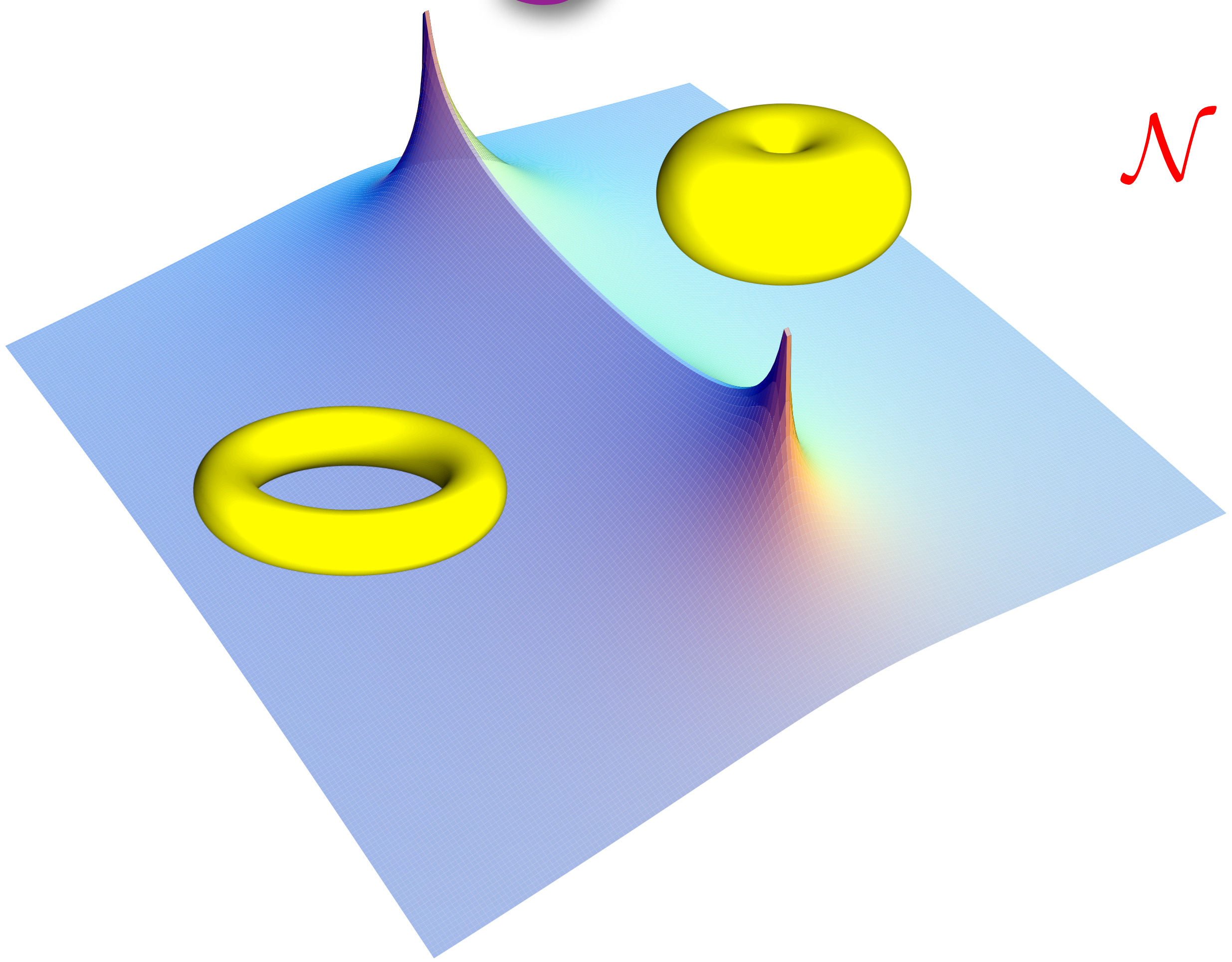
Seiberg-Witten and Massless Monopoles

John Terning

Outline

- Seiberg Duality Warm-up
- Monopoles, Dyons, and Triality
- N=2 SUSY: Seiberg-Witten
- SL(2,Z) duality
- New Anomalies
- Conclusions

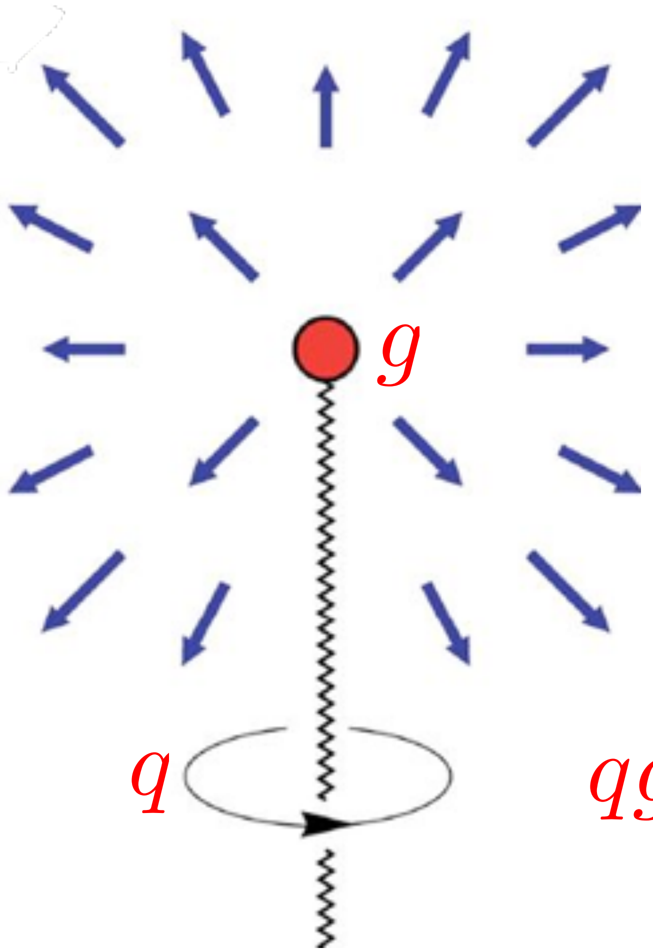
Seiberg-Witten



$\mathcal{N} = 2$ SUSY

exact results and points with massless monopoles and dyons

Dirac

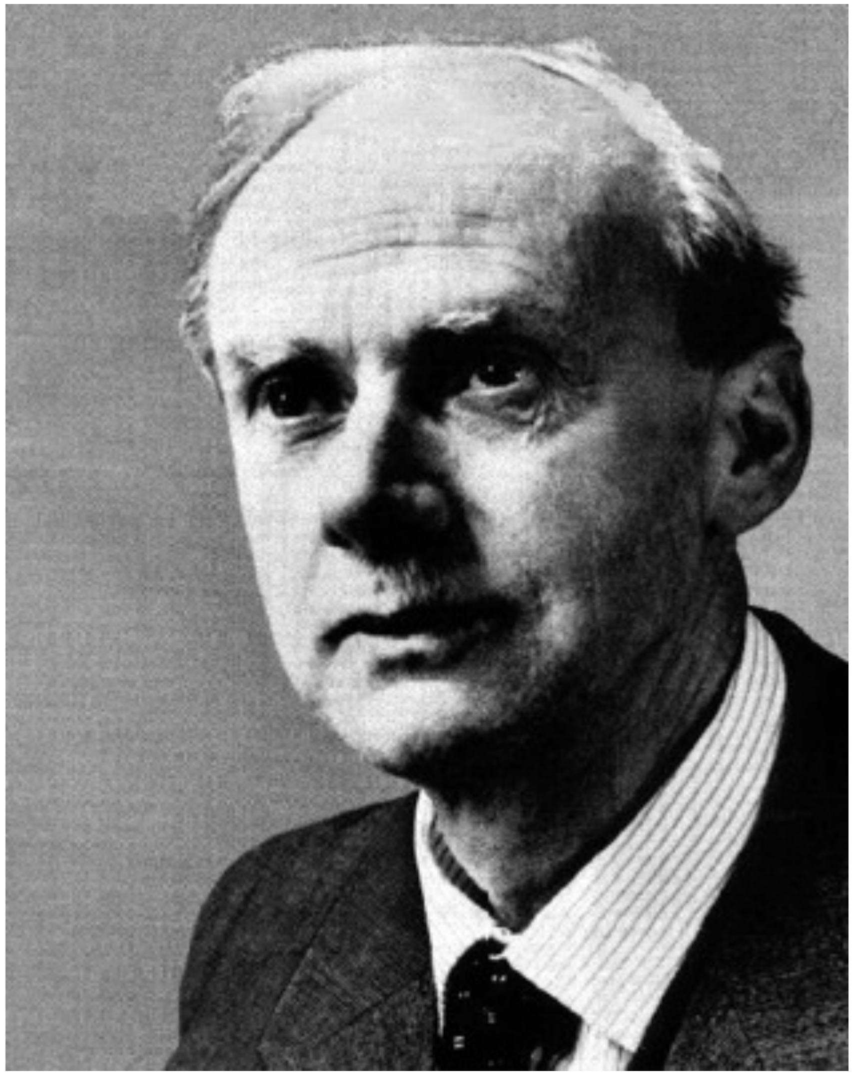


$$qg = \frac{n}{2}$$

charge quantization

Proc. Roy. Soc. Lond. A133 (1931) 60

Dirac



non-local action?

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + {}^*G_{\mu\nu}$$

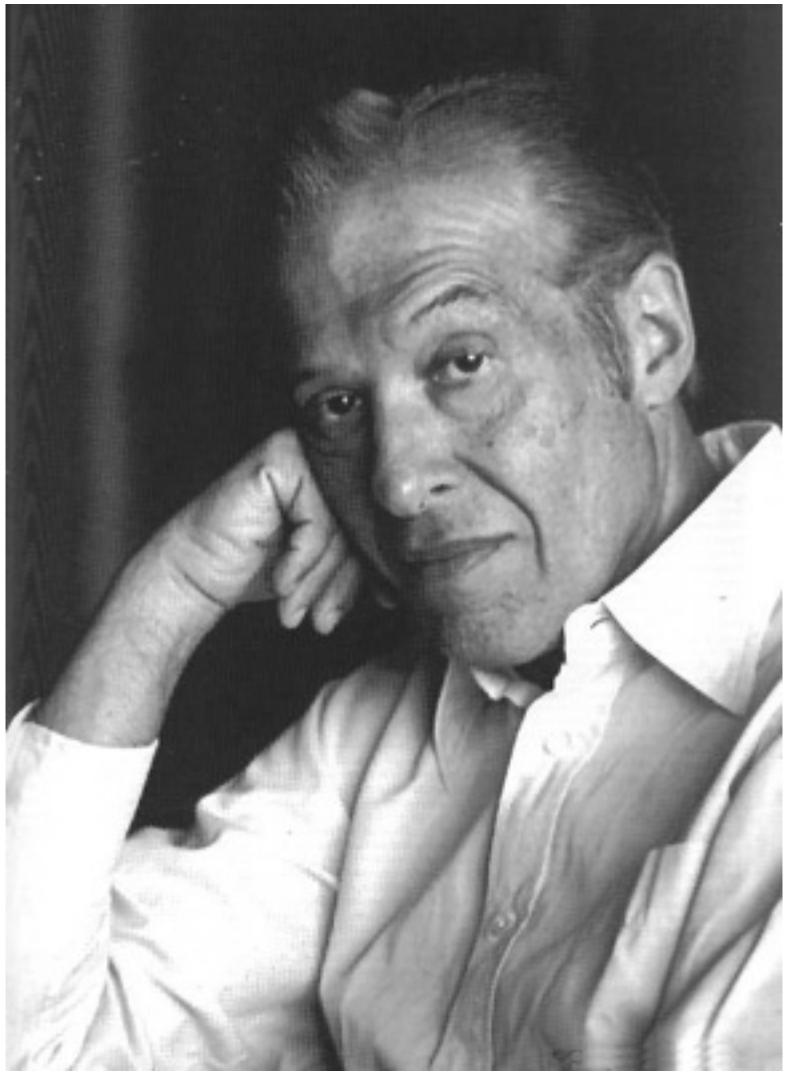
$$\begin{aligned} G_{\mu\nu}(x) &= 4\pi(n\cdot\partial)^{-1}\left[n_\mu K_\nu(x) - n_\nu K_\mu(x)\right] \\ &= \int d^4y\, \left[f_\mu(x-y)K_\nu(y) - f_\nu(x-y)K_\mu(y)\right] \end{aligned}$$

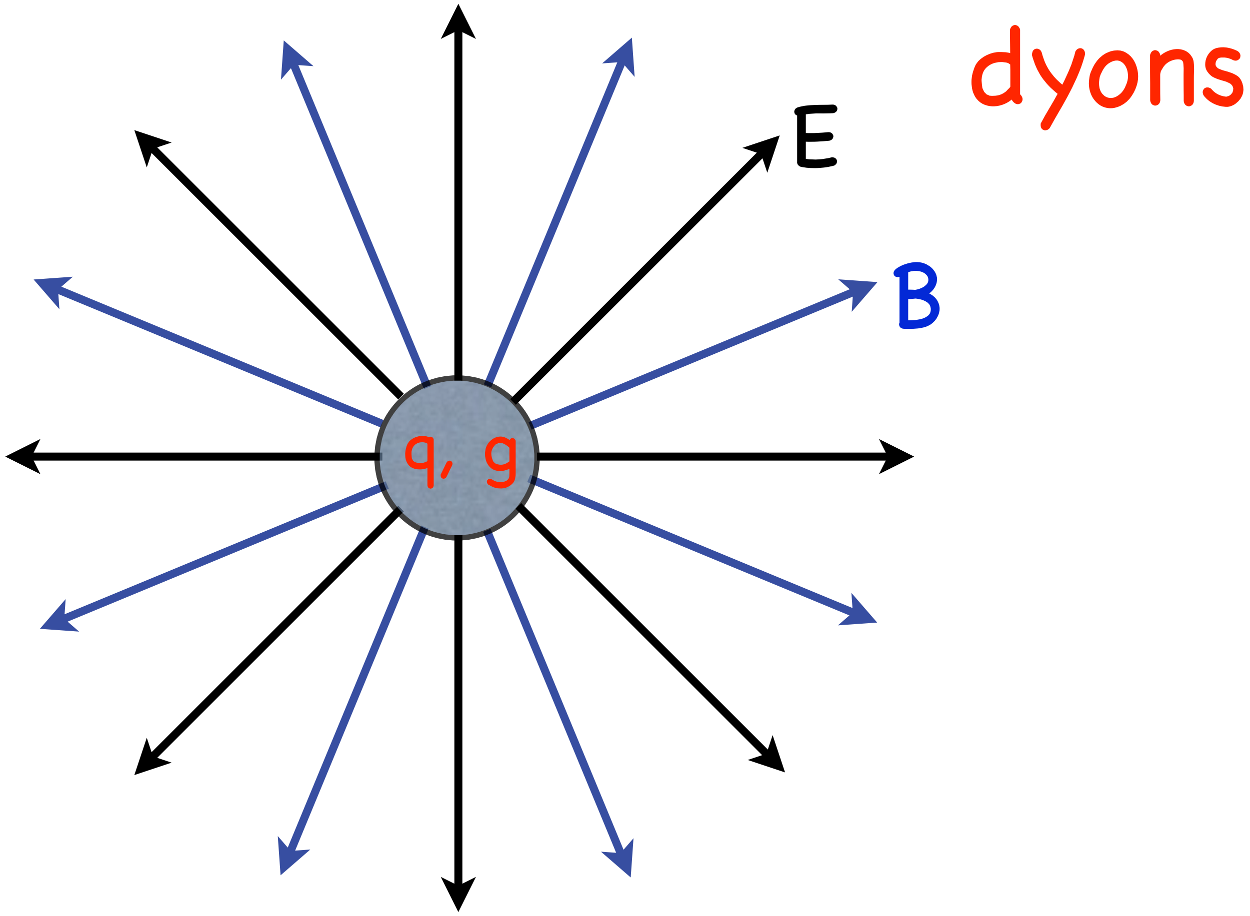
$$\partial_\mu f^\mu(x) = 4\pi\delta(x)$$

$$f^\mu(x) = 4\pi n^\mu\,(n\cdot\partial)^{-1}\,\delta(x)$$

Phys. Rev. 74 (1948) 817

Schwinger

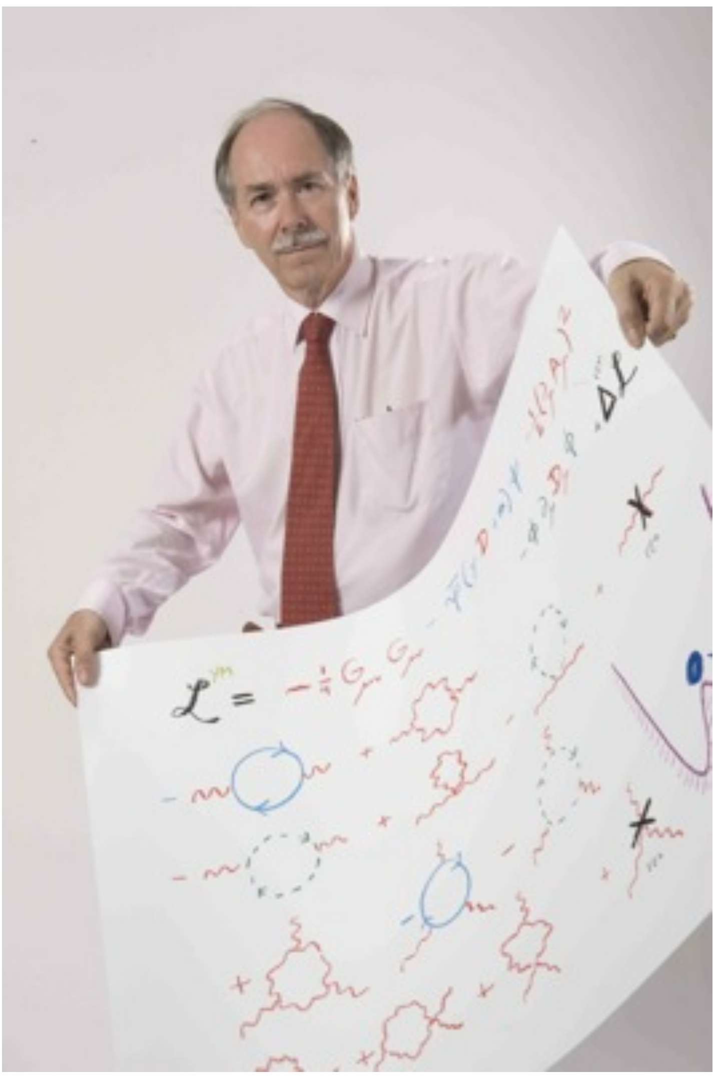


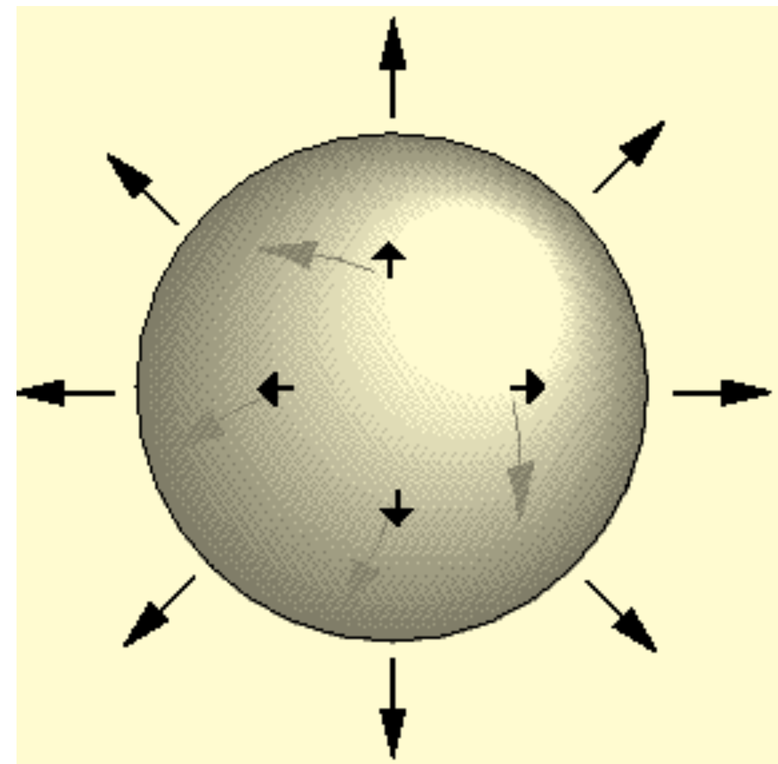


$$q_1 g_2 - q_2 g_1 = \frac{n}{2}$$

Science 165 (1969) 757

`t Hooft-Polyakov





topological monopoles

$$SO(3) \sim SU(2) \rightarrow U(1)$$

Nucl. Phys., B79 1974, 276

JETP Lett., 20 1974, 194

SO(N) Duality

	$SO(N)$	$SU(F)$	$U(1)_R$
Q	$\square$	$\square$	$\frac{F+2-N}{F}$

$$F > N - 2$$

	$SO(F-N+4)$	$SU(F)$	$U(1)_R$
q	$\square$	$\overline{\square}$	$\frac{N-2}{F}$
M	$\mathbf{1}$	$\square\square$	$\frac{2(F+2-N)}{F}$

$F > N - 1$

$$W = \frac{1}{2\mu} M_{ji} q^j q^i$$

SO(N) Duality

$$F = N$$

$$\tilde{N} = F - N + 4$$

	$SO(4)$	$SU(N)$	$U(1)_R$
q	$\square$	$\overline{\square}$	$\frac{N-2}{N}$
M	$\mathbf{1}$	$\square\!\square$	$\frac{4}{N}$

$$W = \frac{1}{2\mu} M_{ji} q^j q^i$$

Integrating Out

$$SO(N) \xrightarrow{F=N} SO(4) \sim SU(2) \times SU(2)$$

$$W = m\, Q_F Q_F \qquad\qquad W = \frac{1}{2\mu} M_{ji} q^j q^i + m\, M_{FF}$$

$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

$$SO(N) \xrightarrow{F=N-1} SO(3) \sim SU(2)$$

$$W = 0 \qquad\qquad W = \frac{1}{2\mu} M_{ji} q^j q^i - \frac{\det M}{64\Lambda^{2N-5}}$$

Integrating Out

$$SO(N) \xrightarrow{F=N} SO(4) \sim SU(2) \times SU(2)$$

$$W = m\, Q_F Q_F \qquad\qquad W = \frac{1}{2\mu} M_{ji} q^j q^i + m\, M_{FF}$$

$$SO(N) \xrightarrow{F=N-1} SO(3) \sim SU(2)$$

$$W = 0 \qquad\qquad W = \frac{1}{2\mu} M_{ji} q^j q^i - \frac{\det M}{64 \Lambda^{2N-5}}$$

\`t Hooft instanton vertex

SO(N)

$$F = N - 1$$

$$\tilde{N} = F - N + 4$$

	$SO(3)$	$SU(N-1)$	$U(1)_R$
q	$\Box$	$\overline{\Box}$	$\frac{N-2}{N-1}$
M	$\mathbf{1}$	$\Box\Box$	$\frac{2}{N-1}$

$$W = \frac{1}{2\mu} M_{ji} q^j q^i - \frac{1}{64\Lambda^{2N-5}} \det M$$

`t Hooft instanton vertex

Dual of the Dual

	$SO(N)$	$SU(F=N-1)$	$U(1)_R$
d	$\square$	$\square$	$\frac{1}{N-1}$

$$W = \frac{M_{ji}N^{ij}}{2\mu} - \frac{N^{ij}}{2\mu}d_j d_i - \frac{\det M}{64\Lambda^{2N-5}} \pm \frac{\det d_j d_i}{64\Lambda^{2N-5}}$$

$$M_{ji} = d_j\, d_i$$

extra discrete axial symmetry of SO(3)

Triality

	$SO(N)$	$SU(N-1)$	$U(1)_R$
Q	$\square$	$\square$	$\frac{1}{N-1}$

$$W = 0$$

	$SO(3)$	$SU(N-1)$	$U(1)_R$
q	$\square$	$\overline{\square}$	$\frac{N-2}{N-1}$
M	$\mathbf{1}$	$\square\square$	$\frac{2}{N-1}$

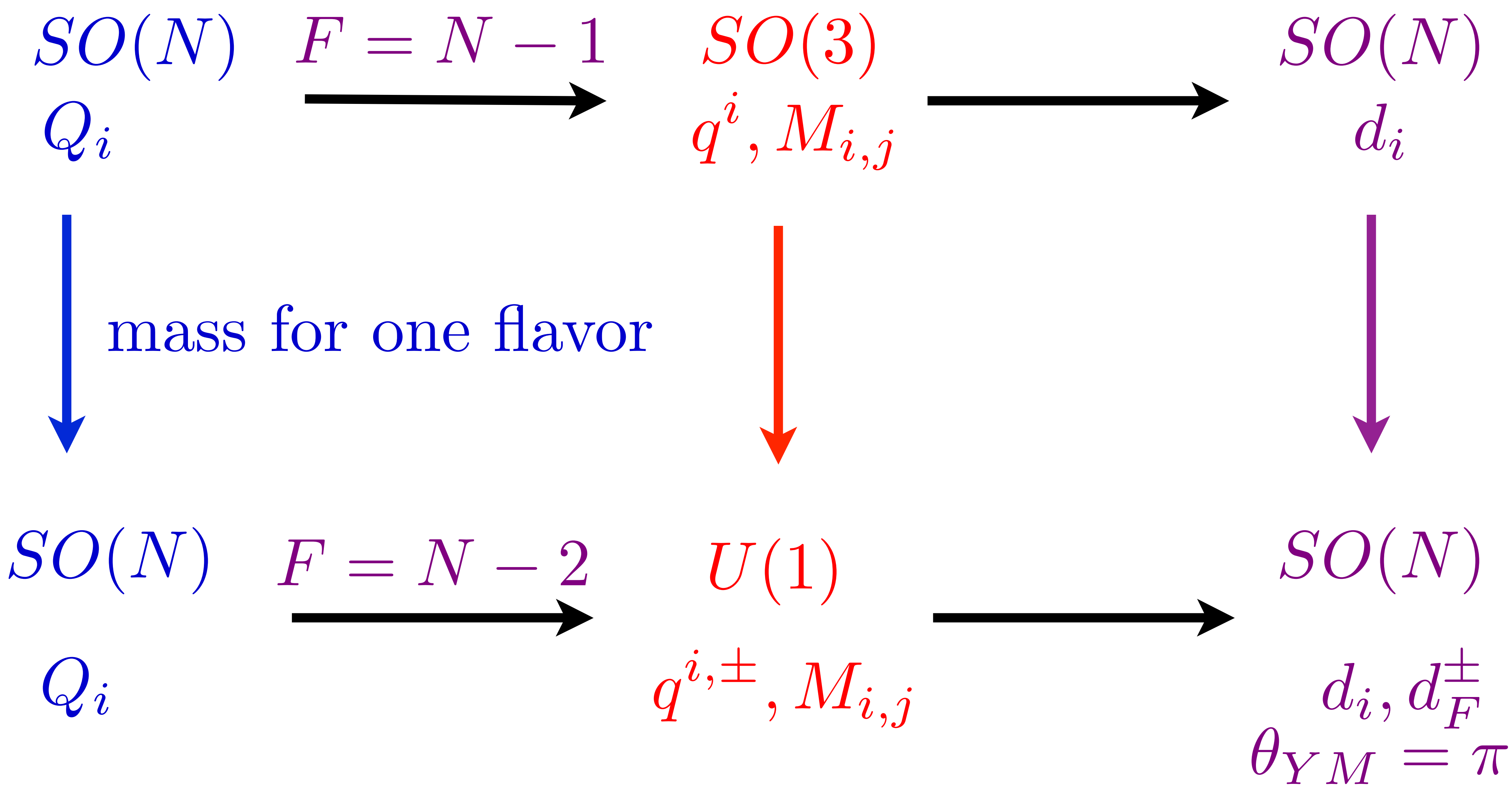
$$W = \frac{1}{2\mu} M_{ji} q^j q^i - \frac{\det M}{64\Lambda^{2N-5}}$$

	$SO(N)$	$SU(N-1)$	$U(1)_R$
d	$\square$	$\square$	$\frac{1}{N-1}$

$$W = -\frac{\det(d_i d_j)}{32\Lambda^{2N-5}}$$

$$\theta_{YM} = \pi$$

Integrating Out



Weak Coupling

$SO(N)$ $\quad F = N-2 \quad$ $SO(3)$ $\quad\longrightarrow\quad$ $SO(N)$

Q_i $\quad\longrightarrow\quad$ $q^{i,\pm}, M_{i,j}$ $\quad$ $d_i,\ d_F$

large $\langle Q \rangle$ $\quad$ $\det M \approx 0$ $\quad$ $\det d_i d_j = 16\Lambda^{2N-4}$

$\downarrow$ $\quad$ $\downarrow$ $\quad$ $\downarrow$

$U(1)$ $\quad$ $U(1)$ $\quad$ $U(1)$

$Q^{\pm}$ $\quad$ $q^{i,\pm}$ $\quad$ $d_F^{\pm}$ $\theta_{YM} = \pi$

"electrons" $\quad$ monopoles $\quad$ dyons

EM Duality

Witten Charge

effective charge shifted

$$\mathcal{L} = -\frac{1}{4e^2} F^{\mu\nu} F_{\mu\nu} - \frac{\theta_{YM}}{32\pi^2} F^{\mu\nu}\, {}^{*}F_{\mu\nu}$$

$$q_{\rm eff} = q + g\frac{\theta_{YM}}{2\pi}$$

Phys. Lett. B86 (1979) 283

E-M Duality

$$\mathcal{L} = -\frac{1}{4g^2} F^{\mu\nu} F_{\mu\nu} - \frac{\theta_{YM}}{32\pi^2} F^{\mu\nu}\, {}^*F_{\mu\nu}$$

$$\mathcal{L} = -\mathrm{Im} \frac{\tau}{32\pi} \left(F^{\mu\nu} + i\, {}^*F^{\mu\nu} \right)^2$$

$$\mathcal{L}_c = \frac{1}{4\pi} A_D^{\mu}\, \partial^{\nu}\, {}^*F_{\mu\nu}$$

Bianchi

$$\mathcal{L}_D = \mathrm{Im} \frac{1}{32\pi\tau} \left(F_D^{\mu\nu} + i\, {}^*F_D^{\mu\nu} \right)^2$$

$$F_D^{\mu\nu} = \partial^{\mu} A_D^{\nu} - \partial^{\nu} A_D^{\mu}$$

$$S : \tau \to -\frac{1}{\tau}$$

Monodromy

$$F = N - 2$$

$$\tau = \frac{\theta_{\rm YM}}{2\pi} + \frac{4\pi i}{g^2}$$

$$z = \det(QQ)$$

large $\langle Q \rangle$ $\qquad \tau \approx \frac{i}{\pi} \ln\left(\frac{z}{\Lambda^b}\right)$

$$b = 3(N-2) - F = 2(N-2)$$

loop in z around ∞: $z \to e^{2\pi i} z$

monodromy: $\tau \to \tau - 2$

Monodromy

$$\tau = \frac{\theta_{\mathrm{YM}}}{2\pi} + \frac{4\pi i}{g^2}$$

large $\langle Q \rangle$ monodromy: $\tau \to \tau - 2$

$$T : \tau \to \tau + 1$$

$$\mathcal{M}_\infty = T^{-2}$$

Monodromy

$\tau = \frac{\theta_{\rm YM}}{2\pi} + \frac{4\pi i}{g^2}$ is not a single valued function

$\frac{4\pi}{g^2} = \mathrm{Im}\,\tau$ is single-valued at weak coupling

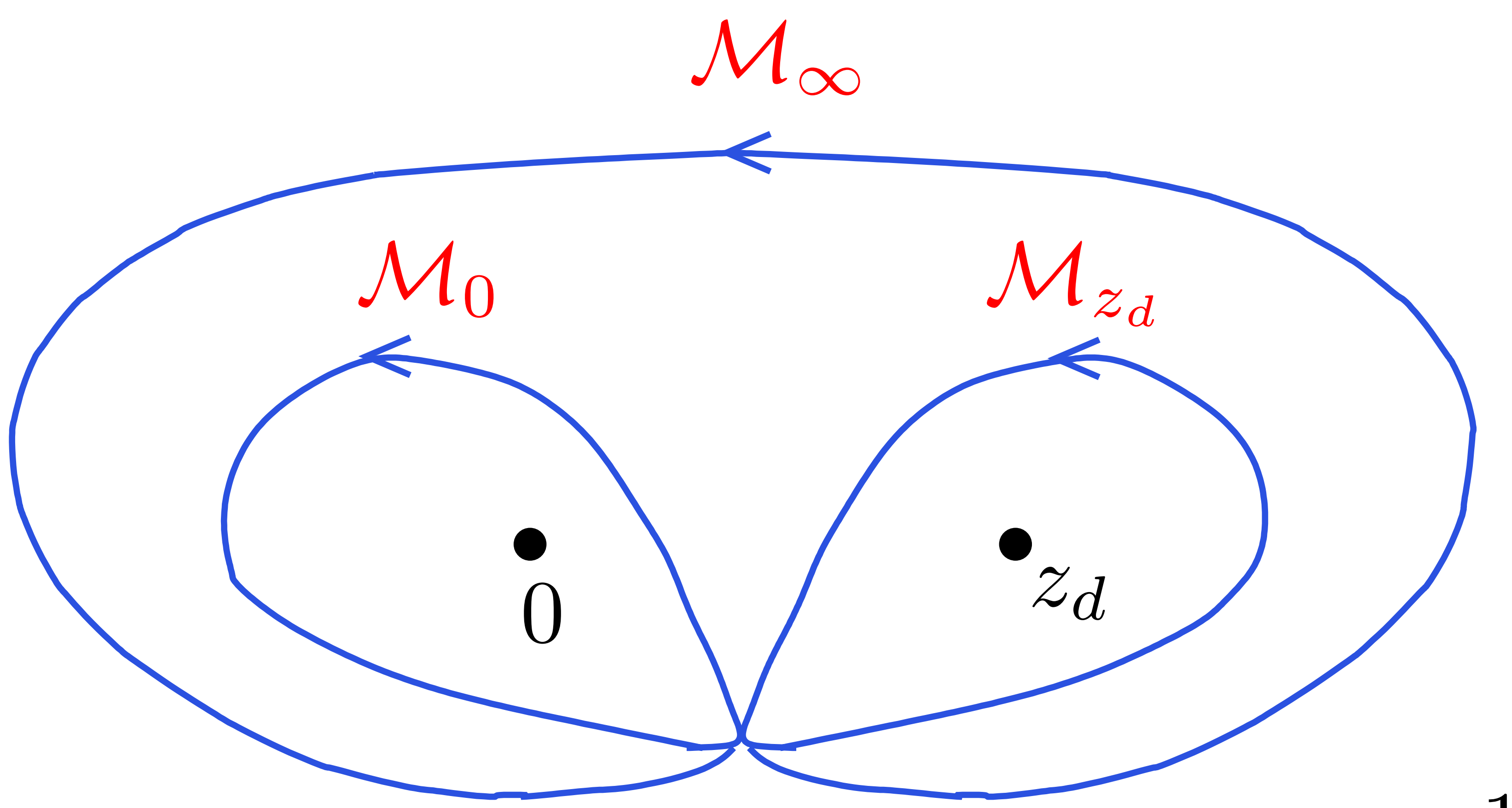
If $\mathrm{Im}\,\tau$ was single-valued everywhere it would be *harmonic*

then it would negative somewhere

way out: there are at least two other singularities
with monodromies that do not commute with $\mathcal{M}_\infty$

$\frac{4\pi}{g^2} = \mathrm{Im}\,\tau$ not single valued

Monodromy



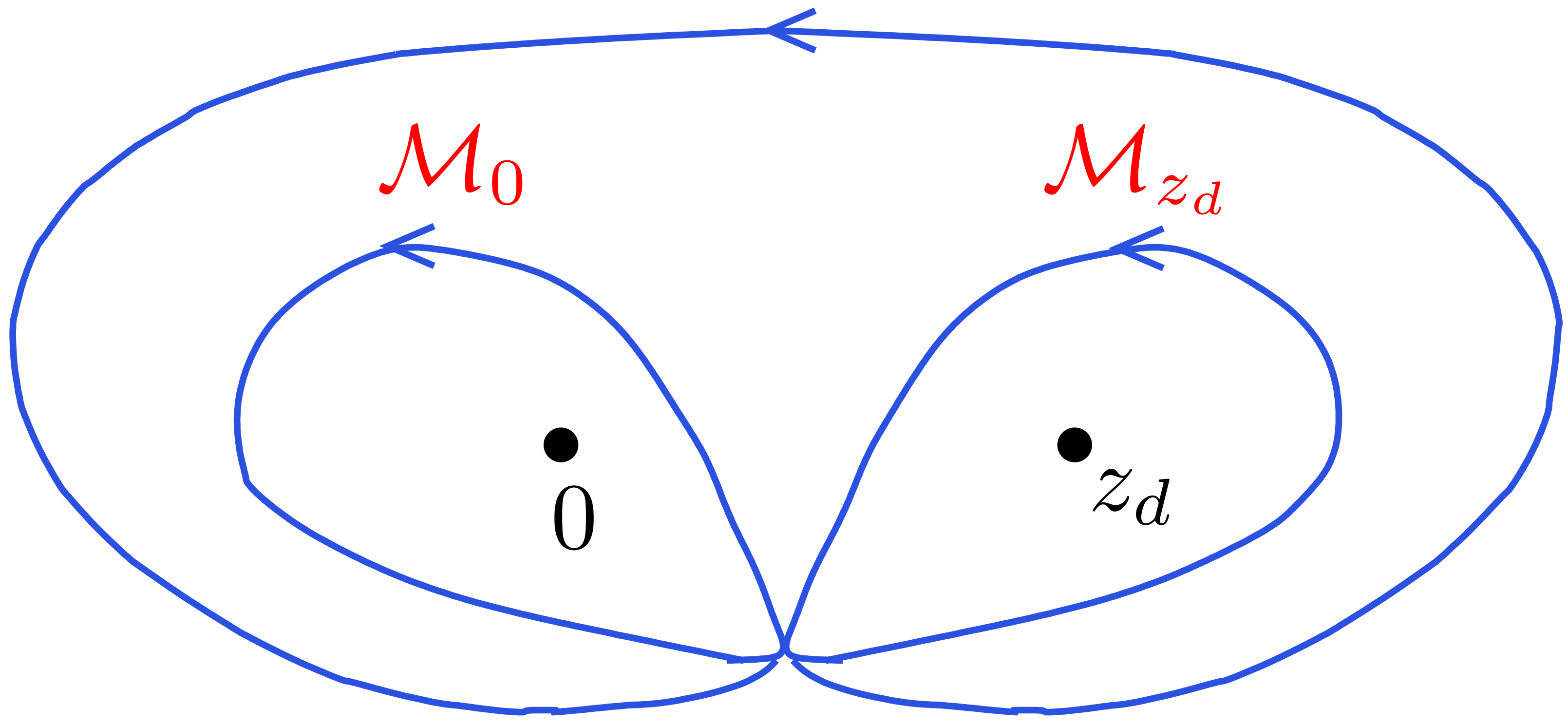
$z_d \equiv 16\Lambda^{2N-4}$

$$\mathcal{M}_0\mathcal{M}_{z_d} = \mathcal{M}_\infty$$

non-commuting

Monodromy

$$\mathcal{M}_\infty = T^{-2}$$



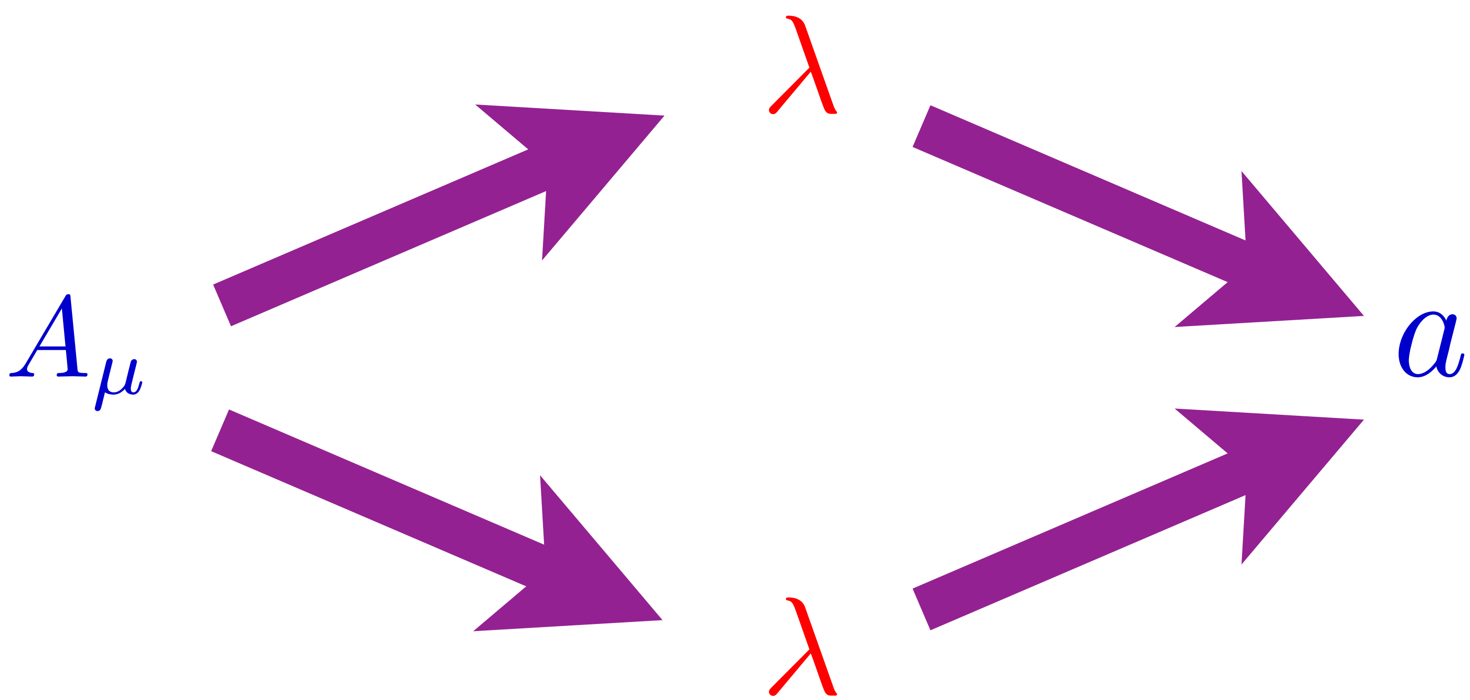
$$\mathcal{M}_0 = D_0^{-1} T^2 D_0 \qquad \mathcal{M}_{z_d} = D_{z_d}^{-1} T^2 D_{z_d}$$

D_i must contain odd power of S

Seiberg-Witten

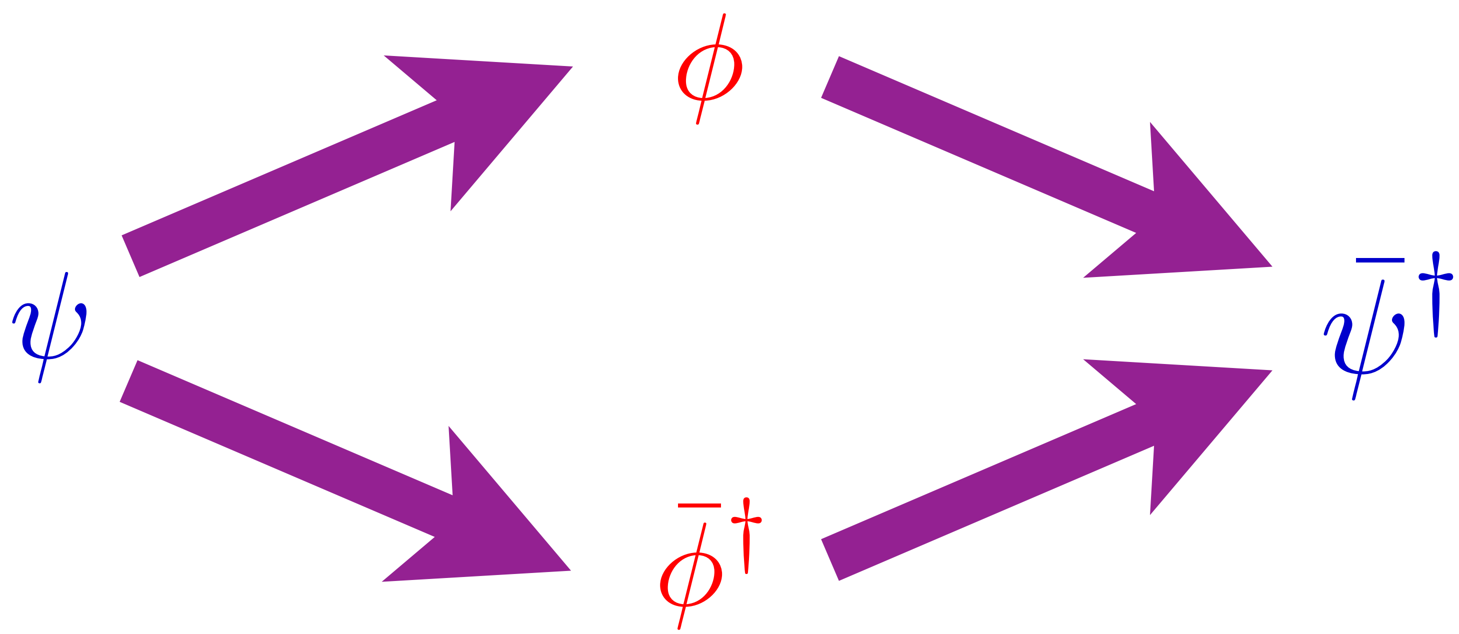

hep-th/9407087

N=2 SUSY



gluon	gluinos	sgluino
spin 1	spin 1/2	spin 0

N=2 SUSY



quark	squark	quark
left-handed		right-handed
spin 1/2	spin 0	spin 1/2

N=1, SO(3), F=1 aka N=2 SO(3)

$$u = \mathrm{Tr}\, a^2 = \mathrm{Tr}\, a^b a^c\, T^b T^c$$

Z_2 takes $u \to -u$

$\langle u \rangle \neq 0$

$U(1)$

Low-Energy N=2

leading terms determined by pre-potential

$$P(a) = \frac{i}{2\pi} a^2 \ln \frac{a^2}{\Lambda^2} + a^2 \sum_{k=1}^{\infty} p_k \left(\frac{\Lambda}{a}\right)^{4k}$$

perturbative non-perturbative

$$\tau = \frac{\theta_{\text{YM}}}{2\pi} + \frac{4\pi i}{g^2} = \frac{\partial^2 P}{\partial a \partial a}$$

Seiberg Phys. Lett. B206 (1988) 75

SL(2,Z) Duality

$$\tau = \frac{\theta_{\rm YM}}{2\pi} + \frac{4\pi i}{g^2}$$

$$S : \tau \rightarrow -\frac{1}{\tau} \qquad T : \tau \rightarrow \tau + 1$$

$$\tau' = \frac{a\tau + b}{c\tau + d}$$

$$ad - bc = 1$$

not a symmetry

SL(2,Z)

$$\tau \equiv \frac{\theta}{2\pi} + \frac{4\pi i}{e^2} \qquad S : \tau \to -\frac{1}{\tau} \qquad T : \tau \to \tau + 1$$

$$\tau' = \frac{a\tau + b}{c\tau + d}$$

$$K^\mu \to aK'^\mu + cJ'^\mu \ , \quad J^\mu \to bK'^\mu + dJ'^\mu$$

$$ad - bc = 1$$

not a symmetry

Witten Charge

effective charge shifted

$$\mathcal{L} = -\frac{1}{4e^2} F^{\mu\nu} F_{\mu\nu} - \frac{\theta_{YM}}{32\pi^2} F^{\mu\nu} \, {}^*F_{\mu\nu}$$

$$q_{\text{eff}} = q + g \frac{\theta_{YM}}{2\pi}$$

Phys. Lett. B86 (1979) 283

SL(2,Z)

$$\frac{\mathrm{Im}\,(\tau)}{4\pi}\,\partial_\mu\left(F^{\mu\nu}+i\,{}^*F^{\mu\nu}\right)=J^\nu+\tau K^\nu$$

$$K^\mu\rightarrow aK'^\mu+cJ'^\mu\ ,\quad J^\mu\rightarrow bK'^\mu+dJ'^\mu$$

$$\left(F^{\mu\nu}+i\,{}^*F^{\mu\nu}\right)\rightarrow\frac{1}{c\tau^*+d}\left(F'^{\mu\nu}+i\,{}^*F'^{\mu\nu}\right)$$

$$\frac{\mathrm{Im}\,(\tau')}{4\pi}\,\partial_\nu\left(F'^{\mu\nu}+i\,{}^*F'^{\mu\nu}\right)=J'^\mu+\tau' K'^\mu$$

N=2 and SL(2,Z)

$$a_D \equiv \frac{\partial P}{\partial a} \qquad \tau = \frac{\partial a_D}{\partial a}$$

$$\frac{-1}{\tau(a)} = \frac{-1}{\frac{\partial a_D}{\partial a}} = -\frac{\partial a}{\partial a_D} = \tau_D(a_D)$$

on vector (a_D, a)

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

N=2 and SL(2,Z)

state with charge n_e

N=2: $$W_{\text{hyper}} = \sqrt{2} n_e a Q \overline{Q}$$

$$M = \sqrt{2}\,|n_e\, a|$$

monopole

SL(2,Z): $$M = \sqrt{2}\,|n_m\, a_D|$$

in general:

$$M = \sqrt{2}\,|n_e\, a + n_m\, a_D|$$

Weak Coupling

large $|a|$: $\quad a = \sqrt{2u}$

$$a_D = \frac{\partial P}{\partial a} = \frac{2ia}{\pi} \ln\left(\frac{a}{\Lambda}\right) + \frac{2ia}{\pi}$$

loop in u around ∞

$$\ln u \to \ln u + 2\pi i$$

$$\ln a \quad \to \quad \ln a + i\pi$$

$$a \quad \to \quad -a$$

$$a_D \quad \to \quad -a_D + 2a$$

monodromy: $\quad \mathcal{M}_\infty = -T^{-2} = \begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix}$

Monodromy

τ is not a single-valued

$\frac{4\pi}{g^2} = \mathrm{Im}\,\tau$ is single-valued at weak coupling

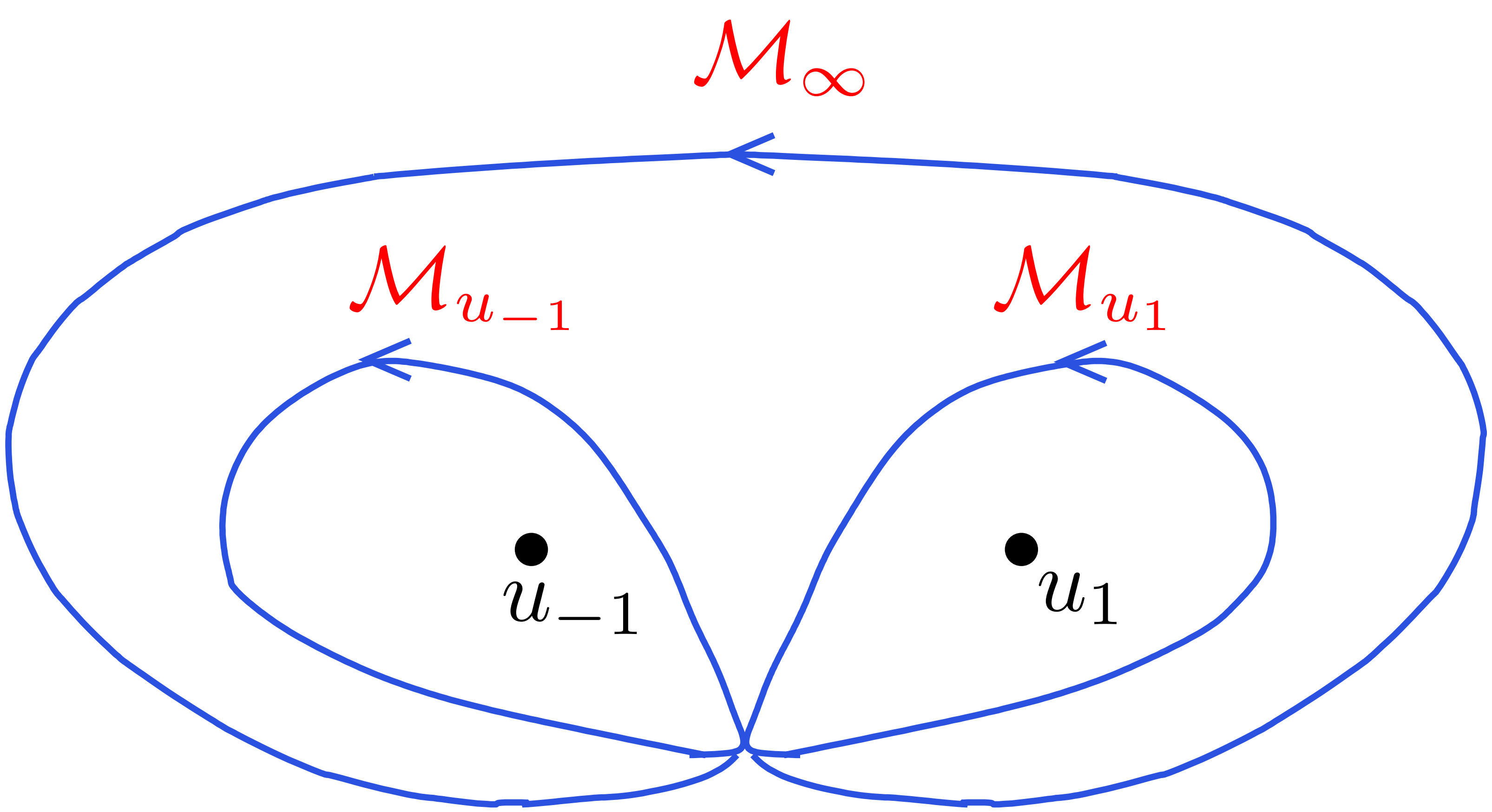
If $\mathrm{Im}\,\tau$ was single-valued everywhere it would be *harmonic*

then it would negative somewhere

way out: there are at least two other singularities
with monodromies that do not commute with $\mathcal{M}_\infty$

$\frac{4\pi}{g^2} = \mathrm{Im}\,\tau$ not single valued

Monodromy



$$\mathcal{M}_{u_1}\mathcal{M}_{u_{-1}} = \mathcal{M}_\infty$$

Monodromy Example

singular point u_j where a state with $(n_m, n_e) = (0, 1)$ becomes massless

$$a(u) \approx c_j (u - u_j)$$

$$\tau(a(u)) \approx \frac{-i}{\pi} \ln \frac{a(u)}{\Lambda}$$

$$(u - u_j) \to e^{2\pi i}(u - u_j)$$

$$a_D(u) \to a_D(u) + 2a(u) \ , \ \ a(u) \to a(u)$$

$$\mathcal{M}_{u_j} = T^2$$

Monodromy

dyon with charge (n_m, n_e) which becomes massless at $u = u_k$

find D_{u_k} that maps this to charge $(0,1)$

$$\begin{pmatrix} a_D(u) \\ a(u) \end{pmatrix} = D_{u_k} \begin{pmatrix} a_D \\ a \end{pmatrix} = \begin{pmatrix} \alpha a_D + \beta a \\ \gamma a_D + \delta a \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} = D_{u_k}^{-1} \begin{pmatrix} n_m \\ n_e \end{pmatrix} = \begin{pmatrix} \delta n_m - \gamma n_e \\ -\beta n_m + \alpha n_e \end{pmatrix}$$

$$\mathcal{M}_{u_k} = D_{u_k}^{-1} T^2 D_{u_k} = \begin{pmatrix} 1 + 2n_e n_m & 2n_e^2 \\ -2n_m^2 & 1 - 2n_e n_m \end{pmatrix}$$

Monodromy

Assuming a monopole with charge $(1,0)$ becomes massless at the point u_1

$$\mathcal{M}_{u_1}\mathcal{M}_{u_{-1}} = \mathcal{M}_\infty$$

$$\mathcal{M}_{u_1} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}, \quad \mathcal{M}_{u_{-1}} = \begin{pmatrix} -1 & 2 \\ -2 & 3 \end{pmatrix}$$

$\mathcal{M}_{u_1} = S^{-1}T^2S$, and $\mathcal{M}_{u_{-1}} = (ST^{-1})^{-1}T^2ST^{-1}$

state at u_{-1} is a dyon with charge $(-1,1)$ or $(1,-1)$

Check

near u_1, where a_D vanishes

break to $\mathcal{N} = 1$ by adding $m \, \mathrm{Tr} \, a^2$

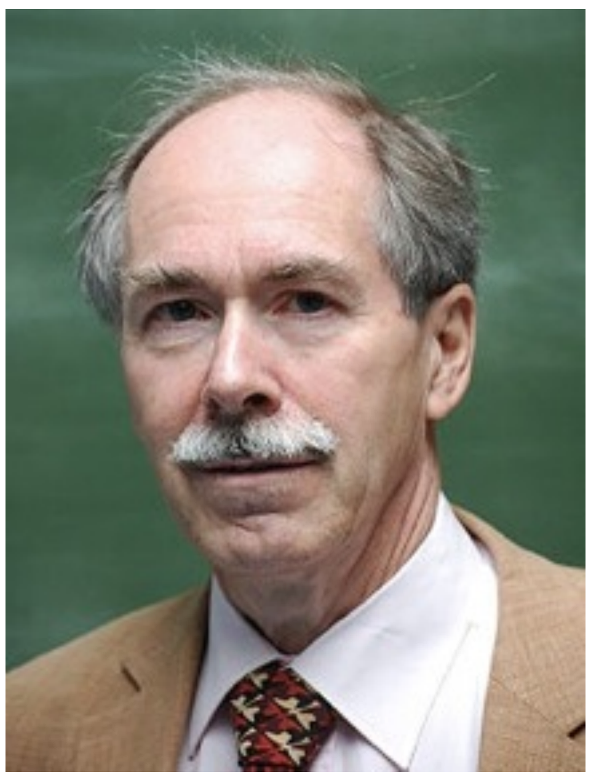
$$W_{\text{eff}} = \sqrt{2} a_D M \overline{M} + m \, f(a_D)$$

$$\sqrt{2} M \overline{M} + m f'(a_D) = 0 \ , \quad a_D M = 0 \ , \quad a_D \overline{M} = 0$$

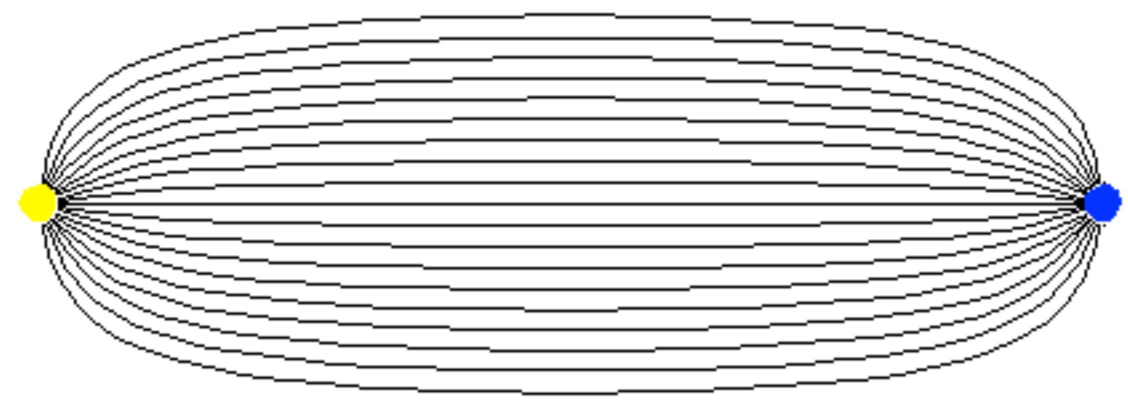
electric charge confinement
through dual Meissner effect

in agreement with Seiberg duality

`t Hooft-Mandelstam




magnetic condensate
confines electric charge



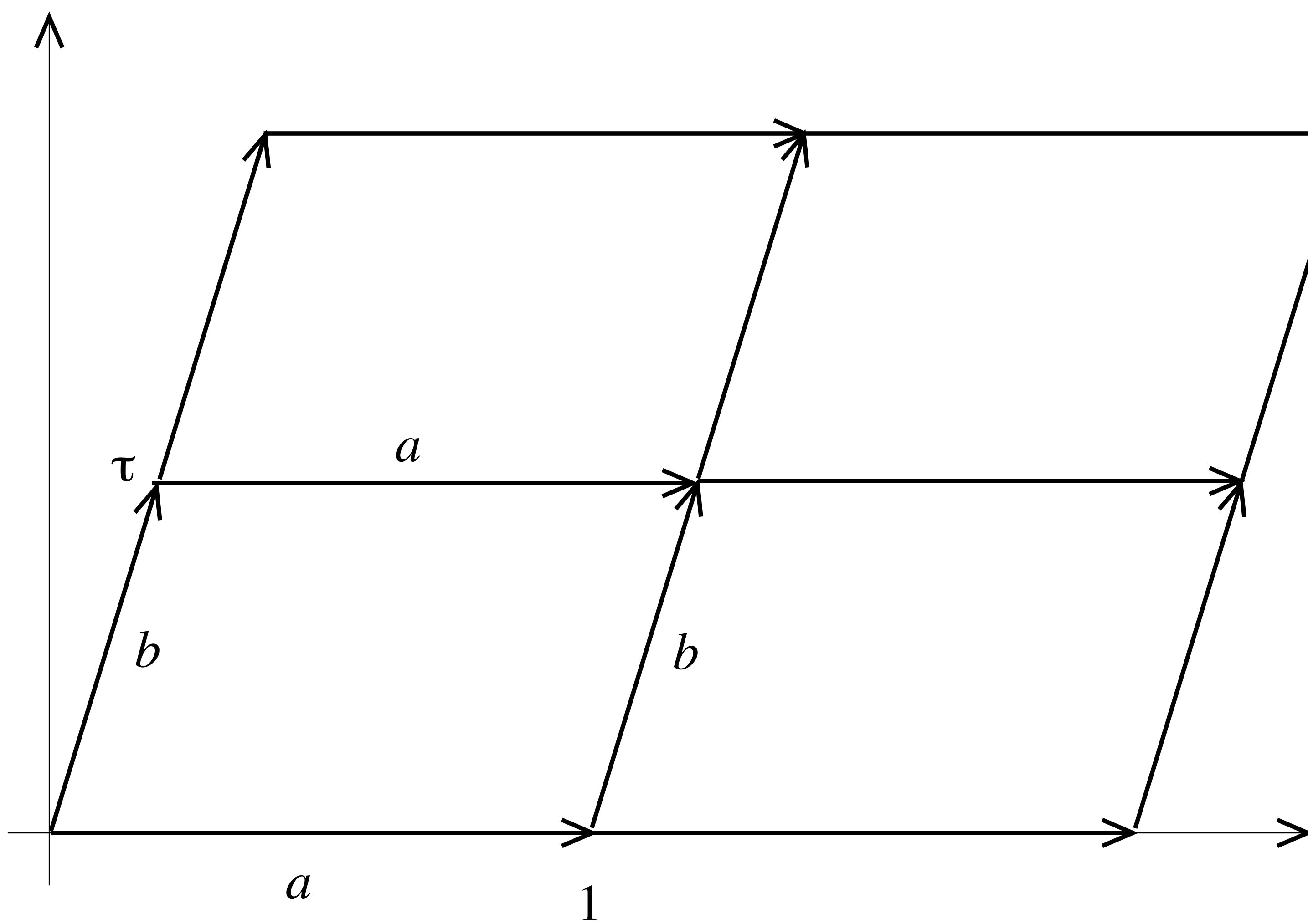
High Energy Physics Ed. Zichichi, (1976) 1225
Phys. Rept. 23 (1976) 245

Torus and SL(2,Z)

τ is a section of an $SL(2,Z)$ bundle

$SL(2,Z)$ is the *modular symmetry* group of a torus

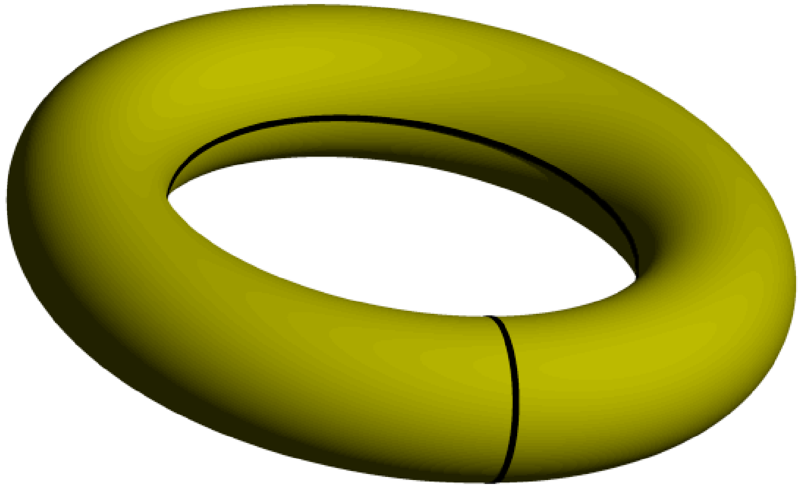
τ represents the *modular parameter* of a torus



Elliptic Curves

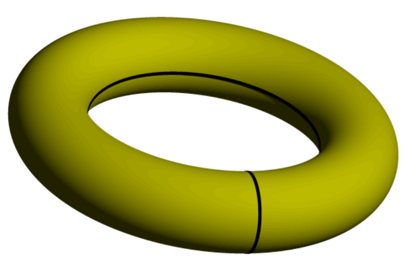
a torus is the solution of an *elliptic curve* in two complex dimensions

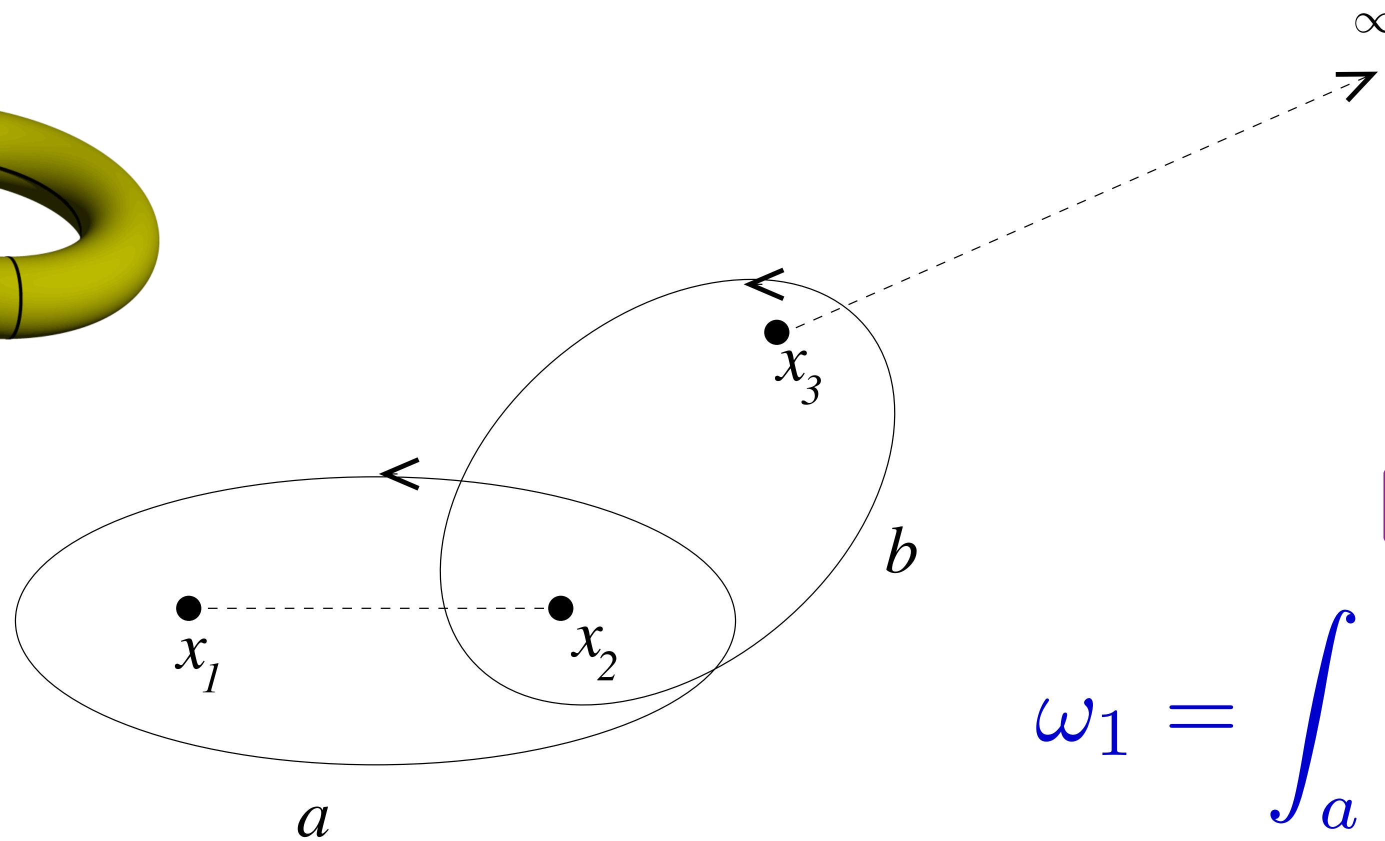
$$y^2 = x^3 + Ax^2 + Bx + C$$



Elliptic Curve

$$y^2 = x^3 + Ax^2 + Bx + C \equiv (x - x_1)(x - x_2)(x - x_3)$$



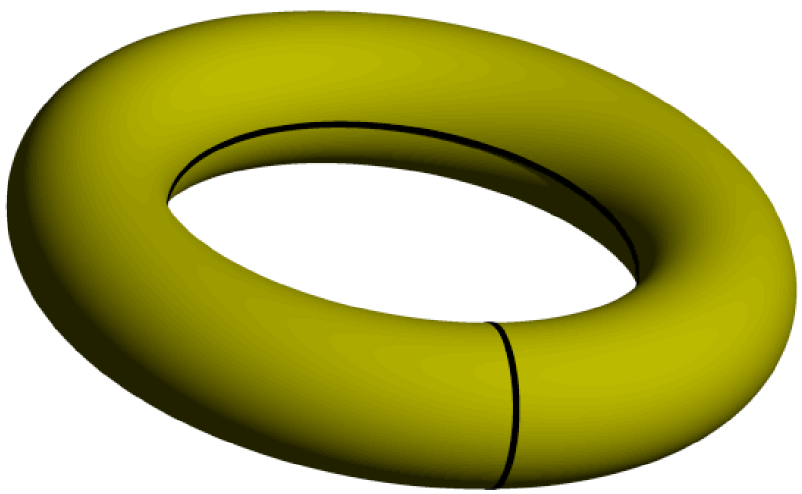


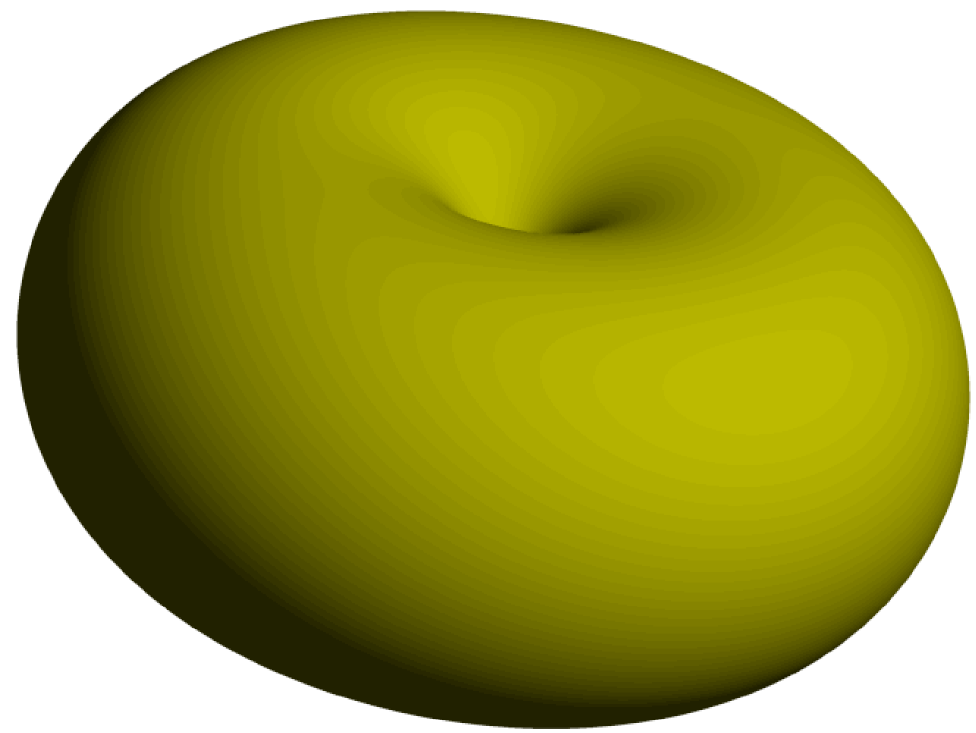
periods:

$$\omega_1 = \int_a \frac{dx}{y} \ , \quad \omega_2 = \int_b \frac{dx}{y}$$

$$\tau(A, B, C) = \frac{\omega_2}{\omega_1}$$

Degenerate Torus





$$\tau(A, B, C) = \frac{\omega_2}{\omega_1}$$

Seiberg-Witten Curve

$$y^2 = (x - \Lambda^2)(x + \Lambda^2)(x - u)$$

torus degenerates at $u = \Lambda^2, -\Lambda^2, \infty$

$$\mathcal{M}_\infty = -T^{-2}$$

$$\mathcal{M}_{u_1} = S^{-1}T^2S, \text{ and } \mathcal{M}_{u_{-1}} = (ST^{-1})^{-1}T^2ST^{-1}$$

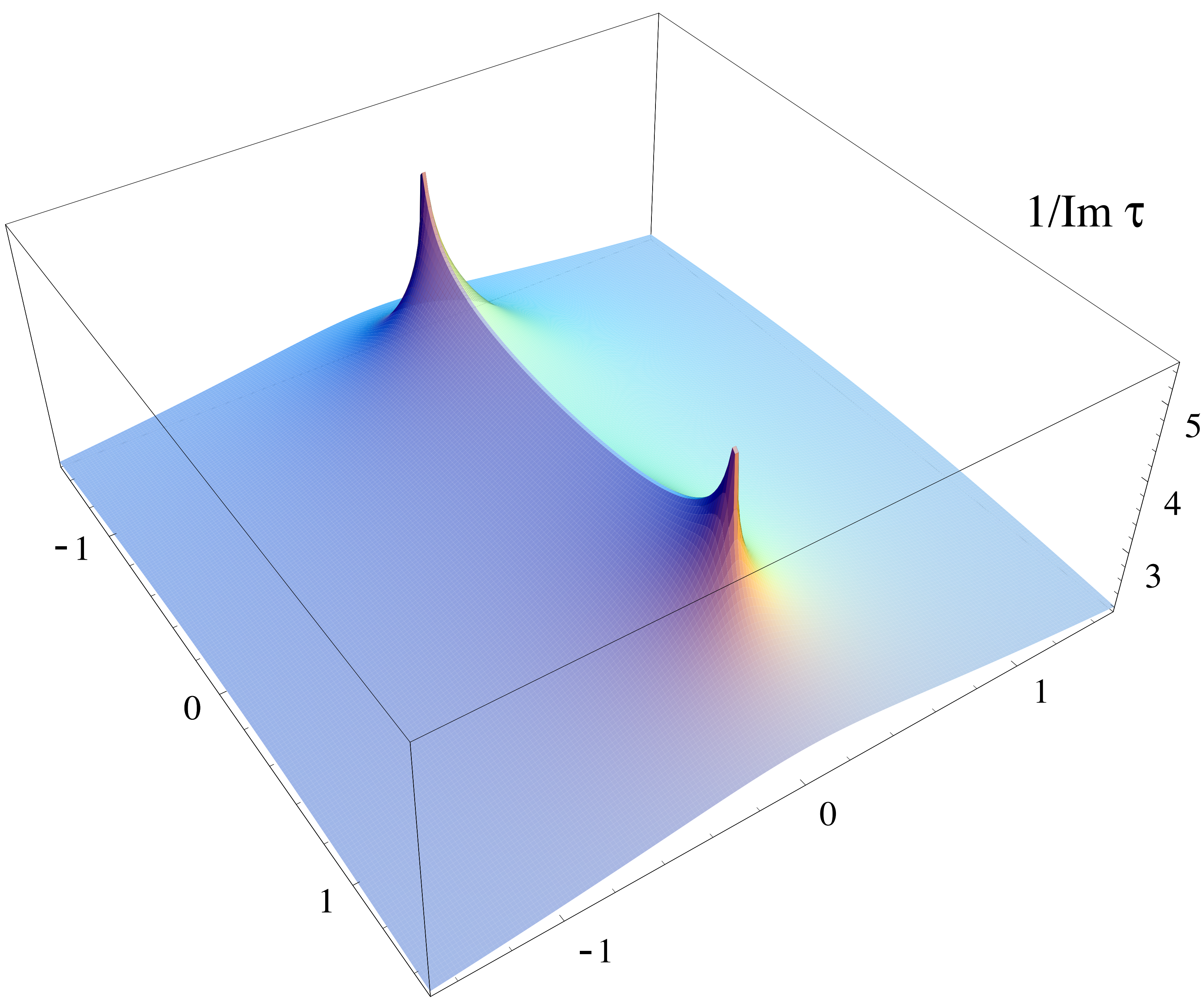
Seiberg-Witten Curve

$$y^2 = (x - \Lambda^2)(x + \Lambda^2)(x - u)$$

$$\omega_1 \quad = \quad 2\int_{-\Lambda^2}^{\Lambda^2} \frac{dx}{\sqrt{y}} = \frac{2\pi}{\Lambda\sqrt{1+\frac{u}{\Lambda^2}}} F\left(\frac{1}{2}, \frac{1}{2}, 1; \frac{2}{1+\frac{u}{\Lambda^2}}\right)$$

$$\omega_2 \quad = \quad 2\int_{u}^{\Lambda^2} \frac{dx}{\sqrt{y}} = \frac{-\pi i}{\sqrt{2}\Lambda} F\left(\frac{1}{2}, \frac{1}{2}, 1; \frac{1}{2}(1 - \frac{u}{\Lambda^2})\right)$$

Gauge coupling

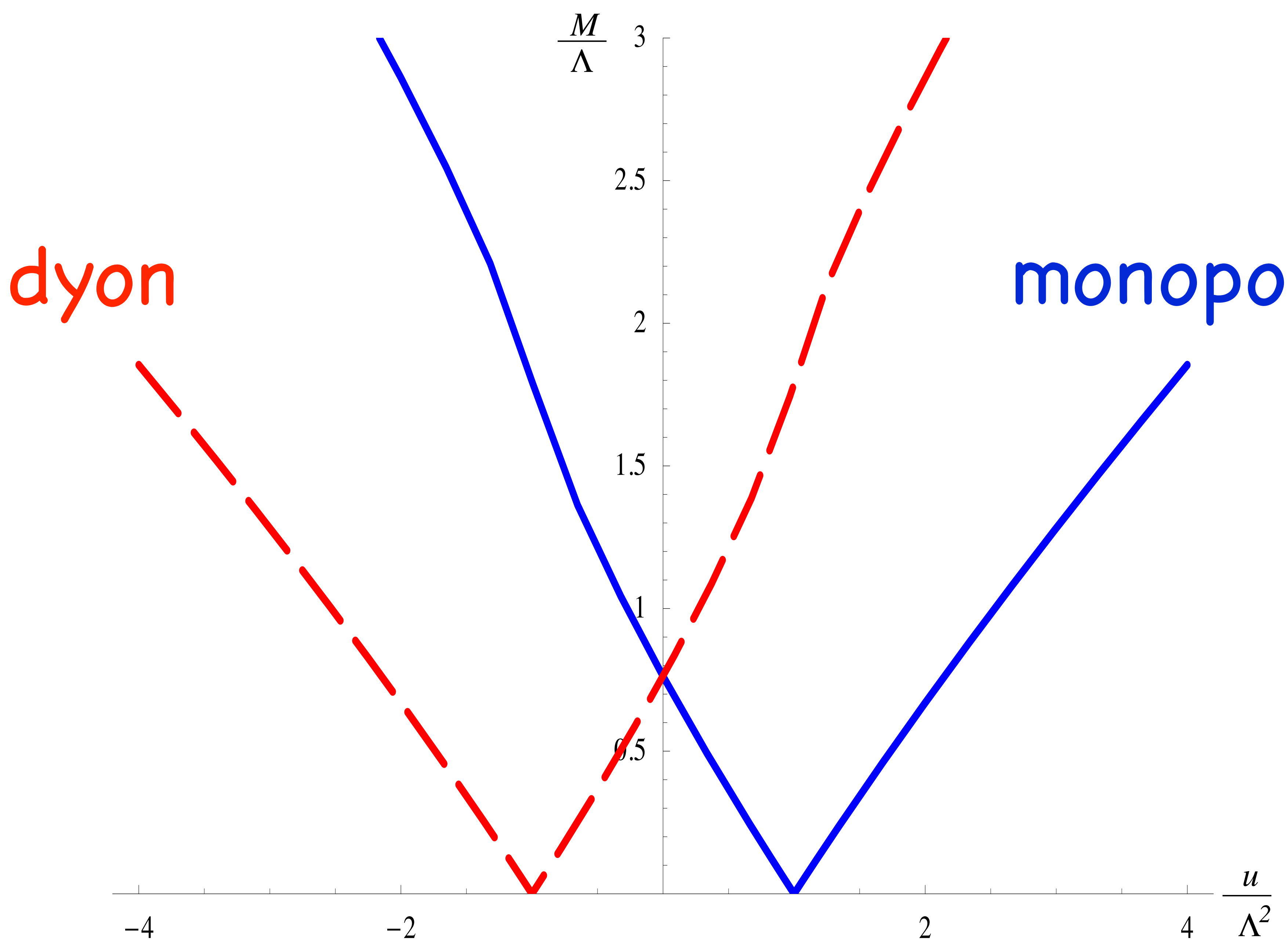


Seiberg-Witten Curve

$$y^2 = (x-\Lambda^2)(x+\Lambda^2)(x-u)$$

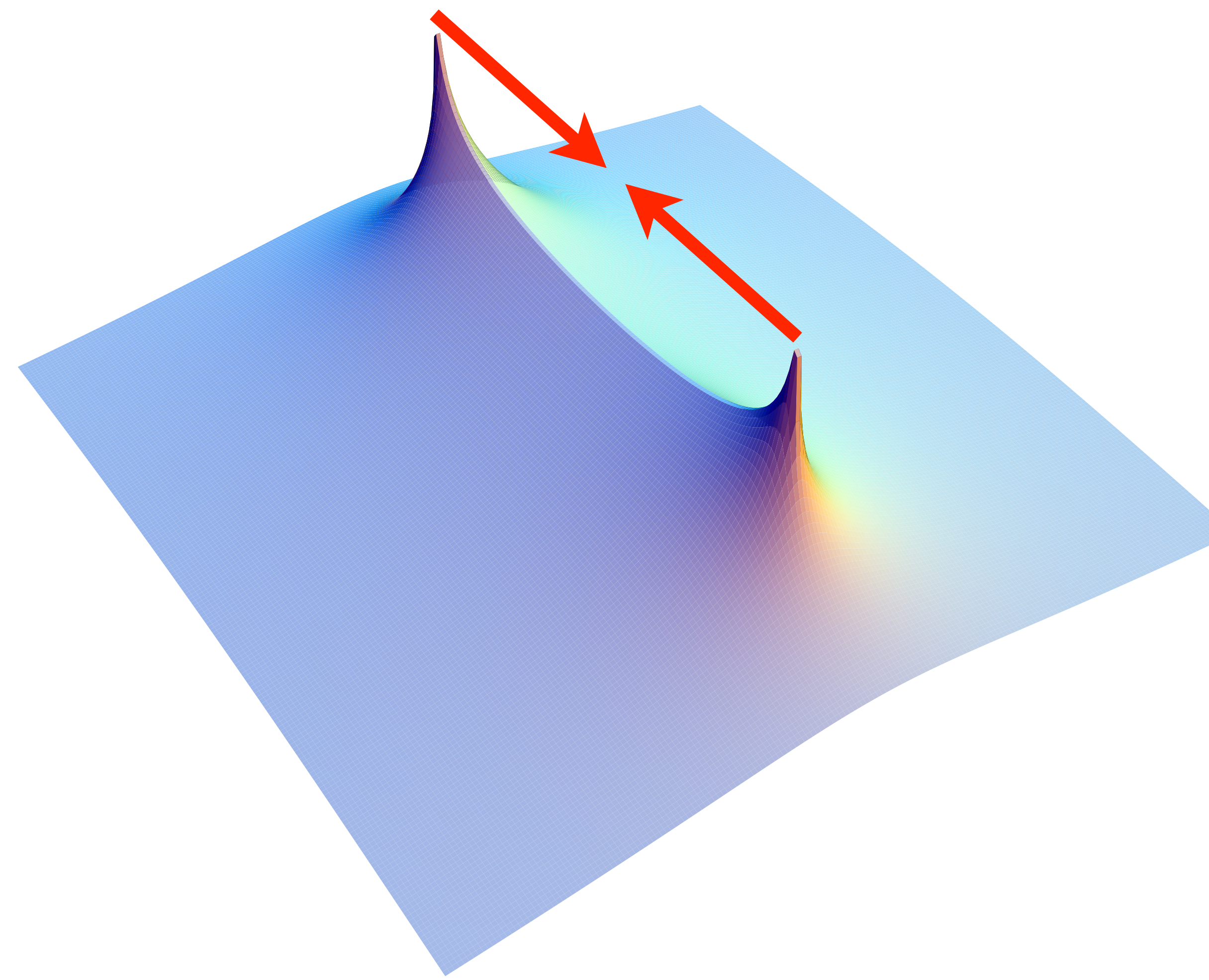
$$\begin{aligned}
a(u) &= -\frac{\sqrt{2}}{\pi}\int_{-\Lambda^2}^{\Lambda^2}\frac{dx\,\sqrt{x-u}}{\sqrt{(x-\Lambda^2)(x+\Lambda^2)}} \\
&= -\sqrt{2(\Lambda^2+u)}\,F\left(-\frac{1}{2},\frac{1}{2},1;\frac{2}{1+\frac{u}{\Lambda^2}}\right) \\
a_D(u) &= -\frac{\sqrt{2}}{\pi}\int_{u}^{\Lambda^2}\frac{dx\,\sqrt{x-u}}{\sqrt{(x-\Lambda^2)(x+\Lambda^2)}} \\
&= -i\frac{1}{2}\left(\frac{u}{\Lambda}-\Lambda\right)F\left(\frac{1}{2},\frac{1}{2},2;\frac{1}{2}\left(1-\frac{u}{\Lambda^2}\right)\right)
\end{aligned}$$

Monopole Mass



Argyres-Douglas





CFT with massless electric and magnetic charges

hep-th/9505062

N=2, SU(2), F=1

mass

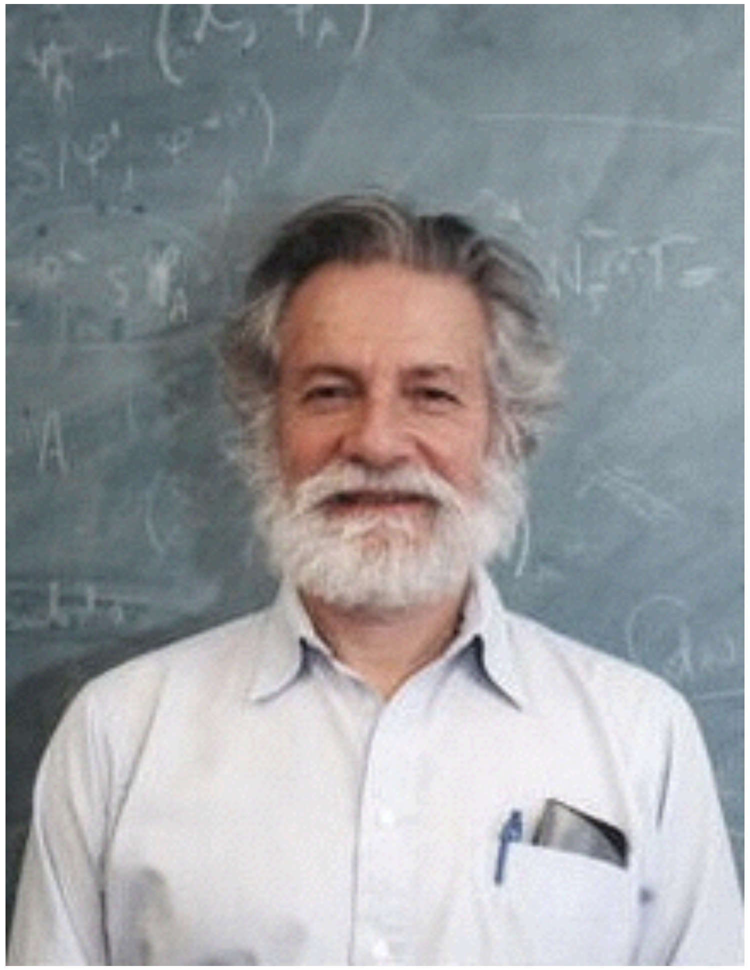
$$y^2 = x^3 - ux^2 + \frac{m}{4}\Lambda_1^3 x - \frac{1}{64}\Lambda_1^6$$

$$m = 3\Lambda_1/4 \qquad u = 3\Lambda_1^2/4$$

$$y^2 = \left(x - \frac{\Lambda_1^2}{4}\right)^3$$

massless monopoles and dyons

Zwanziger



non-Lorentz invariant, local action?

$$\mathcal{L} = -\frac{1}{2n^2e^2}\{[n\cdot(\partial\wedge A)]\cdot[n\cdot^*(\partial\wedge B)] - [n\cdot(\partial\wedge B)]\cdot[n\cdot^*(\partial\wedge A)] + [n\cdot(\partial\wedge A)]^2 + [n\cdot(\partial\wedge B)]^2\} - J\cdot A - \frac{4\pi}{e^2}K\cdot B.$$

electric magnetic

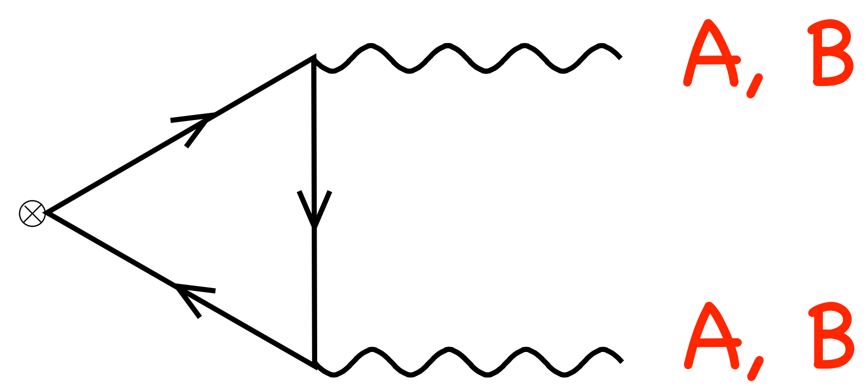
$$F = \frac{1}{n^2}\left(\{n\wedge[n\cdot(\partial\wedge A)]\} - {}^*\{n\wedge[n\cdot(\partial\wedge B)]\}\right)$$

two propagating polarizations

Phys. Rev. D3 (1971) 880

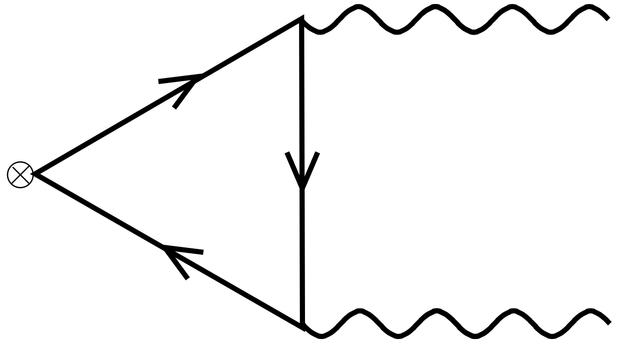
Anomalies

$$\mathcal{L} = -\frac{1}{2n^2e^2}\{[n\cdot(\partial\wedge A)]\cdot[n\cdot^*(\partial\wedge B)] - [n\cdot(\partial\wedge B)]\cdot[n\cdot^*(\partial\wedge A)] + [n\cdot(\partial\wedge A)]^2 + [n\cdot(\partial\wedge B)]^2\} - J\cdot A - \frac{4\pi}{e^2}K\cdot B.$$



Axial Anomaly from SL(2,Z)

$$(q,g) \to (n,0)$$



$$\begin{aligned}\partial_\mu j^\mu_A(x) &= \frac{n^2}{16\pi^2} F'^{\mu\nu}\,{}^*F'_{\mu\nu} \\ &= \frac{n^2}{32\pi^2} \mathrm{Im}\left(F'^{\mu\nu} + i\,{}^*F'^{\mu\nu}\right)^2\end{aligned}$$

Axial Anomaly

$$
\begin{aligned}
\partial_\mu j_A^\mu(x) &= \frac{n^2}{32\pi^2}\mathrm{Im}\,(c\tau^* + d)^2\,(F^{\mu\nu} + i\,{}^*F^{\mu\nu})^2 \\
&= \frac{1}{16\pi^2}\mathrm{Re}\,(q + \tau^* g)^2\,F^{\mu\nu}\,{}^*F_{\mu\nu} + \frac{1}{16\pi^2}\mathrm{Im}\,(q + \tau^* g)^2\,F^{\mu\nu}\,F_{\mu\nu} \\
&= \frac{1}{16\pi^2}\left\{\left[\left(q + \frac{\theta}{2\pi}g\right)^2 - g^2\frac{16\pi^2}{e^4}\right]F^{\mu\nu}\,{}^*F_{\mu\nu}\right. \\
&\qquad \left. + \left[qg + \frac{\theta}{2\pi}g^2\right]F^{\mu\nu}\,F_{\mu\nu}\right.
\end{aligned}
$$

$U(1)^3$ Anomaly

$$\sum_j q_j^3 = 0$$

$$\sum_j q_j \, g_j^2 = 0$$

$$\sum_j q_j^2 \, g_j = 0$$

$$\sum_j g_j^3 = 0$$

Conclusions

the Seiberg-Witten analysis
gives exact results in
strongly coupled theories with
monopoles and dyons

there are theories with
massless monopoles interacting
with massless dyons