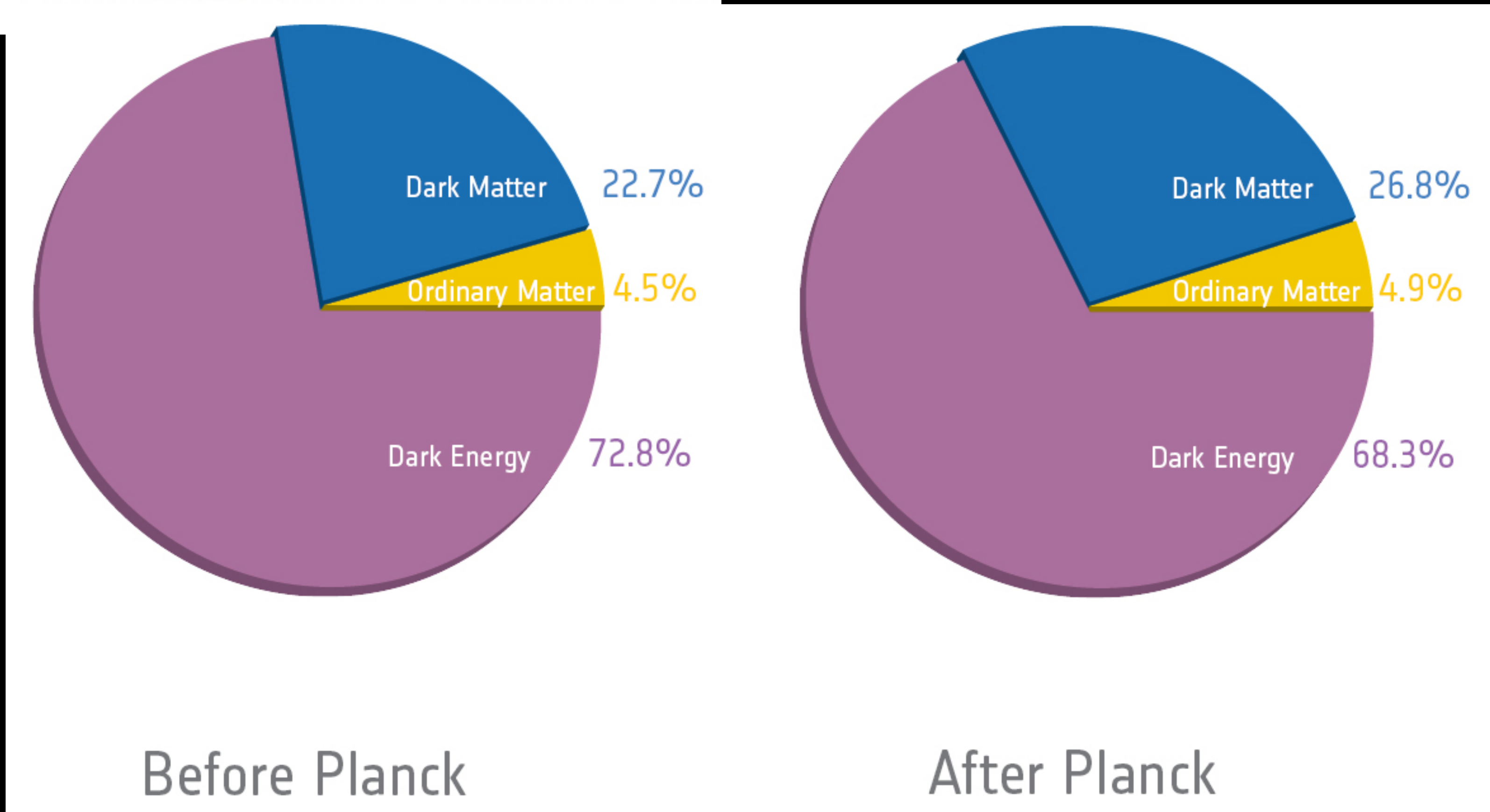
Barcelona, 28 November 2014

Plan of the talk

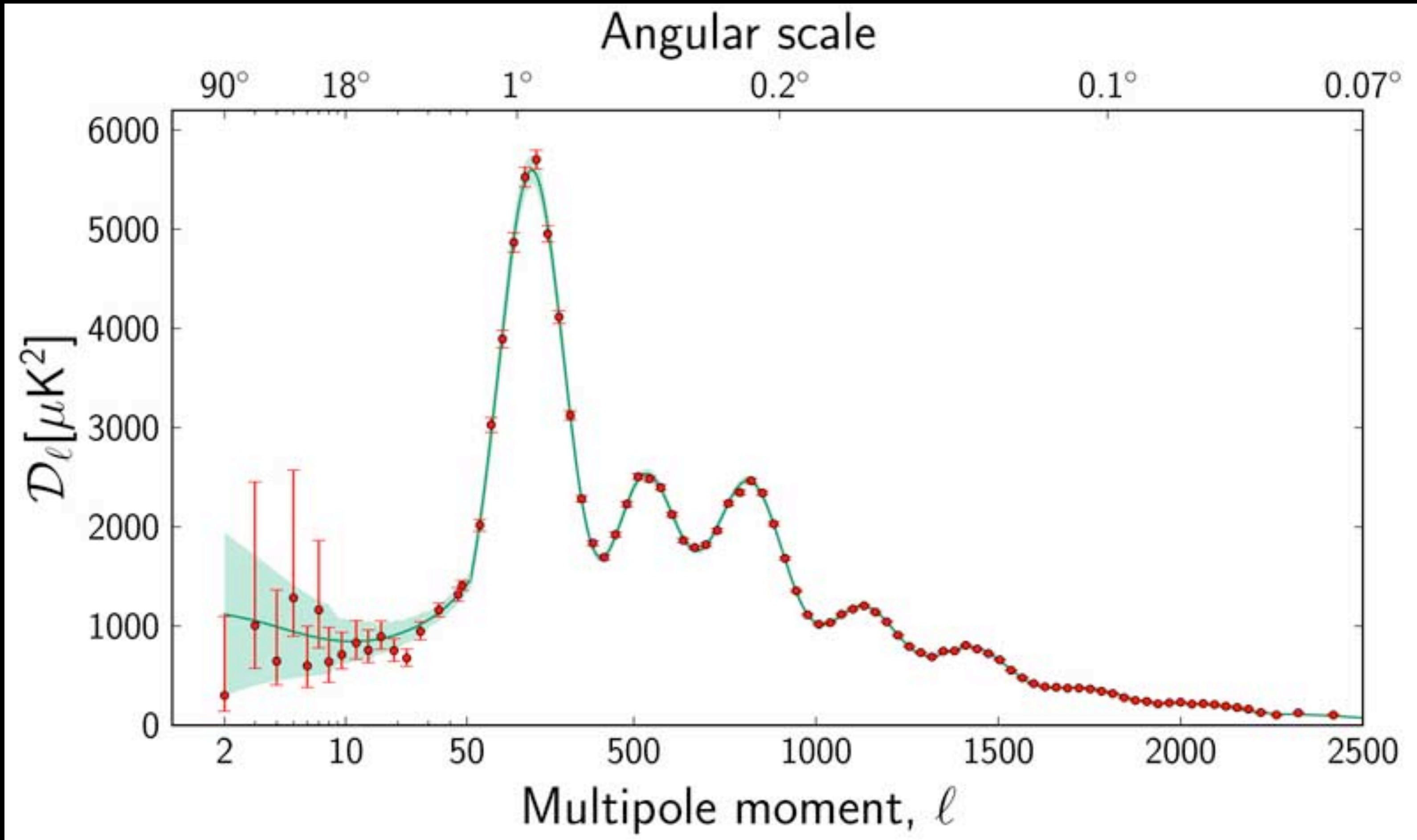
- Very short introduction of LSS dynamics
- The role of symmetries in the large-scale structure
- Going back to fundamentals



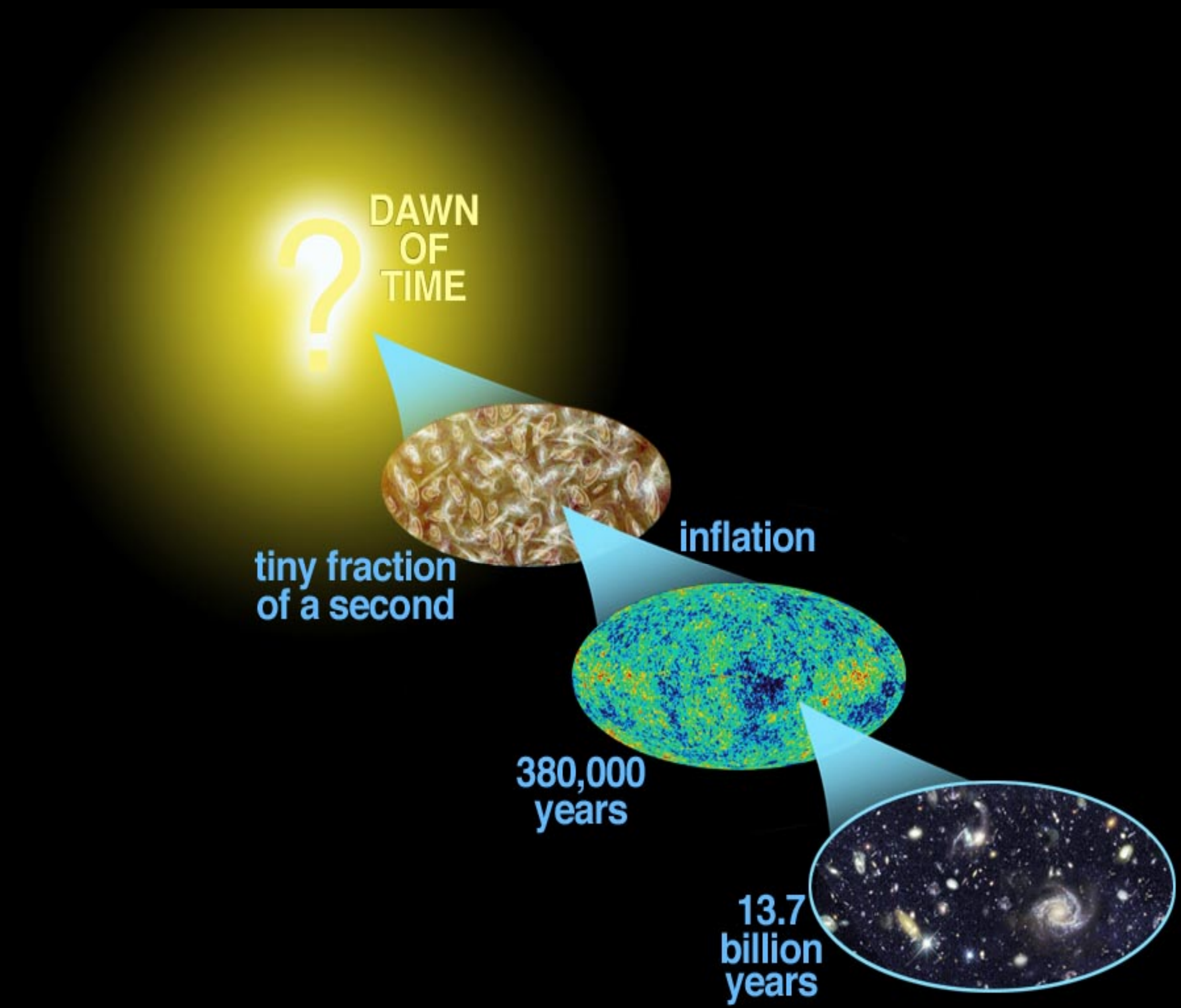


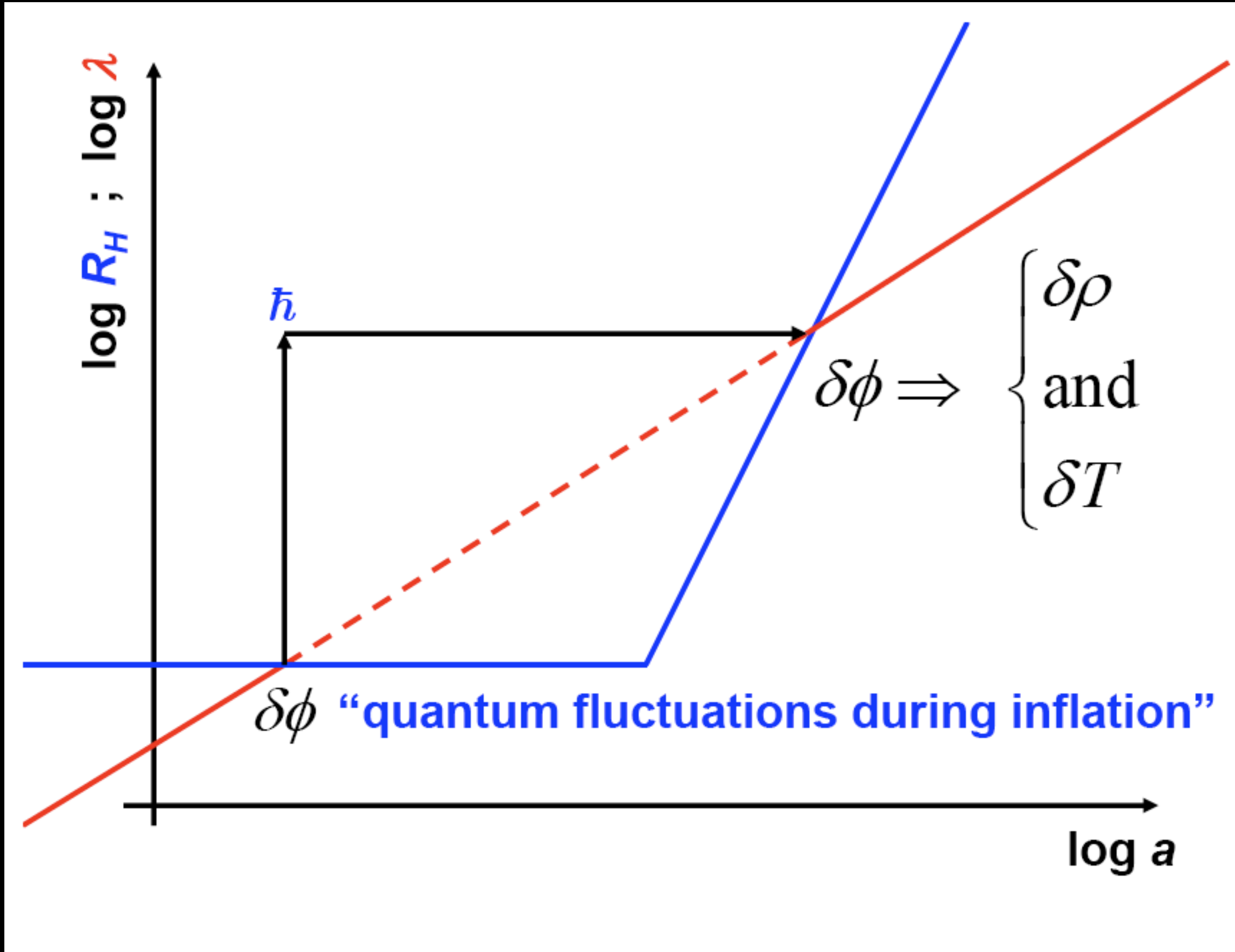

Dark Matter
22.7%
Ordinary Matter
4.5%
Dark Energy
72.8%
Dark Matter
26.8%
Ordinary Matter
4.9%
Dark Energy
68.3%
Before Planck
After Planck

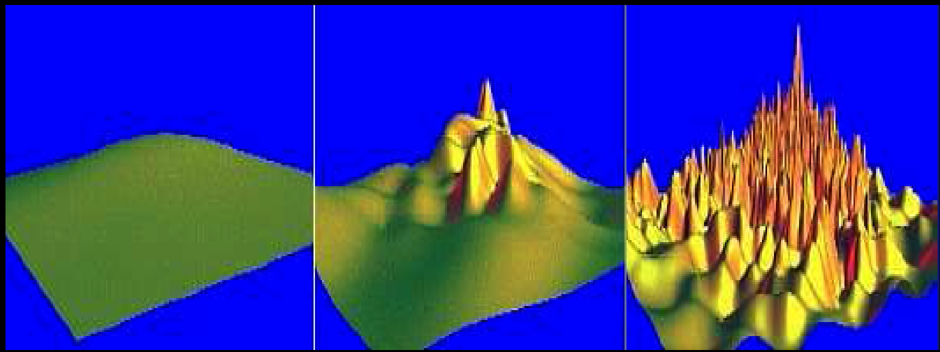
The ΛCDM model matches Planck data



The Inflationary Cosmology

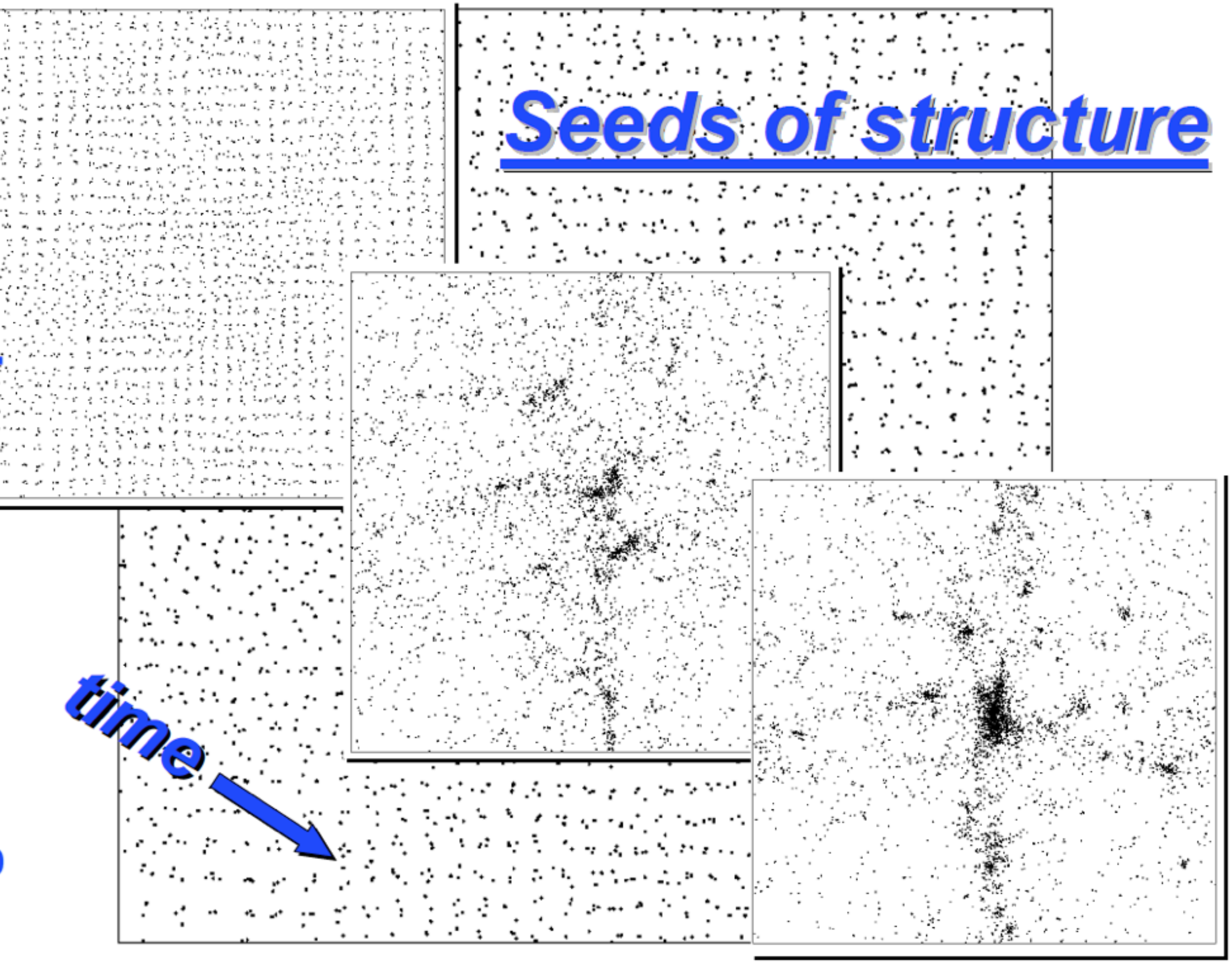



log R_H ; log λ
$\hbar$
$\delta\phi \Rightarrow \begin{cases} \delta\rho \\ \text{and} \\ \delta T \end{cases}$
$\delta\phi$ "quantum fluctuations during inflation"
log a

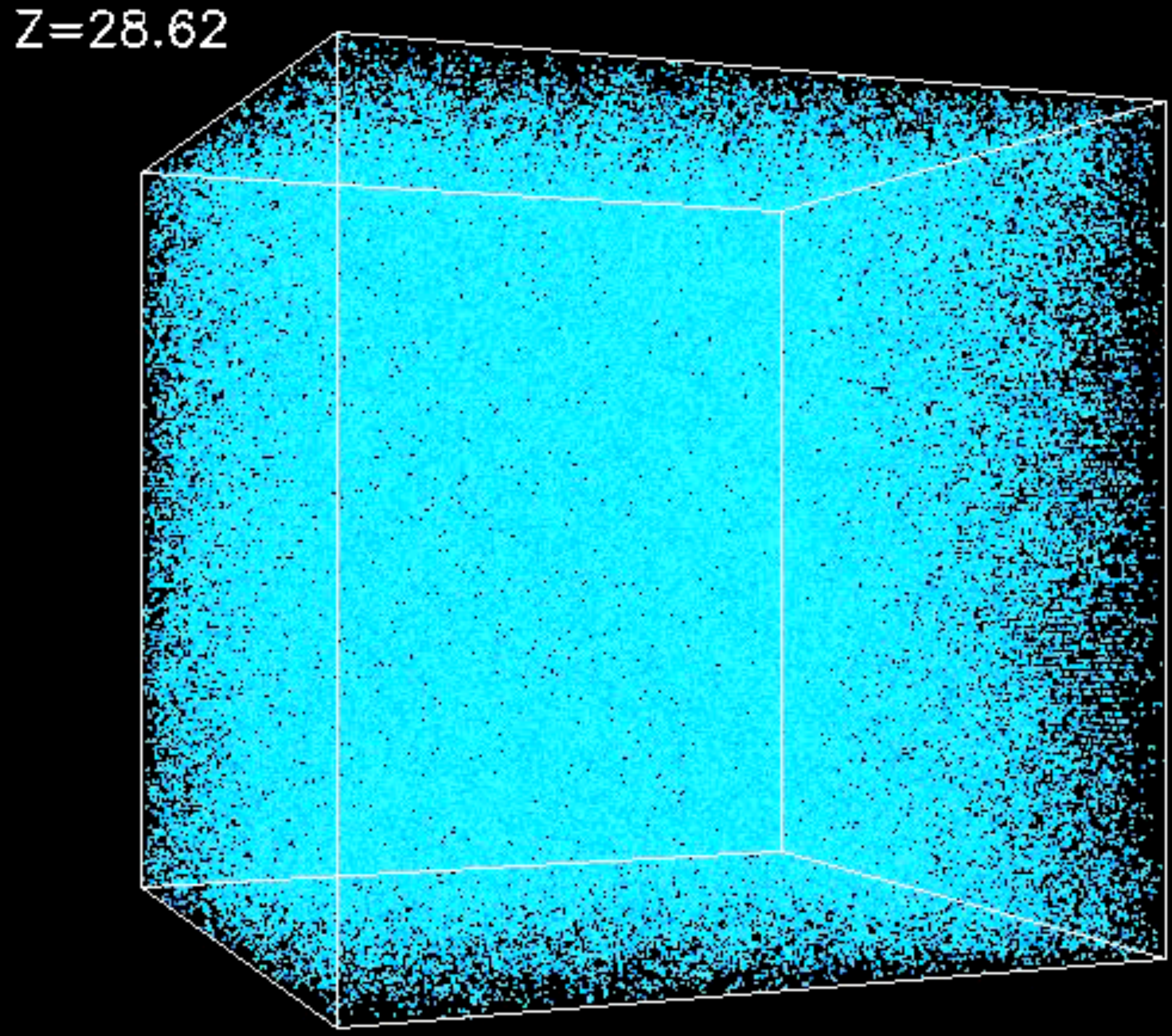


From Quantum Fluctuations to the Large Scale Structure

Seeds of structure



origin of small initial perturbations



The Millenium Simulation Project:

http://www.mpa-garching.mpg.de/galform/virgo/millennium/

The structure in the Universe

Perturbing around the average energy density we may define the density contrast

$$\delta(\vec{x},t) \equiv \frac{\rho(\vec{x},t)-\overline{\rho}}{\overline{\rho}} = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\,\delta_{\vec{k}}(t)\,e^{-i\,\vec{k}\cdot\vec{x}}$$

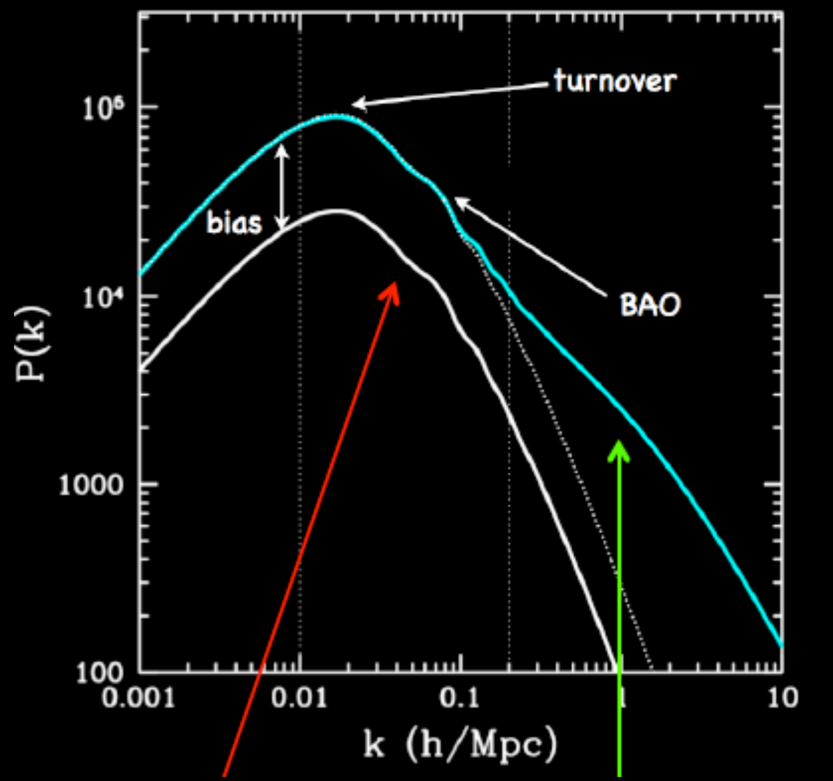
The power spectrum is defined by

$$\langle \delta_{\vec{k}}\,\delta_{\vec{k}'} \rangle = (2\pi)^3\,P_\delta(k)\,\delta^{(3)}(\vec{k}-\vec{k}')$$

$$\boxed{\Delta_\delta(k) = \frac{k^3\,P_\delta(k),}{2\pi^2},\;\; P_\delta = A\,k^n\,T(k)}$$

$$n \simeq 1,\; T(k) = \text{transfer function}$$

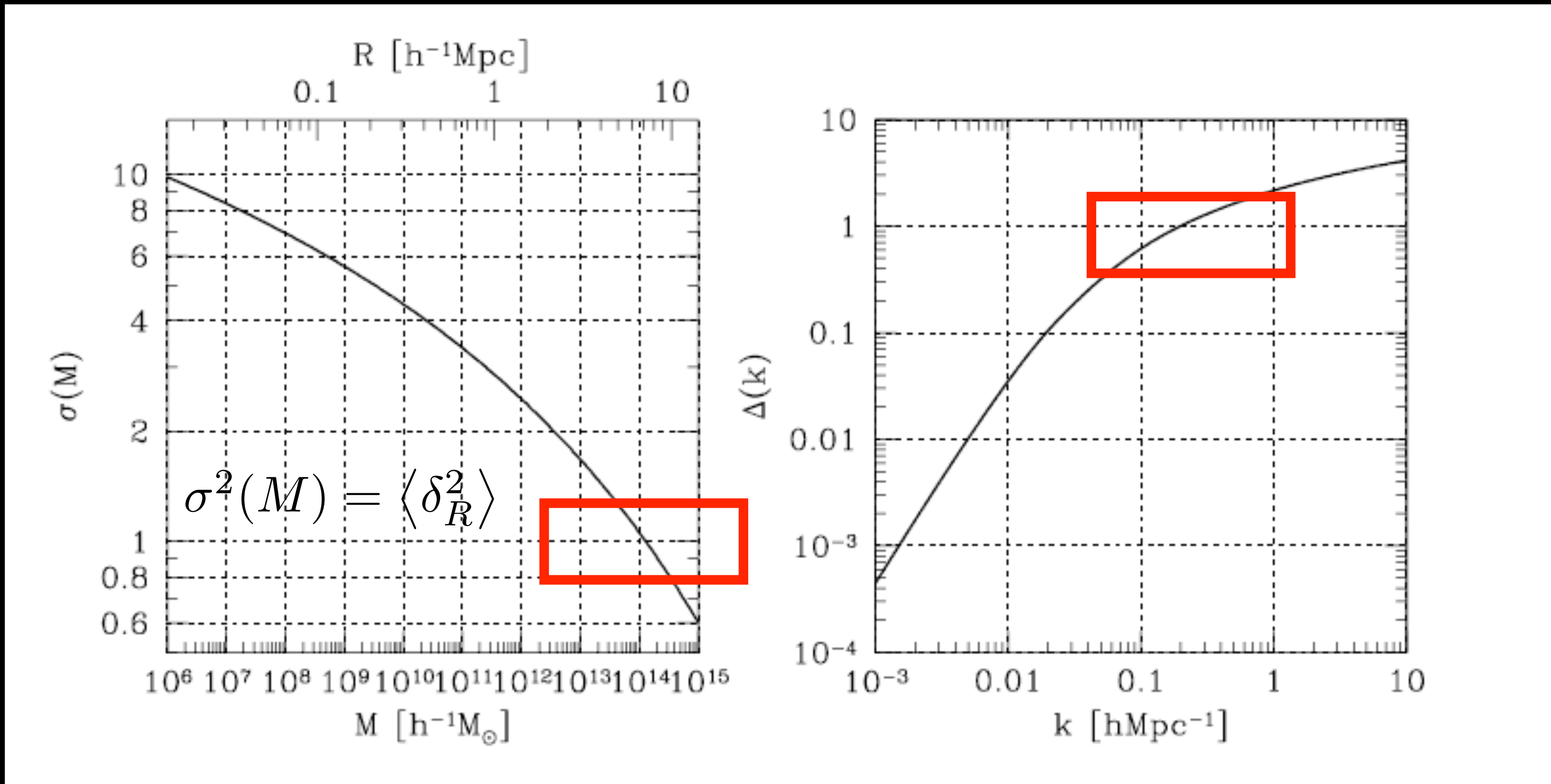
Structures are non-linear objects



Linear: $\ddot{\delta}_{\vec{k}} + 2H\dot{\delta}_{\vec{k}} = 4\pi G_{\rm N}\,\overline{\rho}\,\delta_{\vec{k}} \Rightarrow \delta_{\vec{k}} \propto D(a)(\sim a \text{ in MD})$

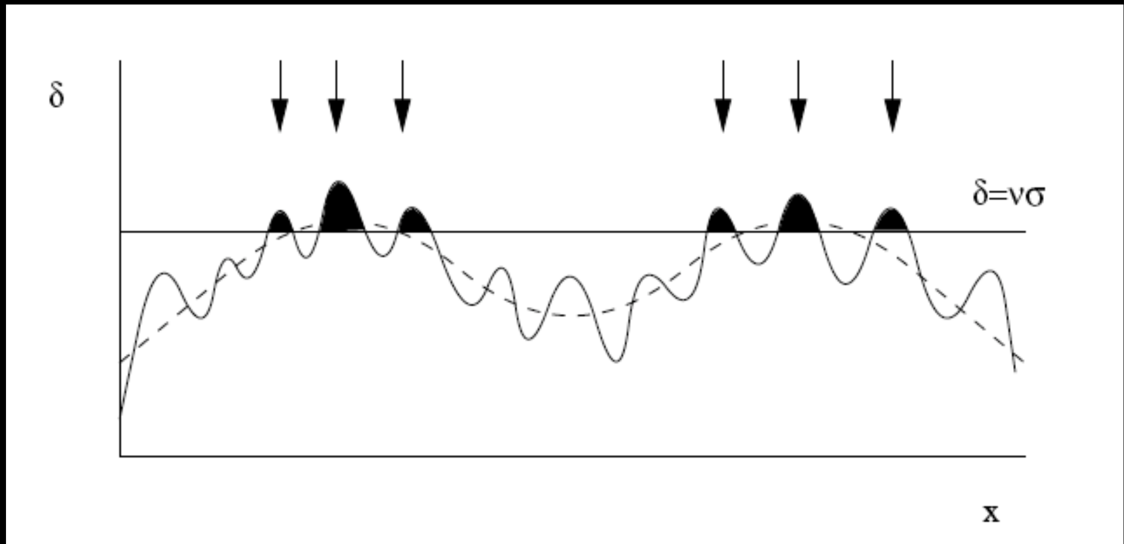
Perturbation theory fails: $\delta = \sum_n \delta_n,\ \delta_n \sim D^n(a)$

Analytical methods fail typically at $k \sim 0.1 h/\mathrm{Mpc}$



R [h⁻¹Mpc]
0.1
1
10
σ(M)
10
8
6
4
2
1
0.8
0.6
$\sigma^2(M) = \langle \delta_R^2 \rangle$
10⁶ 10⁷ 10⁸ 10⁹ 10¹⁰ 10¹¹ 10¹² 10¹³ 10¹⁴ 10¹⁵
M [h⁻¹M⊙]
Δ(k)
10
1
0.1
0.01
10⁻³
10⁻⁴
10⁻³
0.01
0.1
1
10
k [hMpc⁻¹]

Galaxies are interpreted as peaks in the DM distribution



$$\delta_{\rm g}(\vec{x}) \underset{\rm bias}{\overset{?}{=}} \mathcal{F}\left[\delta_{\rm m}(\vec{x})\right]$$

Is it possible to get informations on the LSS/galaxy correlation functions on non-linear scales by using symmetries?

If exact consistency relations do exist, they are useful for

- Checking (semi)-analitycal methods or N-body
- Testing observationally theories of bias, modified gravity, non-Gaussianity, ...

Similar to Ward identities in QFT

Galaxy and DM fluid equations

$$\frac{\partial \delta_{\rm g}(\vec{x},\tau)}{\partial \tau} + \vec{\nabla} \cdot [(1+\delta_{\rm g}(\vec{x},\tau))\vec{v}_{\rm g}(\vec{x},\tau)] = A\, j(\delta),$$

$$\frac{\partial \vec{v}_{\rm g}(\vec{x},\tau)}{\partial \tau} + \mathcal{H}(\tau)\vec{v}_{\rm g}(\vec{x},\tau) + [\vec{v}_{\rm g}(\vec{x},\tau)\cdot\vec{\nabla}]\vec{v}_{\rm g}(\vec{x},\tau) = -\vec{\nabla}\Phi(\vec{x},\tau),$$

$$\frac{\partial \delta(\vec{x},\tau)}{\partial \tau} + \vec{\nabla} \cdot [(1+\delta(\vec{x},\tau))\vec{v}(\vec{x},\tau)] = 0,$$

$$\frac{\partial \vec{v}(\vec{x},\tau)}{\partial \tau} + \mathcal{H}(\tau)\vec{v}(\vec{x},\tau) + [\vec{v}(\vec{x},\tau)\cdot\vec{\nabla}](\vec{x},\tau) = -\vec{\nabla}\Phi(\vec{x},\tau),$$

$$\nabla^2\Phi(\vec{x},\tau) = \frac{3}{2}\Omega_{\rm m}\mathcal{H}^2(\tau)\delta(\vec{x},\tau)$$

Galaxy and DM fluid equations

Mass conservation

$$\frac{\partial \delta_{\rm g}(\vec{x},\tau)}{\partial \tau} + \vec{\nabla}\cdot[(1+\delta_{\rm g}(\vec{x},\tau))\vec{v}_{\rm g}(\vec{x},\tau)] = A\, j(\delta),$$

$$\frac{\partial \vec{v}_{\rm g}(\vec{x},\tau)}{\partial \tau} + \mathcal{H}(\tau)\vec{v}_{\rm g}(\vec{x},\tau) + [\vec{v}_{\rm g}(\vec{x},\tau)\cdot\vec{\nabla}]\vec{v}_{\rm g}(\vec{x},\tau) = \; -\vec{\nabla}\Phi(\vec{x},\tau),$$

$$\frac{\partial \delta(\vec{x},\tau)}{\partial \tau} + \vec{\nabla}\cdot[(1+\delta(\vec{x},\tau))\vec{v}(\vec{x},\tau)] = 0,$$

$$\frac{\partial \vec{v}(\vec{x},\tau)}{\partial \tau} + \mathcal{H}(\tau)\vec{v}(\vec{x},\tau) + [\vec{v}(\vec{x},\tau)\cdot\vec{\nabla}](\vec{x},\tau) = \; -\vec{\nabla}\Phi(\vec{x},\tau),$$

$$\nabla^2\Phi(\vec{x},\tau) = \frac{3}{2}\Omega_{\rm m}\mathcal{H}^2(\tau)\delta(\vec{x},\tau)$$

Galaxy and DM fluid equations

Momentum conservation

$$\frac{\partial \delta_{\rm g}(\vec{x},\tau)}{\partial \tau} + \vec{\nabla}\cdot[(1+\delta_{\rm g}(\vec{x},\tau))\vec{v}_{\rm g}(\vec{x},\tau)] = A\, j(\delta),$$

$$\frac{\partial \vec{v}_{\rm g}(\vec{x},\tau)}{\partial \tau} + \mathcal{H}(\tau)\vec{v}_{\rm g}(\vec{x},\tau) + [\vec{v}_{\rm g}(\vec{x},\tau)\cdot\vec{\nabla}]\vec{v}_{\rm g}(\vec{x},\tau) = -\vec{\nabla}\Phi(\vec{x},\tau),$$

$$\frac{\partial \delta(\vec{x},\tau)}{\partial \tau} + \vec{\nabla}\cdot[(1+\delta(\vec{x},\tau))\vec{v}(\vec{x},\tau)] = 0,$$

$$\frac{\partial \vec{v}(\vec{x},\tau)}{\partial \tau} + \mathcal{H}(\tau)\vec{v}(\vec{x},\tau) + [\vec{v}(\vec{x},\tau)\cdot\vec{\nabla}](\vec{x},\tau) = -\vec{\nabla}\Phi(\vec{x},\tau),$$

$$\nabla^2\Phi(\vec{x},\tau) = \frac{3}{2}\Omega_{\rm m}\mathcal{H}^2(\tau)\delta(\vec{x},\tau)$$

Galaxy and DM fluid equations

$$\frac{\partial \delta_{\rm g}(\vec{x},\tau)}{\partial \tau} + \vec{\nabla}\cdot[(1+\delta_{\rm g}(\vec{x},\tau))\vec{v}_{\rm g}(\vec{x},\tau)] = A\, j(\delta),$$

$$\frac{\partial \vec{v}_{\rm g}(\vec{x},\tau)}{\partial \tau} + \mathcal{H}(\tau)\vec{v}_{\rm g}(\vec{x},\tau) + [\vec{v}_{\rm g}(\vec{x},\tau)\cdot\vec{\nabla}]\vec{v}_{\rm g}(\vec{x},\tau) = -\vec{\nabla}\Phi(\vec{x},\tau),$$

$$\frac{\partial \delta(\vec{x},\tau)}{\partial \tau} + \vec{\nabla}\cdot[(1+\delta(\vec{x},\tau))\vec{v}(\vec{x},\tau)] = 0,$$

$$\frac{\partial \vec{v}(\vec{x},\tau)}{\partial \tau} + \mathcal{H}(\tau)\vec{v}(\vec{x},\tau) + [\vec{v}(\vec{x},\tau)\cdot\vec{\nabla}](\vec{x},\tau) = -\vec{\nabla}\Phi(\vec{x},\tau),$$

$$\nabla^2\Phi(\vec{x},\tau) = \frac{3}{2}\Omega_{\rm m}\mathcal{H}^2(\tau)\delta(\vec{x},\tau) \longleftarrow \text{Poisson}$$

Linearly realized symmetries of the LSS

$$\tau' = \lambda^z \tau, \quad \vec{x}' = \lambda \vec{x},$$

Lifshitz symmetry (during matter-domination)

$$\delta'(\vec{x}, \tau) = \delta(\vec{x}', \tau'),$$
$$\delta'_{\rm g}(\vec{x}, \tau) = \delta_{\rm g}(\vec{x}', \tau'),$$
$$\vec{v}'_{\rm g}(\vec{x}, \tau) = \lambda^{z-1} \vec{v}(\vec{x}', \tau'),$$
$$\vec{v}'(\vec{x}, \tau) = \lambda^{z-1} \vec{v}_{\rm g}(\vec{x}', \tau'),$$
$$\Phi'(\vec{x}, \tau) = \lambda^{2(z-1)} \Phi(\vec{x}', \tau')$$

Non-linearly realized symmetries of the LSS

$$\tau' = \tau, \quad \vec{x}' = \vec{x} + \vec{n}(T)$$

$$T(\tau) = \frac{1}{a(\tau)} \int^{\tau} \mathrm{d}\eta \, a(\eta)$$

$$\delta'_{\mathrm{g}}(\vec{x}, \tau) = \delta_{\mathrm{g}}(\vec{x}', \tau'),$$
$$\vec{v}'_{\mathrm{g}}(\vec{x}, \tau) = \vec{v}_{\mathrm{g}}(\vec{x}', \tau') - \dot{\vec{n}}(T),$$
$$\delta'(\vec{x}, \tau) = \delta(\vec{x}', \tau'),$$
$$\vec{v}'(\vec{x}, \tau) = \vec{v}(\vec{x}', \tau') - \dot{\vec{n}}(T),$$
$$\Phi'(\vec{x}, \tau) = \Phi(\vec{x}', \tau') - \left(\mathcal{H}\dot{\vec{n}}(T) + \ddot{\vec{n}}(T)\right) \cdot \vec{x}$$

Non-linearly realized symmetries of the LSS

$$\tau' = \tau, \quad \vec{x}' = \vec{x} + \vec{n}(T)$$

$$T(\tau) = \frac{1}{a(\tau)} \int^{\tau} \mathrm{d}\eta \, a(\eta)$$

$$\delta'_{\mathrm{g}}(\vec{x},\tau) = \delta_{\mathrm{g}}(\vec{x}',\tau'),$$
$$\vec{v}'_{\mathrm{g}}(\vec{x},\tau) = \vec{v}_{\mathrm{g}}(\vec{x}',\tau') - \dot{\vec{n}}(T),$$
$$\delta'(\vec{x},\tau) = \delta(\vec{x}',\tau'),$$
$$\vec{v}'(\vec{x},\tau) = \vec{v}(\vec{x}',\tau') - \dot{\vec{n}}(T),$$
$$\Phi'(\vec{x},\tau) = \Phi(\vec{x}',\tau') - \Big(\mathcal{H}\dot{\vec{n}}(T) + \ddot{\vec{n}}(T)\Big) \cdot \vec{x}$$

Soft Pion

Galilean transformations

$$\tau' = \tau, \;\; \vec{x}' = \vec{x} + \vec{u}\,\tau$$

Acceleration transformations

$$\tau' = \tau, \;\; \vec{x}' = \vec{x} + \frac{1}{2}\vec{a}\,\tau^2$$

True in phase-space as well

$$\frac{\partial f}{\partial \tau} + \frac{p^i}{am}\frac{\partial f}{\partial x^i} - am\partial_i\Phi\,\frac{\partial f}{\partial p^i} = 0$$

$$\nabla^2\Phi = 4\pi G_{\mathrm{N}}\,\bar{\rho}a^2\delta$$

$$\tau \to \tau' = \int^{\tau} g(\eta)\mathrm{d}\eta, \quad \vec{x} \to \vec{x}' = \vec{n}(\tau) + g(\tau)\vec{x},$$

$$a'(\tau) = g(\tau)a(\tau'),$$

$$\bar{\rho}'(\tau) = \frac{1}{g^3(\tau)}\bar{\rho}(\tau'),$$

$$\delta'(\vec{x},\tau) = g^3(\tau)\delta(\vec{x},\tau) + g^3(\tau) - 1,$$

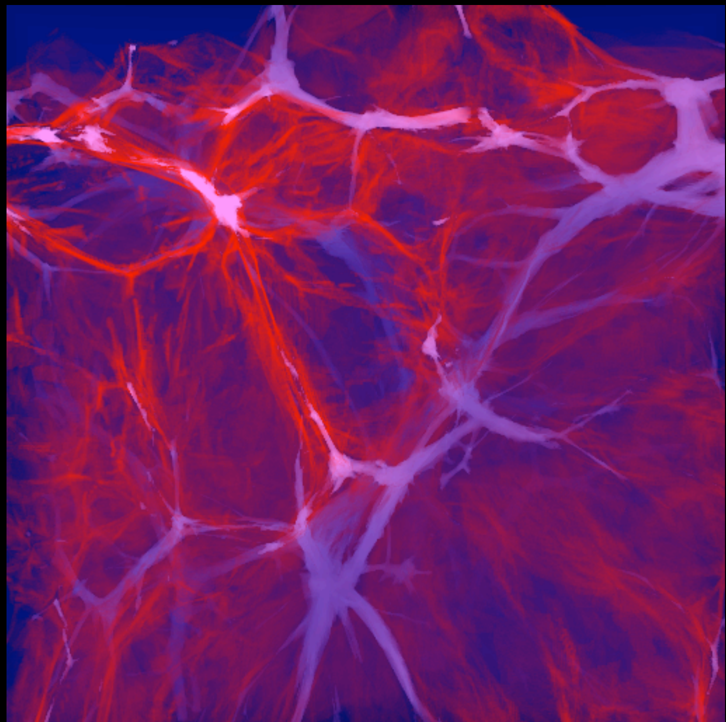
$$\vec{p} \to \vec{p}'(\vec{x},\tau) = \frac{1}{g(\tau)}\vec{p} + \frac{ma(\tau')}{g(\tau)}\left(\dot{\vec{n}} + \dot{g}\vec{x}\right),$$

$$\Phi'(\vec{x},\tau) = \Phi(\vec{x}',\tau') - \int^{x} \dot{p}^{i\prime}\mathrm{d}x^i,$$

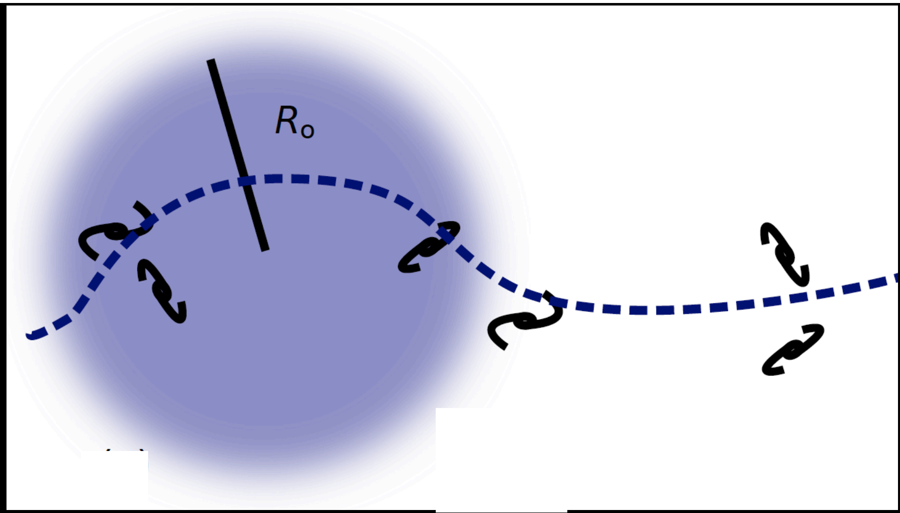
True in phase-space as well

$$\frac{\partial f}{\partial \tau} + \frac{p^i}{am}\frac{\partial f}{\partial x^i} - am\partial_i \Phi \frac{\partial f}{\partial p^i} = 0$$

$$\nabla^2 \Phi = 4\pi G_{\mathrm{N}}\, \bar{\rho} a^2 \delta$$



R. Kaehler et al. ., 1208.3206



Remove (or generate) a long-wavelength velocity mode by a coordinate transformation, essentially remove homogeneous gravitational force (zero mode and one gradient)

$$\vec{n}(\tau) = -\int^{\tau} \mathrm{d}\eta\, \vec{v}_L(\eta, \vec{0}) + \mathcal{O}(qR_0 v_L^2) = \frac{1}{6}\tau^2 \vec{\nabla}\Phi_L(\eta, \vec{0})$$

$$\Big\langle \delta_\mathrm{g}(\vec{x}_1) \cdots \delta_\mathrm{g}(\vec{x}_n) \Big\rangle_{v_L} = \Big\langle \delta'_\mathrm{g}(\vec{x}_1) \cdots \delta'_\mathrm{g}(\vec{x}_n) \Big\rangle = \Big\langle \delta_\mathrm{g}(\vec{x}'_1) \cdots \delta_\mathrm{g}(\vec{x}'_n) \Big\rangle$$

Consistency relations
or
LSS Ward Identities

$$\left\langle \delta_{\rm g}(\vec{q},\tau)\delta_{\rm g}(\vec{k}_1,\tau_1)\cdots\delta_{\rm g}(\vec{k}_n,\tau_n)\right\rangle_{q\to 0} = \left\langle \delta_{\rm g}(\vec{q},\tau)\left\langle \delta_{\rm g}(\vec{k}'_1,\tau_1)\cdots\delta_{\rm g}(\vec{k}'_n,\tau_n)\right\rangle\right\rangle$$

$$\delta_{\rm g}(\vec{k}'_i,\tau_i) = e^{i\vec{k}_i\cdot\vec{n}(\tau_i)}\,\delta_{\rm g}(\vec{k}_i,\tau_i)$$

$$\frac{\left\langle \delta_{\rm g}(\vec{q},\tau)\delta_{\rm g}(\vec{k}_1,\tau_1)\cdots\delta_{\rm g}(\vec{k}_n,\tau_n)\right\rangle'_{q\to 0}}{\left\langle \delta^L_{\rm g}(\vec{q},\tau)\delta_L(\vec{q},\tau)\right\rangle'} = -\sum_{a=1}^{n}\frac{D(\tau_a)}{D(\tau)}\frac{\vec{q}\cdot\vec{k}_a}{q^2}\left\langle \delta_{\rm g}(\vec{k}_1,\tau_1)\cdots\delta_{\rm g}(\vec{k}_n,\tau_n)\right\rangle$$

Similar relations for other quantities (DM overdensity, velocities, gravitational potential)
No primordial non-Gaussianity assumed

A. Kehagias and A.R., 1302.0130; M. Pietroni and M. Peloso, 1302.0223

Consistency Relations of the LSS

- They vanish at equal time because displacement of objects on short scales due to the long mode is homogeneous
- Valid at any short non-linear scales
- Valid for DM and galaxies, independently from the bias
- Eventually violated if symmetries are not valid
- Can be used to check analytical results (done at one-loop order for the galaxy three-point correlator)

Residual Gauge Symmetry

The coordinate transformation is the non-relativistic limit of a series of conformal transformations which add or remove the zero mode and the gradient of the metric long wavelength mode staying in the SAME gauge

$$
\begin{aligned}
\mathrm{d}s^2 &= \left(\frac{\tau_0}{\tau}\right)^{2q}\left[-(1+2\Phi_L)\mathrm{d}\tau^2+(1-2\Phi_L)\mathrm{d}\vec{x}^2\right] \\
&= \left(\frac{\tau_0}{\tau}\right)^{2q}(1+2\Phi_L)\left[-\mathrm{d}\tau^2+(1-4\Phi_L)\mathrm{d}\vec{x}^2\right]
\end{aligned}
$$

$$
\Phi_L(\vec{x}) \simeq \Phi_L(\vec{0})+\partial_i\Phi_L(\vec{0})\,x^i+\mathcal{O}(\partial_i\partial_j\Phi_L(\vec{0}))
$$

P. Creminelli, J. Norena, M. Simonovic and F. Vernizzi 1309.3557; A. Kehagias and A.R., 1309.3671

1. Dilatation plus special conformal transformation

$$
\begin{aligned}
y^i &= x^i(1+\lambda) - 2(\vec{x}\cdot\vec{b})x^i + b^i\vec{x}^2 \\
\mathrm{d}\vec{y}^2 &= \left(1+2\lambda-4\vec{b}\cdot\vec{x}\right)\,\mathrm{d}\vec{x}^2 \\
\lambda &= -2\Phi_L(\vec{0}), \quad b^i = \partial^i\Phi_L(\vec{0})
\end{aligned}
$$

$$
\mathrm{d}s^2 = \tau^2\left(\frac{\tau_0}{\tau}\right)^{2q}(1+2\Phi_L)\left(\frac{-\mathrm{d}\tau^2+\mathrm{d}\vec{y}^2}{\tau^2}\right)
$$

2. Isometry of de Sitter

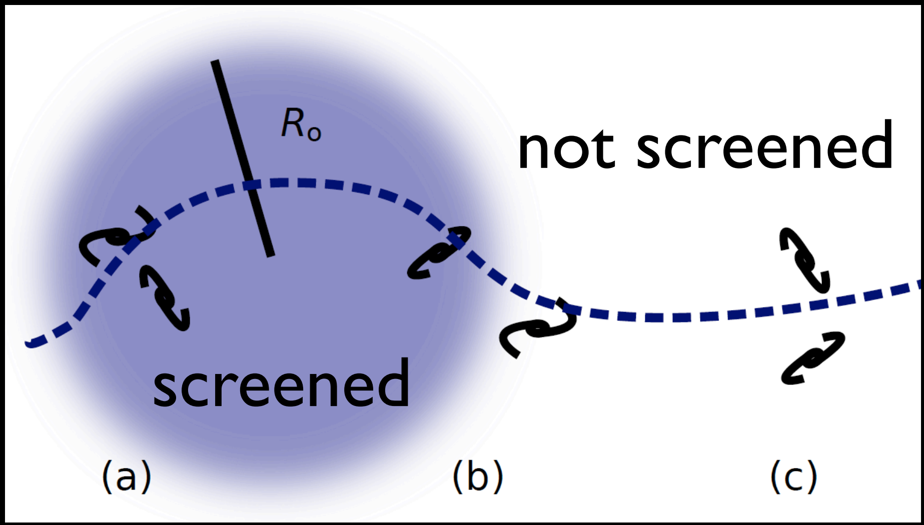
$$
\begin{aligned}
z^i &= y^i(1+\alpha) - 2(\vec{y}\cdot\vec{a})y^i + a^i(-\tau^2+\vec{y}^2), \quad \eta = \tau(1+\alpha-2\vec{y}\cdot\vec{a}) \\
\alpha &= \frac{1}{1-q}\Phi_L(\vec{0}), \quad a^i = \frac{1}{2(q-1)}\partial^i\Phi_L(\vec{0})
\end{aligned}
$$

$$
\mathrm{d}s^2 = \eta^2\left(\frac{\tau_0}{\eta}\right)^{2q}\left(\frac{-\mathrm{d}\eta^2+\mathrm{d}\vec{z}^2}{\eta^2}\right)
$$

The matter-dominated case is recovered for $q = -2$

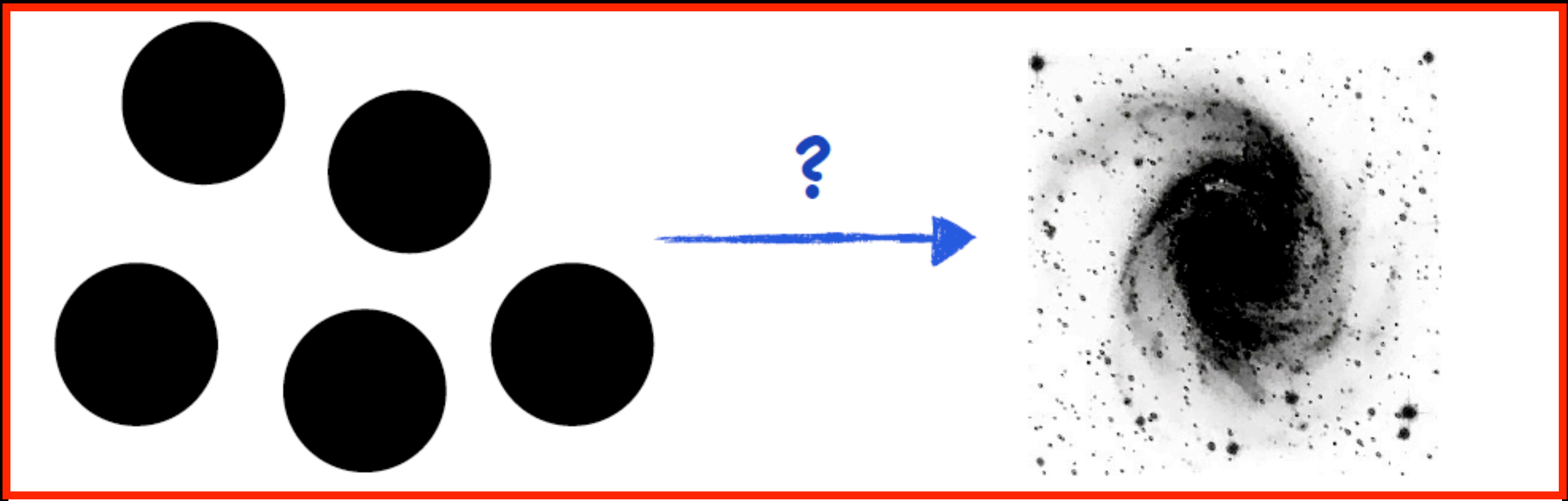
$$z^i = x^i \left(1 - \frac{5}{3}\Phi_L(\vec{x})\right) + \left(\frac{1}{6}\tau^2 + \frac{5}{6}\vec{x}^2\right) \partial^i \Phi_L(\vec{x}),$$
$$\eta = \tau \left(1 + \frac{1}{3}\Phi(\vec{x})\right)$$

In Modified Gravity models correlators may not vanish at equal time because objects feel an extra inhomogeneous force



The problem of bias

In 3D surveys we observe galaxies, not DM



Is it possible to get informations on the halo bias on non-linear scales by using symmetries?

Like in QFT, if symmetries allow the presence of a given set of operators, they can be generated at a given order in perturbation theory

A. Kehagias, J. Norena, H. Perrier and A.R., 1311.0786

Invariant operators: non-local bias

$$\delta(\vec{x},\tau) \qquad \text{I}$$

$$s_{ij}(\vec{x},\tau) = \frac{2}{3\mathcal{H}^2}\partial_i\partial_j\Phi(\vec{x},\tau) - \frac{\delta_{ij}}{2}\delta(\vec{x},\tau) \qquad \text{I}$$

$$t_{ij}(\vec{x},\tau) = -\frac{1}{\mathcal{H}}\left(\partial_i v_j(\vec{x},\tau) - \frac{\delta_{ij}}{3}\vec{\nabla}\cdot\vec{v}(\vec{x},\tau)\right) - s_{ij}(\vec{x},\tau) \qquad \text{II}$$

$$\psi(\vec{x},\tau) = -\frac{\vec{\nabla}\cdot\vec{v}(\vec{x},\tau)}{\mathcal{H}} - \delta(\vec{x},\tau) - \frac{2}{7}s^2(\vec{x},\tau) + \frac{4}{21}\delta^2(\vec{x},\tau) \qquad \text{III}$$

$$\delta_{\rm h}(\vec{x}) = \underbrace{b_{10}\delta(\vec{x})}_{\text{local}} - \underbrace{b_{01}\nabla^2\delta(\vec{x})}_{\text{non-local}} + \underbrace{\frac{1}{2!}b_2\delta^2(\vec{x})}_{\text{local}} + \underbrace{\frac{1}{3!}b_3\delta^3(\vec{x})}_{\text{local}}$$

$$+ \underbrace{\frac{1}{2}b_{s^2}\, s^2(\vec{x})}_{\text{non-local}} + \underbrace{b_{st}\, s\cdot t(\vec{x})}_{\text{non-local}} + \underbrace{b_\psi\psi(\vec{x})}_{\text{non-local}} + \cdots\,.$$

P. Mc Donald and A. Roy, 0902.0991

A. Kehagias, J. Norena, H. Perrier and A.R., 1311.0786

Are these operators generated by the dynamics?

1. Solve the dynamics up to third-order

$$\frac{\partial \delta_{\rm h}(\vec{x},\tau)}{\partial \tau} + \vec{\nabla}\cdot[(1+\delta_{\rm h}(\vec{x},\tau))\vec{v}_{\rm h}(\vec{x},\tau)] = 0,$$

$$\frac{\partial \vec{v}_{\rm h}(\vec{x},\tau)}{\partial \tau} + \mathcal{H}(\tau)\vec{v}_{\rm h}(\vec{x},\tau) + [\vec{v}_{\rm h}(\vec{x},\tau)\cdot\vec{\nabla}]\vec{v}_{\rm h}(\vec{x},\tau) = -\vec{\nabla}\Phi(\vec{x},\tau),$$

$$\frac{\partial \delta(\vec{x},\tau)}{\partial \tau} + \vec{\nabla}\cdot[(1+\delta(\vec{x},\tau))\vec{v}(\vec{x},\tau)] = 0,$$

$$\frac{\partial \vec{v}(\vec{x},\tau)}{\partial \tau} + \mathcal{H}(\tau)\vec{v}(\vec{x},\tau) + [\vec{v}(\vec{x},\tau)\cdot\vec{\nabla}](\vec{x},\tau) = -\vec{\nabla}\Phi(\vec{x},\tau),$$

$$\nabla^2\Phi(\vec{x},\tau) = \frac{3}{2}\Omega_{\rm m}\mathcal{H}^2(\tau)\delta(\vec{x},\tau)$$

$$b_{10} = 1 + b_{10}^{\rm L},\ b_{01} = -R_v^2 + b_{01}^{\rm L},\ b_2 = b_{20}^{\rm L} + \frac{8}{21}b_{10}^{\rm L},$$

$$b_{s^2} = -\frac{4}{7}b_{10}^{\rm L}, b_\psi = -\frac{1}{2}b_{10}^{\rm L},\ b_{st} = -\frac{5}{7}b_{10}^{\rm L}$$

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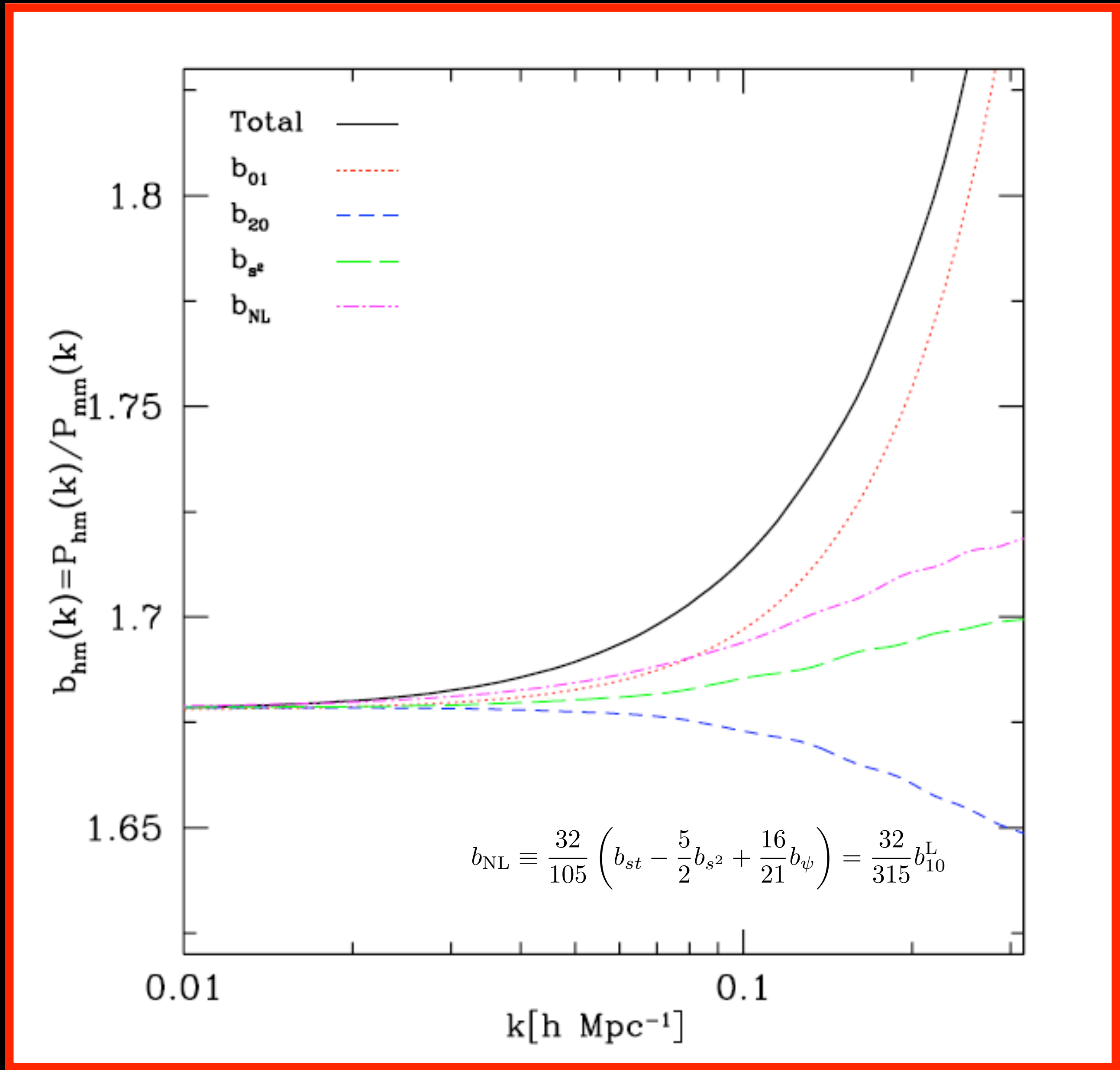
2. Solve for the renormalized halo-matte power spectrum

$$P_{\rm hm}(k) = \left(b_{10} + b_{01}k^2\right) P_{\rm cm}^{\rm NL}(k) + \Delta P_{\rm hm}(k) + P_{\rm cc}(k) I_3(k)$$

$$\begin{aligned}
\Delta P_{\rm hm}(k) &= b_{20} \int \frac{{\rm d}^3 q}{(2\pi)^3} P_{\rm cc}(q) P_{\rm cc}(|\vec{k} - \vec{q}|) F_2(\vec{q}, \vec{k} - \vec{q}) \\
&+ b_{s^2} \int \frac{{\rm d}^3 q}{(2\pi)^3} P_{\rm cc}(q) P_{\rm cc}(|\vec{k} - \vec{q}|) F_2(\vec{q}, \vec{k} - \vec{q}) S(\vec{q}, \vec{k} - \vec{q}), \\
I_3(k) &= \frac{32}{105} \left(b_{st} - \frac{5}{2} b_{s^2} + \frac{16}{21} b_\psi \right) \int {\rm d}\ln r\, \Delta^2_{\rm cc}(kr)\, I_R(r), \\
I_R(r) &= I(r) + \frac{5}{6}, \\
I(r) &= \frac{105}{32} \int_{-1}^{1} {\rm d}\mu\, D_2(\vec{q}, \vec{k}) S(\vec{q}, \vec{k} - \vec{q}), \\
F_2(\vec{q}_1, \vec{q}_2) &= \frac{5}{7} + \frac{1}{2} \frac{\vec{q}_1 \cdot \vec{q}_2}{q_1 q_2} \left(\frac{q_1}{q_2} + \frac{q_2}{q_1} \right) + \frac{2}{7} \left(\frac{\vec{q}_1 \cdot \vec{q}_2}{q_1 q_2} \right)^2, \\
S(\vec{q}_1, \vec{q}_2) &= \left(\frac{\vec{q}_1 \cdot \vec{q}_2}{q_1 q_2} \right)^2 - \frac{1}{3}, \\
D_2(\vec{q}_1, \vec{q}_2) &= \frac{2}{7} \left[S(\vec{q}_1, \vec{q}_2) - \frac{2}{3} \right]
\end{aligned}$$



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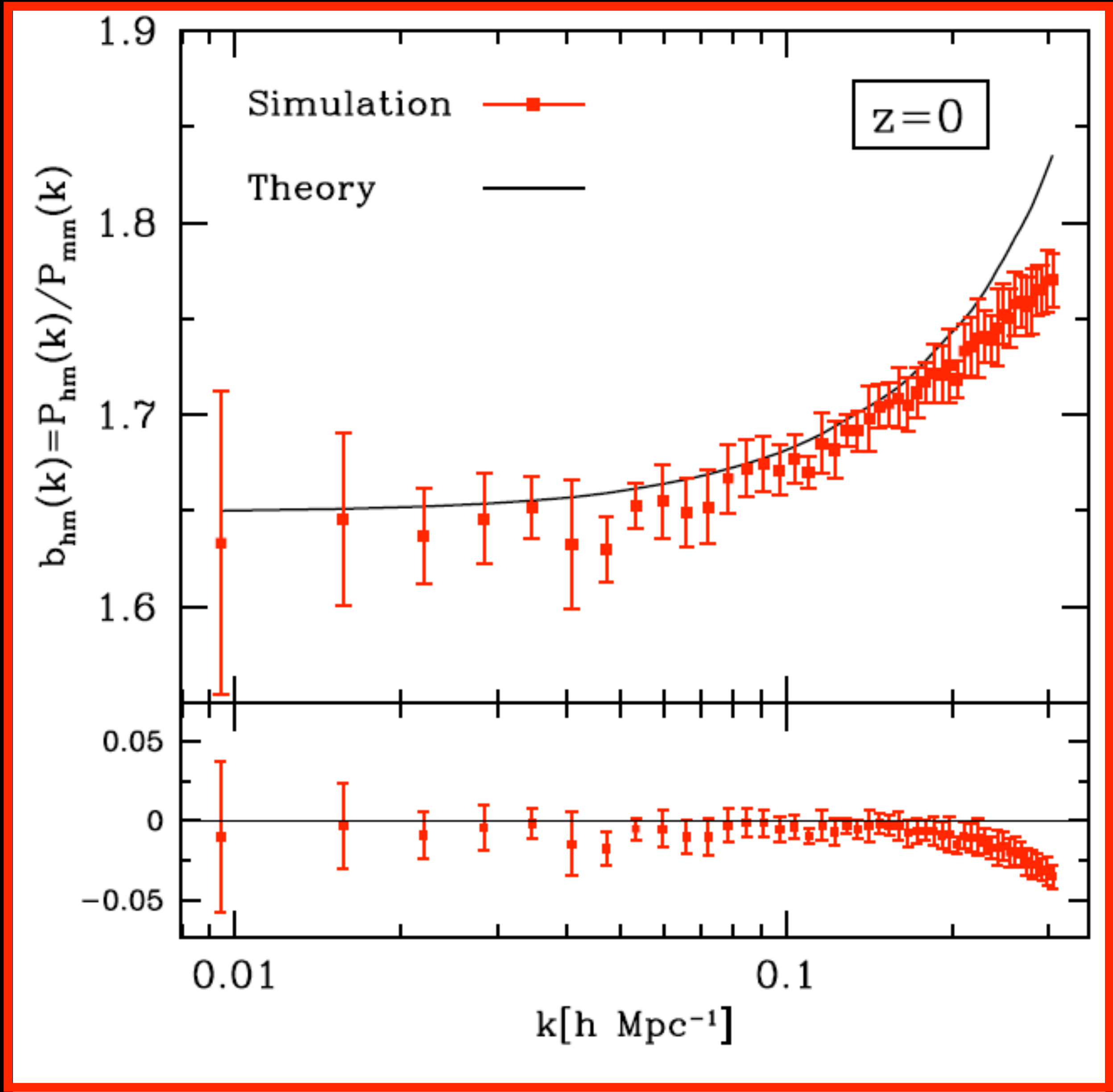

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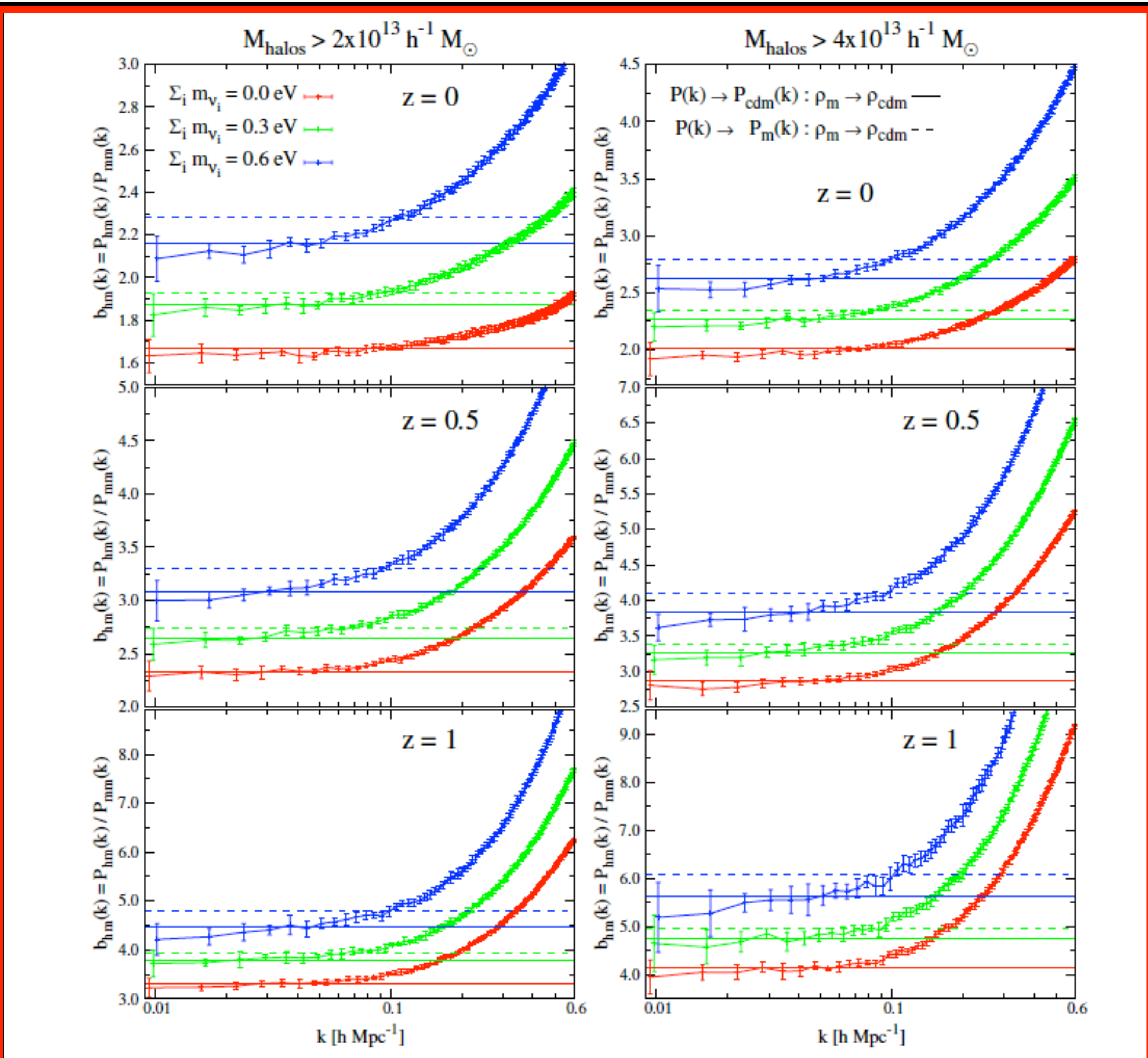
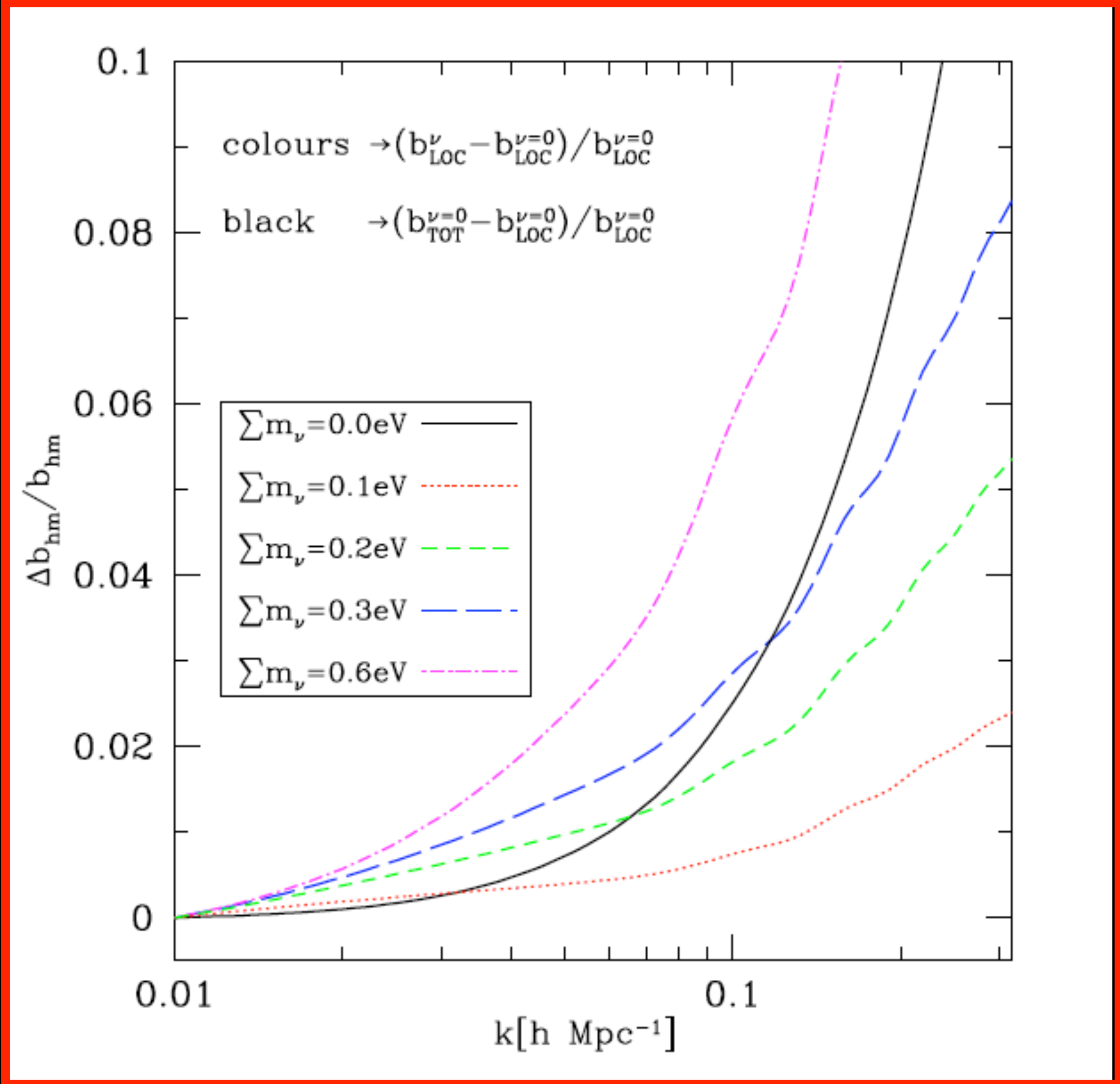
When is non-local halo bias particularly important?
One example: Massive neutrinos



M.Viel et al., 1311.1212



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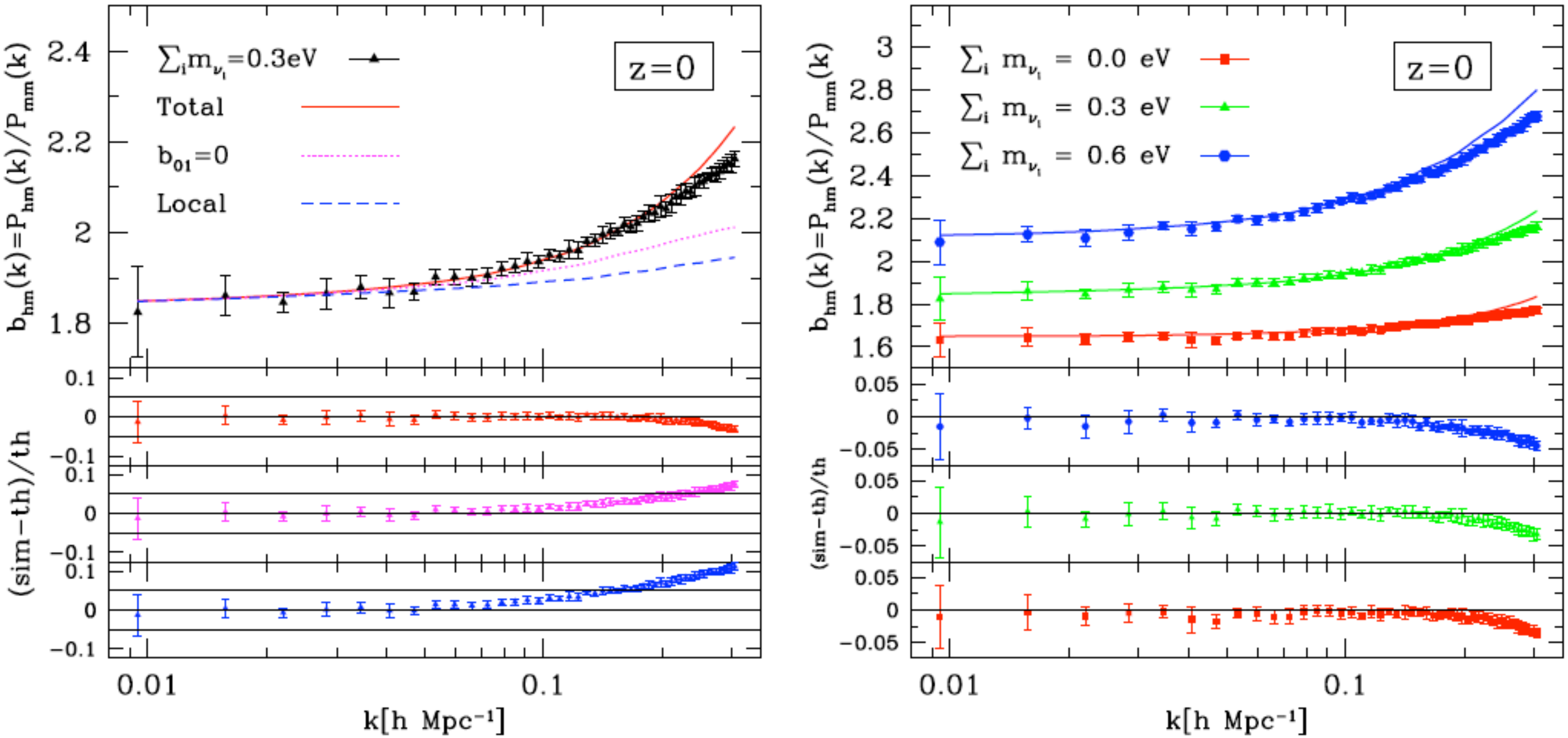

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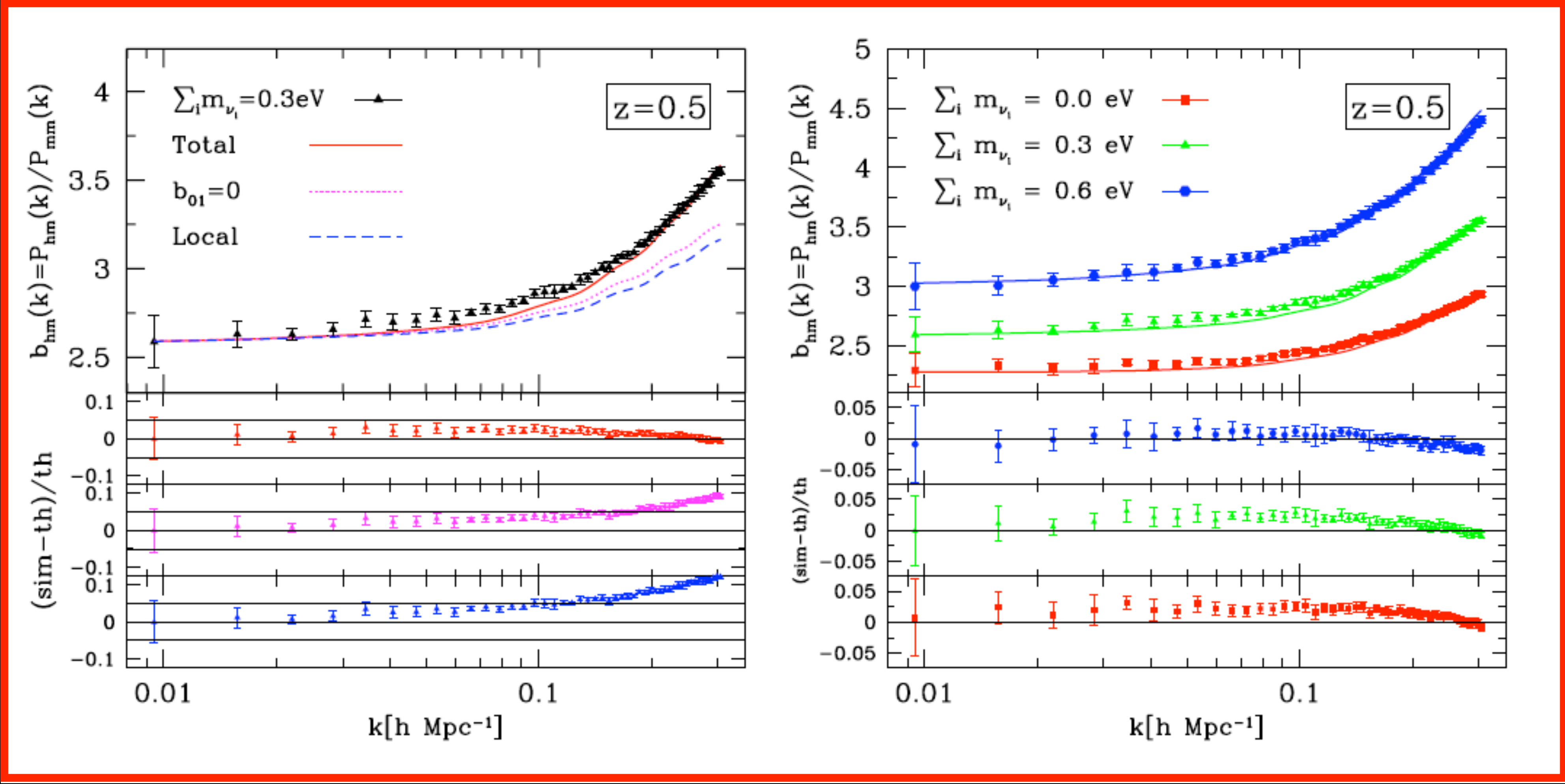

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Is the starting point correct?

In phase-space one starts with the Klimontovich density

$$f_{\rm K}(\vec{x},\vec{p},t)=\sum_i \delta_{\rm D}\left[\vec{x}-\vec{x}_i(t)\right]\delta_{\rm D}\left[\vec{p}-\vec{p}_i(t)\right]$$

satisfying the phase-space equation

$$\frac{\partial f_{\rm K}}{\partial t}+\vec{p}\cdot\frac{\partial}{\partial\vec{r}}f_{\rm K}-\vec{\nabla}\Phi_{\rm K}\cdot\frac{\partial}{\partial\vec{p}}f_{\rm K}=0$$

$$\vec{\nabla}\Phi_{\rm K}=G_{\rm N}\int {\rm d}^3x'{\rm d}^3p'\, f_{\rm K}(\vec{x}',\vec{p}',t)\frac{(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3}$$

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However, in cosmology one considers averages (coarse-graning)

$$\left\langle f_{\mathrm{K}}(\vec{r},\vec{p},t)\right\rangle = \left\langle \sum_i \delta_{\mathrm{D}}\left[\vec{r}-\vec{r}_i(t)\right]\delta_{\mathrm{D}}\left[\vec{p}-\vec{p}_i(t)\right]\right\rangle = f(\vec{r},\vec{p},t)$$

The phase-space equation becomes

$$\frac{\partial f}{\partial t} + \vec{p}\cdot\frac{\partial f}{\partial \vec{r}} - \langle\vec{\nabla}\Phi\rangle\cdot\frac{\partial f}{\partial \vec{p}} + \cdots = 0$$

$$\langle\vec{\nabla}\Phi\rangle = G_{\mathrm{N}}\int \mathrm{d}^3x'\mathrm{d}^3p'\, f(\vec{x}',\vec{p}',t)\frac{(\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3}$$

Halos feel a different averaged force
than the underlying smooth DM

For halos, one needs to compute the average force given the peak constraint (maxima of the matter distribution)

$$\left\langle \vec{\nabla}\Phi \middle| \vec{\nabla}\delta_{\rm dm} \right\rangle = \frac{\left\langle \vec{\nabla}\Phi\vec{\nabla}\delta_{\rm dm} \right\rangle}{\left\langle \left(\vec{\nabla}\delta_{\rm dm}\right)^2 \right\rangle} \vec{\nabla}\delta_{\rm dm} = -\frac{3}{2}H^2 \frac{\sigma_0^2}{\sigma_1^2} \vec{\nabla}\delta_{\rm dm}$$

$$\left\langle \vec{\nabla}\Phi \otimes \vec{\nabla}\Phi \right\rangle = \frac{\alpha^2}{3}\sigma_{-1}^2 \, \mathbf{1}_{3\times3}, \quad \left\langle \vec{\nabla}\Phi \otimes \vec{\nabla}\delta_{\rm dm} \right\rangle = -\frac{\alpha}{3}\sigma_0^2 \, \mathbf{1}_{3\times3},$$

$$\left\langle \vec{\nabla}\delta_{\rm dm} \otimes \vec{\nabla}\Phi \right\rangle = -\frac{\alpha}{3}\sigma_0^2 \, \mathbf{1}_{3\times3}, \quad \left\langle \vec{\nabla}\delta_{\rm dm} \otimes \vec{\nabla}\delta_{\rm dm} \right\rangle = \frac{1}{3}\sigma_1^2 \, \mathbf{1}_{3\times3},$$

$$\sigma_j^2 = \int \frac{{\rm d}^3k}{(2\pi)^3} k^{2j} P_{\rm dm}(k) W^2(KR)$$

$$\vec{\nabla}\Phi_{\rm eff} = \vec{\nabla}\Phi + \frac{3}{2}H^2\Omega_{\rm dm} \frac{\sigma_0^2}{\sigma_1^2} \vec{\nabla}\delta_{\rm dm}$$

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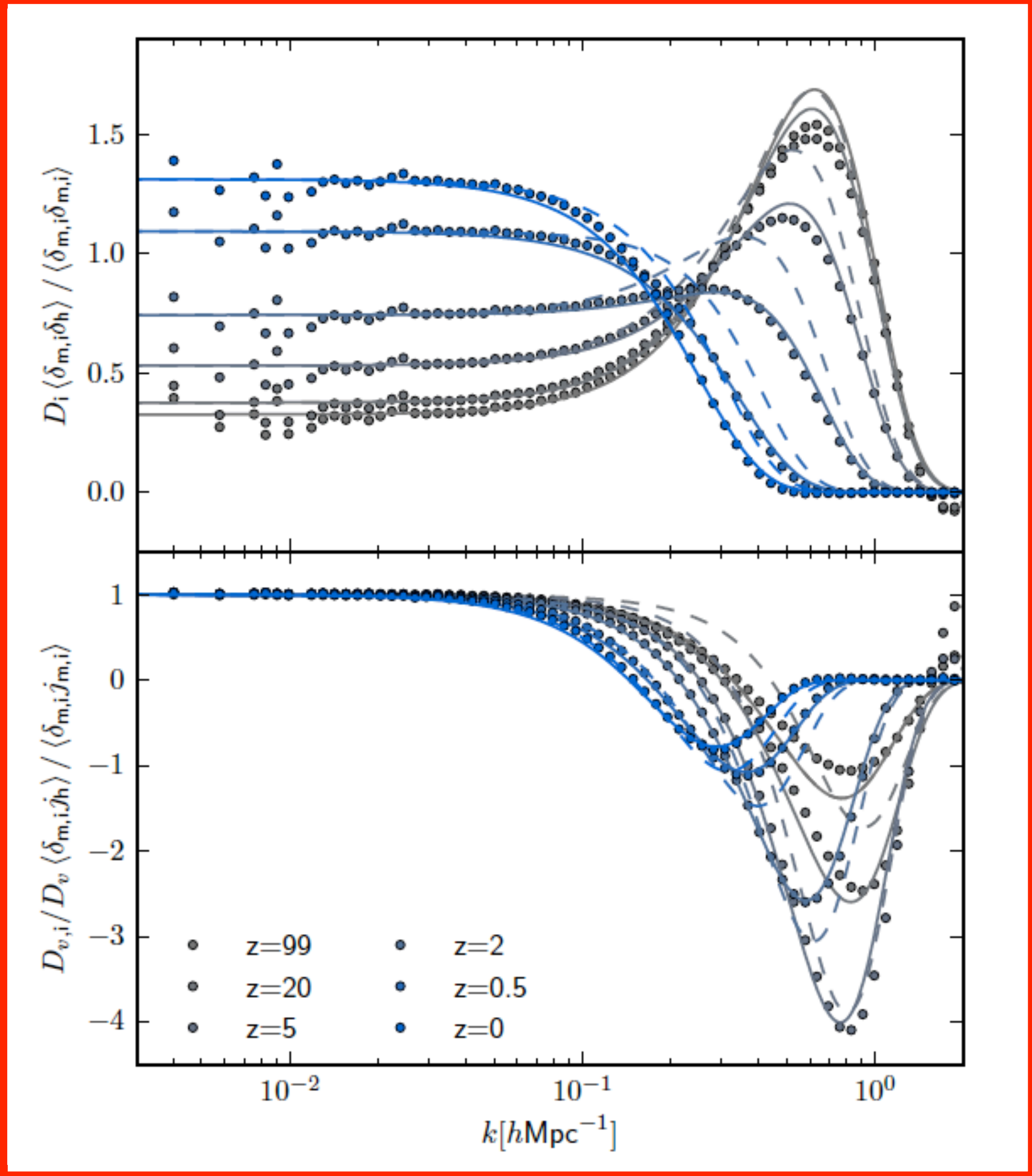

T. Baldauf ,V. Desjacques and U. Seljak, 1405.5885

Conclusions

- Symmetries are a powerful tool to characterize the cosmological perturbations, both in the early and in the late universe
- The symmetries of the large-scale structure allow to derive useful consistency relations and to characterize observables like the galaxy bias
- A correct description of the real observed quantities, i.e. galaxies, calls for a rethinking of our basic tools